Quantum size effect in semiconductor transport

Vijay K. Arora and Fawzi G. Awad*
Department of Physics, University of Riyadh, P.O. Box 2455, Riyadh, Saudi Arabia
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A quantum transport theory for electric conductivity in semiconducting thin films is presented when the de Broglie wavelength of a thermal electron becomes comparable to the thickness of the film and size quantization is important. The theoretical results obtained are analyzed for the electron scattering by acoustic phonons and point defects represented by a δ-function potential. In the ultra-thin limit (UTL), the resistivity in the longitudinal configuration (when current flows parallel to the plane of the thin film) is found to be inversely proportional to the thickness of the film. In the transverse configuration (when current flows perpendicular to the plane of the thin film) the resistivity in UTL is inversely proportional to d^3 when $3\epsilon_0 >> \hbar/\tau$, where d is the thickness of the film, ϵ_0 is the ground-state energy of the electron confined to a box of length d, and τ is the relaxation time. When $3\epsilon_0 << \hbar/\tau$, the resistivity is found to be inversely proportional to d.

I. INTRODUCTION

In semiconductors, when the de Broglie wavelength λ_D of an electron is much smaller than the size of the sample d ($\lambda_D << d$), the electron is considered to be a classical particle and hence the transport properties are normally calculated by using the semiclassical Boltzmann transport equation[1] (BTE) and reasonable agreement is obtained with the experimental results. The validity of this transport equation has been questioned by many workers in the past. A review paper by Dresden[2] discusses in detail various assumptions and approximations implicit in a semiclassical equation of the Boltzmann type, including the relaxation-time approximation, which are not always satisfied. Classical arguments are not valid in the domain where quantum effects are important. For example, when λ_D of an electron becomes comparable to its radius in a magnetic field, a large range of quantum phenomenon has been observed.[3] Similarly, when λ_D becomes comparable to the dimensions in which electron is confined, the wave character of the electron as obtained from the solution of the Schrodinger equation should be taken into account. In this paper, we investigate the quantum size effect (QSE) when $\lambda_D \gtrsim d$, and study the transport properties of quasi-two-dimensional (QTD) gas confined to a thin film of thickness d.

QSE and perspectives of its practical applications have been discussed in a review article by Elinson *et al.*[4] These authors have indicated that QSE reveals itself in the fact that microscopic characteristics of the carrier gas (electron or hole) in films, such as thermodynamical coefficients, kinetic coefficients, optical properties, etc., depend on the film thickness. In the case of a degenerate gas, these functions are of an oscillatory type; while in the nondegenerate case, a monotonically increasing or decreasing behavior on film thickness may be observed.[5–7] Larsen[5] has reviewed the classical and the quantum size effects and their relationship with the transport coefficients in metals. Fivaz and Schmid[6] have presented the scattering effects in two- and three-dimensional limits and have studied the anisotropy of the mobility of a layered structure in terms of an overlap integral. Sandomirskii[7] has considered QSE in a semimetal film and has obtained formula for the carrier density, electric conductivity, the Hall coefficient, and the magnetoresistance for the case of electrons being scattered by randomly distributed centers with δ potential. A theoretical study of QSE describing the electric field effect in thin films of bismuth is presented by Freeman and Gettys,[8] who have shown that the film conductance shows abrupt changes with applied field for thickness which have the Fermi level close to a step in the electronic density of states. Dorda[9] has outlined the analogy of QSE in thin semiconductor films to the quantum effect in surface inversion layers, where electrons are shown to behave like a two-dimensional (2D) gas.

In spite of the large amount of work on metal and semimetal films, little attention has been devoted to the transport properties of electrons confined to semiconducting thin films and its relationship to the bulk properties. We will, therefore, in this paper study some of these properties. In Sec. II, we indicate transport properties of a quasi-two-dimensional (QTD) gas when electrons are constrained to move in a plane, i.e., their motion perpendicular to the plane is neglected. We will call this QTD-I model. In Sec. III, we indicate quantum properties of a QTD gas when electronic motion perpendicular to the film is quantized like in a one-dimensional box. This model is characterized as QTD-II. Also, we study in

Sec. III the electric conductivity in the longitudinal configuration for a QTD-II gas, when current flows parallel to the plane of the film by using the BTE. QTD-II gas is shown to behave somewhat like a QTD-I gas under the assumption of a nondegenerate semiconductor in the ultra-thin limit (UTL), when all the electrons are assumed to populate the ground-state energy level. In Sec. IV, we use the more sophisticated approach, that of the density matrix, to study electronic transport properties. It is indicated that this quantum-statistical approach gives results equivalent to those obtained by the BTE for the longitudinal configuration only. But, this approach also gives results in the transverse configuration which otherwise could not be obtained from the BTE. The use of the eigenfunction in QTD-II model may be questionable in transverse configuration near the contacts, although they may give reasonably good description away from the contacts. The analysis of the final results in UTL is presented.

II. QTD-I MODEL

When electrons are constrained to move in two dimensions their two-dimensional behavior is characterized by eigenfunctions ψ_k and eigenvalues ϵ_k:

$$\psi_k = (1/A)^{1/2}\exp(ik_x x + ik_y y) \ , \tag{2.1}$$

$$\epsilon_k = \hbar^2(k_x^2 + k_y^2)/2m^* \ , \tag{2.2}$$

where $k = (k_x, k_y)$ is a 2D wave vector characterizing an electron with effective mass m^*, and $A = ab$ is the area of the rectangular plane of length a and breadth b. In the above representation, the matrix elements of the velocity components v_x and v_y are

$$\langle k'|v_x|k\rangle = (\hbar k_x/m^*)\delta_{k'k} \ , \tag{2.3}$$

$$\langle k'|v_y|k\rangle = (\hbar k_y/m^*)\delta_{k'k} \ . \tag{2.4}$$

The density of states function $N_0(\epsilon)$ describing the number of states per unit energy interval is given by

$$N_0(\epsilon) \equiv \sum_{k,s}\delta(\epsilon - \epsilon_k) = Am^*/\pi\hbar^2 \ , \tag{2.5}$$

where s stands for spin, giving a degeneracy factor of 2 for an electron.

The equilibrium statistical energy distribution $f_0(\epsilon_k)$ of electrons in QTD-I model, in the general case, is described by Fermi-Dirac distribution function:

$$f_0(\epsilon_k) = 1/\{\exp[(\epsilon_k - \zeta)/k_B T] + 1\} \ , \tag{2.6}$$

with ζ, the Fermi energy, as obtained from the normalization condition $\sum_{ks} f_0(\epsilon_k) = N$, where N is the total number of electrons, given by

$$\zeta = k_B T \ln[\exp(\pi\hbar^2 n_s/m^* k_B T) - 1] \ , \tag{2.7}$$

where $n_s \equiv N/A$ is the surface density of carriers and T is the temperature. Equation (2.7) can be approximated for strongly degenerate and nondegenerate cases as follows:

$$\zeta \approx \begin{cases} \pi\hbar^2 n_s/m^* & \text{for } \pi\hbar^2 n_s/m^* k_B T \gg 1 \text{ (degenerate case)} \quad (2.8) \\ k_B T \ln(\pi\hbar^2 n_s/m^* k_B T) & \text{for } \pi\hbar^2 n_s/m^* k_B T \ll 1 \text{ (nondegenerate case)} \ . \quad (2.9) \end{cases}$$

The steady-state Boltzmann transport function for an electric field $\mathcal{E}_x$ applied in the x direction, as obtained from BTE, is given by[7]

$$f(\epsilon_k) = f_0(\epsilon_k) + e\mathcal{E}_x \frac{\hbar k_x}{m^*}\frac{\partial f_0}{\partial \epsilon_k}\tau(\epsilon_k) \ , \tag{2.10}$$

where $-e$ is the electronic charge, and $\tau(\epsilon_k)$ as a function of energy is obtained from Fermi's golden rule:

$$\tau^{-1}(\epsilon_k) = \left(\frac{2\pi}{\hbar}\right)\sum_{k'}|\langle k|V|k'\rangle|^2\delta(\epsilon_k - \epsilon_{k'}) \ , \tag{2.11}$$

for isotropic scattering. V describes the interaction of an electron with lattice imperfections. For an electron interacting with acoustic phonons, $\tau^{-1}(\epsilon_k)$ is given by

$$\tau^{-1}(\epsilon_k) = \left(\frac{\pi E_1^2}{\rho U A d}\right)\sum_{\alpha,\vec{q}}|\langle\alpha'|\exp(i\vec{q}\cdot\vec{r})|\alpha\rangle|^2 q[(N_q+1)\delta(\epsilon_{\alpha'} - \epsilon_\alpha + \hbar\omega_q) + N_q\delta(\epsilon_{\alpha'} - \epsilon_\alpha - \hbar\omega_q)] \ , \tag{2.12}$$

where E_1 is the deformation potential constant, ρ is the bulk density of the crystal, U is the sound velocity in the bulk crystal, d is the thickness of the plate in which electrons are confined, $\vec{q} = (q_x, q_y)$ is the momentum of the phonon, $\hbar\omega_q$ is the energy of phonon, and N_q is the number of phonons with wave vector q. $|\alpha\rangle$ stands for $|k\{N_q\}\rangle$ where $\{N_q\}$ describes the lattice eigenfunctions. In the elastic limit ($\hbar\omega_q \ll \epsilon_\alpha$) at high temperatures ($N_q + 1 \approx N_q \approx k_B T/\hbar U q$), τ_{as}^{-1} evaluated as follows:

$$\tau_{as}^{-1}(\epsilon_k) = m^* E_1^2 k_B T/\rho U^2 \hbar^3 d \tag{2.13}$$

(QTD–I acoustic phonons) ,

a result which agrees with that obtained by Kawaji.[10] For a δ-function potential (point-defect scattering), the result obtained is

$$\tau_{ds}^{-1} = n_i V_0^2 m^*/\hbar^3 d \qquad (2.14)$$

(defect scattering in QTD–I) ,

where $n_i = N_i/V$ is the volume density of the defects. τ is independent of energy for both types of scattering in the QTD-I model.

The electric current density, as obtained from

$$J_x = -\frac{e}{Ad}\sum_{\vec{k}s} f(\epsilon_k)(\hbar k_x/m^*)$$

$$= -\frac{e^2}{AD}\mathcal{E}_x \sum_{\vec{k}s}\left[\frac{\hbar k_x}{m^*}\right]^2 \frac{\partial f_0}{\partial \epsilon_k}\tau(\epsilon_k) \ , \qquad (2.15)$$

leads to an expression for conductivty $\sigma_s (J_x = \sigma_s \mathcal{E}_x)$

$$\sigma_s = n_s e^2 \tau/m^* d \qquad (2.16)$$

for energy-independent τ given by Eq. (2.13) or Eq. (2.14), which can be written in final form as

$$\sigma_s = \begin{cases} n_s e^2 \rho U^2 \hbar^3/m^{*2} E_1^2 k_B T & \text{(acoustic phonons)} \qquad (2.17) \\ n_s e^2 \hbar^3/m^{*2} n_i V_0^2 & \text{(point defects)} \ . \qquad (2.18) \end{cases}$$

Comparing the above expression with the bulk conductivity

$$\sigma_b = \begin{cases} 4ne^2\rho U^2 \pi\hbar^4/3(2\pi m^* k_B T)^{1/2} m^{*2} E_1^2 k_B T & \text{(acoustic phonons)} \qquad (2.19) \\ 4ne^2\pi\hbar^4/3(2\pi m^* k_B T)^{1/2} m^{*2} n_i V_0^2 & \text{(point defects)} \ , \qquad (2.20) \end{cases}$$

we obtain the ratio of surface resistivity, ρ_s to bulk resistivity ρ_b, in both cases, equal to

$$\rho_s/\rho_b = \sigma_b/\sigma_s = 4\sqrt{\pi}\lambda_D/3d \ , \qquad (2.21)$$

with

$$\lambda_D = \hbar/(2m^* k_B T)^{1/2} \ . \qquad (2.22)$$

The ratio ρ_s/ρ_b is, therefore, proportional to λ_D/d and hence is a linearly increasing function of inverse thickness d^{-1}. No conductivity is obtained in the transverse configuration (when electric field is applied perpendicular to the film) as the motion of the electrons is constrained in XY plane only.

III. QTD-II MODEL

In QTD-II model, the energy of electron gas is quantized in the direction of the thin film of thickness d. Assuming the plane of the thin film to be perpendicular to z axis with its boundaries at $z = 0$ and $z = d$, the eigenfunctions ψ_{lk} and eigenvalues ϵ_{lk} are obtained as[7,8]

$$\psi_{lk} = (2/\Omega)^{1/2} \exp(ik_x x + ik_y y)\sin(l\pi z/d) \ , \qquad (3.1)$$

$$\epsilon_{lk} = \hbar^2(k_x^2 + k_y^2)/2m^* + l^2\epsilon_0, \quad l = 1, 2, 3, \ldots , \qquad (3.2)$$

$$\epsilon_0 = \pi^2\hbar^2/2m^* d^2 \ , \qquad (3.3)$$

where $\Omega = abd$ is the volume of the film, and ϵ_0 is the ground-state energy of the electron confined to one-dimensional box of thickness d. The matrix elements of the velocity components v_x, v_y, and v_z are given by

$$\langle l'k'|v_x|lk\rangle = (\hbar k_x/m^*)\delta_{l'l}\delta_{k'k} \ , \qquad (3.4)$$

$$\langle l'k'|v_y|lk\rangle = (\hbar k_y/m^*)\delta_{l'l}\delta_{k'k} \ , \qquad (3.5)$$

$$\langle l'k'|v_z|lk\rangle = 2\hbar l l'\{1-\exp[i\pi(l'-l)]\}/im^* d(l'^2 l^2)\delta_{k'k}$$
$$= 0 \quad \text{when } l' = l \ . \qquad (3.6)$$

We see from Eqs. (3.4)–(3.6) that the matrix elements of v_x and v_y are diagonal in the representation of Eq. (3.1), but for v_z, the diagonal components vanish. Thus, as we will see, BTE can be applied to obtain expectation values of v_x and v_y when electrons are driven out of equilibrium by the application of electric field parallel to the film (longitudinal configuration). But, BTE cannot be used for the transverse configuration when current flows perpendicular to the film. As stated in Sec. I, eigenfunctions of Eq. (3.1) are not good near the contacts in transverse case, they may give reasonably good description away from the contacts.

The density of states function $N(\epsilon)$ for QTD-II is given by

$$N(\epsilon) = N_0[\sqrt{\epsilon/\epsilon_0}] \ , \qquad (3.7)$$

where $N_0 = m^* A/\pi\hbar^2$ is the constant density of states for 2D gas in QTD-I model, and $[\sqrt{\epsilon/\epsilon_0}]$ is integral part of $\sqrt{\epsilon/\epsilon_0}$. This function, along with that in QTD-I and that for the bulk semiconductor is indicated graphically in Fig. 1. We see from that plot, that for a 3D gas, this function is a monotonically increasing function $[N(\epsilon) \sim \epsilon^{1/2}]$ in energy, whereas for QTD-II this function has discontinuities for energies corresponding to the bottom of each subband characterized by quantum number l. The constant density of states in QTD-I model is equal to that corresponding to the ground-state energy ϵ_0 of QTD-II gas.

Fermi energy ζ for QTD-II gas can be calculated from the following equation obtained from the normalization condition $[\sum_{lks} f(\epsilon_{lk}) = N]$:

$$N_0 k_B T \sum_{l=1}^{\infty} \ln\left[1 + \exp\left(\frac{\zeta - l^2\epsilon_0}{k_B T}\right)\right] = N \ . \qquad (3.8)$$

For a nondegenerate semiconductor $[\exp(\zeta - \epsilon_0)/k_B T$

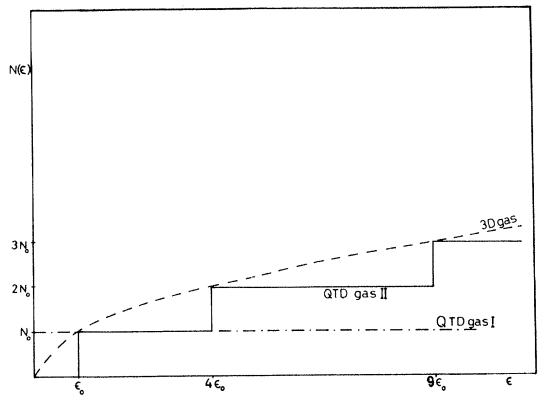

FIG. 1. The density of states per unit energy interval, $N(\epsilon)$ as a function of ϵ. $N_0 = m^* A/\pi\hbar^2$ is the density of states for QTD-I gas (shown by broken line). Solid line is for QTD-II gas exhibiting quantum character, and dashed line is for 3D gas.

$\ll 1$], this equation yields, for the Fermi energy ζ, the expression

$$\zeta = k_B T \ln[n_s \pi\hbar^2/\gamma(\epsilon_0, T) m^* k_B T] \ , \tag{3.9}$$

with

$$\gamma(\epsilon_0, T) \equiv \sum_{l=1}^{\infty} \exp\left[-\frac{l^2\epsilon_0}{k_B T}\right] = \begin{cases} \left(\dfrac{\pi k_B T}{4\epsilon_0}\right)^{1/2} & \text{for } \epsilon_0 \ll k_B T \ , \\ \exp\left[-\dfrac{\epsilon_0}{k_B T}\right] & \text{for } \epsilon_0 \gg k_B T \ . \end{cases} \tag{3.10}$$

The above expression for ζ agrees with that in QTD-I model if $\gamma(\epsilon_0, T)$ is taken unity.

The electrical conductivity of QTD-II gas in longitudinal configuration, when electric field is applied parallel to the film, obtained from BTE as described in the previous section is given by

$$\sigma_Q = -\frac{e^2}{2\Omega} \sum_{lks} \left(\frac{\hbar k_\perp}{m^*}\right)^2 \frac{\partial f_0}{\partial \epsilon} \tau(\epsilon_{lk}) \ , \tag{3.11}$$

where $k_\perp = (k_x^2 + k_y^2)^{1/2}$, and $\tau(\epsilon_{lk})$ can be obtained from Eq. (2.12) with $\alpha = lk\{N_q\}$. It can be shown as in Appendix, that when basis states are those of QTD-II,

$$\sum_q |\langle \alpha' | \exp(i\vec{q}\cdot\vec{r}) | \alpha \rangle|^2 = (1 + \tfrac{1}{2}\delta_{l'l}) \ . \tag{3.12}$$

The inverse relaxation time is then evaluated both for acoustic and point-defect scattering, as follows:

$$\tau_Q^{-1}(\epsilon_{lk}) = \tau_s^{-1}([\sqrt{\epsilon_{lk}/\epsilon_0}] + \tfrac{1}{2}) \ , \tag{3.13}$$

where $\tau_s = \tau_{as}$ or τ_{ds} as given by Eqs. (2.13) and (2.14) for acoustic phonon scattering or point-defect scattering, respectively. Thus in QTD-II model, the relaxation time is not constant but depends on energy whose value as a function of energy has quantum jumps similar to those obtained in the density of states. In UTL, when most of the electrons occupy the lowest subband, τ_Q^{-1} reduces to τ_{UTL}^{-1} given by

$$\tau_{UTL}^{-1} = \tfrac{3}{2}\tau_s^{-1} \ , \tag{3.14}$$

which indicates that in UTL, τ_Q^{-1} is constant, but its constant value differs from that in QTD-I model by a factor 1.5.

For a nondegenerate semiconductor in the UTL, with τ^{-1} as given by Eq. (3.14), the conductivity σ_Q of Eq. (3.11) reduces to a simple analytical result:

$$\sigma_Q = \tfrac{2}{3}\sigma_s \ , \tag{3.15}$$

where σ_s is given by Eqs. (2.17) and (2.18). Therefore, the conductivity in QTD-II model is smaller than that in QTD-I model by a factor of $\frac{2}{3}$. The ratio of resistivity $\rho_Q = \sigma_Q^{-1}$ to the bulk resistivity is given by

$$\rho_Q/\rho_b = 2\sqrt{\pi}\lambda_D/d \ . \tag{3.16}$$

In the bulk limit ($\lambda_D \ll d$), when spacing between two adjacent levels is very small, the summation over 1 can be replaced by an integral, and $\tau_Q^{-1}(\epsilon_{lk}) \alpha \epsilon^{1/2}$ as in 3D case, the σ_Q obtained by Eq. (3.11) becomes equal to σ_b given by Eq. (2.20). Thus QTD-II gas behaves like a 3D gas when $\lambda_D \ll d$ and somewhat like QTD-I gas when $\lambda_D \gg d$.

IV. QUANTUM TRANSPORT THEORY

It was noticed, in the last section, that BTE can be successfully used in calculating transport coefficients in the longitudinal configuration. For transverse configuration, the expectation value of the current vanishes if BTE is used, because the matrix elements of velocity operator as given by Eq. (3.6) are nondiagonal. In such cases, it is useful to use the Liouville's equation for the density matrix as used by Arora and co-workers[11] to describe the electronic transport. In the quantum transport theory (QTT), the expectation value of the current is obtained from the statistical mechanics recipe given by

$$\langle \vec{J} \rangle = \mathrm{Tr}(\rho \vec{J}_{op})/\mathrm{Tr}(\rho) = -\frac{e}{\Omega} \sum_{\alpha\alpha'} \langle \alpha | \rho | \alpha' \rangle \langle \alpha' | \vec{v} | \alpha \rangle \ , \tag{4.1}$$

where $\langle \alpha' | \vec{v} | \alpha \rangle$ are given by Eqs. (3.4)–(3.6) and $\langle \alpha | \rho | \alpha' \rangle$ is evaluated from the solution of Liouville's equations for the density matrix:

$$i\hbar \frac{\partial \rho}{\partial t} = [H, \rho] \ , \tag{4.2}$$

where $H = H_0 + V + F$ is the Hamiltonian of the system, which consists of unperturbed part H_0, electron-lattice interaction V, and electron-electric field interaction $F = e\vec{\mathcal{E}}\cdot\vec{r}$. The eigenfunctions and

eigenvalues of the electronic part of H_0 are those given by Eqs. (3.1) and (3.2), respectively. A steady state linearized solution of Eq. (4.2) in the representation $\alpha=(lk)$ of Eq. (3.1), by following the method developed earlier,[11] is given by

$$\langle\alpha'|\rho|\alpha\rangle=f_0(\epsilon_\alpha)\delta_{\alpha'\alpha}+\frac{\langle\alpha'|[\rho_0,F]|\alpha\rangle}{\epsilon_{\alpha'}-\epsilon_\alpha-i\hbar\tau^{-1}_{\alpha'\alpha}} \ , \tag{4.3}$$

with

$$\tau^{-1}_{\alpha'\alpha}=\tfrac{1}{2}\tau^{-1}_{\alpha'}+\tfrac{1}{2}\tau^{-1}_{\alpha} \ , \tag{4.4}$$

where $\tau^{-1}_\alpha=\tau^{-1}(\epsilon_\alpha)$ is that given by Eq. (2.11) and evaluated as in Eq. (3.13). The matrix elements of commutator $[\rho_0,F]$ for an electric field $\vec{\mathcal{E}}=(\mathcal{E}_x,\mathcal{E}_y,\mathcal{E}_z)$ are given by

$$\langle\alpha'|[\rho_0,F]|\alpha\rangle=\frac{df}{d\epsilon_{lk}}\frac{e\hbar^2}{im^*}(k_x\mathcal{E}_x+k_y\mathcal{E}_y)\delta_{l'l}\delta_{k'k}-\frac{2e\hbar^2ll'(1-e^{i\pi(l'-l)})}{m^*d(l'^2-l^2)^2\epsilon_0}[f_0(\epsilon_{l'k'})-f_0(\epsilon_{lk})]\mathcal{E}_z\delta_{k'k} \ . \tag{4.5}$$

Using Eqs. (4.3) to (4.5) along with Eq. (4.1) and using $\langle\vec{J}\rangle=\sigma\cdot\vec{\mathcal{E}}$, we get for the components of σ, the expressions

$$\sigma_{xx}=\sigma_{yy}=\sigma_Q \ , \tag{4.6}$$

$$\sigma_{zz}=-\frac{16\hbar^3e^2}{m^*d^2\epsilon_0\Omega}\,i\sum_{ll'k}\frac{(ll')^2\sin^2[\pi(l'-l)/2][f_0(\epsilon_{l'k})-f_0(\epsilon_{lk})]i\hbar\tau^{-1}_{l'k',lk}}{(l'^2-l^2)^3[(l'^2-l^2)\epsilon_0^2+\hbar^2\tau^{-2}_{l'k,lk}]} \ , \tag{4.7}$$

$$\sigma_{ij}=0, \quad i\neq j \ . \tag{4.8}$$

The results of QTT thus agree with those obtained from BTE for σ_{xx} and σ_{yy}, when electric field is applied parallel to the thin film. But, for the transverse case, the QTT gives conductivity σ_{zz} as given by Eq. (4.7) which could not be obtained from BTE. In the UTL, σ_{zz} is given by

$$\sigma_{zz}^{\rm UTL}\approx 128ne^2\hbar^4\tau_{12}^{-1}/27m^{*2}d^2\epsilon_0(q\,\epsilon_0^2+\hbar^2\tau_{12}^{-2}) \ , \tag{4.9}$$

with

$$\tau_{12}^{-1}=\tfrac{1}{2}[\tau^{-1}(\epsilon_{1k})+\tau^{-1}(\epsilon_{2k})]=2\tau_s^{-1} \ . \tag{4.10}$$

Now, when the spacing between two lowest levels is much smaller than the energy due to collision broadening $3\epsilon_0\ll\hbar/\tau_{12}$, σ_{zz} and the ratio $(\rho_Q/\rho_b)_\perp$, for acoustic phonon scattering, is given by

$$\sigma_{zz}=128ne^2\rho U^2\hbar^3d/27m^{*2}E_1^2k_BT \ , \tag{4.11}$$

$$(\rho_Q/\rho_b)_\perp=9\sqrt{\pi}\chi_D/32d, \quad \text{when } 3\epsilon_0\ll\hbar/\tau_{12} \ . \tag{4.12}$$

This resistivity ratio is smaller than that in the longitudinal configuration by a factor of $\frac{81}{128}$.

In the other extreme, when $3\epsilon_0\gg\hbar/\tau_{12}$, σ_{zz} and resistivity ratio $(\rho_Q/\rho_b)_\perp$, for acoustic phonon scattering is given by

$$\sigma_{zz}=256ne^2\hbar^4\tau_s^{-1}/243m^{*2}d^2\epsilon_0^3 \ , \tag{4.13}$$

$$(\rho_Q/\rho_b)_\perp=81\pi^{13/2}\hbar^2\chi_D\tau_s^2/512m^{*2}d^5, \ 3\epsilon_0\gg\hbar/\tau_{12} \ . \tag{4.14}$$

There is a marked difference in the temperature and thickness dependence in the two limiting cases discussed above. In the former case, when $3\epsilon_0\ll\hbar/\tau_{12}$, the resistivity ratio is independent of the scattering parameters, is inversely proportional to the thickness of the film, and varies with temperature as $T^{-1/2}$. When $3\epsilon_0\gg\hbar/\tau_{12}$, the resistivity ratio depends strongly on scattering parameters, is inversely proportional to the cube of the thickness of the film, and varies with temperature as $T^{-5/2}$.

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APPENDIX

In this Appendix, we prove Eq. (3.12) of the text:

$$\sum_{\vec{q}}|\langle\alpha'|\exp(i\vec{q}\cdot\vec{r})|\alpha\rangle|^2=\frac{2}{\pi d}\int_0^d\int_0^d dzdz'\sin\left(\frac{l'\pi z}{d}\right)\sin\left(\frac{l\pi z}{d}\right)\sin\left(\frac{l'\pi z'}{d}\right)\sin\left(\frac{l\pi z'}{d}\right)\int_0^\infty dq_z e^{iq_z(z-z')}$$

Since

$$\int_0^\infty dq_z e^{iq_z(z-z')}=2\pi\delta(z-z') \ ,$$

we have

$$\sum_{\vec{q}}|\langle\alpha'|\exp(i\vec{q}\cdot\vec{r})|\alpha\rangle|^2=\frac{4}{d}\int_0^d dz\,\sin^2(l'\pi z/d)\sin^2(l\pi z/d)=\frac{4}{d}\left(\frac{d}{4}+\frac{d}{8}\delta_{l'l}\right)=1+\tfrac{1}{2}\delta_{l'l} \ ,$$

which proves Eq. (3.12).

*On leave of absence from Department of Physics, University of Damascus, Damascus, Syria.

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