Magnetoresistance of copper, gold, and indium

J. E. Huffman, M. L. Snodgrass,* and F. J. Blatt
Physics Department, Michigan State University, East Lansing, Michigan 48824
(Received 11 August 1980)

We have measured the transverse magnetoresistance of high-purity polycrystalline wires of copper and gold between 4.2 and 50 K, and of indium between 4.2 and 9 K, in fields up to 5 and 10 T, respectively. Kohler's rule is obeyed by the samples of copper and gold in this range of field and temperature. Deviations from Kohler's rule are, however, quite evident in indium. These departures from Kohler's rule can be understood in terms of the same model invoked to account for similar observations in aluminum.

I. INTRODUCTION

In an earlier article[1] we reported on deviations from Kohler's rule in pure aluminum and some dilute aluminum alloys. Those results confirmed and elaborated on earlier work by Fickett[2] which indicated that the magnetoresistance ratio $\Delta\rho/\rho_0$ is not a universal function of H/ρ_0 as demanded by Kohler's rule. Here ρ_0 is the sample resistivity in zero field and $\Delta\rho$ is the increment in resistivity due to the transverse magnetic field.

The deviations observed in aluminum and in the dilute aluminum alloys were successfully interpreted in terms of a simple two-band model used to represent the more complicated Fermi surface and scattering anisotropy of the real metals. Similar observations were subsequently also reported by Krevet and Schauer on aluminum foils.[3]

The purpose of the present investigation was twofold. First, we wished to examine the magnetoresistance of indium in anticipation that this metal may display departures from Kohler's rule similar to those found in aluminum. Despite a small tetragonal distortion of the lattice, the Fermi surface of indium bears a close resemblance to that of aluminum; hence, the transport properties of the two metals could be expected to show some similarities. Since indium has a substantially lower Debye temperature than aluminum, departures from Kohler's rule, if they exist, should manifest themselves at correspondingly lower temperatures. Concurrent work by Thaler and Fletcher[4] at temperatures below 4.2 K and lower fields supported these expectations. In this work we performed measurements to fields as high as 10 T and covered the temperature range in which the deviations from Kohler's rule are pronounced.

Second, we wished to investigate the anomalous temperature dependence of the magnetoresistance of copper reported by Schwartz and Stangler.[5] The effect described by those workers appeared to bear some similarity to the departures from Kohler's rule that we had found in aluminum. Since the Fermi surface of copper and of other noble metals is nonspherical, departures from Kohler's rule could well occur, although earlier work in our laboratory had given no hint of unusual behavior in copper. We repeated these earlier, unreported measurements to see if we could reproduce the results of Schwartz and Stangler. We also measured the magnetoresistance of high-purity gold wires that were at hand. No deviations from Kohler's rule were observed for either copper or gold.

The remainder of this paper is organized into three sections. The first contains a brief description of the apparatus and a summary of the procedures employed in preparing the samples used in this study. Next, we present the results for indium and their interpretation on the basis of the same two-band model used with moderate success in analyzing the results on aluminum. The last section contains our results on copper and gold and a discussion of them and of the work of Schwartz and Stangler.

II. EXPERIMENTAL DETAILS

The indium sample used in the measurements came from a roll of 0.045-inch-diameter polycrystalline wire of nominal purity of 99.999% supplied by Indium Corporation of America. The wire was used as received without further treatment.

The pure gold from which our sample was prepared was in the form of a small ingot of 99.9999% nominal purity. It was swaged and drawn to 10-mil-diameter wire using a nonferrous swaging mill and diamond dies. During the drawing process the wire was vacuum annealed several times. Following the last draw, the wire sample was folded and wound as a bilfilar helix on a pyrolytic graphite cylinder which had the same diameter as the sample holder in the cryostat. The wire was then annealed near 980 °C in vacuum for five hours and then for an additional six hours in oxygen at a pressure of 0.05-mm Hg. Essentially the same procedure was employed in the preparation of our copper sample.

The sample holder consisted of a cylindrical copper block into which two grooves had been machined and which had been drilled out to admit a half-inch diameter copper form on which a heater had been wound. The heater unit was swabbed with vacuum grease to make good thermal contact with the sample holder. Two Au-Fe versus Chromel-P thermocouples were attached to the sample holder by cementing them in the grooves of the cylinder with Ge 7031 varnish. The other end of the thermocouple was sunk to the 4.2-K heat bath. The thermocouples were made from the same spools of wire that had previously been used by Chiang and for which we had calibration data to 10 T.[6] One thermocouple was used as the sensing element in the heater control circuit, the other to measure the temperature of the sample holder. Temperature measurements were within 0.1 K at lower temperatures with the uncertainty increasing to 0.2 K at the higher temperatures and strong magnetic fields.

The sample holder was surrounded by a copper heat shield and the entire unit was then enclosed in a vacuum chamber.

The measurements on copper and gold were performed using a conventional 5-T superconducting solenoid. The data on indium were obtained by replacing the 5-T magnet with a 10-T solenoid. The field homogeneity over the sample region was better than 1% with the 5-T magnet, and better than 1.5% with the 10-T magnet.

III. MAGNETORESISTANCE OF INDIUM

Two methods are conventionally employed for displaying magnetoresistance data. The first is the so-called Kohler plot, where $\Delta\rho/\rho_0$ is plotted against H/ρ_0. If the sample obeys Kohler's rule, all data, taken at any temperature and magnetic field, fall on one universal curve. To the extent that small concentrations of impurities do not alter the band structure, dilute alloy data should also fall on the same curve.[7]

Since our interest is in the temperature dependence of the magnetoresistance, it is more informative to plot $\Delta\rho/\rho_0$ as a function of temperature under conditions of constant magnetic field. Such curves should show a monotonic decrease with increasing temperature if Kohler's rule is obeyed, since the zero-field resistivity ρ_0 increases as the temperature is raised, and, therefore, H/ρ_0 is reduced.

Figure 1 shows results on a high-purity single crystal of indium, which were previously reported

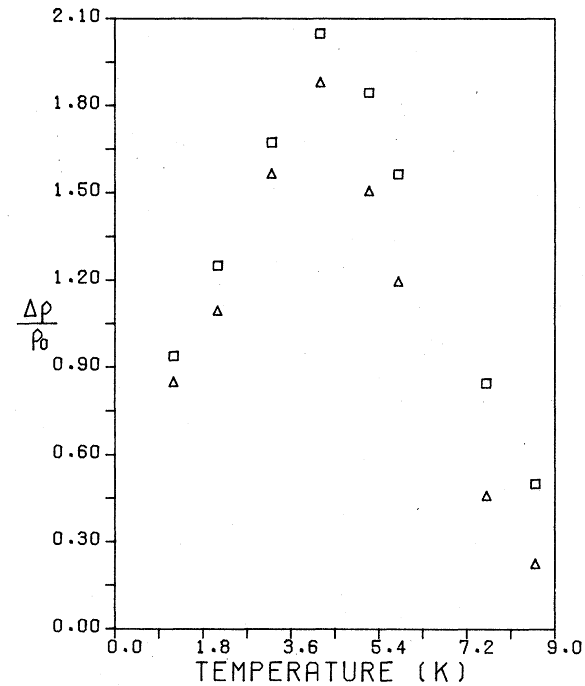

FIG. 1. Magnetoresistance ratio as a function of temperature for a single-crystal sample of indium (RRR = 91 000). Two values for magnetic field are shown: □ = 0.5 T, △ = 0.25 T. Taken from data published by Thaler and Fletcher (Ref. 4).

by Thaler and Fletcher.[4] The data, presented in their paper in the form of a traditional Kohler plot, was clearly indicative of substantial deviations from Kohler's rule. We have replotted their data for fields of 0.5 and 0.25 T, which shows the peak in magnetoresistance at a temperature of about 4.5 K. Our data, which extend to somewhat higher temperatures, are shown in Fig. 2 and show a pronounced peak at about 6 K in $\Delta\rho/\rho_0$, reminiscent of that observed in aluminum. Moreover, the temperature at the peak, between 4.5 and 6 K, is about $\Theta_D/20$; in aluminum, the peak in magnetoresistance also appears at about $\Theta_D/20$.

The close similarity between indium and aluminum, both with respect to the experimental results obtained here and to their band structures, suggests that the same simple model, capable of providing an interpretation of the results on aluminum, should also suffice here. The detailed arguments leading to the results given below are contained in the earlier paper[1] and are not reproduced now.

For two quasi-free-electron bands, identified here by the subscripts e and h, the magnetoresistance ratio is given by

$$\frac{\Delta\rho}{\rho_0} = \frac{\rho_H - \rho_0}{\rho_0} = \frac{A}{1 + B[(\rho_0/H)Nec]^2}, \qquad (1)$$

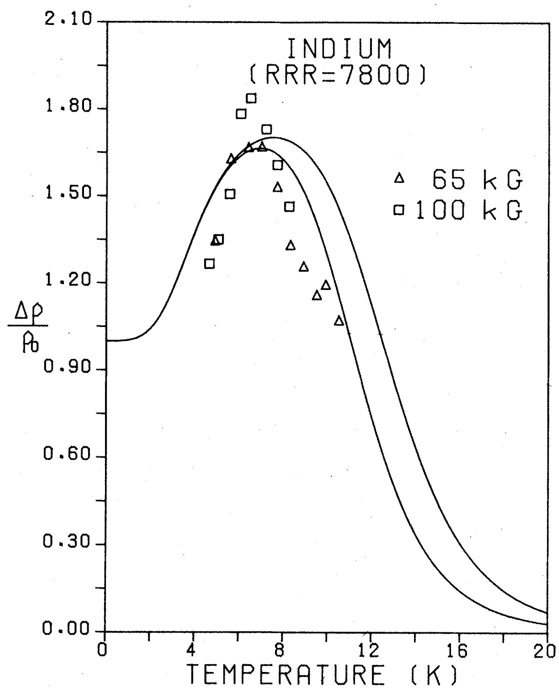

FIG. 2. Magnetoresistance ratio versus temperature for a polycrystalline indium sample (RRR = 7800) at 6.5 and 10 T. The solid curves shown are theoretical plots of the magnetoresistance ratio for these two fields using the two-band model. The parameters used in Eqs. (1) and (6) to generate the curves are listed in Table I. (1 T = 10 kG).

where

$$A = \frac{[(n_e/n_h) + (\rho_h/\rho_e)]^2}{(\rho_h/\rho_e)(1-n_e/n_h)^2}, \tag{2}$$

$$B = \frac{(n_e/N)^2}{(1-n_e/n_h)^2}\left(\frac{\rho_e+\rho_h}{\rho_0}\right)^2, \tag{3}$$

$$N = n_h - n_e, \tag{4}$$

and

$$\sigma_i = 1/\rho_i = n_i e^2 \tau_i / m_i. \tag{5}$$

Kohler's rule fails even in this simple model unless the temperature dependences of the relaxation times for the two bands are proportional to each other over the entire temperature range. That is, for Kohler's rule to hold, σ_e/σ_h must be temperature independent. We shall assume that the resistivities associated with the two bands exhibit temperature dependences of the form

$$\rho_e = a_e + b_e T^x, \quad \rho_h = a_h + b_h T^x. \tag{6}$$

Our procedure for determining the various parameters was as follows. The low-temperature, zero-field conductivity (which must be estimated by extrapolation, since indium becomes superconducting below 3.4 K) is equal to $1/a_e + 1/a_h = 1/a$. Next, a fit of the zero-field temperature-dependent part of the resitivity to

$$\rho(T) = \left(\frac{1}{b_e} + \frac{1}{b_h}\right)^{-1} T^x = bT^x \tag{7}$$

gives the values of b and x. Lastly, from the low-field values of the Hall coefficient in the low-temperature limit one can deduce the ratio a_e/a_h.[8] This leaves only one adjustable parameter, the ratio b_e/b_h, which is used in arriving at a best fit using the two-band model. The values of these parameters are listed in Table I. Figure 3 shows the calculated and measured zero-field resistivities, and in Fig. 2 the continous solid lines show the calculated magnetoresistance for the two appropriate field values.

There are several points that should be made here. First, we find that we cannot obtain even an approximate fit to the zero-field resistivity with a function of the form $a + bT^5$, as was found by Bressan *et al.*[9] for temperatures between 1.2 and 4.2 K. However, because of indium's low Debye temperature, one does not expect a T^5 dependence to be valid for intermediate temperatures, above about 5 K. For these higher temperatures the Bloch-Grüneisen expression

$$\rho_L \sim \left(\frac{T}{\Theta_D}\right) J_5\left(\frac{\Theta_D}{T}\right) \tag{8}$$

may be more appropriate.

If one attempts to represent $\rho_i(T)$ by a simple power law it is evident that the exponent will be less than 5, and the fit cannot be expected to be completely satisfactory. While we believe that the reason for the lower exponent is the one just given, it should be pointed out that the residual resistance ratio of our sample does not approach that of the samples used by Bressan *et al.* and it is conceivable that deviations from Matthiessen's rule have influenced the value of the exponent x in Eq. (7).

Second, as is evident from Fig. 2, the agreement between the calculated curves and experimental data is considerably poorer than in the

TABLE I. Parameters used in Eqs. (1) and (6) to generate the theoretical curves shown in Figs. 2 and 3.

a_e	1.75 nΩ cm
a_h	0.438 nΩ cm
b_e	1.10×10^{-2} nΩ cm/$K^{3.65}$
b_h	9.90×10^{-3} nΩ cm/$K^{3.65}$
x_e	3.65
x_h	3.65
n_e	7.66×10^{21} cm^{-3}
n_h	4.60×10^{22} cm^{-3}

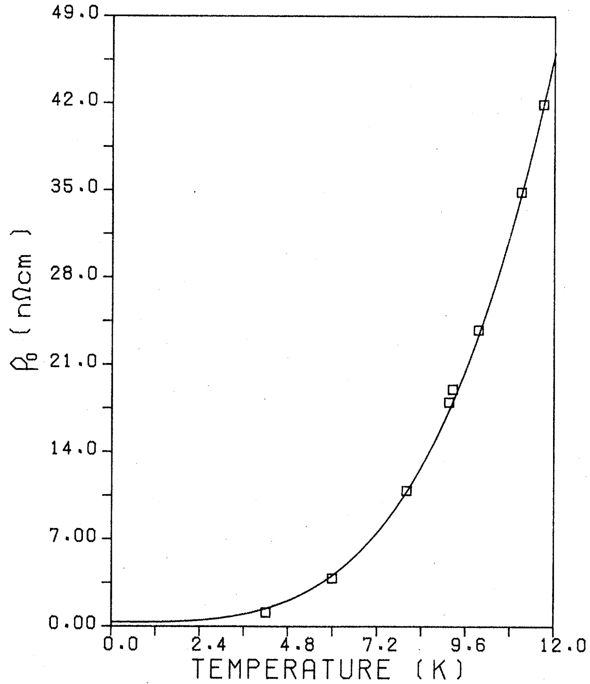

FIG. 3. Zero-field resistivity versus temperature for indium (RRR = 7800). The solid line represents the theoretical curve given by the two-band model, using the parameters of Table I in Eq. (6).

case of aluminum. We suspect that this may be a reflection of the fact that the simple two-band model is even less valid for indium than for aluminum. Not only is the crystal structure of indium not of cubic symmetry, but the anisotrophy of the hole relaxation time in indium is nearly three times as great as it is for aluminum.[10] To take account of anisotropic scattering, a generalized form of Kohler's rule was derived by Jones and Sondheimer[11]:

$$\frac{\Delta\rho}{\rho_0}=f\left(\frac{H}{\rho_0},\ \Delta_\infty\right), \tag{9}$$

where $\Delta_\infty = \Delta\rho_\infty/\rho_0$ is a constant. Bressan *et al.* employed a more restrictive form of this equation in analyzing their data on indium,[12] namely,

$$\frac{\Delta\rho}{\rho_0}=\Delta_\infty f\left(\frac{H}{\rho_0}\Delta_\infty\right). \tag{10}$$

However, the simple two-band model used by us [Eq. (1)] leads to an expression of the form

$$\frac{\Delta\rho}{\rho_0}=\frac{A\left(\frac{1}{Nec}(H/\rho_0A)\right)^2}{\left(\frac{1}{Nec}(H/\rho_0A)\right)^2+\frac{B}{A^2}},\quad A=\Delta_\infty \tag{11}$$

which, though consistent with the Jones and Sondheimer result, is not equivalent to Eq. (10).

Third, the somewhat higher temperature of the peak in Fig. 2 as compared to that of Fig. 1 is, we believe, due to two factors. First, according to Eq. (11), the peak temperature should shift to higher values with increasing magnetic field. Data on aluminum show this dependence quite clearly.[1] The results of Fletcher and Thaler were obtained in fields below 2 T. Second, the peak should also shift to lower temperatures with increasing sample purity; the single-crystal sample of Thaler and Fletcher had a residual resistivity ratio (RRR) substantially greater than our sample.

Significant improvement in fitting experimental data could have been achieved by allowing the exponent x to take on different values for the electron and hole bands. It is our opinion that the model does not justify such refinement.

IV. MAGNETORESISTANCE OF COPPER AND GOLD

Our results on these two noble metals are illustrated in Figs. 4 and 5. As best as we can determine, the magnetoresistance in both gold and copper appears to obey Kohler's rule. In particular, for neither metal does the magnetoresistance at constant field exhibit a peak when plotted as a function of temperature. Instead, $\Delta\rho/\rho_0$ is a monotonically diminishing function of tem-

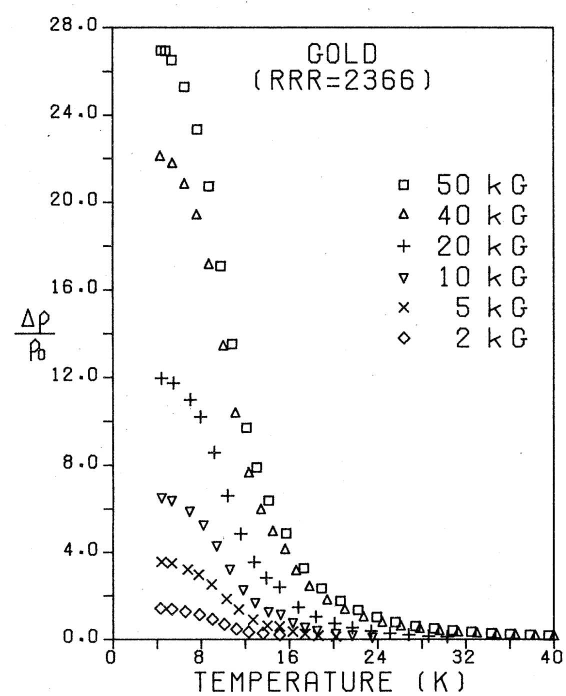

FIG. 4. Magnetoresistance ratio versus temperature for a polycrystalline sample of gold (RRR = 2366). Six values of magnetic field are shown. The data show a monotonic decrease with increasing temperature, consistent with Kohler's rule.

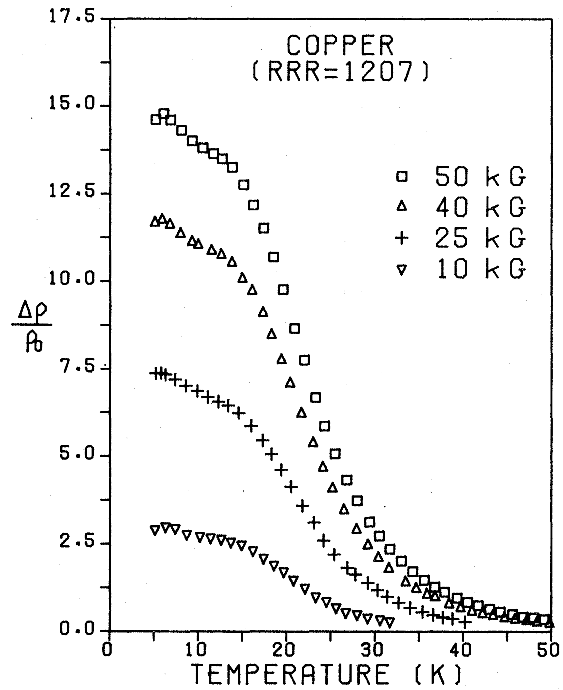

FIG. 5. Magnetoresistance ratio versus temperature for polycrystalline copper (RRR = 1207). The data show a monotonic decrease with increasing temperature, consistent with Kohler's rule.

perature as one would expect.

On contrasting our results with those of Schwartz and Stangler,[5] two features should be noted. First, and probably of lesser significance, is the fact that their results were obtained from measurements on thin foils, approximately 13 microns thick, in which size effects play a significant role. Although Schwartz and Stangler did make appropriate size-effect corrections in arriving at their zero-field resistances and resistance ratios, it is not evident from their paper that size-effect corrections also were applied at elevated fields. As was demonstrated by Sondheimer,[13] the magnetoresistance of thin films depends on film thickness, so that size-effect corrections are both field and temperature dependent, and must be applied before thin-film magnetoresistance results can be compared with bulk theories.

Second, and of greater significance, is the fact that Schwartz and Stangler do not show the magnetoresistance ratio as is customarily done, but the change in resistance $\Delta\rho$. This, we are convinced, is in fact the primary reason for the apparent anomaly which they report. A peak in $\Delta\rho$ does not necessarily reflect a peak in the magnetoresistance ratio $\Delta\rho/\rho_0$, and it is the latter that is of concern when comparing experimental results with standard theory. The same

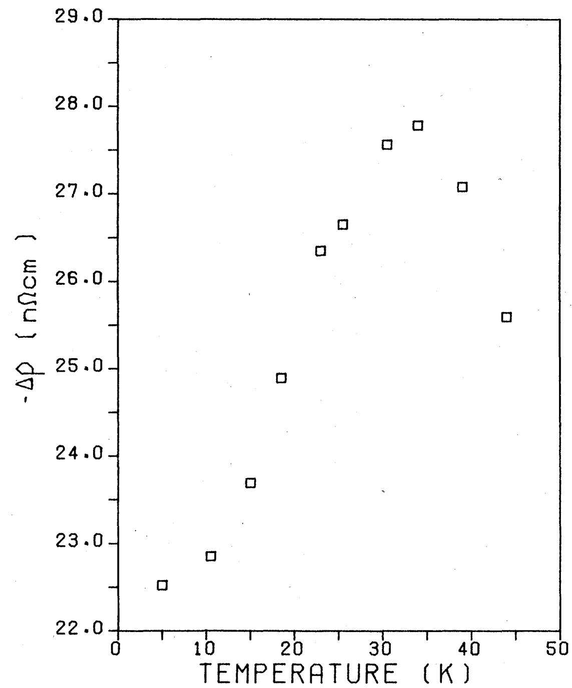

FIG. 6. The change in resistivity versus temperature for one value of field (5 T), using the same data as appear in Fig. 5. The peak, which occurs at about 35 K, corresponds to the peak reported by Schwartz and Stangler (Ref. 5).

data that are shown in Fig. 5 for $B = 5$ T have been replotted in Fig. 6, where the points for the latter are derived from Fig. 5 by multiplying by the zero-field resistivity at each temperature. A peak is clearly evident, and this peak appears at roughly the same temperature as the one reported by Schwartz and Stangler.

V. CONCLUSION

The transverse magnetoresistance of polycrystalline indium wires exhibits a pronounced departure from Kohler's rule. The simple two-band model that was fairly satisfactory in reproducing the corresponding results in aluminum gives only qualitative agreement when applied to indium. The magnetoresistance data of the two metals do, however, appear to scale approximately in the ratio of their Debye temperatures.

Measurements of the magnetoresistance of pure gold and copper wires gave no indications of an anomalous behavior. An earlier report of an anomaly in the magnetoresistance of copper is traced to an inappropriate presentation of experimental data.

ACKNOWLEDGMENT

This work was supported by the National Science Foundation Grant No. DMR 78-07892-02.

*Present address: Bell Laboratories, 555 Union Boulevard, Allentown, Pennsylvania 18103.

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