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Magnetoplasma polaritons at the interface between a semiconductor and a metallic screen. II. The Faraday geometry

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Magnetoplasma polariton modes at the interface between a highly doped semiconductor and a metallic screen are studied in the Faraday geometry, namely, the waves propagate along the static magnetic field in a direction parallel to the interface. One bona fide mode is found for every value of the applied field, and it terminates at the cyclotron frequency in the limit of very large wave vectors. The possibility of experimental detection of the predicted mode is discussed.

I. INTRODUCTION

In a recent article Halevi and Guerra-Vela[1] (I) have studied magnetoplasma polaritons at the interface between a semiconductor and a metallic screen in the Voigt geometry; specifically, the wave was assumed to propagate at a right angle to the static magnetic field, itself parallel to the interface. The present work differs from I in that we consider wave propagation parallel to the magnetic field, the Faraday configuration. The dispersion relation for this case has been stated (without proof) by Davydov and Zakharov[2] and applied to the low-frequency region $\omega \ll \nu$ (where ω is the circular frequency of the wave and ν is the collisional frequency of the electrons) in connection with the experiments of Baibakov and Datsko.[3] By contrast, our main concern here is the high-frequency or polariton region $\omega \gg \nu$. As discussed recently by Halevi and Quinn[4] the existence of bona fide *low*-frequency modes in the Faraday geometry is highly questionable. On the other hand, such modes are known to exist[5-7] in the high-frequency region for a free semiconducting surface. As we shall see in the present work, bona fide polaritons also exist for a semiconductor bounded by a highly conducting metallic screen.

In the experiments of Baibakov and Datsko[3] the metallic screen was separated by an air gap from the semiconductor surface. This three-media geometry was studied quite recently by Yi, Quinn, and Halevi.[8] We also wish to quote a related work by Rao and Uberoi[9] who investigated an interface between two polar semiconductors. Depending on the strength of the magnetic field, they found up to four types of oscillations. Additional information may be found in a review article by the author.[10]

II. THE DISPERSION RELATION

Our starting point is the wave equation of an anisotropic dielectric medium, characterized by a dielectric tensor $\underline{\epsilon}$,[1,10]

$$\nabla^2\vec{E} - \vec{\nabla}(\vec{\nabla}\cdot\vec{E}) - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\underline{\epsilon}\cdot\vec{E} = 0\,. \tag{1}$$

For a plane-wave solution this equation becomes

$$q^2\vec{E} - \vec{q}(\vec{q}\cdot\vec{E}) - q_0^2\underline{\epsilon}\cdot\vec{E} = 0\,, \tag{2}$$

where $\vec{q}$ is the wave vector in the medium (the semiconductor) and $q_0 = \omega/c$ is the wave vector in vacuum. If we choose the z axis along the static magnetic field $\vec{B}_0$ then, for a local theory, the nonvanishing elements of $\underline{\epsilon}$ are ϵ_{xx}, ϵ_{yy}, ϵ_{zz}, ϵ_{xy}, and ϵ_{yx}. Moreover, they possess the symmetry $\epsilon_{xx} = \epsilon_{yy}$ and $\epsilon_{xy} = -\epsilon_{yx}$.

We choose the y axis perpendicular to the interface and assume that the wave propagates parallel to $\vec{B}_0$, i.e., $q_x = 0$. Then the vector equation (2) reduces to the following set of equations:

$$(q_y^2 + q_z^2 - q_0^2\epsilon_{xx})E_x + q_0^2\epsilon_{yx}E_y = 0\,, \tag{3a}$$

$$-q_0^2\epsilon_{yx}E_x + (q_z^2 - q_0^2\epsilon_{xx})E_y - q_y q_z E_z = 0\,, \tag{3b}$$

$$-q_y q_z E_y + (q_y^2 - q_0^2\epsilon_{zz})E_z = 0\,. \tag{3c}$$

The determinant of this set of linear equations must vanish and from here we find that

$$\epsilon_{xx}q_y^4 - [(\epsilon_{xx} + \epsilon_{zz})(q_0^2\epsilon_{xx} - q_z^2) + q_0^2\epsilon_{yx}^2]q_y^2 + \epsilon_{zz}[(q_0^2\epsilon_{xx} - q_z^2)^2 + q_0^4\epsilon_{yx}^2] = 0\,. \tag{4}$$

Basically the same formula was derived by Wallis *et al.*[6] for a dielectric surface bounded by vacuum. For a given propagation mode $q_z(\omega)$, this equation has two physically acceptable solutions for q_y which we label q_{y1} and q_{y2}. Therefore we must postulate a field composed of two plane waves,

$$\vec{E}(y,z) = (\vec{E}_1 e^{iq_{y1}y} + \vec{E}_2 e^{iq_{y2}y})e^{i(q_z z - \omega t)}\,. \tag{5}$$

Clearly, $-iq_{y1}$ and $-iq_{y2}$ have to be identified with the decay constants α_1 and α_2 in the semiconductor. (The decay constant in the highly conducting screen is infinite.)

We proceed to derive the dispersion relation by eliminating E_y from Eqs. (3b), and (3c). Writing out the result explicitly for the two partial waves we have

$$q_{yk}q_z\epsilon_{yx}E_{xk}+(q_{yk}^2\epsilon_{xx}+q_z^2\epsilon_{zz}-q_0^2\epsilon_{xx}\epsilon_{zz})E_{zk}=0\,,\quad k=1,2\,. \tag{6}$$

Because all fields are screened out by the metal bounding the semiconductor, we have, in particular, $E_x=E_z=0$ in the metallic screen. Now these components of the electric field are continuous at the interface $y=0$. Then by Eq. (5) we have

$$E_{x1}+E_{x2}=0\,, \tag{7}$$

$$E_{z1}+E_{z2}=0\,. \tag{8}$$

Equations (6)–(8) are a set of four homogeneous equations and their determinant must vanish. The algebra involves canceling out a factor $(q_{y1}-q_{y2})$ which is consistent with our assumption of *two* partial waves.[11] The result is

$$\epsilon_{zz}(q_0^2\epsilon_{xx}-q_z^2)+\epsilon_{xx}q_{y1}q_{y2}=0\,. \tag{9}$$

The product $q_{y1}q_{y2}$ may be readily calculated from Eq. (4). Eliminating q_z^2 we find

$$q_z^2=q_0^2\left[\epsilon_{xx}\pm\epsilon_{yx}\left(\frac{\epsilon_{xx}}{\epsilon_{zz}-\epsilon_{xx}}\right)^{1/2}\right]. \tag{10}$$

This formula was first given (without proof) by Davydov and Zakharov.[2] With the substitution of the dielectric tensor elements one obtains the dispersion relation for polaritons at the interface between a semiconductor and a metallic screen.

We note that, at finite frequencies, the boundary conditions (7) and (8) are applicable only in the limit $\omega_p'\to\infty$, where ω_p' is the plasma frequency of the metallic screen. In practice this frequency is finite, however much greater than the frequencies which characterize the semiconductor. Therefore the penetration of the fields into the metallic half-space is extremely small and Eqs. (7) and (8) are excellent approximations.

III. POLARITON SOLUTIONS

We neglect the effect of phonons, which is legitimate for frequencies ω which are much greater than the longitudinal phonon frequency ω_L. We also neglect damping effects $(\omega\gg\nu)$, thus limiting the discussion to the "high-frequency" or polariton region. Then our parameters are the plasma frequency $\omega_p=(4\pi ne^2/m^*\epsilon_\infty)^{1/2}$, the cyclotron frequency $\omega_c=eB_0/m^*c$, and the high-frequency (or background) dielectric constant ϵ_∞. The elements of the dielectric tensor required in Eq. (10) are

$$\epsilon_{xx}=\epsilon_\infty\left(1-\frac{\omega_p^2}{\omega^2-\omega_c^2}\right), \tag{11a}$$

$$\epsilon_{yx}=-i\epsilon_\infty\frac{\omega_c\omega_p^2}{\omega(\omega^2-\omega_c^2)}, \tag{11b}$$

$$\epsilon_{zz}=\epsilon_\infty\left(1-\frac{\omega_p^2}{\omega^2}\right). \tag{11c}$$

The substitution of Eqs. (11) into Eq. (10) results in the following explicit dispersion relation:

$$q_z^2=\epsilon_\infty\frac{\omega^2}{c^2}\frac{\omega_H^2-\omega^2\pm\omega_p\sqrt{\omega_H^2-\omega^2}}{\omega_c^2-\omega^2}, \tag{12}$$

where $\omega_H=(\omega_p^2+\omega_c^2)^{1/2}$ is the hybrid plasmon cyclotron frequency. [This equation is inapplicable for $\omega_c=0$ because in this case $\epsilon_{xx}=\epsilon_{zz}$ and Eq. (10) does not hold.] It is evident from Eq. (12) that propagation of electromagnetic waves at the semiconductor-screen interface is *not* possible for $\omega>\omega_H$. This means that there are no very high-frequency modes, as were found in the Voigt geometry.[1] At *low* frequencies $(\omega\ll\omega_c)$ we see that $q_z\propto\omega$. Thus the dispersion curve rises linearly from the origin, a behavior corresponding to a "fast" wave. Another important feature is that, as $\omega\to\omega_c$, q_z goes to infinity as $(\omega_c-\omega)^{-1/2}$ for the positive sign in Eq. (12). Thus we have a resonance at the cyclotron frequency ω_c. This behavior is also borne out by a numerical calculation, whose results are shown in Fig. 1. The dispersion relations ω versus q_z are presented in terms of dimensionless variables for four values of the parameter ω_c/ω_p. Invariably, the dispersion relation is linear at low frequencies and terminates at the cyclotron frequency. It is interesting that the frequency range of propagation $(0,\omega_c)$ coincides with that of volume helicons.

It is imperative to investigate the behavior of the decay constants $\alpha_1=-iq_{y1}$ and $\alpha_2=-iq_{y2}$ in order to determine whether the dispersion curves in Fig. 1 correspond to potentially observable normal modes of the semiconductor-screen interface. These decay constants are calculated from the biquadratic equation (4), with q_z given by Eq. (12). We find that the upper parts of the dispersion curves in Fig. (1) (continuous lines) describe bona fide or pure interface modes, i.e., their decay constants α_1 and α_2 are both real and positive and $\alpha_1\neq\alpha_2$. The lower parts of the dispersion curves (broken lines) correspond to "generalized modes,"[6] whose decay constants are complex conjugates $(\alpha_2=\alpha_1^*)$ with positive real parts. It turns out that the transition from generalized to pure interface modes takes place at a point defined by the intersection of the $\omega(q_z)$ dispersion curve and the curve $\alpha_1(\omega,q_z)=\alpha_2(\omega,q_z)$.[8,12] The latter

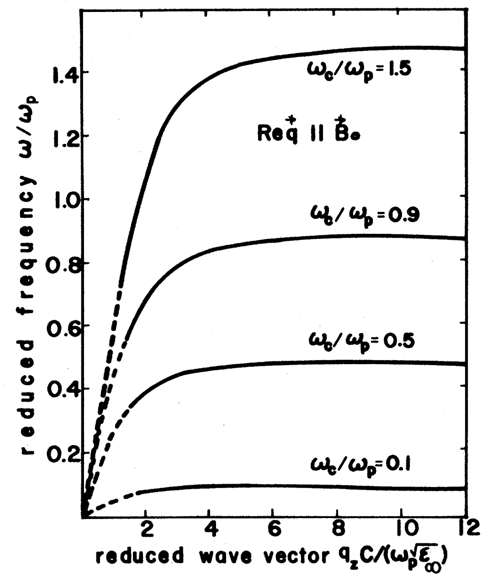

FIG. 1. Dispersion relations of magnetoplasma modes at the interface between a highly doped semiconductor and a metallic screen in the Faraday geometry. For every value of the static magnetic field B_0 there is one mode, and it terminates at the cyclotron frequency ω_c. Continuous lines indicate a bona fide interface mode whose decay constants are both real and positive. Broken lines indicate a generalized interface mode whose decay constants are complex conjugates.

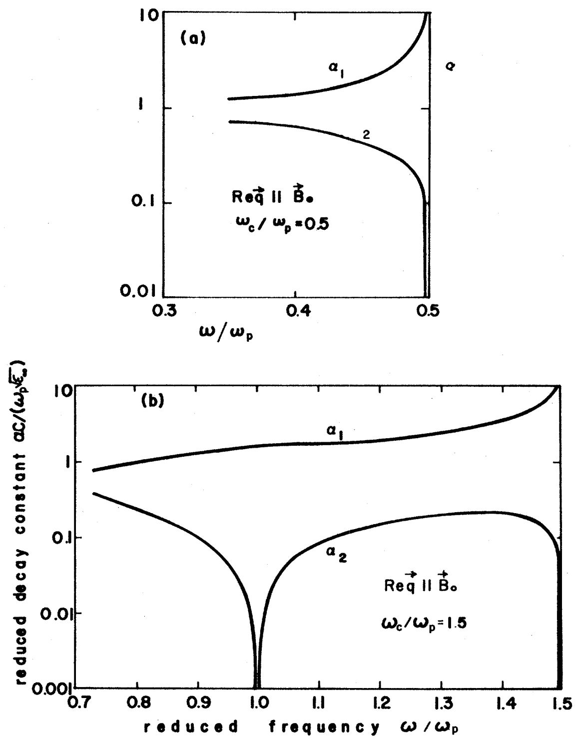

FIG. 2. The decay constants α_1 and α_2 of the interface modes shown in Fig. 1 for two values of the parameter ω_c/ω_p: (a) 0.5, (b) 1.5. Only the frequency region of bona fide modes is shown. Note the logarithmic scale.

curve forms a loop; the generalized modes are located inside the loop and the pure modes are outside the loop.

In Fig. 2(a) we plot the normalized decay constants α_1 and α_2 as a function of the normalized frequency for $\omega_c/\omega_p = 0.5$. This logarithmic plot is restricted to the frequency region of pure interface modes. At the threshold frequency α_1 and α_2 are of the order of far-infrared wavelengths (~100 μm). As the frequency increases α_1 increases monotonically, while α_2 decreases monotonically. As we approach the resonant frequency ω_c, $\alpha_1 \to \infty$ and $\alpha_2 \to 0$. We see from Eq. (5) that in this limit

$$\vec{E}(y,z) \to \vec{E}_2 e^{i(q_z z - \omega t)} .$$

Therefore, in the retardationless limit ($q_z \to \infty$) the interface magnetoplasmon is represented by a *single* plane-wave solution of constant amplitude.

The behavior of the decay constants for $\omega_c/\omega_p = 0.1$ and 0.9 is similar to Fig. 2(a). On the other hand, for $\omega_c/\omega_p = 1.5$ the behavior of α_2 drastically changes. As shown in Fig. 2(b), α_2 goes to zero at the plasma frequency ω_p. At this frequency, $\epsilon_{zz} = 0$. Then from an analysis of Eq. (4) one can readily show that α_1 is finite and $\alpha_2 = 0$. For other values of the ratio ω_c/ω_p we also find[13] that the behavior shown in Fig. 2(a) is exhibited for $\omega_c < \omega_p$, while the behavior in Fig. 2(b) occurs for $\omega_c > \omega_p$.

IV. DISCUSSION

Effect of phonons. If ω is comparable to or smaller than the phonon frequencies ω_T and ω_L then the "one" in the parentheses of Eqs. (11a) and (11c) must be replaced by $(\omega^2 - \omega_L^2)/(\omega^2 - \omega_T^2)$. Thus, in addition to the pole at ω_c, ϵ_{xx} also has a pole at ω_T. At both frequencies $q_z \to \infty$, so now there are two resonant frequencies, ω_T and ω_c. We expect that the dispersion curves of Fig. 1 will split into a "phonon branch" and a "plasmon branch" whose precise behavior will depend on the parameters ω_c/ω_p, ω_T/ω_p, and ω_L/ω_p. It is readily seen that the lower branch will preserve the linearity ($\omega \propto q_z$) for low frequencies. A numerical study of Eq. (4) would be required to determine the frequency regions of bona fide interface modes. The element $\epsilon_{zz}(\omega)$ now has two zeros in place of the single zero at ω_p. In analogy to Fig. 2(b) we expect that $\alpha_2 \to 0$ at both of these zeroes.

Effect of damping. As discussed in I, allowance for a finite damping frequency ν will cause two qualitative changes in the dispersive properties. One occurs for very low frequencies, $\omega \ll \nu$. It has been shown in Ref. 4 that, in addition to a "fast mode" (which seems to be the same as the one studied here), Eq. (10) also gives a "slow mode" with the dispersion

$$q_z \simeq (1+i)\frac{\omega_p}{\omega_c}\frac{\sqrt{\omega\nu}}{c} .$$

This, however, is *not* a bona fide mode because it is damped out in a distance of the order of the wavelength. The other change that we expect is a backbending of the dispersion relation as the resonant frequencies ω_c and ω_T are approached. This is known to happen in many cases[10] provided that the wave vector is taken as a complex quantity while the frequency is real. We also expect that α_2 will display minima—rather than go to zero—at the frequencies ω_c and ω_p.

Possibility of experimental detection. We expect that the *pure* interface modes predicted in the present work might be observable by attenuated-total-reflection (ATR) spectroscopy.[7] It seems that the best geometry would be sandwiching a metallic film (any metal) between a high-index prism and a semiconductor such as InSb.[1,10] The desirable thickness of the film would be $\sim c/\omega_p'$ (where ω_p' is the plasma frequency of the metal), i.e., a few hundred Å. Otherwise, if one sandwiches the semiconductor between the prism and the metal, the optimum thickness of the film would strongly depend on the frequency.

The major difficulty from the experimental point of view seems to be the complex polarization of the magnetoplasma modes in the Faraday geometry. Both for a free semiconductor surface[5-7] and for a semiconductor-screen interface, the electric and magnetic fields of the wave possess longitudinal components E_z and B_z, transverse components E_x and B_x which are parallel to the interface, and transverse components E_y and B_y which are perpendicular to the interface. This amounts to elliptical polarization in a plane which is *not* parallel to the plane of incidence. As a consequence p-polarized light does not seem to excite the proper normal modes of the system. In fact the reflected (observed) light has an s-polarized as well as a p-polarized component.[7]

As for the *generalized modes,* we wish to comment on a difficulty. The decay constants α and the normal components of the wave vector q_y are related by the equation $\alpha = -iq_y$. In terms of real and imaginary parts (primed and double-primed letters, respectively),

$$\alpha' + i\alpha'' = -i(q_y' + q_y'') = q_y'' - iq_y' . \tag{13}$$

The two partial waves of a generalized mode differ only in the signs of α''. Therefore, Eq. (13) gives $q_y' = \pm |\alpha''|$. This means that one of the partial waves approaches the interface at an angle $\tan^{-1}(q_z/|\alpha''|)$ and the other wave recedes from the interface at the same angle with respect to the interface normal. Thus we have the simple picture of an incident and a reflected ray, however with one important difference: The amplitudes of both rays decrease exponentially with distance from the interface, the decay constants being given by α'. It is not at all clear that this kind of wave can be excited in the semiconductor by conventional ATR spectroscopy.[14] In fact, it seems that a theory of excitation of magnetoplasma modes has not yet been published. These remarks hold equally well for a free semiconductor surface and for a semiconductor-screen interface.

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[12]C. Uberoi, private communication.

[13]P. Halevi, unpublished.

[14]The author has been informed by M. V. Ortenberg that generalized modes have been actually excited by the "strip-line technique," which allows for an air gap between the semiconductor and the metallic screen. It turns out that Poynting's vector is always oriented *outwards*, even though one of the partial waves *approaches* the semiconductor surface from the outside. See M. V. Ortenberg, in *Infrared and Millimeter Waves*, Vol. 3, edited by K. J. Button (Academic, New York, 1980) and references therein.