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Two-dimensional level broadening in the extreme quantum limit

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Broadening of Landau levels in a two-dimensional electronic system under a strong, perpendicular magnetic field is obtained within a theory in which the broadening and the impurity screening are treated self-consistently. The random-phase approximation is used for the electronic response, and the self-consistent Born approximation is employed for the electron-impurity scattering. The zero-temperature, extreme quantum limit is considered, revealing narrowing of the Landau level width with decreasing filling factor. Possible relevance of these theoretical results to recent experiments on the narrowing of the cyclotron resonance linewidth in space-charge layers is discussed.

Recently, there has been a considerable amount of interest in the properties of two-dimensional electronic systems under very high magnetic fields.[1] A magnetic field B normal to the plane of confinement of the electrons quantizes the system into discrete Landau levels. If the magnetic field is strong enough and the electron density and temperature are not very large, it is possible for the electrons to occupy only the lowest Landau level. This is the so-called extreme quantum limit. Electronic motion is then completely quantized and the lowest Landau level has a macroscopic degeneracy of $(geB/2\pi c\hbar)$ where g accounts for the spin and valley degeneracy of the system. Under such high fields the spin and valley degeneracy is expected to be lifted and g is then unity.

Such a system has attracted the attention of experimentalists[2–5] and theorists[6–11] for a variety of reasons. In particular, the singular density of states (in the absence of damping) of such a system implies novel transport properties[2,3,6,11] not observable in three-dimensional systems. In principle, such a system should be completely insulating in the absence of any level broadening. In addition, the kinetic energy of the system is effectively quenched in the extreme quantum limit and this gives rise to strong electron-correlation effects leading to the possibility of a highly correlated, possibly even a crystalline electronic ground state, at least in the limit of low density and temperature.[7–10]

Recently, two experiments[4,5] on semiconducting space-charge layers (which have been experimental models for two-dimensional electron systems for quite some time) have been reported which pertain to this extreme quantum behavior of two-dimensional systems. Both study cyclotron resonance of two-dimensional electrons, either confined near the Si-SiO_2 interface in n-type Si inversion layer[4] or near the GaAs-GaAlAs interface in a modulation-doped heterojunction.[5] The experiments were carried out in the extreme quantum limit, with only the lowest Landau level occupied. Both of these experiments reveal an extreme narrowing of the cyclotron resonance linewidth. The linewidth when plotted as a function of the filling factor $\nu = (2\pi c\hbar/eB)N$ (the fractional occupancy of the ground Landau level for a magnetic field strength of B and electron density N), shows a remarkable decrease in magnitude as ν changes from 1.0 (completely filled) to 0.5 (half filled). This decrease could be as large as by a factor of 5. The resonance for $\nu < 0.5$ is thus extremely sharp. The nature of the resonance does not change very much below $\nu \simeq 0.5$. In addition the Si sample shows an apparent shift of cyclotron mass in this extreme quantum limit whereas the GaAs heterojunction does not. These observations have been interpreted[4] in terms of a highly correlated ground state of the electron system, possibly a charge-density wave or a Wigner crystal state which is stabilized by the high magnetic field due to the quenching of the kinetic energy in the extreme quantum limit. Even though no definitive theory for these experiments really exists and the Wigner crystallization (in the absence of any magnetic field) would occur at much lower electron density,[12] indications are that the strong magnetic field would raise the transition temperature and/or the critical density for transition into a more ordered electronic ground state.[7–10] Such a collective ground state is expected to lead to a level narrowing and, if pinned by impurities at the interface, could lead to a shift in mass as well.[13] But the fact remains that no semiquantitative theory exists that would explain these experiments, particularly the extreme line nar-

rowing in the quantum limit.

Motivated by these experiments, we have studied the nature of level broadening in two-dimensional systems in the extreme quantum limit without assuming the existence of a correlated ground state. The formalism used is in terms of self-consistent screening and Landau level broadening in the extreme quantum limit in an interacting electron-impurity system where the screening and the level broadening determine and are determined by each other. In a two-dimensional system, this self-consistency is essential since the singularity of density of states is suppressed precisely by this level broadening. A similar problem was considered by Ando[11] who obtained level broadening for the realistic silicon inversion layer system. Results for the GaAs system and also the dependence of screening on filling factor as well as of level broadening on impurity concentration are reported here for the first time. Also our emphasis, unlike Ando's, is on the extreme quantum limit with the recent experiments[4,5] as the motivation.

Since we are more interested in obtaining qualitative trends rather than quantitative estimates applicable to realistic systems, we choose a purely two-dimensional system as our model. To simulate different materials we take the appropriate lattice dielectric constants (entering the definition of Coulomb interaction) and effective masses (defining the cyclotron frequency) for the respective materials. Thus in our model we have a system of two-dimensional electrons interacting with a system of fixed but randomly distributed charged impurity centers confined in the same two-dimensional plane. This two-dimensional plane is assumed to be submerged in a system with an effective background dielectric constant and an effective mass appropriate for the (100) Si-SiO_2 system or the GaAs system as the case may be. We may, however, point out that in the limit of low electron density the two-dimensional model is a reasonably good approximation to the actual Si-SiO_2 inversion layer system and has been widely used in the literature.[7–10] To calculate the effect of scattering of electrons by the charged impurities we use the self-consistent Born approximation (SCBA), which is the simplest approximation free from divergence.[6] We refer to Ref. 6 for the details of SCBA. Neglecting coupling between adjacent Landau levels the broadening of the nth Landau level (for our model) in the SCBA is given by[6]

$$\Gamma_n^2 = N_i \sum_{\vec{q}} |u(\vec{q})|^2 e^{-q^2 l^2/2} [L_n^0(\tfrac{1}{2} q^2 l^2)]^2 \ , \qquad (1)$$

where L is the associated Laguerre polynomial and $l = (c\hbar/eB)^{1/2}$ is the radius of the ground Landau orbit. In writing down Eq. (1) we have assumed an impurity concentration of N_i per unit area interacting with the electrons through an effective interaction $u(\vec{q})$ in Fourier space.

The effective interaction $u(\vec{q})$ is derived from a bare Coulomb interaction $u_0(\vec{q})$ within random-phase approximation (RPA). The bare interaction itself has the well-known form in two dimensions, $u_0(\vec{q}) = -2\pi e e_i/\kappa q$, where e, e_i are the electronic and the impurity charges, respectively, and κ is the effective background dielectric constant given by $\frac{1}{2}(\kappa_{Si} + \kappa_{SiO_2})$ for the silicon system and by κ_{GaAs} for the GaAs system. The effective interaction $u(\vec{q}) = u_0(\vec{q})/\epsilon(\vec{q})$ is determined by the static dielectric constant $\epsilon(\vec{q})$ which is given by the following expression within the RPA:

$$\epsilon(\vec{q}) = 1 - v(\vec{q})\Pi(\vec{q}) \ . \qquad (2)$$

In Eq. (2), $v(\vec{q})$ is the electron-electron interaction and $\Pi(\vec{q})$ is the polarizability of the electron gas in the presence of a magnetic field. The polarizability $\Pi(\vec{q})$ is calculated in the "bare bubble" approximation in the spirit of RPA. It is the same as the one calculated by Ando and Uemura,[14] but without the vertex correction. For our purpose we can write it as

$$\Pi(\vec{q}) = \sum_{n,n'} C_{nn'}(q) \chi_{nn'} \ . \qquad (3)$$

In Eq. (3) the coefficients $C_{nn'}(q)$ are given by

$$C_{nn'}(q) = \frac{A}{\pi}\left(\frac{eB}{c\hbar}\right)\frac{n!}{n'!}\left(\frac{l^2q^2}{2}\right)^{n'-n} \times e^{-l^2q^2/2}\left[L_n^{n'-n}\left(\frac{l^2q^2}{2}\right)\right]^2 \ , \qquad (4)$$

where A is the area of the system. The function $\chi_{nn'}$ involves the electronic Green's function $G_n(z)$ and is given by

$$\chi_{nn'} = -(\pi\hbar^2)^{-1}\int_{-\infty}^{+\infty} dz\, f(z)\,\mathrm{Im}[G_n(z)G_{n'}(z)] \ . \qquad (5)$$

In Eq. (5), $f(z) = (1 + e^{\beta\hbar z})^{-1}$, where $\beta = (k_B T)^{-1}$. The Green's function is obtained by solving Dyson's equation with the broadening Γ_n entering through the complex self-energy. Solving Dyson's equation one gets[6]

$$G_n(z) = \hbar\{\tfrac{1}{2}(z - E_n + \mu) + \tfrac{1}{2}[(z - E_n + \mu)^2 - 4\Gamma_n^2]^{1/2}\}^{-1} \ , \qquad (6)$$

where $E_n = (n + \frac{1}{2})\hbar\omega_c$ with $\omega_c = eB/mc$ as the cyclotron frequency. In Eq. (6) μ is the chemical potential of the system.

The question of self-consistency now arises since $\epsilon(\vec{q})$ determines Γ_n through Eq. (1) and is itself determined by the broadening through Eqs. (2)–(6). Thus impurity scattering affects the screening behavior of the electron gas. The only other quantity needed to complete the self-consistent loop is the

chemical potential μ which is obtained by demanding that the total electron density of the system equal a given number N per unit area. The above set of equations has been solved numerically in a self-consistent fashion at zero temperature and in the extreme quantum limit ($0 \leq \nu \leq 1$), assuming negligible mixing between adjacent Landau levels. We have chosen the following values of the parameters: $e_i = e$, $\kappa_{Si} = 11.7$, $\kappa_{SiO_2} = 3.9$, $\kappa_{GaAs} = 12.35$, $m_{Si} = 0.190 m_e$, $m_{GaAs} = 0.07 m_e$.

In Fig. 1 the self-consistent level broadening Γ is plotted as a function of ν (Γ has been scaled with respect to ω_0, the cyclotron energy for a magnetic field B_m such that $\nu = 1$) for the silicon inversion layer taking $N = 10^{11}$ cm^{-2} and $N_i = 10^{10}$ cm^{-2}. Also shown is the dimensionless screening factor $q_s l$ as a function of ν where q_s is defined by writing $\epsilon(q)$ as $q\epsilon(q) = q + q_s F(q)$ with $F(q) \to 1$ as $q \to 0$. Thus q_s is the analog of Thomas-Fermi wave number in the extreme quantum limit. It is seen that (Γ/ω_0) decreases rapidly in the region $0.5 \lesssim \nu \leq 1$; the line narrowing is by a factor of 4 or 5. Below $\nu \lesssim 0.5$, (Γ/ω_0) remains essentially unchanged, increasing slightly at first and then decreasing again for very high magnetic fields ($\nu \lesssim 0.1$). The behavior of self-consistent screening factor $q_s l$ gives a clue to this strong line narrowing, $q_s l$ is small around $\nu \sim 1.0$ (the completely filled level behaves as an "insulator" with no screening), but it increases rapidly with decrease in ν becoming a maximum around $\nu \sim 0.46$ (the half filled level is like a "metal" with strong screening). Thus the effective electron-impurity interaction changes its behavior from a long-range potential to a short-ranged one as the magnetic field increases at a fixed density thus diminishing ν. Note that previous non-self-consistent model potential calculations[6] give rise to entirely different behavior for Γ as a function of ν; a short-range model potential gives rise to $\sqrt{B}$ dependence of Γ whereas a long-range potential implies a B-independent Γ. The actual interaction, however, changes its nature with the increase in magnetic field. To emphasize the aspect of self-consistency we have shown the variation of $q_{TF} l$ with ν Fig. 1 where q_{TF} is semiclassical limit for the screening constant in two dimensions.[15] The difference between the self-consistent and the non-self-consistent screening is obvious from the figure.

In Fig. 2(a) we show the variation in Γ with ν for the GaAs-GaAlAs heterojunction at $N = 3 \times 10^{11}$

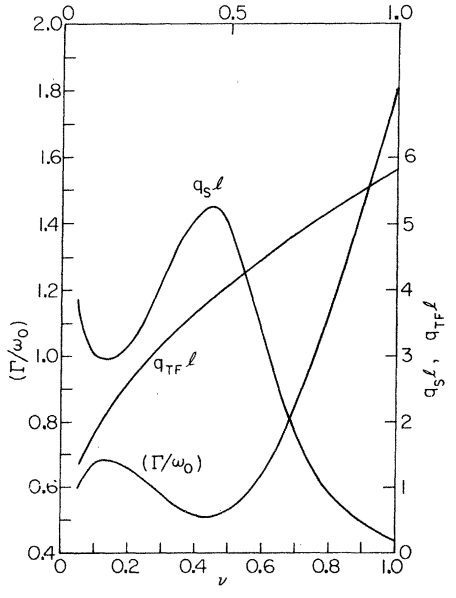

FIG. 1. Shows the variation of level broadening (Γ/ω_0), screening factor $q_s l$, and the Thomas-Fermi screening constant $q_{TF} l$ as functions of the filling factor ν for the Si inversion layer at $N = 10^{11}$ cm^{-2} and $N_i = 10^{10}$ cm^{-2}. ω_0 is the cyclotron resonance energy for $\nu = 1$.

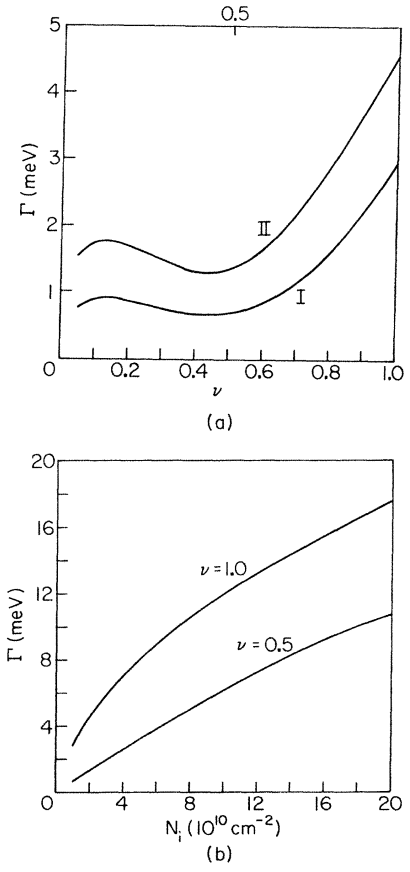

FIG. 2. (a) Shows the variation of Γ with ν in the GaAs-GaAlAs heterojunction at two values of impurity concentration, $N_i = 10^{10}$ and 2×10^{10} cm^{-2} (N is 2×10^{11} cm^{-2}). (b) Shows the variation of Γ with the impurity concentration N_i in the GaAs-GaAlAs heterojunction for $N = 2 \times 10^{11}$ cm^{-2}, and for two values of ν.

cm^{-2} and for two different impurity concentrations $N_i = 10^{10}$ and 2×10^{10} cm^{-2}. Clearly the functional dependence of Γ on ν is much the same as it is for silicon inversion layers. The level broadening increases in magnitude with N_i as one would expect. However, unlike non-self-consistent theories,[6] where $\Gamma \propto \sqrt{N_i}$, the present self-consistent theory indicates that the dependence of Γ on N_i is of the form $\Gamma \propto N_i^n$ where n varies between 0.5 and 1 depending on the value of ν (or the magnetic field value at a fixed density). In particular n is approximately equal to 0.9 and 0.6 for ν around 0.5 and 1.0, respectively. This behavior is shown in Fig. 2(b) for $\nu = 0.5$ and 1 where Γ is a minimum and a maximum, respectively. Note that the strong line-narrowing effect is suppressed for higher impurity concentrations. The same is true for silicon inversion layers as well. The actual magnitudes of the level broadening are in the few meV region for $\nu \sim 1$ and less than 1 meV for $\nu \sim 0.5$. This gives a scattering time τ of about $0.1 \sim 0.9 \times 10^{-12}$ sec which compares well with lifetimes extracted from the experiment.[4,5] It is to be emphasized that in this self-consistent theory we have no adjustable parameter (unlike model approximations[6] where strength and range of the interaction are parametrized).

In comparing the results of the present theory with experiments, a word of caution is in order. What we have obtained here is the broadening of single-particle Landau levels due to collisional damping whereas in magnetotransport experiments[2,3] one measures the transport lifetime and in magneto-optical experiments[4,5] one obtains the resonance lifetime. Even though there is in general a one-to-one correspondence between single-particle lifetimes and these experimental lifetimes, the connection between the two has not yet been firmly established in strongly interacting systems. For short-range scatterers, the single-particle broadening and the resonance linewidth in cyclotron resonance experiments have been shown to be the same.[6] But according to Ando,[11] for long-range scatterers the cyclotron resonance width could be much smaller than the Landau level broadening. Nevertheless it is tempting to compare the level broadening of Fig. 1 with the cyclotron resonance linewidth as a function of ν reported in Ref. 4. We find that the qualitative behavior of Γ is exactly the same as that observed for the resonance linewidth in the extreme quantum limit. In particular, the calculated Γ drops by a factor of about 4 as ν changes from 1 to 0.5, consistent with experimental results. In addition, our values for the actual broadening in the Si inversion layer for the experimental parameters used in Ref. 4 are 4.6 meV at $\nu = 1.0$ and 1.3 meV at $\nu = 0.5$, which compares quite favorably with the corresponding experimental numbers of about 4 and 1 meV, respectively. In view of Ando's result,[11] one is forced, however, to conclude that this remarkable agreement between calculated level broadening and measured resonance linewidth should not be taken as an explanation for the experimental data since the cyclotron resonance linewidth does not reflect the same behavior. We would like to point out, however, that there is no adjustable parameter in the theory.

The mass shift reported in Ref. 4 cannot be understood within the context of present theory alone since electron-impurity interaction by itself would not produce any shift in the cyclotron mass. Higher-order electron-electron interactions coupled with electron-impurity scattering would produce a very small mass correction[16] incompatible with experimental observation for the Si inversion layer. We note, however, that because of the neglect of short-range electron-correlation effects, our theory is strictly valid for low r_s values. Thus this theory is more appropriate for the GaAs system[5] where the small effective mass gives rise to small r_s values which make RPA quite a valid approximation. In addition the samples of Ref. 5 show no electron localization for $B = 0$. Thus we are more confident in applying the theory to the GaAs system where no mass shift is observed. Finally, the experimentally observed[4] increase in line broadening with the increase in substrate bias in the Si inversion layer can be understood within this theory through an increase of the effective N_i as the two-dimensional electron system is brought closer and closer to the Si-SiO_2 interface. Again the theory predicts the line broadening increasing as N_i^n where $n \sim 1$ for $\nu \sim 0.5$. Inclusion of temperature in the theory would give rise to a thermal broadening in addition to the collisional broadening. For the GaAs system the effective Fermi temperature is much higher than the experimental temperature and temperature effects (as long as $T << T_F$) should not be too large. But for the Si inversion layers T_F is smaller and thermal broadening in the experimental temperature range of 1–25 K is expected to be substantial. For $T \gtrsim T_F$ one does not expect to see the extreme line narrowing effect as ν changes from 1 to 0.5 due to the thermal broadening effects.

We must emphasize that the theory developed here is the most simple one that includes the effect of impurity scattering on electron screening under a strong magnetic field in a self-consistent fashion. It does not include scattering effects beyond SCBA. As such multiple scattering effects, important at high impurity concentrations (where the effect of self-consistency considered here is also more crucial), are completely ignored. Another important drawback of the calculation presented here is its neglect of Landau level mixing. This is particularly severe around $\nu = 1$, where $\Gamma/\omega_0 \sim 1.5$. Clearly the adjacent Landau levels ($n = 0$ and 1) are overlapping in that situation, and one should explicitly take into account the effect of Landau level mixing. Thus the results presented

here (particularly around $\nu \sim 1$) should be taken as only indicative of approximate qualitative trends. Obviously screening around $\nu \sim 1$ would not be as weak as it seems from Fig. 1, if there is substantial overlap between adjacent Landau levels. However, in view of the fact that Γ decreases rapidly for $\nu < 1$, the approximation of negligible Landau level mixing is at least consistent. But the actual numerical value of Γ (or of q_s) around $\nu \sim 1$ should not be taken too seriously. Work which includes the effects of Landau level mixing as well as of finite temperatures is now in progress.

In conclusion we have developed a theory of self-consistent screening and Landau level broadening in a two-dimensional electronic system, in the quantum limit. We find that the calculated level broadening Γ behaves very similar to the observed[4,5] variation of cyclotron resonance linewidth with the filling factor ν We also find that Γ for $\nu \sim \frac{1}{2}$ is affected much more by an increase in impurity concentration N_i compared with Γ for $\nu \sim 1$, giving possibly a clue to the substrate bias behavior in Ref. 4. Since Ando's calculation[11] shows that the dramatic increase in Γ near $\nu \sim 1$ is not reflected in the corresponding cyclotron resonance linewidth, we *cannot* claim the result presented in this paper to be an *explanation* for the experimental observations of Refs. 4 and 5. But we could possibly speculate that the interesting behavior of Γ with ν in the extreme quantum limit may be playing some role in the recently observed experimental phenomena.[4,5] The use of RPA restricts the theory to small r_s values, making it quite valid for the GaAs-$Al_xGa_{1-x}As$ system studied in Ref. 5. Since experience in metals has revealed that RPA works reasonably well outside its expected realm of validity, the expectation is that even for the Si inversion layer where r_s values involved are rather large (but *not* too large, actually in the range of 8–10), this theory may have some applicability and the observed line narrowing may partly be due to the one-electron physics described herein.

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