Theory of thin proximity-effect sandwiches. II. Effects of s-wave elastic scattering

Gerald B. Arnold
Department of Physics, University of Notre Dame, Notre Dame, Indiana 46556
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We show that Anderson's theorem concerning *s*-wave elastic scattering is inapplicable to certain types of inhomogeneous superconductors. The example of a proximity-effect sandwich consisting of a thin normal (*N*) metal in perfect contact with a thick superconducting (*S*) metal is considered in detail. The proper treatment of *s*-wave elastic scattering in the *N* metal is developed. Previously inexplicable experimental results on tunneling into *NS* sandwiches are found to be understandable in terms of the theory presented.

I. INTRODUCTION

In accounting for the effects of elastic *s*-wave scattering on the density of states observed by tunneling into the *N* side of a thin *NS* sandwich, the Anderson theorem[1] is often assumed to be valid. This theorem states that elastic *s*-wave scattering in a bulk superconductor has no effect on the superconducting pair potential. The theorem may be generalized to the mathematical statement that as long as the Hamiltonian of the system possesses time-reversal invariance, the introduction of a perturbation which does not break that invariance will not affect the pair potential.[2] This generalization applies only to a bulk system, i.e., a system which possesses translational symmetry in the absence of the perturbations in question (i.e., elastic scattering centers).

However, the *NS* sandwich (shown in Fig. 1) is not a bulk system. Nonetheless, previous work on *NS* sandwiches[3] has implicitly relied on the Anderson theorem in treating elastic scattering, presumably because the Hamiltonian of the entire system is time-reversal invariant. However, the criterion of translational symmetry (homogeneity) in the absence of the perturbation is clearly *not* satisfied by the *NS* sandwich.

In this paper we shall derive some consequences of the inapplicability of the Anderson theorem to elastic *s*-wave scattering in thin *NS* sandwiches. This represents an extension of our work in Ref. 4, where we dealt with clean systems. In the latter, we showed that assuming a thin *N* metal in perfect contact with an *S* metal, and assuming self-energies which are spatially local but equal to their average value in the appropriate (*N* or *S*) region we could solve the Bogolyubov equations exactly to find the Green's function for the *NS* double layer. Using this Green's function, we obtained equations for the local self-energies arising from the electron-phonon interaction. These had the form [cf. Eqs. (6.5) and (6.6) of Ref. 4]:

$$\phi^{ph}(E,x)=\int_0^\infty dE'f(E',x)K_+(E,E',x) \ , \quad (1.1)$$

$$Z^{ph}(E,x)E=E-\int_0^\infty dE'N(E',x)K_-(E,E',x) \ , \quad (1.2)$$

where ϕ^{ph} is the "pairing self-energy" and Z^{ph} is the "renormalization function." The local electron-phonon interaction kernels are denoted by $K_\pm(E,E',x)$ [cf. Eq. (6.2), Ref. 4]. The local normalized density of states is $N(E',x)$. This function is just the local density of states at x normalized by its normal state value. By analogy, we have defined

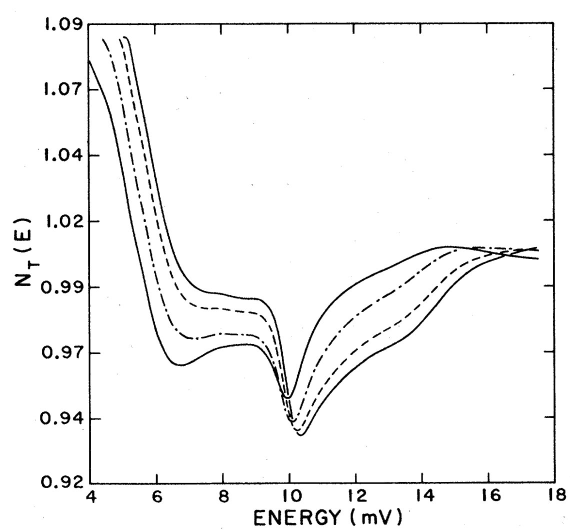

FIG. 1. Tunneling density of states for $R=0.02$, $\Delta_N^{ph}=0$, $Z_N^{ph}=1$. The solid line for the curve which is smallest in value from 10 to 16 mV is for $d/l=0$. The dashed line: $d/l=0.25$. Dash-dotted line: $d/l=1$. Second solid line: $d/l=5$.

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$f(E',x)$ as the local "pair density of states." The significance of this function will be made apparent below. In a bulk superconductor, one has

$$\phi^{ph}(E,x)=\phi(E) \ , \tag{1.3}$$

$$Z^{ph}(E,x)=Z(E) \ , \tag{1.4}$$

$$\Delta(E)=\phi(E)/Z(E) \ , \tag{1.5}$$

$$N(E',x)=\frac{E'}{[E'-\Delta(E')^2]^{1/2}} \ , \tag{1.6}$$

$$f(E',x)=\frac{\Delta(E')}{[E'-\Delta(E')^2]^{1/2}} \ . \tag{1.7}$$

As a function of x, $K_{\pm}(E,E',x)$ varies as the electron-phonon interaction, hence it is quite local. To an excellent approximation, this function is spatially constant for x in N, and is also spatially constant for x in S, with a sharp change (over a lattice distance or so) at the NS interface.

In order to clarify the nature of the pair density, $f(E',x)$, we consider the BCS approximation, where

$$K_{-}(E,E',x)=0 \ , \tag{1.8}$$

$$K_{+}(E,E',x)=\lambda^*(x)\Theta(E_c-E') \ , \tag{1.9}$$

where $\Theta(x)$ is the unit step function and E_c is a cutoff energy for the local BCS interaction energy $\lambda^*(x)$. In this limit one finds

$$\phi^{ph}(E,x)=\lambda^*(x)\int_0^{E_c}dE'f(E',x)=\lambda^*(x)F(x) \ , \tag{1.10}$$

where $F(x)$ is the order parameter at x. Since the latter varies over distances of the order of a coherence length in a metal, we observe that the pair density has the same scale of spatial variation. If the region of interest is much thinner than a coherence length, then $f(E',x)$ is essentially constant in this region. It is therefore appropriate to consider the spatial average of the self-energy over the thin N region. Returning to Eqs. (1.1) and (1.2) we average over the N region to find

$$\langle\phi^{ph}(E,x)\rangle_N=\phi_N^{ph}(E) =\int_0^\infty dE'\langle f(E',x)\rangle_N K_{+}(E,E')_N \ , \tag{1.11}$$

$$\langle Z^{ph}(E,x)\rangle_N E=Z_N^{ph}(E)E =E-\int_0^\infty dE'\langle N(E',x)\rangle_N K_{-}(E,E')_N \ , \tag{1.12}$$

where $K_{\pm}(E,E')_N$ are the values taken by the interaction kernels in the N region. These expressions yield the contribution of *inelastic* electron-phonon scattering to the self-energy in the N region. Elastic s-wave scattering can be treated in the same fashion. One merely sets

$$K_{\pm}(E,E')_N=\pm\frac{i\hbar}{2\tau}\delta(E-E') \ , \tag{1.13}$$

where τ is the elastic scattering lifetime.

Including elastic scattering, the total spatial average self-energy in the N region is

$$\phi_N(E)=\phi_N^{ph}(E)+\frac{i\hbar}{2\tau}\langle f(E,x)\rangle_N \ , \tag{1.14}$$

$$Z_N(E)E=Z_N^{ph}(E)E+\frac{i\hbar}{2\tau}\langle N(E,x)\rangle_N \ . \tag{1.15}$$

We note at this point that these expressions will apply to any region N which is thin compared to a coherence length. These results may be extended to encompass situations such as long thin rods of N material embedded in S metal, or small particles of N metal embedded in S metal. Such extensions merely involve recognizing that the pair density will vary with two (or three) coordinates, so that the spatial average occurs over more coordinates. Thus, Eqs. (1.14) and (1.15) may be used to establish the influence of elastic scattering for a *variety* of inhomogeneous systems, not just for the NS double layer.

Let us first determine under what conditions the Anderson theorem can hold. The theorem requires

$$\Delta_N(E)=\phi_N^{ph}(E)/Z_N^{ph}(E) \ . \tag{1.16}$$

By definition

$$Z_N(E)\Delta_N(E)=\phi_N(E) \ . \tag{1.17}$$

The latter definition coupled with Eqs. (1.14) and (1.15) implies that if Eq. (1.16) is true, then

$$\langle f(E,x)\rangle_N=\frac{\Delta_N(E)}{E}\langle N(E,x)\rangle_N \ . \tag{1.18}$$

Thus, the validity of the Anderson theorem in inhomogeneous systems of the type mentioned above requires a simple relationship between the average pair density and the average normalized quasiparticle density of states. We will show that such a simple relationship does not exist in the case of a thin NS sandwich. It is reasonable to suppose that the other inhomogeneous systems mentioned will also not satisfy Eq. (1.18).

The inapplicability of the Anderson theorem has two primary consequences for a thin NS sandwich:

(1) The superconducting pair potential is "homogenized" to some extent over the NS layers. By this we mean that for very thin, "dirty" N layers $\Delta_N(E)$ approaches $\Delta_S(E)$.

(2) The effective path length for quasiparticle interference arising from Andreev reflection (which gives rise to Tomasch[5] and Rowell-McMillan[6] oscillations in the tunneling density of states) appears to increase.

These two effects are related. Andreev reflection involves scattering of quasiparticles from the step in the pair potential, $\Delta_S - \Delta_N$, at the *NS* boundary. As Δ_N approaches Δ_S, this step height decreases, affecting the interference phenomena arising from Andreev reflection in the indicated fashion.

The second consequence appears to have a more intuitive basis. Successive elastic scatterings of quasiparticles from randomly-located imperfections cause the average path traversed to be greater than that which is traversed in a clean layer. Some effects arising from the effective increase in path length for interference have possibly been observed by Bermon and So[7] in their experiments on thin Cu-Pb sandwiches.

The homogenization of the pair potential occurs in virtue of the fact that in a very thin, "dirty" material, the elastic scattering contribution to the self-energy dominates the electron-phonon interaction contribution. From Eqs. (1.14) and (1.15) one finds

$$\Delta_N = \frac{\phi_N^{ph} + \frac{i\hbar}{2\tau}\langle f(E,x)\rangle_N}{Z_N^{ph} + \frac{i\hbar}{2\tau}\frac{\langle N(E,x)\rangle_N}{E}} . \tag{1.19}$$

For large $\hbar/\tau$ this is

$$\Delta_N \approx E\frac{\langle f(E,x)\rangle_N}{\langle N(E,x)\rangle_N} . \tag{1.20}$$

The ratio of the pair density to the quasiparticle density may be interpreted as the fraction of quasiparticles of energy E paired in the N metal. Both $f(E,x)$ and $N(E,x)$ are related to quasiparticle wave functions, and so are continuous at the *NS* boundary. Because the dimensions of the N region are small compared to a coherence length, $\langle f(E,x)\rangle_N$ is essentially equal to its value at the *NS* boundary. This value, by continuity, is equal to the value of the pair density in the S material just on the other side of the boundary. As demonstrated in Ref. 4 for the thin *NS* sandwich, the quasiparticle density of states in N continuously approaches the value it has just inside the boundary of the S metal. For an S region of size large compared to a coherence distance the averages in Eq. (1.20) take on their bulk values [Eqs. (1.6) and (1.7) with $\Delta = \Delta_S$], to a good approximation. Hence Δ_N is approximately equal to Δ_S, by Eq. (1.20). That is, in this limit, the fraction of paired quasiparticles at energy E in the N metal is approximately equal to the fraction of paired quasiparticles at energy E in the S metal.

In Sec. II, we shall present a more detailed derivation of the contribution of *s*-wave elastic scattering to the N metal self-energy. In Sec. III, some simple limits will be investigated in order to make contact with known results. In Sec. IV, we present the results of calculations of the effects of elastic scattering on the observed tunneling density of states. We consider specular tunneling[8] only. The density of states which we will use is just that which was derived in Ref. 4, Eq. (4.5), since this expression was obtained without making assumptions on the energy dependence of $\phi(E)$ or $Z(E)$ for either N or S metal. In the final section, we discuss the effects of elastic scattering on the tunneling density of states for low energies.

II. SELF-ENERGY IN THE *N* METAL

The self-energy functions obtained in Ref. 4 do not include the effects of elastic scattering from impurities. We shall invoke the effects of elastic scattering in a way which is analogous to the treatment of such effects in a bulk system.[9]

As discussed in Ref. 4, for thin N metals it is appropriate to replace the local matrix self-energy by its average over the thickness of the N region. In such an average, quantities which oscillate like $e^{\pm 2ik_Fx}$ are negligible. Thus, we may employ Eq. (3.12) of Ref. 4 for the retarded Green's functions in the N metal

$$G(x,x,E)_{11} = \frac{m}{\hbar^2 k_{Fx}}\left[\frac{E}{\Omega_N}\chi_1(E) + \frac{\Delta_N}{\Omega_N}\chi_2(E,x)\right] , \tag{2.1}$$

$$G(x,xE)_{12} = \frac{m}{\hbar^2 k_{Fx}}\left[\frac{\Delta_N}{\Omega_N}\chi_1(E) + \frac{E}{\Omega_N}\chi_2(E,x)\right] , \tag{2.2}$$

where

$$\chi_1(E) = \frac{iF(E)\cos(\Delta k^N d) + \sin(\Delta k^N d)}{iF(E)\sin(\Delta k^N d) - \cos(\Delta k^N d)} , \tag{2.3}$$

$$\chi_2(E,x) = \frac{i\cos[\Delta k^N(x+d)]G(E)}{iF(E)\sin(\Delta k^N d) - \cos(\Delta k^N d)} , \tag{2.4}$$

with

$$\Omega_{N,S} = (E^2 - \Delta_{N,S}^2)^{1/2} , \tag{2.5}$$

$$F(E) = \frac{(E^2 - \Delta_S\Delta_N)}{\Omega_N\Omega_S} , \tag{2.6}$$

$$G(E) = \frac{E(\Delta_S - \Delta_N)}{\Omega_S\Omega_N} , \tag{2.7}$$

$$\Delta k^N = \frac{2Z_N(E)}{\hbar v_F \cos\Theta}\Omega_N . \tag{2.8}$$

The quantity k_{FX} is equal to $k_F\cos\Theta$ where $\cos\Theta = (1 - k_\parallel^2/k_F^2)^{1/2}$ so that Θ is the angle between k and the normal to the *NS* interface. The renormalization function $Z_N(E)$ occurring in Eq. (2.8) is the function averaged over the thickness of the N layer.

Since the large parentheses in Eqs. (2.1) and (2.2) are equal to $-iN(E,X)$ and $-if(E,X)$, respectively [cf. Eqs. (1.1) and (1.2) and Eqs. (6.5) and (6.6) of Ref. 4], we obtain [from Eqs. (1.13)–(1.15)]:

$$Z_N(E)=Z_N^{ph}(E)-\frac{\hbar}{2\tau}\int_0^1 d(\cos\Theta)\left[\frac{\chi_1(E)}{\Omega_N}+\frac{\Delta_N}{E\,\Omega_N}\int_{-d}^{0}\frac{dx}{d}\chi_2(E,x)\right], \tag{2.9}$$

$$\Delta_N(E)=\Delta_N^{ph}(E)-\frac{\hbar}{2Z_N^{ph}(E)\tau}\int_0^1 d(\cos\Theta)\frac{\Omega_N}{E}\int_{-d}^{0}\frac{dx}{d}\chi_2(E,x), \tag{2.10}$$

where $\Delta_N^{ph}=\phi_N^{ph}/Z_N^{ph}$.

III. SIMPLE LIMITING CASES

It is useful to consider limits of Eqs. (2.9) and (2.10) which correspond to known results. First we consider the limit in which $\Delta_S=\Delta_N$. In this case we have a homogeneous superconductor. One readily verifies that $F(E)=1$ and $G(E)=0$ so that

$$\chi_1(E)=-i, \tag{3.1}$$

$$\chi_2(E,x)=0, \tag{3.2}$$

and hence

$$Z_N(E)=Z_N^{ph}(E)+\frac{i\hbar}{2\tau}\frac{1}{\Omega_N}, \tag{3.3}$$

$$\Delta_N(E)=\Delta_N^{ph}(E). \tag{3.4}$$

This is the expected result for a bulk superconductor with elastic s-wave scattering.

Now consider the limit as the thickness of the N metal (d) tends to zero. Since

$$\lim_{d\to 0}\chi_1(E)=-iF(E), \tag{3.5}$$

$$\lim_{d\to 0}\chi_2(E,x)=-iG(E), \tag{3.6}$$

we find that Eqs. (2.9) and (2.10) become

$$Z_N(E)=Z_N^{ph}(E)+\frac{i\hbar}{2\tau}\frac{1}{\Omega_S}, \tag{3.7}$$

$$\Delta_N(E)=\Delta_N^{ph}(E)+\frac{i\hbar}{2Z_N^{ph}(E)\tau}\frac{[\Delta_S(E)-\Delta_N(E)]}{\Omega_S}. \tag{3.8}$$

The second equation is readily solved for $\Delta_N(E)$

$$\Delta_N(E)=\frac{\Delta_N^{ph}(E)+i\Gamma_N(E)\Delta_S/\Omega_S}{1+i\Gamma_N(E)/\Omega_S}, \tag{3.9}$$

where we define

$$\Gamma_N(E)=\frac{\hbar}{2Z_N^{ph}(E)\tau}. \tag{3.10}$$

The solution for Δ_N in this limit bears a striking resemblance to the result for Δ_N in the McMillan tunneling model[10] of the proximity effect. The McMillan model was designed to treat NS double layers which are separated by a tunnel barrier. The N and S metals are assumed to be of comparable thickness, each being thinner than a coherence length. This is remarkable, because we have obtained Eq. (3.9) for a *strongly-coupled* double layer system, where the S layer is semi-infinite, and the N layer is of *negligible* thickness.

In the McMillan tunneling model, $\Gamma_N(E)$ is equal to

$$\frac{T^2 A d_S N_S(0)}{Z_N(E)^{ph}},$$

where T is the tunneling matrix element between the N and S metals, A is the NS interface area, d_S is the thickness of the S metal, and $N_S(0)$ is the normal electron density of states in S at the Fermi level. In contrast, our expression for Γ_N [Eq. (3.10)] is given by the lifetime of a normal electron in the N metal due to *elastic scattering*. The correspondence between our result and that of McMillan is due to the fact that both Γ_N's arise from lifetime effects in the N metal. In McMillan's model, Γ_N is inversely proportional to the average time which a quasiparticle spends in the N metal. Other remarks on the solution in this limit are presented in Ref. 11.

The next limit which we can easily investigate is that in which the imaginary part of $\Delta k^N d/\cos\Theta$ is large. For convenience, we define $R=2d/\hbar v_F$ so that, using Eq. (2.8)

$$\Delta k^N d = RZ_N\Omega_N/\cos\Theta. \tag{3.11}$$

Assuming that

$$\cos(\Delta k^N d)\approx -i\sin(\Delta k^N d), \tag{3.12}$$

$$\chi_1(E)\approx -i, \tag{3.13}$$

we find

$$\int_{-d}^{0}\frac{dx}{d}\chi_2(E,x)\approx\frac{\cos\Theta}{RZ_N\Omega_N}\frac{E(\Delta_S-\Delta_N)}{E^2-\Delta_S\Delta_N+\Omega_S\Omega_N}. \tag{3.14}$$

In this limit, Eqs. (2.9) and (2.10) therefore become

$$Z_N = Z_N^{ph} + \frac{i\hbar}{2\tau}\frac{1}{\Omega_N}\left[1 + \frac{i}{RZ_N\Omega_N}\frac{\Delta_N(\Delta_S-\Delta_N)}{(E^2-\Delta_S\Delta_N+\Omega_S\Omega_N)}\right] , \tag{3.15}$$

$$\Delta_N = \Delta_N^{ph} - \frac{\hbar}{4Z_N^{ph}(E)\tau}\frac{1}{RZ_N\Omega_N}\frac{(\Delta_S-\Delta_N)\Omega_N}{E^2-\Delta_S\Delta_N+\Omega_S\Omega_N} . \tag{3.16}$$

It is convenient to remove a potentially divergent factor from Eq. (3.15) by constructing the equation for $RZ_N\Omega_N$. The latter is a more physically significant quantity, because it determines the interference phenomena discussed in Ref. 4. Thus

$$RZ_N\Omega_N = RZ_N^{ph}\Omega_N + i\left[\frac{d}{l}\right] - \left[\frac{d}{l}\right]\frac{1}{RZ_N\Omega_N}\frac{\Delta_N(\Delta_S-\Delta_N)}{(E^2-\Delta_S\Delta_N)+\Omega_S\Omega_N} , \tag{3.17}$$

where we define the mean-free path due to elastic scattering, $l = v_F\tau$. Similarly, Eq. (3.16) may be written

$$\Delta_N = \Delta_N^{ph} - \frac{d}{2l}\frac{1}{RZ_N\Omega_N RZ_N^{ph}}\frac{(\Delta_S-\Delta_N)\Omega_N}{E^2-\Delta_S\Delta_N+\Omega_S\Omega_N} . \tag{3.18}$$

Equation (3.17) is a simple quadratic equation for $RZ_N\Omega_N$ in terms of $E, d/l, \Delta_N$, and $RZ_N^{ph}\Omega_N$. Its solution is

$$RZ_N\Omega_N = \frac{1}{2}\left[RZ_N^{ph}\Omega_N + i\frac{d}{l}\right] + \frac{1}{2}\left[\left[RZ_N^{ph}\Omega_N + i\frac{d}{l}\right]^2 + 2\left[\frac{d}{l}\right]A\right]^{1/2} , \tag{3.19}$$

where

$$A = \frac{\Delta_N(\Delta_S-\Delta_N)}{E^2-\Delta_S\Delta_N+\Omega_S\Omega_N} . \tag{3.20}$$

Finally, Eqs. (2.9) and (2.10) also simplify in the limit $E >> \Delta_S, \Delta_N$, where to order E/Δ_S:

$$\chi_1(E) \cong -i , \tag{3.21}$$

$$\int_{-d}^{0}\frac{dx}{d}\chi_2(E,x) \cong -i\frac{(\Delta_S-\Delta_N)}{E}\frac{\sin(\Delta K^N d)}{\Delta K^N d}e^{i\Delta K^N d} . \tag{3.22}$$

Using $\Delta K^N d = RZ_N\Omega_N/\cos\Theta$, we find

$$RZ_N\Omega_N \cong RZ_N^{ph}\Omega_N + id/l , \tag{3.23}$$

$$\Delta_N \cong \frac{\Delta_N^{ph} + iI(E)\Delta_S}{1+iI(E)} , \tag{3.24}$$

where

$$I(E) \equiv \left[\frac{d}{l}\right]\left[\frac{1}{RZ_N^{ph}E}\right]\int_0^1 d(\cos\Theta)\frac{\sin(RZ_N\Omega_N/\cos\Theta)}{(RZ_N\Omega_N/\cos\Theta)}\exp\left[\frac{iRZ_N\Omega_N}{\cos\Theta}\right] . \tag{3.25}$$

Note that the interference factor, or phase difference, $RZ_N\Omega_N$, is just that which would be expected from the usual theory for elastic scattering. However, the equation for Δ_N is decidedly different from the usual assumption $\Delta_N = \Delta_N^{ph}$, based (incorrectly) on Anderson's theorem. Indeed, the magnitude of $I(E)$ is governed by the prefactor

$$\frac{d}{l}\frac{1}{RE} = \frac{\hbar v_F}{2lE} \tag{3.26}$$

so that deviations of Δ_N from Δ_N^{ph} are pronounced when the "energy-dependent coherence length," $\hbar v_F/2E$, is greater than the mean-free path due to elastic scattering. In the limit wherein the above ratio is large and $RZ_N^{ph}E$ is small, $\Delta_N(E)$ is approximately equal to $\Delta_S(E)$, so that the superconducting properties of the thin NS double layer have effectively been "homogenized" by elastic scattering. The fact that this homogenization effect may be present at the phonon energies of the S metal has implications for tunneling experiments. We shall discuss these in detail in Sec. IV.

IV. TUNNELING DENSITY OF STATES AT PHONON ENERGIES

In Sec. III we discussed the solutions for Δ_N and Z_N in the limit $E >> \Delta_S, \Delta_N$. In the limit of large d/l with small $RZ_N^{ph}E$, one finds that Eq. (3.24) reduces to

$$\Delta_N(E) \approx \Delta_S(E)\left[\frac{1}{1-4iRZ_N^{ph}E}\right] \approx \Delta_S(E)e^{i4RZ_N^{ph}E} . \quad (4.1)$$

Thus, as $RZ_N^{ph}E$ approaches zero, $\Delta_N(E)$ approaches $\Delta_S(E)$, and we obtain the "homogenization" referred to in the Introduction. In addition, in this limit the tunneling density of states [Eq. (4.3) below] becomes

$$N_T(E) \approx \text{Re}\left[\frac{E}{[E^2-\Delta_N(E)^2]^{1/2}}\right] \approx 1+\text{Re}\left[\frac{\Delta_S(E)^2}{2E^2}e^{8iRZ_N^{ph}E}\right] . \quad (4.2)$$

Thus in the limit of large d/l and $RZ_N^{ph}E << 1$, we observe that the tunneling density of states approaches that which is expected for McMillan-Rowell (4d) interference oscillations (which go as $e^{2iRZ_N^{ph}E}$), but with a path length which is FOUR TIMES the *expected* path length of 4d. This illustrates the interference path lengthening mentioned in the Introduction. The factor of 4 should be regarded as a "saturation" value. To determine in detail the effect of elastic scattering on the tunneling density of states, we choose to consider an NS sandwich where S = Pb, and use the $\Delta_S(E)$ obtained in Ref. 12.

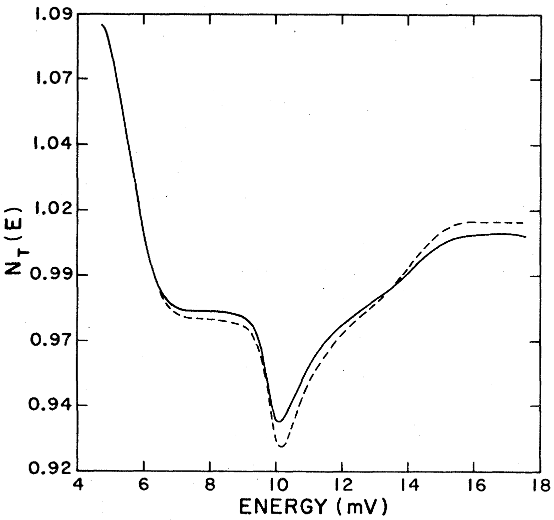

FIG. 2. Tunneling density of states for $\Delta_N^{ph}=0, Z_N^{ph}=1$, illustrating saturation effect. Solid line: $R=0.01$ and $d/l=5$. Dashed line: $R=0.04$ and $d/l=0$.

From Eqs. (4.2) and (4.5) of Ref. 4, we have the tunneling density of states at 0 K for specular tunneling (we will not consider the "random tunneling" case):

$$N_T(E) = \text{Im}\left[\frac{(E/\Omega_N)[iF(E)\cos(\Delta K^N d)+\sin(\Delta K^N d)]+i(\Delta_N/\Omega_N)G(E)}{[iF(E)\sin(\Delta K^N d)-\cos(\Delta K^N d)]}\right] . \quad (4.3)$$

$Z_N(E)$ and $\Delta_N(E)$ were found by solving Eqs. (2.21) and (2.22) by numerical iteration at each energy for given $\Delta_S(E), R, d/l, Z_N^{ph}$, and Δ_N^{ph}.

In Fig. 1 we plot results for $R=0.02$ and $d/l=0$, 0.25, 1, and 5. Qualitatively, one may compare this to Fig. 6 of Ref. 4, which shows the effect of increasing thickness (R) on the tunneling density of states. The comparison indicates that, roughly speaking, increasing the amount of elastic scattering (increasing d/l) has an effect which is similar to increasing thickness. As discussed in the Introduction, behavior of this sort was observed in the experiments of Berman and So.[7]

In Fig. 2 we illustrate the phenomenon of "saturation," wherein the tunneling density of states approaches Eq. (4.2) for large d/l, $RZ_N^{ph}E$ approaching zero. We see that the curves for $R=0.01$, $d/l=5$ and for $R=0.04$, $d/l=0$, are quite close to one another, as expected.

Thus, it appears that elastic scattering (from stacking faults, vacancies, dislocations, for example) can account for the previously unexplained results of Ref. 7. The inclusion of such scattering is certainly justified in this case because the films of Ref. 7 were quench-condensed, and subsequently annealed at 77 K. It is certain that many imperfections (though, apparently, few impurities) in the N metal remain even after the annealing process.

V. LOW ENERGIES

We begin our consideration of the low-energy regime with equations for the limit $d/l >> RZ_N^{ph}\Delta_N$. The solution for $RZ_N\Omega_N$ in this limit is

$$RZ_N\Omega_N \approx id/l . \quad (5.1)$$

To obtain Δ_N, we must solve Eq. (3.18). After some straightforward algebraic manipulation, we obtain a cubic polynomial in Δ_N:

$$[1-4(RZ_N^{ph}E)^2-4iRZ_N^{ph}\Omega_S]\Delta_N^3+[4(RZ_N^{ph}E)^2(2\Delta_N^{ph}+\Delta_S)+4iRZ_N^{ph}\Omega_S\Delta_N^{ph}-\Delta_S]\Delta_N^2$$
$$+\{4iRZ_N^{ph}\Omega_S E^2-E^2-4(RZ_N^{ph}E)^2[(\Delta_N^{ph})^2+2\Delta_S\Delta_N^{ph}]\}\Delta_N+[4(RZ_N^{ph}E)^2\Delta_S(\Delta_N^{ph})^2+E^2\Delta_S-4iRZ_N^{ph}\Omega_S E^2\Delta_N^{ph}]=0 \quad (5.2)$$

This equation was solved numerically. The real and imaginary parts of the nonextraneous root Δ_N of Eq. (5.2) are displayed in Fig. 3 for $Z_N^{ph}=1.2$, $\Delta_S=1.4$, $\Delta_N^{ph}=0.3$ with R values of 1, 0.5, 0.25, and 0.1. Note the disappearance of the cusp in ReΔ_N at Δ_S. This is a symptom of the transition to a regime in which Δ_N approaches Δ_S, i.e., the passage to the "homogenized limit." The limit as $R\rightarrow 0$ is just $\Delta_N=\Delta_S$, as mentioned in Sec. III. Thus, it is natural that as R decreases below $R=0.25$, we observe a "flattening" of ImΔ_N about the zero line in Fig. 3.

In Fig. 4 we plot the density of states [Eq. (4.3)] obtained by solving Eqs. (2.9) and (2.10) for $RZ_N\Omega_N$ and Δ_N, with $d/l=3$, $\Delta_N^{ph}=0.3$, $Z_N^{ph}=1.2$, $\Delta_S=1.4$, and $R=1$, 0.5, 0.25, and 0.1. The values obtained for $RZ_N\Omega_N$ and Δ_N were very close to those obtained above for the limit of large d/l. This illustrates that d/l need not be unphysically large in order for the somewhat simpler Eqs. (5.1) and (5.2) to be approximately valid at low energies.

The behavior illustrated in Fig. 4 is qualitatively like that observed by Freake in Ref. 13. In Fig. 2 of that reference, the tunneling density of states is plotted for Mg-oxide-Cu-Pb specimens with Pb thickness of 7000 Å and Cu thicknesses ranging from 250 to 1200 Å. The similarity between these experimental results and Fig. 4 is striking. One quantitative disagreement is present, however. The dip in the experimental conductance appears to occur at successively lower values of energy, whereas the dip in the theoretical plot always occurs near $\Delta_S=1.4$. This is easily explained, however, because, as we have remarked,[14] the value of Δ_S at the NS interface is depressed below its bulk value to a value of (approximately) $\Delta_S^{\rm bulk}(1-\pi R\,\Delta_S^{\rm bulk})$ for small R values. For larger R values ($\pi R\,\Delta_S^{\rm bulk}>0.1$) this expression is no

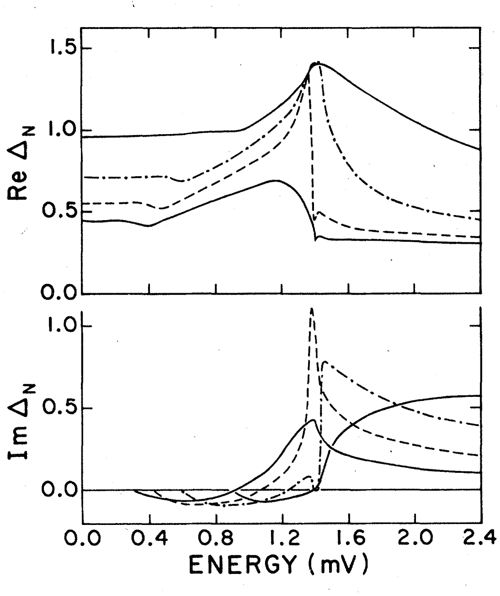

FIG. 3. The real and imaginary parts of solutions to Eq. (5.2), with $\Delta_N^{ph}=0.3$, $Z_N^{ph}=1.2$, and $\Delta_S=1.4$. In the top figure, the lowest solid line is for $R=1$. The corresponding curve in the bottom figure is that which passes through 0.4 near $E=1.4$ mV. The second solid lines in each plot are for $R=0.1$. The dashed line: $R=0.5$. The dash-dotted line: $R=0.25$.

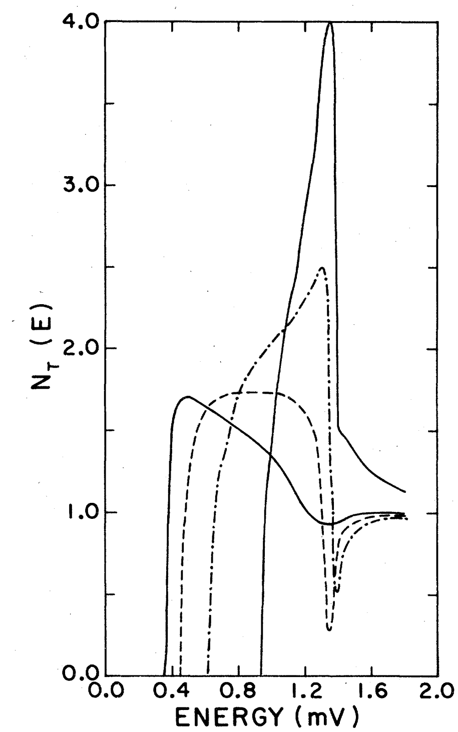

FIG. 4. The tunneling density of states [Eq. (4.3)] at low energies, $\Delta_N^{ph}=0.3$, $Z_N^{ph}=1.2$, $\Delta_S=1.4$, and $d/l=3$. Solid line which is nonzero at $E=0.4$ mV is for $R=1$. Dashed line: $R=0.5$. Dash-dotted line: $R=0.25$. Second solid line: $R=0.1$.

longer accurate, but it is clear that the depression of Δ_S at the NS interface increases. Since it is the value of Δ_S at the NS interface which comes into the density of states, for the larger R values we should use smaller Δ_S values. This should correct the quantitative disagreement mentioned above.

VI. CONCLUSION

The proper treatment of elastic scattering in an NS sandwich appears to explain (at least qualitatively) some previously inexplicable experimental results from tunneling in NS double layers. Since the author himself has invoked effective mean-free paths in the N metal layer to carry out analyses of proximity effect tunneling data on NS sandwiches,[15] some discussion of the relevance of the theory of this paper to that analysis is in order.

In Ref. 15, the effective mean-free path was included merely as an imaginary constant, id/l, in the phase difference $RZ_N\Omega_N$. In the thinner films, it was assumed that this mean-free path was due to diffuse scattering at interfaces, *not* to elastic scattering within the N metal layer. If this assumption is correct, it is questionable that the above work is directly applicable to such scattering. Because diffuse scattering is an effect which is localized at a boundary plane, it may not be susceptible to the same treatment as elastic scattering from imperfections located at random within the N metal. Because the treatment of diffuse scattering is phenomenological, however, a definitive statement on the difference between the effects of diffuse scattering and bulk elastic scattering must await future work.

If, however, the scattering which is apparent in the data of Ref. 15 is predominantly bulk elastic scattering, then the above results are of direct consequence. The reanalysis of the data, assuming that all the scattering may be treated as if it were bulk elastic scattering, is currently proceeding.

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