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Comment on the observation of order-parameter enhancement by a change in the magnetic flux

O. Entin-Wohlman

*Department of Physics and Astronomy, Tel-Aviv University, Tel-Aviv, Israel**
and Physics Department, University of California, Los Angeles, California 90024

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In a recent experiment a microwave-induced order-parameter enhancement was observed by measuring the change in the magnetic flux trapped in an Al cylinder. It is shown here that the data are in quantitative agreement with Eliashberg's theory, contrary to what is concluded by the experimenters.

I. INTRODUCTION

Eliashberg[1,2] and co-workers[3] have predicted that microwave radiation may cause enhancement in the superconducting order parameter due to redistribution of the quasiparticles away from the gap edge. In a recent article, Pals and Dobben[4] reported a beautiful experiment which demonstrates the enhancement. They measured variations in the magnetic flux frozen in an Al cylinder, as a function of temperature and the radiation power. They observed that application of a microwave power changes the flux in the same direction as a temperature decrease (in the absence of the radiation). Pals and Dobben tried to use their data in order to extract the values of the enhanced order-parameter and concluded that their results do not fit quantitatively with Eliashberg's theory. It is shown here that they did not use the correct expressions resulting from the theory; when those are applied, their data are in good agreement with the prediction of Eliashberg's theory.

II. MAGNETIC FLUX IN AN IRRADIATED CYLINDER

The measurements of Pals and Dobben are based on the fluxoid quantization condition for a closed contour inside a superconductor. The expression for the magnetic flux ϕ trapped in a long superconductor of radius r and thickness d (d is much smaller than the penetration depth) is[4–6]

$$\phi = n\phi_0\left[1+\frac{c^2}{e^2}\frac{1}{2\pi^2 rd}\frac{3}{V_F l}\frac{1}{TN(0)}\times\left(\frac{T}{\Delta}\right)^2\frac{1}{1+(4T/\Delta)\delta f(\Delta)}\right]^{-1} . \quad (1)$$

Here ϕ_0 denotes the flux quantum, n is an integer, V_F is the Fermi velocity, l is the mean free path, $N(0)$ is the density of states at the Fermi level, and Δ is the microwave-affected order parameter. In Eq. (1), $\delta f(\Delta)$ denotes the deviation of the quasiparticle distribution from equilibrium at energy $E=\Delta$, caused by the microwave radiation. That is, the quasiparticle distribution $f(E)$ in the presence of the radiation is given by

$$f(E)=f_0(E)-\delta f(E) , \quad (2)$$

where $f_0(E)$ is the Fermi function. The term $1+(4T/\Delta)\delta f(\Delta)$ appears in Eq. (1) because the supercurrent density is modified by the radiation.[5,6] This term is missing in the expression for the magnetic flux given in Ref. 4. It can be shown[6] that for low radiation powers, δf is proportional to the power. For the low powers and frequency applied by Pals and Dobben, $(4T/\Delta)\delta f(\Delta)$ is small compared to 1, and may be neglected. However, it might be important when the power or the frequency are increased.[6]

The value of Δ as a function of the microwave power and frequency, and the temperature, is found from the self-consistency equation. The self-consistency equation reads[3,6,7]

$$\left[\frac{T_c-T}{T_c}-\beta_0\left(\frac{\Delta}{T}\right)^2-\frac{\pi\alpha}{2T}-\beta_0\frac{\pi}{2}\left(\frac{\omega}{T}\right)^2\alpha\tau_E+\alpha\tau_E\frac{\omega}{4T}g\left(\frac{2\Delta}{\omega}\right)\right]\Delta=0 , \quad (3)$$

where T_c is the thermodynamic transition temperature

$$\beta_0 = 7\zeta(3)/8\pi^2 = 0.106 \ ,$$

$$g(x) = \frac{4}{1+x}\Theta(x-1)\left[K\left(\frac{1-x}{1+x}\right) - x^2\Pi\left((1-x)^2, \frac{1-x}{1+x}\right)\right] + \Theta\left(1-\frac{x}{2}\right) 2\pi\left(\frac{x^2}{4-x^2}\right)^{1/2} \ , \tag{4}$$

and K, Π are elliptic integrals of the first and third kind, respectively. ω is the radiation frequency, τ_E is the inelastic phonon scattering time of the electrons,

$$\alpha = \left(\frac{e}{c}\right)^2 |\vec{A}_\omega|^2 \frac{V_F l}{3} \ , \tag{5}$$

and $\vec{A}_\omega$ describes the vector potential of the radiation. The quantity $\alpha\tau_E$ is related to the microwave power.[6,7] We have solved Eq. (3) numerically, using the experimental parameters of Ref. 4 ($T_c = 1.296$ K, $\omega = 9$ GHz) to find the enhanced Δ as a function of temperature. Figure 1 portrays $(\Delta - \Delta_0)/\Delta_0$, where Δ_0 is the equilibrium value of the order-parameter,

$$\left(\frac{\Delta_0}{T}\right)^2 = \frac{1}{\beta_0}\frac{T_c - T}{T_c} \ , \tag{6}$$

as a function of temperature, for various values of $\alpha\tau_E$. [These were chosen to fit the magnitude of $(\Delta - \Delta_0)/\Delta_0$ found in Ref. 4.] The slope of the straight lines is ~ 1.3, in agreement with the finding of Pals and Dobben. The remark of Pals and Dobben that they measure a steeper slope than the linear dependence predicted by Eliashberg's theory is therefore incorrect, as the self-consistency equation (3) does not yield a linear dependence. Figure 2(a) depicts the variation of the magnetic flux ϕ for $\alpha\tau_E = 0$ as a function of temperature (full line) and the behavior of ϕ as a function of $\alpha\tau_E$ at $(T_c - T)/T_c = 0.024$ (dotted line). Both curves are in accordance with Fig. 1 of Ref. 4 (curves a and b there). Figure 2(a) shows $(\phi - \phi_0)/\phi_0$, where ϕ_0 is the flux in the absence of the radiation (at the same temperature), as a function of temperature. To compute the curves in Fig. 2 we use the solution of the self-consistency equation (3) in Eq. (1). [We used the experimental parameters of Ref. 4: $r = 0.4$ mm, $d = 50$ nm, $l = 37$ nm, $V_F = 1.29 \times 10^6$ m/sec, and $N(0) = 1.1 \times 10^{47}$ m^{-3}J^{-1}].

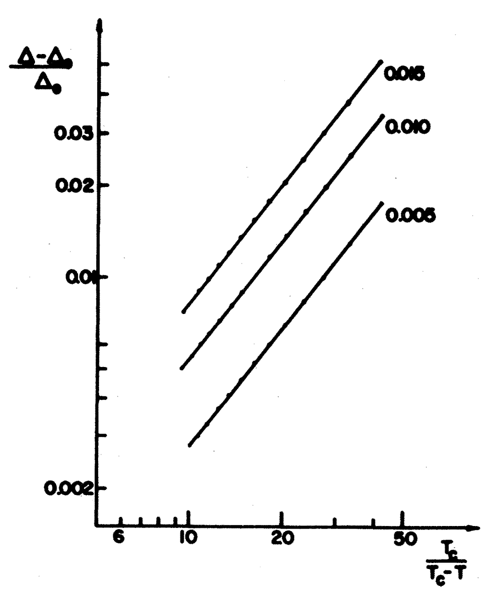

FIG. 1. Relative increase of the order parameter as a function of temperature for various values of $\alpha\tau_E$ (marked on the curves).

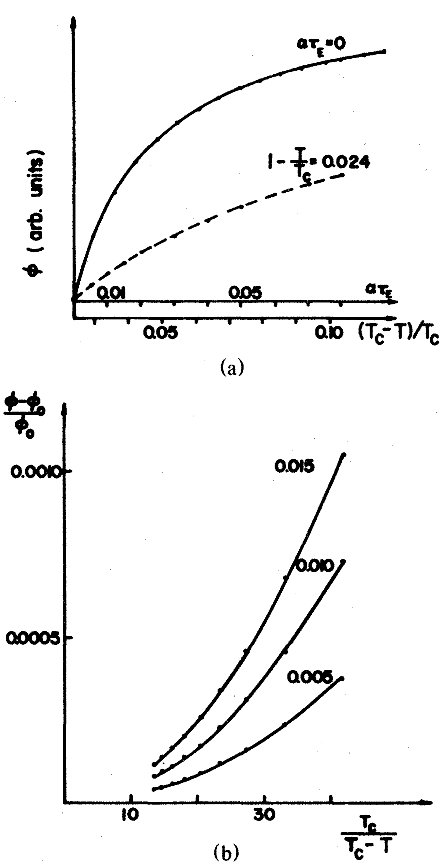

FIG. 2. (a) Magnetic flux in the absence of the radiation as a function of temperature (full line), and magnetic flux as a function of $\alpha\tau_E$, at $(T_c - T)/T_c = 0.024$ (dotted line). (b) Relative increase of the magnetic flux as a function of temperature, for various values of $\alpha\tau_E$ (marked on the curves).

III. DISCUSSION

Measurements of the microwave-induced order-parameter enhancement have been mostly carried out by critical current experiments,[7–9] which are difficult to interpret.[6,7,9] A direct observation of gap enhancement in a tunneling experiment was presented by Kommers and Clarke.[10] The experiment of Pals and Dobben[4] confirms the idea of the enhancement, and is relatively easy to analyze. It was shown here that at low microwave powers the data fitted well with Eliashberg's theory. The reason is that the expression we used for $\delta f(E)$, the deviation caused by the radiation in the quasiparticle distribution, is valid for $\alpha \tau_E < 1$ (i.e., low microwave powers). It should be interesting to measure the magnetic flux as a function of temperature at higher powers, and for different frequencies, and compare them with Eqs. (1) and (3).

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*Permanent address.

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