Coupling of order-parameter modes with $l \geq 1$ to zero sound in ^{3}He-B

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The frequencies of B-phase order-parameter modes are calculated with $l \geq 1$ pairing interactions and all Landau parameters included. The two modes that have been observed are modified by v_3 and the $l=2$ Landau parameters. The corrections from v_3 and F_2^a can explain the observed frequency and temperature dependence of the 2+ mode (real squashing mode). A new mode with $J=4$, whose frequency depends on v_3, v_5, and F_4^s, may be observable in the zero-sound dispersion.

Weak-coupling collisionless kinetic equations, including Fermi-liquid interactions and a single angular momentum component ($l=1$) of the pairing interaction, have been used extensively to study the order-parameter collective modes coupled to zero sound in superfluid ^{3}He-B: this work was summarized and completed by Wölfle in Ref. 1. In this paper we extend the theory to include all angular momentum components v_l of the pairing interaction. We find that v_3 modifies the frequencies of the two modes already observed in ultrasound experiments. We also find a new mode whose interaction with zero sound could be strong enough to be seen.

Inconsequence of the spontaneously broken spin-orbit symmetry of the equilibrium order parameter, the total angular momentum J (but not the orbital angular momentum l) is an appropriate quantum number to characterize the collective modes of the order parameter $\vec{d}(\hat{p};\vec{q},\omega)$ at $q=0$. A spin-triplet mode with total angular momentum J can have components with orbital angular momentum $l=J\pm 1$. Only modes with even J are coupled to the density, and hence with a pure $l=1$ pairing interaction the relevant modes have $J=0,2$. Assuming particle-hole symmetry, one finds that the density couples to modes of $\vec{d}^{(-)}(\vec{q},\omega)=\frac{1}{2}[\vec{d}(\vec{q},\omega)-\vec{d}^*(-\vec{q},-\omega)]$, but not to modes of $\vec{d}^{(+)}(\vec{q},\omega)=\frac{1}{2}[\vec{d}(\vec{q},\omega)+\vec{d}^*(-\vec{q},-\omega)]$. The $J=0$ mode of $\vec{d}^{(-)}$ (henceforth the "0− mode") is part of the zero-sound mode, while the 2− modes have frequency $\omega_{2-}=\sqrt{12/5}\times\Delta(T)$ for $F_2^s=v_3=0$. Wölfle named the 2− mode which couples to sound in the absence of transverse magnetic fields the squashing mode; it has $J_z=0$ along $\hat{q}$. With particle-hole asymmetry included, the $J=0$ and modes of $\vec{d}^{(+)}$ also couple to the density. The 0+ mode has $\omega_{0+}=2\Delta(T)$, independent of Fermi-liquid corrections, and the 2+ modes have $\omega_{2+}=\sqrt{8/5}\Delta(T)$ for $F_2^a=v_3=0$. Again the mode with $J_z=0$ along $\hat{q}$ interacts with sound in zero transverse field; Wölfle calls this the real squashing mode. Both $J=2$ modes have now been seen in sound-propagation experiments.[2–4] Theoretically the effect on the zero-sound dispersion from the real squashing (2+) mode should be smaller than that from the squashing (2−) mode by the square of a particle-hole asymmetry parameter which Koch and Wölfle[5] estimate to be of order $(\Delta/\epsilon_F)\ln(0.1\epsilon_F/\Delta)$, and the experiments appear to be roughly consistent with this predictions.[6] The 0+ mode has not yet been identified experimentally, but it should have a coupling strength intermediate between that of the 2− and 2+ modes.

To include the pairing interactions with $l\neq 1$, we first note that spin-singlet modes[7] cannot couple to the density. Gauge invariance requires that only the combinations $\delta\Delta\Delta^\dagger \pm \Delta\delta\Delta^\dagger$ enter observables, but these combinations cannot contain any scalar components from a triplet Δ and a singlet $\delta\Delta$. Thus we will treat the coupled equations for the diagonal and off-diagonal mean fields

$$\delta\epsilon_{\alpha\beta}(\hat{p})=\delta\epsilon(\hat{p})\delta_{\alpha\beta}+\delta\vec{\epsilon}(\hat{p})\cdot\vec{\sigma}_{\alpha\beta} \tag{1}$$

and

$$\delta\Delta_{\alpha\beta}(\hat{p})=i\vec{d}(\hat{p})\cdot(\vec{\sigma}\sigma_y)_{\alpha\beta} \ . \tag{2}$$

We take $q=0$ and neglect particle-hole asymmetry, which is sufficient to determine the frequencies of the modes coupled to zero sound with negligible error, but not to calculate the coupling constants. The coupled equations for $\delta\epsilon(\hat{p})$ and $\vec{d}^{(-)}(\hat{p})$ are

$$\delta\epsilon(\hat{p}) - \delta\epsilon_{\rm ext}(\hat{p}) = \int \frac{d\Omega'}{4\pi} F^s(\hat{p}\cdot\hat{p}')[-\lambda\delta\epsilon(\hat{p}') + \tfrac{1}{2}\omega\Delta\bar{\lambda}\hat{p}'\cdot\vec{\rm d}^{(-)}(\hat{p}')] \ , \tag{3}$$

$$\vec{\rm d}^{(-)}(\hat{p}) = \int \frac{d\Omega'}{4\pi} v(\hat{p}\cdot\hat{p}') \left[-\tfrac{1}{2}\omega\Delta\bar{\lambda}\delta\epsilon(\hat{p}')\hat{p}' + \frac{1}{v_1}\vec{\rm d}^{(-)}(\hat{p}') + \tfrac{1}{4}(\omega^2 - 4\Delta^2)\bar{\lambda}\,\vec{\rm d}^{(-)}(\hat{p}') + \lambda\hat{p}'\cdot\vec{\rm d}^{(-)}(\hat{p}')\hat{p}' \right] \ , \tag{4}$$

where the pairing pseudointeraction is

$$v(\hat{p}\cdot\hat{p}') = \sum_{{\rm odd}\ l} (2l+1) v_l P_l(\hat{p}\cdot\hat{p}') \ , \tag{5}$$

the function λ is given by

$$\lambda = \Delta^2\bar{\lambda} = 4\Delta^2 \int_\Delta^\infty dE \frac{\tanh(E/2T)}{(4E^2-\omega^2)(E^2-\Delta^2)^{1/2}} \ , \tag{6}$$

and we have used the $J=0$ equilibrium order parameter $A_{i\mu} = \Delta(T)\delta_{i\mu}$. To solve these equations we first note that if $\vec{\rm d}^{(-,l)}$ is the component of $\vec{\rm d}^{(-)}$ with angular momentum l, then $\hat{p}\cdot\vec{\rm d}^{(-,l)}$ contains only $J = l\pm 1$ components, and hence can be represented as

$$\hat{p}\cdot\vec{\rm d}^{(-,l)}(\hat{p}) = B^{(l+1,l)}_{\mu_1\cdots\mu_{l+1}} p_{\mu_1}\cdots p_{\mu_{l+1}} + B^{(l-1,l)}_{\mu_1\cdots\mu_{l-1}} p_{\mu_1}\cdots p_{\mu_{l-1}} \ , \tag{7}$$

where $B^{(l+1,l)}$ and $B^{(l-1,l)}$ are symmetric and traceless in all pairs of their indices.[8] Hence if we decompose Eq. (4) into its angular momentum components, take the dot product of each with $\hat{p}$, and use the recursion formula[9]

$$(\hat{p}\cdot\hat{p}')P_l(\hat{p}\cdot\hat{p}') = \frac{l+1}{2l+1}P_{l+1}(\hat{p}\cdot\hat{p}') + \frac{l}{2l+1}P_{l-1}(\hat{p}\cdot\hat{p}') \tag{8}$$

we can immediately read off the coupled equations satisfied by the $l = J\pm 1$ components of the mode with total angular momentum J:

$$\frac{J}{2J+1}\omega\Delta E^{(J)} = \frac{2J}{2J+1}\Delta^2 B^{(J,J+1)} + \left[2\bar{X}_{J-1} + \tfrac{1}{2}\left(\omega^2 - \frac{4(J+1)}{2J+1}\Delta^2 \right) \right] B^{(J,J-1)} \ , \tag{9}$$

$$\frac{J+1}{2J+1}\omega\Delta E^{(J)} = \left[2\bar{X}_{J+1} + \tfrac{1}{2}\left(\omega^2 - \frac{4J}{2J+1}\Delta^2 \right) \right] B^{(J,J+1)} + \frac{2(J+1)}{2J+1}\Delta^2 B^{(J,J-1)} \ . \tag{10}$$

In these equations we have introduced

$$\bar{X}_l = X_l/\bar{\lambda} = \left(\frac{1}{v_1} - \frac{1}{v_l} \right) \Big/ \bar{\lambda} \ . \tag{11}$$

which is independent of the cutoff used to define the pairing pseudointeractions v_l. $E^{(J)}$ is the totally traceless and symmetric tensor representation of the angular momentum J component of $\delta\epsilon(\hat{p})$,

$$\delta\epsilon^{(J)}(\hat{p}) = E^{(J)}_{\mu_1\cdots\mu_J} p_{\mu_1}\cdots p_{\mu_J} \ , \tag{12}$$

and can be calculated immediated from Eqs. (3) and (7),

$$E^{(J)} = \frac{E^{(J)}_{\rm ext} + [F^s_J/2(2J+1)]\omega\Delta\bar{\lambda}(B^{(J,J+1)} + B^{(J,J-1)})}{1 + [F^s_J/(2J+1)]\lambda} \ . \tag{13}$$

Equations (9), (10), and (13) determine the frequencies of the $J-$ modes. For the $2-$ modes we find[10]

$$X_3[\omega^2 - \tfrac{12}{5}\Delta^2 + \tfrac{3}{25}F^s_2(\omega^2 - 4\Delta^2)\lambda] + \tfrac{1}{4}\omega^2(\omega^2 - 4\Delta^2)\bar{\lambda} = 0 \ , \tag{14}$$

and for the $4-$ modes,

$$4X_3X_5(1 + \tfrac{1}{9}F^s_4\lambda) + X_3\bar{\lambda}[\omega^2 - \tfrac{16}{9}\Delta^2 + \tfrac{4}{81}F^s_4(\omega^2 - 4\Delta^2)\lambda] + X_5\bar{\lambda}[\omega^2 - \tfrac{20}{9}\Delta^2 + \tfrac{5}{81}F^s_4(\omega^2 - 4\Delta^2)\lambda] + \tfrac{1}{4}\omega^2(\omega^2 - 4\Delta^2)(\bar{\lambda})^2 = 0 \ . \tag{15}$$

Equation (14) agrees with the result given by Wölfle in the limit $v_3 \to 0$ and $|F^s_2| \ll 1$. A positive F^s_2 increases the eigenfrequency ω_{2-}, while a positive (attractive) v_3 lowers ω_{2-}. Because $\lambda \rightarrow 0$ for $T \rightarrow T_c$, ω_{2-} always approaches $\sqrt{12/5}\Delta(T)$ in this limit.

The equations satisfied by $\delta\vec{\epsilon}(\hat{p})$ and $\vec{d}^{(+)}(\hat{p})$ are

$$\delta\vec{\epsilon}(\hat{p}) - \delta\vec{\epsilon}_{\rm ext}(\hat{p}) = \int \frac{d\Omega'}{4\pi} F^a(\hat{p}\cdot\hat{p}')\{-\lambda[\delta\vec{\epsilon}(\hat{p}') - \hat{p}'\cdot\delta\vec{\epsilon}(\hat{p}')\hat{p}'] - \tfrac{1}{2} i\omega\Delta\bar{\lambda}\hat{p}'\times\vec{d}^{(+)}(\hat{p}')\} \ , \qquad (16)$$

$$\vec{d}^{(+)}(\hat{p}) = \int \frac{d\Omega'}{4\pi} v(\hat{p}\cdot\hat{p}')\left[\tfrac{1}{2} i\omega\Delta\bar{\lambda}\hat{p}'\times\delta\vec{\epsilon}(\hat{p}') + \frac{1}{v_l}\vec{d}^{(+)}(\hat{p}') + \tfrac{1}{4}\omega^2\bar{\lambda}\,\vec{d}^{(+)}(\hat{p}') - \lambda\hat{p}'\cdot\vec{d}^{(+)}(\hat{p}')\hat{p}'\right] \ . \qquad (17)$$

These are slightly more difficult to solve than Eqs. (3) and (4) because of the cross products, but if we proceed just as for the $\vec{d}^{(-)}$ equation, we can eventually obtain equations for the $l = J \pm 1$ components $B^{(J,J\pm 1)}$ of the mode of $\vec{d}^{(+)}$ with total angular momentum J:

$$\frac{J}{2J+1} i\omega\Delta\check{E}^{(J)} = \frac{2J}{2J+1}\Delta^2 B^{(J,J+1)} - \left[2\bar{X}_{J-1} + \tfrac{1}{2}\left(\omega^2 - \frac{4J}{2J+1}\Delta^2\right)\right] B^{(J,J-1)} \ , \qquad (18)$$

$$\frac{J}{2J+1} i\omega\Delta\check{E}^{(J)} = \left[2\bar{X}_{J+1} + \tfrac{1}{2}\left(\omega^2 - \frac{4(J+1)}{2J+1}\Delta^2\right)\right] B^{(J,J+1)} - \frac{2(J+1)}{2J+1}\Delta^2 B^{(J,J-1)} \ . \qquad (19)$$

The new tensor $\check{E}^{(J)}$ is defined by[11]

$$\check{E}^{(J)}_{\mu_1\cdots\mu_J} = \frac{1}{J}(\epsilon_{\mu_1 jk}E^{(J)}_{k,j\mu_2\cdots\mu_J} + \cdots + \epsilon_{\mu_J jk}E^{(J)}_{k,j\mu_1\cdots\mu_{J-1}}) \ , \qquad (20)$$

where in analogy to Eq. (12) we represent the angular momentum J component of $\delta\vec{\epsilon}(\hat{p})$ by

$$\delta\epsilon_i^{(J)}(\hat{p}) = E^{(J)}_{i,\mu_1\cdots\mu_J} p_{\mu_1}\cdots p_{\mu_J} \ . \qquad (21)$$

From Eq. (16) for $\delta\vec{\epsilon}(\hat{p})$ we find

$$\check{E}^{(J)} = \frac{\check{E}^{(J)}_{\rm ext} - [F^a_J/2(2J+1)]i\omega\Delta\bar{\lambda}\{B^{(J,J+1)} - [(J+1)/J]B^{(J,J-1)}\}}{1 + [F^a_J/(2J+1)]\lambda} \qquad (22)$$

which together with Eqs. (18) and (19) determines the frequency of the $J+$ mode. For the $2+$ mode the dispersion relation is

$$X_3[\omega^2 - \tfrac{8}{5}\Delta^2 + \tfrac{2}{25}F^a_2(\omega^2 - 4\Delta^2)\lambda] + \tfrac{1}{4}\omega^2(\omega^2 - 4\Delta^2)\bar{\lambda} = 0 \ . \qquad (23)$$

In the limit $v_3 \rightarrow 0$ and $|F^a_2| \ll 1$ this reduces to

$$\omega^2_{2+} \simeq \tfrac{8}{5}\Delta(T)^2(1 + \tfrac{3}{25}\lambda F^a_2) \ , \qquad (24)$$

which differs from the result given by Wölfle,[1] who found the coefficient of F^a_2 to be three times larger than it is in Eq. (24).

Figure 1 shows the dispersion relations for $\omega_{2+}(T)$ calculated from Eq. (23) in the two extreme cases $v_3 = 0$ and $F^a_2 = 0$, with the remaining parameter in each case adjusted to give[3] $\omega_{2+}(T=0) = 1.075\Delta(T=0)$. For $v_3 = 0$ this requires $F^a_2 = -1.56$, while $F^a_2 = 0$ implies $X_3 = -2.31$, which corresponds to $v_3 = 0.14$ if we take $v_l = 0.20$. Although the temperature dependence of $\omega_{2+}(T)$ changes very little between these two cases, precision measurements of $\omega_{2+}(T)$ can in principle be used to determine both F^a_2 and v_3 with no other experimental input except T_c. However, existing measurements of $\omega_{2+}(T)$ alone cannot be used to determine either F^a_2 or v_3; an analogous ambiguity exists between the corrections to $\omega_{2-}(T)$ from F^s_2 and from v_3. Measurements of $\chi_B(T=0)$ can give F^a_2 (at least at low pressures where the nontrivial strong-coupling corrections are negligible) once current uncertainties over F^s_1 and F^a_0 have been resolved, so another determination of v_3 from $\omega_{2+}(T)$ should be possible eventually. Similarly, since F^s_2 can be determined from the difference between the first- and zero-sound velocities (given an accurate value for F^s_1), an independent value for v_3 can be obtained from accurate measurements of $\omega_{2-}(T)$. Any significant discrepancy between these two values for v_3 could then be interpreted as experimental evidence for nontrivial strong-coupling corrections to the collective mode frequencies. In Fig. 1 we have also shown experimental results for $\omega_{2+}(T)/\Delta_{\rm BCS}(T)$ from Refs. 3 and 4. The agreement between these data and Eq. (23) is excellent, given that some discrepancy is expected due to the strong-coupling corrections to $\Delta(T)$.

We can estimate the maximum possible effect on zero sound from modes with $J > 2$ by a simple argument. When the order-parameter oscillations are driven by zero sound, the order-parameter fluctuation tensor must be constructed from powers of $\delta_{\mu\nu}$ and of $q_\mu q_\nu$. Furthermore, a tensor of the form in Eq. (7) with total angular momentum J must contain the tensor $q_{\mu_1}\cdots q_{\mu_J}$, and the associated dimension-

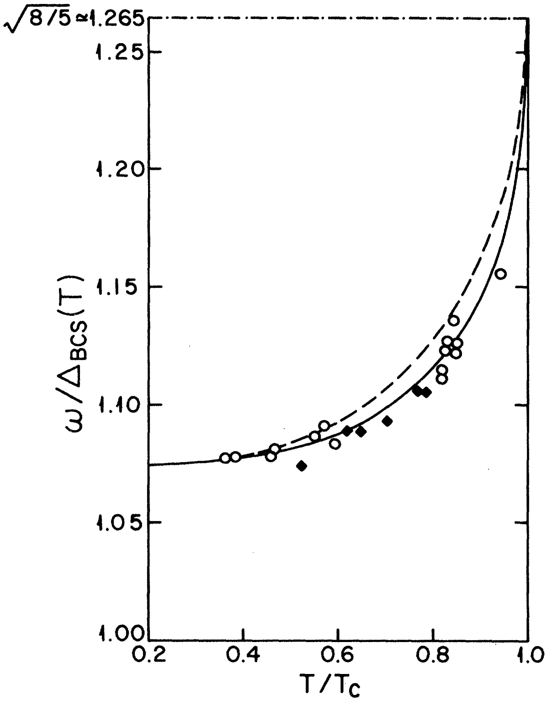

FIG. 1. Temperature dependence of the new collective-mode resonant frequency normalized to $\Delta_{\text{BCS}}(T)$. The open circles are the data of Ref. 3 obtained at a pressure of 13.0 bars. The diamonds are the data of Ref. 4 obtained at pressures between 0.8 and 3.5 bars. The solid (dashed) curve is the calculated temperature dependence of the 2+ mode from Eq. (23) with $X_3=-2.31$ and $F_2^q=0$ ($v_3=0$ and $F_2^q=-1.56$). The interaction parameters were chosen to fit the $T=0$ K value of $\omega_{2+}(T=0)=1.075\ \Delta(T=0)$ reported by Ref. 3. The data of Ref. 4 show that the same temperature dependence of $\omega_{2+}/\Delta_{\text{BCS}}$ exists at low pressure where strong-coupling effects are negligible.

less coupling constant is $(qv_F/\omega)^J=(v_F/c)^J=1/s^J$. To couple this order-parameter oscillation back into the ($J=0$) density oscillation requires another tensor with angular momentum J, and hence the correction to the zero-sound dispersion relation from oscillations of $\vec{d}^{(-)}$ with total angular momentum J is proportional to $1/s^{2J}$. The 4− mode thus appears to be the only additional order-parameter mode which might be observable with zero sound. In Fig. 2 we

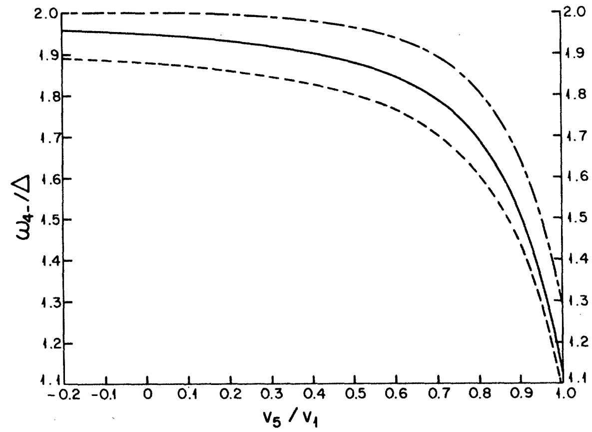

FIG. 2. $T=0$ solutions to the 4− mode equation. The solid (dashed) curve corresponds to $F_4^s=0$ ($F_4^s=-1.0$) and $X_3=-2.31$, the value which gives $\omega_{2+}(T=0)/\Delta(T=0)=1.075$ for $F_2^q=0$. The dash-dotted curve corresponds to $X_3=-15.0$ (a smaller v_3/v_1) and $F_4^s=0$. The abscissa is v_5/v_1 with $v_1=0.2$.

show the frequency $\omega_{4-}(T=0)/\Delta(T=0)$ from Eq. (15) as a function of X_5 and F_4^s taking $X_3=-2.31$, the value corresponding to $\omega_{2+}(T=0)=1.075\Delta(T=0)$ and $F_2^q=0$. For $X_3=-15.0$, corresponding to a smaller v_3/v_1, and $F_4^s=0$, $\omega_{4-}(T=0)$ lies closer to 2Δ. If, as seems likely, ω_{4-} falls close to 2Δ, the 4− mode may be difficult to distinguish from the 0+ mode at 2Δ. However, the 4− mode will split in a transverse magnetic field, while the 0+ mode will not.

In summary, we emphasize that corrections from v_3 and F_2^q can explain the observed frequency of the 2+ (real squashing) mode, that the properties of this mode at low pressures should allow an experimental determination of v_3, and that a new mode with $J=4$ may be observable in the zero-sound dispersion.

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[11]For $J=0$, $\tilde{E}^{(J)}=B^{(J,J-1)}=0$.