Dielectric function at metallic densities with nonlocal interactions

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A nonlocal type of interaction with a separable form has been proposed to obtain the dielectric function for the degenerate electron gas. The corresponding vertex function is then solved exactly and the expressions for compressibility ratio κ_F/κ and the dispersion parameter $\tilde{\beta}/\tilde{\beta}_{\text{RPA}}$ have been obtained. The calculated results of $\tilde{\beta}/\tilde{\beta}_{\text{RPA}}$ are in reasonable agreement with the available experimental data. Further, the model-dependent calculations not only satisfy the Ward identity for the wave-function renormalization $Z_{\vec{p}}$, but also show no compressibility divergence.

I. INTRODUCTION

A number of attempts have been made over the last two decades to arrive at a suitable form for the dielectric function of a degenerate electron gas prevalent at metallic densities. Several different approaches have been tried for the dielectric function $\epsilon(\vec{k},\omega)$. One of the approaches is the many-body theory approach in which, in the static approximation, the resulting equation for $\epsilon(\vec{k},\omega)$ is given as

$$\epsilon(\vec{k},\omega)=1+2V_k\sum_{\vec{p}'} g_{\vec{k}\omega}(\vec{p}')\tilde{\Lambda}_{\vec{k}\omega}(\vec{p}'), \tag{1}$$

where V_k is the Coulomb interaction $4\pi e^2/k^2$, and the proper vertex function $\tilde{\Lambda}_{k\omega}(\vec{p})$ satisfies the following integral equation[1]:

$$\tilde{\Lambda}_{\vec{k}\omega}(\vec{p})=1-\sum_{\vec{p}'}\tilde{I}(\vec{p},\vec{p}')g_{\vec{k}\omega}(\vec{p}')\tilde{\Lambda}_{\vec{k}\omega}(\vec{p}'), \tag{2}$$

$$g_{\vec{k}\omega}(\vec{p})=(f_{\vec{p}+\vec{k}}-f_{\vec{p}})/(\omega+i\eta+\epsilon_{\vec{p}}-\epsilon_{\vec{p}+\vec{k}}), \tag{3}$$

where $\epsilon_{\vec{p}}=p^2/2m+\sum_{\vec{p}'}\tilde{I}(\vec{p},\vec{p}')f_{p'}$, η is a positive infinitesimal quantity, $f_{\vec{p}}$ is the Fermi function, and $\sum_{\vec{p}}$ is shorthand for $(2\pi)^{-3}\int d^3p$. $\tilde{I}(\vec{p},\vec{p}')$ is the static interaction term, characterizing an effective interparticle interaction.

Most of the theoretical attempts to find $\epsilon(\vec{k},\omega)$ have been to use different approximate forms for $\tilde{I}(\vec{p},\vec{p}')$. One common approximation originally due to Hubbard[2] and used widely is the screened Coulomb interaction of the Yukawa form, namely

$$\tilde{I}(\vec{p},\vec{p}')=-\{4\pi e^2/[(\vec{p}-\vec{p}')^2+K_S^2]\}, \tag{4}$$

where K_S is the screening parameter usually taken to be the inverse Thomas-Fermi length. Hubbard, however, solved the integral equation (2) making a further approximation for $\tilde{I}$, that the $(\vec{p}-\vec{p}')^2$ term in $\tilde{I}$ could be replaced by $k^2+k_F^2$ (k_F being the Fermi wave number). Kleinman[3] and Overhauser[3] have independently pointed out that this further approximation of Hubbard[2] is incorrect for large k dependence and could lead to large effects, in particular on the correlation energy at metallic densities which is sensitive to large values of k.

Langreth[1] employed a variational technique to solve the integral equation (2) with $\tilde{I}(\vec{p},\vec{p}')$ given by Eq. (4), and he has shown with the simplest choice of the trial function that the dielectric function is exact in both low and high values of the momentum transfer. Since then, more powerful variational calculations have been done by Shastry *et al.*,[4] who have proposed a self-consistent quasi-static-screening approach in which they replaced the interaction term $\tilde{I}$ by $4\pi e^2/[k^2+k_F^2\xi^2W(k)]$, where $W(k)$ is a slowly decreasing function of k and $\xi^2(r_S)$ is determined self-consistently. Also there exists a numerical solution of Eq. (2) by Woo and Jha.[5] In all these calculations, however, one goes beyond the random-phase approximation (RPA) calculations. In the present paper we have assumed, unlike previous studies, a nonlocal form for the interaction which can be expressed as

$$\tilde{I}(\vec{p},\vec{p}')=-4\pi e^2\lambda^2/[(p^2+\beta^2)(p'^2+\beta^2)], \tag{5}$$

where λ and β are two parameters which are to be suitably chosen. From the point of view of the integral equation this amounts to making a degenerate kernel approximation of the equation. In principle we may take the sum of a large number of such terms as given by Eq. (5), but it then involves a large number of unknown parameters. The present choice immediately leads to an exact solution of Eq. (2), and we can then derive exact expressions for the dielectric function within our nonlocal separable approximation form for the interaction. While obtaining the solution of the integral equation (2), we keep in mind that the proper vertex function $\tilde{\Lambda}_{\vec{k}\omega}(\vec{p})$ must obey the Ward identity, namely

$$\lim_{\vec{k}\to 0,\,\omega\to 0}\tilde{\Lambda}_{k\omega}(\vec{p})=1/Z_{\vec{p}},$$

where $Z_{\vec{p}}$ is the wave-function renormalization and must be such that $0 < Z_{\vec{p}} \leq 1$ for all $\vec{p}$.

In the following section we give the solution of Eq. (2) and obtain an exact expression of the dielectric function. In Sec. III we obtain other related functions, namely the compressibility ratio and the plasma dispersion parameter and show explicitly how our results lead to terms beyond that which one obtains in the usual RPA case. In Sec. IV we give our numerical results and discuss how they compare with available experimental and other theoretical results.

II. SOLUTION OF THE VERTEX INTEGRAL EQUATION AND THE EXPRESSION FOR DIELECTRIC FUNCTION

The integral equation (2) for the proper vertex function $\tilde{\Lambda}_{\vec{k}\omega}(\vec{p})$ with the approximation (5) for the static interaction term $\tilde{I}(\vec{p},\vec{p}')$ can be written as

$$\tilde{\Lambda}_{k\omega}(\vec{p}) = 1 + \frac{4\pi e^2\lambda^2}{(p^2+\beta^2)} \int \frac{d^3p'}{(2\pi)^3} g_{\vec{k}\omega}(\vec{p}')\tilde{\Lambda}_{\vec{k}\omega}(\vec{p}') \frac{1}{p'^2+\beta^2}. \tag{6}$$

Introducing a quantity $C_{k\omega}$ by

$$C_{k\omega} = \int d^3p' g_{\vec{k}\omega}(\vec{p}')\tilde{\Lambda}_{\vec{k}\omega}(\vec{p}') \frac{1}{p'^2+\beta^2}, \tag{7}$$

we can rewrite (6) as

$$\tilde{\Lambda}_{\vec{k}\omega}(\vec{p}) = 1 + \frac{4\pi e^2\lambda^2}{(2\pi)^3(p^2+\beta^2)} C_{k\omega}. \tag{8}$$

Substituting the expression for $\tilde{\Lambda}_{\vec{k}\omega}(\vec{p}')$ from Eq. (8) into the right-hand side of Eq. (7) we easily find

$$C_{k\omega} = \frac{\int d^3p' g_{\vec{k}\omega}(\vec{p}')/(p'^2+\beta^2)}{1 - 4\pi e^2\lambda^2 \int \frac{d^3p'}{(2\pi)^3} g_{\vec{k}\omega}(\vec{p}')/(p'^2+\beta^2)^2}. \tag{9}$$

Hence the solution of Eq. (6) for $\tilde{\Lambda}_{k\omega}(\vec{p})$ becomes

$$\tilde{\Lambda}_{k\omega}(\vec{p}) = 1 + \frac{\frac{4\pi e^2\lambda^2}{(p^2+\beta^2)} \int \frac{d^3p'}{(2\pi)^3} g_{\vec{k}\omega}(\vec{p}') \frac{1}{p'^2+\beta^2}}{1 - 4\pi e^2\lambda^2 \int \frac{d^3p'}{(2\pi)^3} g_{\vec{k}\omega}(\vec{p}') \frac{1}{(p'^2+\beta^2)^2}}. \tag{10}$$

Thus the expression for the dielectric function $\epsilon(\vec{k},\omega)$ is easily derived from Eqs. (1) and (10) and is given by

$$\epsilon(\vec{k},\omega) = 1 + 2V_k \int \frac{d^3p'}{(2\pi)^3} g_{k\omega}(\vec{p}') + \frac{2V_k 4\pi e^2\lambda^2 \left(\int \frac{d^3p'}{(2\pi)^3} g_{k\omega}(\vec{p}') \frac{1}{p'^2+\beta^2} \right)^2}{1 - 4\pi e^2\lambda^2 \int \frac{d^3p'}{(2\pi)^3} g_{k\omega}(\vec{p}') \frac{1}{(p'^2+\beta^2)^2}}. \tag{11}$$

We may note that the first two terms in the above expression for $\epsilon(\vec{k},\omega)$ in Eq. (11) are the usual RPA results while the last term in Eq. (11) gives us the correction term to the RPA in our present model. In order to obtain the compressibility ratio and plasma dispersion relation, we evaluate the various integrals occuring in the expression for $\epsilon(\vec{k},\omega)$ given in Eq. (11). As is common we replace as usual $\epsilon_{\vec{p}}$ by $p^2/2m$ and consider the system at temperature $T=0$. In this case the Fermi distribution function $f_{\vec{p}}$ becomes unity for $|p| < p_F$ and it vanishes otherwise. The value of the integral I_1, where I_1 stands for

$$I_1 = \int \frac{d^3p'}{(2\pi)^3} g_{\vec{k}\omega}(\vec{p}'), \tag{12}$$

with

$$g_{\vec{k}\omega}(\vec{p}) = (f_{\vec{p}+\vec{k}} - f_{\vec{p}}) \Big/ \left(\frac{p^2}{2m} - \frac{(\vec{p}+\vec{k})^2}{2m} + \omega + i\eta \right), \tag{13}$$

is the usual RPA result, and the results for the real and imaginary parts are well known and are given in the Appendix for ready reference. To have the complete result for $\epsilon(\vec{k},\omega)$, we need to evaluate two more integrals which occur in expression (11), namely,

$$I_2(\beta) = \int \frac{d^3p'}{(2\pi)^3} g_{\vec{k}\omega}(\vec{p}') \frac{1}{p'^2+\beta^2} \tag{14}$$

and

$$I_3(\beta) = \int \frac{d^3p'}{(2\pi)^3} g_{\vec{k}\omega}(\vec{p}') \frac{1}{(p'^2+\beta^2)^2}, \tag{15}$$

where $g_{\vec{k}\omega}(\vec{p})$ is given by Eq. (13). We may note that I_3 is simply obtained as

$$I_3(\beta) = -(\partial/\partial\beta^2) I_2(\beta). \tag{16}$$

The values of real and imaginary parts of $I_2(\beta)$ and $I_3(\beta)$ are displayed in the Appendix. Using Eqs. (14) and (16), we have from Eq. (10) the following expression for $\tilde{\Lambda}_{\vec{k}\omega}(\vec{p})$:

$$\tilde{\Lambda}_{\vec{k}\omega}(\vec{p}) = 1 + \frac{[4\pi e^2\lambda^2/(p^2+\beta^2)] I_2(\beta)}{1 + 4\pi e^2\lambda^2(\partial/\partial\beta^2) I_2(\beta)}. \tag{17}$$

Since the proper vertex function $\tilde{\Lambda}_{\vec{k}\omega}(\vec{p})$ must satisfy the Ward identity, we need to find out $\lim_{k\to 0,\omega\to 0} \tilde{\Lambda}_{\vec{k}\omega}(\vec{p})$. The relevant expression for $Z_{\vec{p}}^{-1}$ becomes

$$Z_{\vec{p}}^{-1} = 1 + Z_1/Z_2, \tag{18}$$

where

$$Z_1 = \frac{k_{FT}^2\lambda^2}{8\pi(p^2+\beta^2)}\, \frac{2\pi(k_F^2-\beta^2)}{k_F^2(\beta^2+k_F^2)} \ln \frac{\beta^2}{\beta^2+k_F^2} + \frac{4\pi}{\beta k_F}\tan^{-1}\frac{2\beta k_F}{\beta^2-k_F^2} - \frac{2\pi(k_F^2+3\beta^2)}{\beta^2(\beta^2+k_F^2)}$$

and

$$Z_2 = 1 - \frac{k_{FT}^2\lambda^2}{\beta^2}\left(\frac{(k_F^2+2\beta^2)}{4\beta^2(\beta^2+k_F^2)} - \frac{(\beta^2+k_F^2)}{2[(\beta^2-k_F^2)^2+4k_F^2\beta^2]} - \frac{\beta^2}{2(\beta^2+k_F^2)^2}\ln\frac{\beta^2}{\beta^2+k_F^2} - \frac{1}{4\beta k_F}\tan^{-1}\frac{2\beta k_F}{\beta^2-k_F^2}\right),$$

$$k_{FT}^2 = 6\pi n e^2/\epsilon_F, \quad \epsilon_F = k_F^2/2m, \text{ and } k_F^3 = 3n\pi^2,$$

where n is the electron density.

We study the behavior of this function $Z_{\vec{p}}$ in Sec. IV, in order to test the Ward identity. In the following section we write down the expressions for the compressibility ratio for the free and interacting electron gas and plasma dispersion relation.

III. COMPRESSIBILITY RATIO AND PLASMA DISPERSION RELATION

The compressibility ratio κ_F/κ of the free and interacting electron gas is given by[6]

$$\frac{\kappa_F}{\kappa} = \lim_{\vec{k}\to 0} \frac{k_{FT}^2}{k^2[\epsilon(\vec{k},0)-1]}\,. \tag{19}$$

In the limit of the long wavelength (i.e., $\vec{k}\to 0$) and $\omega = 0$, our expression for $\epsilon(\vec{k},\omega)$ yields the following:

$$\lim_{\vec{k}\to 0} k^2[\epsilon(\vec{k},0)-1] = k_{FT}^2 + \frac{[(k_{FT}^2)^2\lambda^2/8\pi k_F^2]g_1(\beta)}{1+\frac{1}{4}k_{FT}^2\lambda^2 g_2(\beta)}, \tag{20}$$

where

$$g_1(\beta) = \frac{\pi k_F^2}{\beta^4} + \frac{4\pi k_F^2(2\beta^2+k_F^2)}{\beta^2(\beta^2+k_F^2)^2} - \frac{4\pi k_F(3\beta^2+k_F^2)}{\beta^3(\beta^2+k_F^2)}\tan^{-1}\frac{2\beta k_F}{\beta^2-k_F^2} + \frac{4\pi}{\beta^2}\left(\tan^{-1}\frac{2\beta k_F}{\beta^2-k_F^2}\right)^2 - \frac{4\pi(\beta^2-k_F^2)}{\beta k_F(\beta^2+k_F^2)}\ln\frac{\beta^2}{\beta^2+k_F^2}$$
$$\times\tan^{-1}\frac{2\beta k_F}{\beta^2-k_F^2} + \left(\frac{2\pi(\beta+k_F)^2}{\beta^2(\beta^2+k_F^2)} - \frac{4\pi k_F^2(3\beta^2+k_F^2)}{\beta^2(\beta^2+k_F^2)^2}\right)\ln\frac{\beta^2}{\beta^2+k_F^2} + \left(\frac{\pi}{k_F^2} - \frac{4\pi\beta^2}{(\beta^2+k_F^2)^2}\right)\left(\ln\frac{\beta^2}{\beta^2+k_F^2}\right)^2$$

and

$$g_2(\beta) = \frac{(k_F^2+2\beta^2)}{\beta^4(\beta^2+k_F^2)} - \frac{2(\beta^2+k_F^2)}{\beta^2[(\beta^2-k_F^2)^2+4k_F^2\beta^2]} - \frac{1}{\beta^3 k_F}\tan^{-1}\frac{2\beta k_F}{\beta^2-k_F^2} - \frac{2}{(\beta^2+k_F^2)^2}\ln\frac{\beta^2}{\beta^2+k_F^2}.$$

The collective plasma mode in the electron gas can be found from the vanishing of the real part of the dielectric function. In the present case therefore, the collective plasma mode will be given by

$$\mathrm{Re}\left(1+2V_kI_1+\frac{2V_k4\pi e^2\lambda^2I_2^2(\beta)}{1-4\pi e^2\lambda^2I_3(\beta)}\right)=0. \tag{21}$$

When we expand the real part occurring in Eq. (21) in powers of kV_F/ω, the required plasma dispersion relation in the long-wavelength limit becomes

$$1-\frac{\omega_p^2}{\omega^2}\left(1+\frac{k^2}{k_{FT}^2}\,\frac{9}{5}-\frac{k^2\lambda^2}{2\beta^4}+\ldots\right)=0. \tag{22}$$

If the plasma frequency is written as

$$\omega_p(k)=\omega_p(0)+\tilde{\beta}k^2, \tag{23}$$

then the ratio $\tilde{\beta}/\tilde{\beta}_{\mathrm{RPA}}$ becomes

$$\frac{\tilde{\beta}}{\tilde{\beta}_{\mathrm{RPA}}}=1-\frac{5\lambda^2}{9\beta^4}\,\frac{k_{FT}^2}{2}, \tag{24}$$

where $\tilde{\beta}_{\mathrm{RPA}}$ is obtained from Eqs. (22) and (23) by dropping the λ^2-dependent term.

To write down the relevant expression for the plasma dispersion curve, we have from Eq. (22)

$$\omega_p(k)=\omega_p(0)\left[1+\frac{k^2}{k_{FT}^2}\left(\tfrac{9}{10}-\frac{\lambda^2k_{FT}^2}{4\beta^4}\right)+\cdots\right]. \tag{25}$$

Numerical results both for compressibility ratio and dispersion parameter are discussed in the next section.

IV. NUMERICAL RESULTS AND DISCUSSION

In order to calculate the various physical quantities we must know the values of the screening parameters β^2 and λ^2. However, it may be mentioned that it is not very difficult to guess the approximate values of these parameters, considering the values of similar parameters available from earlier theories. Nevertheless, we have studied the variation of $\tilde{\beta}/\tilde{\beta}_{\mathrm{RPA}}$ for different values of λ^2 and β^2. The results of our calculations are plotted in Fig. 1 along with the experimental[7] results for various values of r_S $[r_S=(\frac{4}{3}\pi n a_0^3)^{-1/3}$,

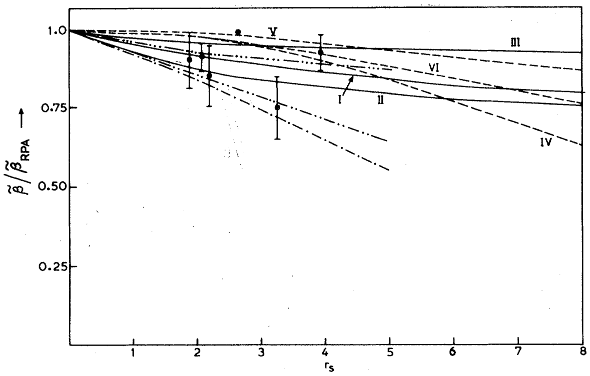

FIG. 1. Variation of dispersion parameter $\tilde{\beta}/\tilde{\beta}_{\mathrm{RPA}}$ with metallic electron densities represented by r_S, as given by experimental results of Raether and various theories. -·- Vashishta and Singwi theory; -··· - Rajagopal, Rath, and Kimball theory; -···· - Shastry, Jha, and Rajagopal theory. Curves I, II, III: present theory calculations when $\lambda^2 = k_F^2 + k_{FT}^2$, $\beta^2 = 2k_F^2 + k_{FT}^2$; $\lambda^2 = \beta^2 = 2k_F^2 + k_{FT}^2$, and $\lambda^2 = k_F^2 + k_{FT}^2$, $\beta^2 = 2k_F^2 + 2k_{FT}^2$, respectively; Curves IV, V, and VI: present theory calculations when $\lambda^2 = 1.8818$ Å^{-2}, $\beta^2 = 2k_F^2 + k_{FT}^2$; $\lambda^2 = 1.8818$ Å^{-2}, $\beta^2 = 2k_F^2 + 2k_{FT}^2$, and $\lambda^2 = 3.3038$ Å^{-2}; $\beta^2 = 2k_F^2 + 2k_{FT}^2$, respectively.

where n is the electron density and a_0 the Bohr radius].

For comparison of our calculations with experimental results of $\tilde{\beta}/\tilde{\beta}_{\mathrm{RPA}}$ we have chosen various sets of values of λ^2 and β^2. For example, we have taken β^2 to be of similar form as used in other theories, namely β^2 given in terms of k_F^2 and k_{FT}^2. For the choice of λ^2 we have considered two possibilities: in one case λ^2 is taken to be a constant independent of r_S and in another case its form is taken to be similar to that of β^2 which is r_S dependent through k_F^2.

For curves IV, V, and VI we have kept λ^2 a constant independent of r_S and $\beta^2 = 2k_F^2 + k_{FT}^2$ for curve IV, and $\beta^2 = 2k_F^2 + 2k_{FT}^2$ for curves V and VI. In curves IV and V λ^2 equals 1.8818 Å^{-2}, while β^2 is taken as $2k_F^2 + k_{FT}^2$ and $2k_F^2 + 2k_{FT}^2$, respectively. Curve VI is plotted with $\beta^2 = 2k_F^2 + 2k_{FT}^2$, but λ^2 is varied to 3.3038 Å^{-2}. While curve V shows agreement with experimental data for $r_S = 1.88$, 2.65, and 3.93, respectively, curves IV and VI are in agreement with the experimental results for Be and Na.

Curves I, II, and III in Fig. 1 correspond to the second possibility of λ^2 and show the plots of $\tilde{\beta}/\tilde{\beta}_{\mathrm{RPA}}$ vs r_S for values of $\beta^2 = 2k_F^2 + k_{FT}^2$, $\lambda^2 = k_F^2 + k_{FT}^2$; $\beta^2 = \lambda^2 = 2k_F^2 + k_{FT}^2$ and $\beta^2 = 2k_F^2 + 2k_{FT}^2$, $\lambda^2 = k_F^2 + k_{FT}^2$, respectively. It is evident from Fig. 1 that for small values of r_S, that is, for r_S lying approximately between 1 to 3.5, the calculated results for $\tilde{\beta}/\tilde{\beta}_{\mathrm{RPA}}$ decrease more rapidly for r_S-dependent λ^2 in comparison with r_S independent. For larger values of r_S, that is r_S greater than approximately 3.5, the trend is reversed; that is, the calculated values of $\tilde{\beta}/\tilde{\beta}_{\mathrm{RPA}}$ decrease much faster with increasing r_S for λ^2 which is r_S independent in comparison with the r_S-dependent λ^2. It may be noted that, when both λ^2 and β^2 are r_S dependent, our calculated results show good agreement with most of the experimental results. In particular, curves I and II are in reasonable agreement with the experimental data. For completeness we have also compared our results with those of Shastry, Jha, and Rajagopal (SJR),[4] Vasishta and Singwi (VS),[8] and Rajagopal, Rath, and Kimball (RRK).[9] Our results are larger than the predictions of VS and RRK theories but closer to the findings of SJR theory.

We now use the values of the parameters employed in curves I and II as well as those in curves IV and V to investigate the behavior of the wave function renormalization $Z_{\vec{p}}$. As mentioned before, we require that $Z_{\vec{p}}$ given by Eq. (18) must satisfy $0 < Z_{\vec{p}} \leq 1$ for all $\vec{p}$. We have evaluated this from $p = 0$ to p_F and we find that $Z_{\vec{p}}$ indeed satisfies the required Ward identity for various choices of the parameters λ^2 and β^2. The values of $Z_{\vec{p}}$ have been evaluated both at high and low metallic densities such as at $r_S = 2$ and 8 and those given in the accompanying Table I.

From our calculations we can conclude that for any given set of λ^2 and β^2 the values of $Z_{\vec{p}}$ is less than 1. Furthermore, as r_S increases, that is, metallic electron density decreases, the value

TABLE I. Values of the wave-function renormalization $Z_{\vec{p}}$ for various choices of the parameters λ^2 and β^2.

	$\lambda^2 = 2k_F^2 + k_{FT}^2$, $\beta^2 = 2k_F^2 + k_{FT}^2$		$\lambda^2 = k_F^2 + k_{FT}^2$, $\beta^2 = 2k_F^2 + k_{FT}^2$		$\lambda^2 = 1.8818$ Å^{-2}, $\beta^2 = 2k_F^2 + k_{FT}^2$		$\lambda^2 = 1.8818$ Å^{-2}, $\beta^2 = 2k_F^2 + 2k_{FT}^2$	
	$r_S = 2$	$r_S = 8$	$r_S = 2$	$r_S = 8$	$r_S = 2$	$r_S = 8$	$r_S = 2$	$r_S = 8$
$Z_{\vec{p}}$ $p = 0$	0.9048	0.8327	0.9146	0.8527	0.9787	0.7819	0.9882	0.8752
$Z_{\vec{p}}$ $p = p_F$	0.9260	0.8532	0.9485	0.8711	0.9820	0.8072	0.9906	0.8853

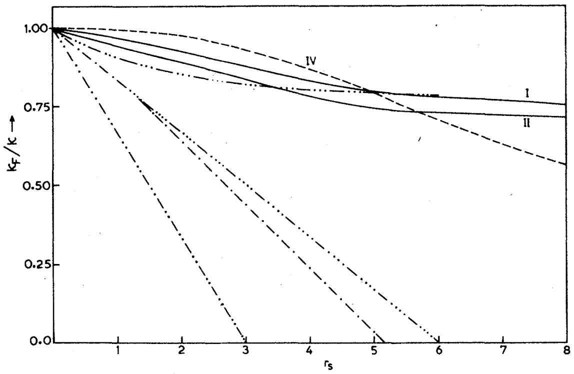

FIG. 2. Variation of compressibility ratio κ_F/κ with metallic densities as given by various theories. -·- VS theory; -··- Hubbard theory; -···- SJR theory; -·····- SJR theory ($\xi^2=0$ limit); curves I and II present theory calculation with $\lambda^2=k_F^2+k_{FT}^2$, $\beta^2=2k_F^2+k_{FT}^2$, and $\lambda^2=\beta^2=2k_F^2+k_{FT}^2$; curve IV: present theory calculation for $\lambda^2=1.8818$ Å^{-2} and $\beta^2=2k_F^2+k_{FT}^2$.

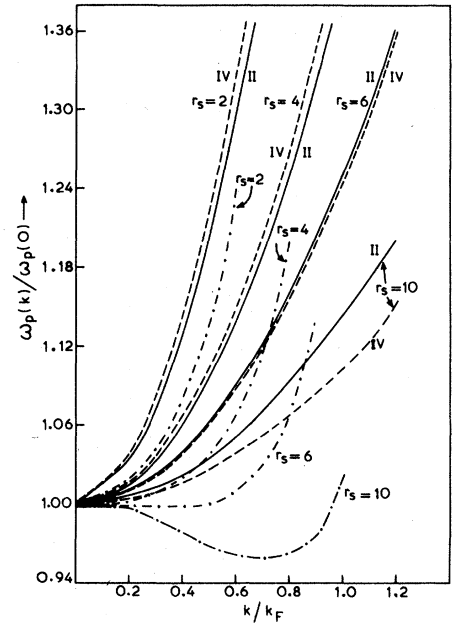

FIG. 3. Comparison of plasma dispersion curves as given by STLS theory with present theory calculations for different values of metallic densities. -·- STLS theory; --- present theory calculations when $\lambda^2=1.8818$ Å^{-2}, $\beta^2=2k_F^2+k_{FT}^2$; — present theory calculations for $\lambda^2=\beta^2=2k_F^2+k_{FT}^2$.

of $Z_{\vec{p}}$ decreases but is not less than 0.7819. We can also see that the value at $p=p_F$ is higher than that at $p=0$. It is interesting to note that at $\vec{p}\to\infty$, $Z_{\vec{p}}\to 1$ as can be easily seen from Eq. (18). Thus the fact that the Ward identity is satisfied along with the good experimental fit of $\tilde{\beta}/\tilde{\beta}_{\rm RPA}$ gives confidence in our choice of the nonlocal type of interaction given in Eq. (5).

We now present the calculated values of the compressibility ratio of the free and interacting electron gas, namely κ_F/κ obtained in Eq. (19). In Fig. 2 are shown the results of our calculations for κ_F/κ for various values of r_S. The curves I and II correspond to values of the parameters $\lambda^2=k_F^2+k_{FT}^2$, $\beta^2=2k_F^2+k_{FT}^2$, and $\lambda^2=\beta^2=2k_F^2+k_{FT}^2$. Both the curves show remarkable similarity to the theoretical values of Shastry *et al.*[4] The present calculation is also compared with the results of Hubbard[2] as well as of Vasishta and Singwi.[8] We find that the divergence in κ/κ_F obtained in these theories is absent in the present model as is also the case in the calculations of Shastry *et al.* Curve IV has been plotted for a fixed value of λ^2 set equal to 1.8818 Å^{-2} with $\beta^2=2k_F^2+k_{FT}^2$. While for small r_S (<2), κ_F/κ stays nearly constant equal to 1; for larger values of r_S, i.e., $r_S>2$, the value of κ_F/κ starts decreasing somewhat sharply, reaching a value of 0.56 for $r_S=8$. However, curve IV still does not present a compressibility divergence at metallic densities as is present in other theories.

We have already pointed out the behavior of the collective plasma mode in calculating $\tilde{\beta}/\tilde{\beta}_{\rm RPA}$. For completeness, we give in Fig. 3 the plasma dispersion curves obtained in our model and compare it with that obtained by Singwi *et al.* (STLS).[6] Their expression corresponding to ours for $\omega_p(k)/\omega_p(0)$ is given by

$$\omega_p(k)=\omega_p(0)\left\{1+\left[\frac{9}{10}-\frac{1}{2}\gamma\left(\frac{k_{FT}}{k_F}\right)^2\right]\left(\frac{k}{k_{FT}}\right)^2+\cdots\right\}, \tag{26}$$

where γ is related to static structure factor.

We have plotted the values of $\omega_p(k)/\omega_p(0)$ vs k/k_F for various values of r_S for $\lambda^2=\beta^2=2k_F^2+k_{FT}^2$ denoted by curve II in Fig. 3 [the other choice of parameters $\lambda^2=k_F^2+k_{FT}^2$ and $\beta^2=2k_F^2+k_{FT}^2$ gives nearly the same values of $\omega_p(k)/\omega_p(0)$ as given by the values of λ^2 and β^2 used in the evaluation of curve II]. For values of r_S ranging from 2 to 4 our results are somewhat similar to the results of Singwi *et al.*,[6] represented by the dash-dot curve in Fig. 3. For higher values of r_S ($r_S>6$), however, their calculated values of $\omega_p(k)/\omega_p(0)$ are less than 1 for small values of k/k_F reaching a minimum for a particular value of k/k_F, and beyond that follows the trend similar to $r_S<6$. For example, for $r_S=10$, their calculations for $\omega_p(k)/\omega_p(0)$ show a minimum 0.96 at $k=0.7k_F$ and it increases for $k>0.7k_F$. Such a trend does not occur in our calculations as is evident from Fig. 3. The curve marked IV in Fig. 3 corresponds to the choice $\lambda^2=1.8818$ Å^{-2} (r_S

independent) and $\beta^2 = 2k_F^2 + k_{FT}^2$.

We thus find from our study that a choice of a nonlocal type of interaction which provides a special type of screening in the potential and yields an exact expression for the dielectric function affords another approach to studying the properties of the degenerate electron gas at metallic densities.

APPENDIX

The values of real and imaginary parts of the integrals I_1, I_2 are given by

$$\mathrm{Re}\, I_1 = \frac{k_F^2}{4\pi^2 V_F} + \frac{k_F^3}{8\pi^2 k V_F}\left[1 - \left(\frac{\omega + k^2/2m}{kV_F}\right)^2\right]\ln\frac{\omega + k^2/2m + kV_F}{\omega + k^2/2m - kV_F}$$

$$- \frac{k_F^3}{8\pi^2 k V_F}\left[1 - \left(\frac{\omega - k^2/2m}{kV_F}\right)^2\right]\ln\frac{\omega - k^2/2m + kV_F}{\omega - k^2/2m - kV_F},$$

$$\mathrm{Im}\, I_1 = \begin{cases} \dfrac{1}{4\pi}\,\dfrac{\omega}{k}\,\dfrac{k_F^2}{V_F^2}, & \omega \leqslant kV_F - k^2/2m \\ \dfrac{1}{8\pi k}\,\dfrac{k_F^3}{V_F}\left[1 - \left(\dfrac{\omega - k^2/2m}{kV_F}\right)^2\right], & -k^2/2m \leqslant \omega - kV_F \leqslant k^2/2m \\ 0, \quad \omega \geqslant kV_F + k^2/2m \end{cases}$$

$$\mathrm{Re}\, I_2 = \frac{k_F}{4\pi^2 k V_F}\left[\frac{1}{2}\ln\frac{\omega + k^2/2m - kV_F}{\omega + k^2/2m + kV_F}\left(\ln\frac{\beta^2 - 2\omega k_F/V_F}{\beta^2 + k_F^2 - 2\omega k_F/V_F} + \frac{(\omega + k^2/2m)^2 k_F^2}{\beta^2 k^2 V_F^2 - 2\omega k_F V_F k^2}\right)\right.$$

$$- \frac{1}{2}\ln\frac{\omega - k^2/2m - kV_F}{\omega - k^2/2m + kV_F}\left(\ln\frac{\beta^2}{\beta^2 + k_F^2} + \frac{(\omega - k^2/2m)^2 k_F^2}{\beta^2 k^2 V_F^2}\right)$$

$$- \frac{(\omega - k^2/2m)k_F^2}{kV_F\beta^2} + \frac{(\omega + k^2/2m)k_F^2}{kV_F\beta^2 - 2\omega k k_F} - \frac{2kk_F}{\beta^2 - 2\omega k_F/V_F}$$

$$+ \frac{2\beta k}{\beta^2 - 2\omega k_F/V_F}\tan^{-1}\frac{2\beta k_F}{\beta^2 + k^2 - k_F^2}$$

$$\left. + \frac{1}{2}\ln\frac{\beta^2 + (k + k_F)^2}{\beta^2 + (k - k_F)^2}\left(\ln\frac{\beta^2 - 2\omega k_F/V_F}{\beta^2 + k_F^2 - 2\omega k_F/V_F} + \frac{k^2 - \beta^2}{\beta^2 - 2\omega k_F/V_F}\right)\right],$$

$$\mathrm{Im}\, I_2 = \begin{cases} \dfrac{k_F}{8\pi k V_F}\ln\dfrac{k_F^2 + \beta^2}{k_F^2 + \beta^2 - 2\omega k_F/V_F}, & \omega \leqslant kV_F - k^2/2m \\ \dfrac{k_F}{8\pi k V_F}\ln\dfrac{k_F^2 + \beta^2}{\left(\dfrac{\omega - k^2/2m}{kV_F/k_F}\right)^2 + \beta^2}, & -k^2/2m \leqslant \omega - kV_F \leqslant k^2/2m \\ 0, \quad \omega \geqslant kV_F + k^2/2m \end{cases}$$

where $V_F = k_F/m$. The values of I_3 can then be obtained following Eq. (16).

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