Raman instability in n-type piezoelectric semiconductors

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Raman instability of the Stokes component of the scattered electromagnetic wave has been investigated in an n-type piezoelectric semiconductor in the presence of a large transverse magnetostatic field. The general dispersion relation has been obtained following the coupled-mode theory and considering that the scattering results not only from the molecular vibrations produced owing to the pump wave at a frequency equal to that of the transverse-optical phonons but also from the electron plasma wave. The analysis has been applied to both cases of isotropic ($B_0=0$) and magnetoactive ($B_0\neq 0$) plasmas. The threshold value of the pump amplitude necessary for the onset of instability and the growth rate of the unstable mode well above the threshold have been obtained analytically for $B_0=0$ and $B_0\neq 0$. It is observed that a large transverse magnetostatic field can reduce the threshold value of the pump amplitude and increase the growth rate of the unstable Raman mode at an electric field amplitude greater than the threshold value. Numerical estimates have been made for n-InSb crystal at 77 K. The crystal has been irradiated with a pulsed 10.6-μm CO_2 laser to obtain the necessary electric field. The analytical as well as the numerical results have been compared with those of Sen while studying Brillouin instability in n-type magnetoactive piezoelectric semiconductors.

I. INTRODUCTION

If the intensity of the incident electromagnetic wave (pump) is very high, the initially scattered wave can enhance further scattering of the pump. Such enhanced scattering is called stimulated scattering and the phenomenon of stimulated Raman scattering (SRS), which is among the earliest-discovered nonlinear optical processes since the advent of laser, occurs when the pump decays into another electromagnetic wave and a Langmuir wave. Such a phenomenon was first observed by Woodburg and Ng[1] in a number of organic and inorganic liquids. Following Kroll's[2] approach for a stimualted Brillouin scattering process, significant studies were made for forward[3,4] as well as backward[5,6] SRS phenomena. Bloembergen[7] and Wang[8] have recently reviewed SRS. A spin-flip Raman laser has been developed by a number of workers[9,10] using pulsed 10.6 – μm CO_2 laser in n-InSb crystal.

Though SRS and the consequent Raman instability have been studied widely in gaseous plasmas[11–13] to obtain the threshold value of the pump amplitude as well as the growth rate of the unstable mode well above threshold for the homogeneous[11–13] and inhomogeneous plasmas,[14,15] little attention has been paid to the semiconductor plasmas. A theory has been developed by Foo and Tzoar[16] to study the first-order Raman effect in magnetoactive narrow-gap semiconductor based on the mechanism of phonon fluctuations. Yariv[17] has given a simplified treatment of SRS and Raman instability in solids. In light of the above review, the present authors have attempted to study the important phenomenon of Raman instability in a narrow-gap n-type piezoelectric semiconductor (viz., n-InSb), where the physical origin of the phenomenon is the nonvanishing nonlinear polarization due to the coupling of the transverse-optical-phonon vibrations at frequency ω_T with the pump at frequency ω_0, as well as the electron plasma wave at frequency ω_p, in the presence of a transverse magnetostatic field such that $\omega_T<\omega_p<\omega_0$ and the electron cyclotron frequency $\omega_c \gg (\omega_T^2+\omega_p^2)^{1/2}/2$. A simplified treatment of the instability has been made by the present authors following the coupled-mode theory developed by Nishikawa.[18] We have investigated analytically the threshold condition for the onset of instability as well as the growth rate of the unstable Raman mode well above the threshold value of the electric-field amplitude of the pump. The Raman instability is discussed using the hydrodynamic model of the one-component (electron) semiconductor plasma when $kl \ll 1$, where k and l are the wave number of the density fluctuation produced and the electron mean free path, respectively. As we are concerned with the instability of the Stokes component of the scattered electromagnetic wave, the selection rules are

$$\omega_0=\omega_1+\omega \quad \text{and} \quad \vec{k}_0=\vec{k}_1+\vec{k}\ ,$$

where $\omega_1(\vec{k}_1)$ and $\omega(\vec{k})$ are the frequencies (wave vectors) of the scattered electromagnetic wave and the density perturbation, respectively, and $\vec{k}_0$ represents the wave vector of the pump.

The analysis is applicable for semiconductors with moderate piezoelectric properties and consequently, the effects of nonlinear-material parameters have been neglected.[19,20] The nonlinear effects associated with the conduction-band

nonparabolicity[21] and the free-carrier absorption have been neglected in the present investigation following Sen.[19] The nonlinearities which have been taken into account are the nonlinear current and the polarization which is the cause of nonlinear coupling between the density fluctuations (electrostatic wave) at frequency ω and the scattered electromagnetic wave at frequency ω_1. The effect of oscillatory Hall drift normal to both the pump and the magnetostatic field has also been incorporated.

Numerical estimates for the threshold value of the pump amplitude necessary for the onset of Raman instability and the growth rate of the unstable Raman mode well above the threshold have been made for both isotropic and magnetoactive n-InSb crystal at 77 K irradiated by a pulsed 10.6-μm CO_2 laser. The present analysis is based on the earlier analysis of Brillouin instability in magnetoactive piezoelectric semiconductors by Sen.[19]

Section II deals with the theoretical formulation of the general dispersion relation in the presence of the transverse magnetostatic field. In Sec. III we have studied the possibility of Raman instability in the isotropic semiconductor plasma and obtained the expressions for the threshold condition as well as the growth rate of the unstable mode well above threshold. Section IV is devoted to the analysis of the case of a magnetoactive plasma when the magnetostatic field applied is so large that the electron cyclotron frequency $\omega_c \sim \omega_0$. In Sec. V the results are discussed and numerical estimations of the threshold electric field as well as the growth rate of the unstable mode in n-InSb crystal at 77 K is made.

II. THEORETICAL FORMULATIONS

A one-component (electron) homogeneous piezoelectric semiconductor plasma has been considered to be immersed in a transverse magnetostatic field $\vec{B}_0$ (along the z-axis) which is normal to the propagation vector $\vec{k}_1$ and $\vec{k}$ as well as the pump wave $\vec{E}_0 \exp[i(\omega_0 t - k_0 x)]$ (applied along the x-axis). In a Raman-active medium, the stimulated scattering of a large-amplitude electromagnetic-pump wave occurs owing to the excitation of a molecular vibrational mode. The model used in the present analysis is as follows: The Raman medium is taken as consisting of N harmonic oscillators per unit volume, each oscillator representing one molecule. The oscillators are independent of each other so that the ensemble of oscillators cannot support a wave motion with a nonvanishing group velocity. Each oscillator is characterized by its position x and normal vibrational coordinate $u(x,t)$. The equation of motion for a single oscillator is then

$$\frac{\partial^2 u(x,t)}{\partial t^2} + \gamma \frac{\partial u(x,t)}{\partial t} + \omega_T^2 u(x,t) = \frac{F(x,t)}{M} , \tag{2.1}$$

where γ is the damping constant equal to the phenomenological phonon-collision frequency ($\sim 10^{-2}\omega_T$) (Refs. 17 and 18), ω_T being the undamped molecular vibrational frequency and is taken to be equal to the transverse-optical-phonon frequency. M is the reduced mass of a single molecule and $F(x,t)$ is the driving force per unit volume which can be obtained by considering the electromagnetic energy in the presence of the molecules. $F(x,t)$ for polarizable material is given as

$$F(x,t) = \tfrac{1}{2}\epsilon_0 \epsilon_\infty \left(\frac{\partial \alpha}{\partial u}\right)_0 \langle E^2 \rangle (x,t) , \tag{2.2}$$

where ϵ_0 and ϵ_∞ are the absolute permittivity and the high-frequency permittivity, respectively; $(\partial\alpha/\partial u)_0$ is the differential polarizability and the angular brackets around E indicate the averaging over a few optical periods as the molecules cannot respond to optical frequencies. This shows that because of the nonvanishing differential polarizability $(\partial\alpha/\partial u)_0$, the molecular vibration can be driven by the electric field. The other basic equations are

$$\frac{\partial \vec{v}_0}{\partial t} + \nu \vec{v}_0 = -\frac{e}{m}(\vec{E}_0 + \vec{v}_0 \times \vec{B}_0), \tag{2.3}$$

$$\frac{\partial E}{\partial x} = -\frac{ne}{\epsilon} - \frac{\beta}{e}\frac{\partial^2 u}{\partial x^2} , \tag{2.4}$$

$$\frac{\partial n}{\partial t} + v_0 \frac{\partial n}{\partial x} + n_0 \frac{\partial v}{\partial x} = 0 , \tag{2.5}$$

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v}_0 \cdot \vec{\nabla})\vec{v} = -\frac{e}{m}(\vec{E} + \vec{v} \times \vec{B}_0) - \frac{k_B T}{m n_0}\vec{\nabla} n - \nu \vec{v} , \tag{2.6}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} , \tag{2.7}$$

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} , \tag{2.8}$$

$$\vec{D} = \epsilon \vec{E} + \vec{P} , \tag{2.9}$$

and

$$\vec{P} = \epsilon N \left(\frac{\partial \alpha}{\partial u}\right)_0 u \vec{E} . \tag{2.10}$$

These equations are for the electron plasma and the notations used have been explained earlier.[22-23]

The molecular vibration at a frequency ω causes a modulation of the dielectric constant ϵ. This leads to the phase modulation of the radiation field and creates sidebands with frequencies $(\omega_0 \pm p\omega)$ where $p = 1, 2, 3, \ldots$. Stated otherwise, a modulation of ϵ at ω can lead to energy exchange be-

tween the electromagnetic fields separated in frequency by multiples of ω.[17] The modes at frequencies $\omega_0+p\omega$ are known as anti-Stokes modes, while those at $\omega_0-p\omega$ are Stokes modes. Our primary interest is in the first Stokes mode at frequency $\omega_0-\omega$. In the present analysis we consider the exchange only between the laser field at frequency ω_0 (which is the pump) and the Stokes field at frequency $\omega_0-\omega(=\omega_1)$. Using Eqs. (2.1) and (2.2), the complex amplitude of the molecular vibration driven at a frequency $\omega(=\omega_0-\omega_1)$ is obtained as

$$u=\frac{\epsilon\left(\frac{\partial\alpha}{\partial u}\right)_0 E_0 E_1^*}{2M(\omega_T^2-\omega^2+i\omega\gamma)}\,, \tag{2.11}$$

where E_1^* represents the complex conjugate of the scattered electromagnetic wave. The use of Eqs. (2.4) and (2.11) yields perturbed electron density (n_T) caused by the molecular vibrations as

$$n_T=\frac{i\epsilon k}{e}\left(\frac{\omega_T^2-\omega^2+i\omega\gamma-\frac{ik\beta}{2M}\left(\frac{\partial\alpha}{\partial u}\right)_0 E_0}{\frac{\epsilon}{2M}\left(\frac{\partial\alpha}{\partial u}\right)_0 E_0}\right)u\,. \tag{2.12}$$

The density perturbation associated with the molecular vibrations at frequency ω beats with the pump at frequency ω_0 and produces fast components of the density perturbation at frequencies $(p\omega_0\pm\omega)$. For the present case of investigation of SRS of the Stokes mode, we consider only the fast component which is associated with a frequency $\omega_0-\omega$ viz., n_s). Using Eqs. (2.3) to (2.6) and following Guha and Sen,[23] one obtains n_s as

$$n_s=\frac{ik\langle E\rangle n_T}{\omega_1^2-\bar{\omega}_R^2-i\omega_1(\nu-ikv_{0x})}\,, \tag{2.13}$$

where suffixes T and s denote the components of the perturbed-carrier concentration associated with the molecular vibrations and the Stokes mode respectively. In Eq. (2.13),

$$\bar{\omega}_R^2=\omega_R^2\left(1+\frac{\omega_c^2}{[(\bar{\omega}-i\nu)^2-\omega_c^2]}\right),\quad \omega_R^2=\omega_p^2\omega_L^2/\omega_T^2+k^2v_\theta^2,$$

$$\omega_L^2/\omega_T^2=\epsilon_1/\epsilon_\infty,\quad \omega_p^2=n_0e^2/m\epsilon_0\epsilon_1,\quad v_\theta=(k_B T/m)^{1/2},$$

$$\bar{\omega}=\omega-kv_{0x},\quad \omega_c=|e|B_0/m,\text{ and }\langle E\rangle=(e/m)E_0-\omega_0v_{0y}.$$

ω_L is the longitudinal-optical-phonon frequency and is given by $\omega_L=k_B\Theta_D/\hbar$, where k_B and Θ_D are Boltzmann's constant and Debye temperature of the lattice, respectively. ϵ_1 is the static dielectric constant of the crystal. Using Eqs. (2.4) and (2.6) to (2.8), the components of $\vec{v}$ are obtained as

$$v_x=\frac{e}{m}\,\frac{(\bar{\omega}-i\nu)}{[(\bar{\omega}-i\nu)^2-\omega_c^2]}\left(ibE_x-\frac{\omega_c}{(\bar{\omega}-i\nu)}E_y\right) \tag{2.14}$$

and

$$v_y=\frac{e}{m}\,\frac{\omega_c}{[(\bar{\omega}-i\nu)^2-\omega_c^2]}\left(bE_x+i\,\frac{(\bar{\omega}-i\nu)}{\omega_c}E_y\right),$$

where

$$b=1+\frac{k^2v_\theta^2}{(\omega_p^2\omega_L^2/\omega_T^2)}\,.$$

Assuming that the pump varies as $\exp[i(\omega_0 t-k_0x)]$, the components of the oscillatory-electron fluid velocity $\vec{v}_0$ in the presence of the pump and the transverse magnetostatic field $\vec{B}_0$ are obtained from Eq. (2.3) as

$$v_{0x}=-\frac{(i\omega_0+\nu)}{[(i\omega_0+\nu)^2+\omega_c^2]}\,\frac{e}{m}E_0 \tag{2.15}$$

and

$$v_{0y}=-\frac{\omega_c}{(i\omega_0+\nu)}\,v_{0x}\,,$$

where we have neglected the spatial dependence of the pump as it does not play any significant role in nonlinear phenomenon.[24,25] Using Eqs. (2.7) to (2.10), we obtain the general wave equation

$$\vec{\nabla}\times(\vec{\nabla}\times\vec{E})=-\mu_0\frac{\partial\vec{J}}{\partial t}-\mu_0\epsilon\frac{\partial^2\vec{E}}{\partial t^2}$$

$$-\mu_0\epsilon N\left(\frac{\partial\alpha}{\partial u}\right)_0\frac{\partial^2}{\partial t^2}(u^*\vec{E})\,, \tag{2.16}$$

where $\epsilon=\epsilon_0\epsilon_\infty$ and the perturbed current density $\vec{J}$ is given by

$$\vec{J}=-e(n_0\vec{v}+n\vec{v}_0)\,, \tag{2.17}$$

the components of which are obtained by using Eqs. (2.12) to (2.15) in Eq. (2.17) as

$$J_x=\frac{-\epsilon\omega_p^2\omega_L^2(\bar{\omega}-i\nu)}{\omega_T^2[(\bar{\omega}-i\nu)^2-\omega_c^2](1-Q)}\left(ibE_x-\frac{\omega_c}{\bar{\omega}-i\nu}E_y\right)$$

$$-i\,\frac{\epsilon Q}{\omega_1(1-Q)}(k^2c_l^2-\omega_1^2)E_x$$

and

$$J_y=\frac{-\epsilon\omega_p^2\omega_L^2\omega_c}{\omega_T^2[(\bar{\omega}-i\nu)^2-\omega_c^2]}\left(bE_x+i\,\frac{(\bar{\omega}-i\nu)E_y}{\omega_c}\right) \tag{2.18}$$

with

$$Q=i\,\frac{\epsilon v_{0x}}{\beta\omega}\left(1+i\,\frac{k\bar{E}}{[\omega_1^2-\bar{\omega}_R^2-i\omega_1(\nu-ikv_{0x})]}\right)$$

$$\times\left(\frac{[\omega_T^2-\omega^2+i\omega\nu-ik\beta(\partial\alpha/\partial u)_0E_0/2M]}{\epsilon(\partial\alpha/\partial u)_0E_0/2M}\right).$$

In deriving Eq. (2.18) we have expressed u in terms of E_x by using Eq. (2.8) as

$$u=-\frac{J_x}{k\omega\beta}+i\,\frac{\epsilon(k^2c_l^2-\omega_1^2)E_x}{k\omega\beta\omega_1}\,. \tag{2.19}$$

The dispersion relation has been obtained by using Eqs. (2.11) and (2.17) to (2.19) in Eq. (2.16) for the scattered electromagnetic wave (ω_1, k_1) as

$$\begin{pmatrix} A_{xx} & A_{xy} \\ A_{yx} & A_{yy} \end{pmatrix} = 0 , \tag{2.20}$$

where

$$A_{xx} = \frac{\bar{\omega}_R^2(\bar{\omega} - i\nu)}{[(\bar{\omega} - i\nu)^2 - \omega_c^2](1-Q)} - \frac{Q}{\omega_1(1-Q)}(k^2 c_l^2 - \omega_1^2) - \omega_1\{1 + [N\epsilon(\partial\alpha/\partial u)_0^2 |E_0|^2]/[2M(\omega_T^2 - \omega^2 + i\omega\nu)]\} ,$$

$$A_{xy} = i\frac{\omega_p^2\omega_L^2\omega_c}{\omega_T^2[(\bar{\omega} - i\nu)^2 - \omega_c^2](1-Q)} ,$$

$$A_{yx} = -i\frac{\omega_p^2\omega_L^2\omega_c\omega_1}{\omega_T^2[(\bar{\omega} - i\nu)^2 - \omega_c^2]} ,$$

and

$$A_{yy} = k^2 c_l^2 - \omega_1^2 + \frac{\omega_p^2\omega_L^2\omega_1(\bar{\omega} - i\nu)}{\omega_T^2[(\bar{\omega} - i\nu)^2 - \omega_c^2]} .$$

In obtaining Eq. (2.20) we have made use of the assumption that $k^2 v_\theta^2 \ll \omega_p^2\omega_L^2/\omega_T^2$. The general dispersion relation given by Eq. (2.20) can be analyzed to investigate the possibility of Raman instability in isotropic (i.e., when $B_0 = 0$) as well as magnetoactive (when $B_0 \neq 0$) n-type piezoelectric semiconductors. Equation (2.20) shows that the two modes represented by $A_{xx} = 0$ and $A_{yy} = 0$ are coupled to each other via the finite transverse magnetostatic field B_0. In the absence of a magnetostatic field, we obtain the simple phenomenon of SRS under a one-dimensional configuration for an isotropic n-type piezoelectric semiconductor. We have studied the possibility of Raman instability for both cases (viz., isotropic and magnetoactive semiconductor plasmas).

III. ISOTROPIC SEMICONDUCTOR PLASMA

In the absence of the magnetostatic field, the plasma becomes isotropic and we get the dispersion relation under this condition from Eq. (2.20) as

$$A_{xx} = 0 ,$$

which is written as

$$(\omega_T^2 - \omega^2 + i\omega\gamma)\left(\frac{\omega_R^2}{\omega_1(\bar{\omega} - i\nu)} - \frac{Q}{\omega_1^2}(k^2 c_l^2 - \omega_1^2) - 1 + Q\right) = (1-Q)D , \tag{3.1}$$

where

$$D = N\epsilon\left(\frac{\partial\alpha}{\partial u}\right)_0^2 E_0^2/2M .$$

Remembering that the present investigation has been made for $\omega_1 \approx \omega_0 (> \omega_p) > \omega$ and $\omega_T > \nu$, one can obtain

$$(\omega_T^2 - \omega^2 + i\omega\gamma)\left(\frac{\omega_R^2}{\bar{\omega}\omega_0} - 1 + Q\right) = (1-Q)D . \tag{3.2}$$

In order to examine the possibility of Raman instability in isotropic n-type piezoelectric semiconducting crystal, Eq. (3.2) is solved for complex values of $\omega (= \omega_r + i\omega_i)$ with real positive k. To make the analysis simplified, we assume $\omega_T < \omega_r < \omega_p$ and $\omega_i \ll \omega_r$. Equating the imaginary parts of Eq. (3.2), one gets

$$\omega_i = -\frac{\gamma}{2}\left[\frac{\epsilon N e(\partial\alpha/\partial u)_0 E_0^2}{m\beta(\omega_R^2 - \omega_0\omega_r)} - 1\right] . \tag{3.3}$$

We are interested in studying Raman instability in n-type piezoelectric semiconductors (viz., n-InSb) irradiated with a pulsed 10.6-μm CO_2 laser. One can achieve the above condition as $\omega_r > kv_{0x}$, ν, and $\omega_1 \approx \omega_0$. The scattered mode becomes unstable only when $\omega_i < 0$; consequently, the condition for Raman instability is obtained as

$$\frac{e}{m\beta}\frac{\epsilon N(\partial\alpha/\partial u)_0 E_0^2}{(\omega_R^2 - \omega_0\omega_r)} > 1 . \tag{3.4}$$

From condition (3.4) one can also notice that $\omega_R^2 > \omega_0\omega_r$ is a precondition for the mode to be unstable. This can be achieved by adjusting the electron concentration in the crystal.

In the absence of E_0 the growth rate disappears and the wave starts attenuating with an attenuation factor equal to $\gamma/2$. Thus one notices from Eq. (3.3) that E_0 must be finite and should have a threshold value for the onset of the Raman instability. The threshold value is obtained by equating ω_i to zero in Eq. (3.3) which gives

$$(E_{0\text{th}})_{B_0=0} = \left(\frac{m\beta(\omega_R^2 - \omega_0\omega_r)}{\epsilon N e(\partial\alpha/\partial u)_0}\right)^{1/2} . \tag{3.5}$$

To obtain the growth rate $|\omega_i|$ of the unstable Raman mode well above the threshold (i.e., $E_0 \gg E_{0\text{th}}$), we take

$$\frac{\epsilon N e(\partial\alpha/\partial u)_0 E_0^2}{m\beta(\omega_R^2 - \omega_0\omega_r)} \gg 1 ;$$

consequently, one gets from Eq. (3.3)

$$|\omega_i|_{B_0=0} = \frac{\epsilon N e\gamma(\partial\alpha/\partial u)_0 E_0^2}{2m\beta(\omega_R^2 - \omega_0\omega_r)} . \tag{3.6}$$

From Eq. (3.6) it may be noted that the growth rate of the unstable mode well above threshold varies as the square of the applied pump amplitude. It can further be concluded from Eq. (3.5) that $(E_{0\text{th}})_{B_0=0}$ can be lowered by increasing the carrier concentration so that $\omega_R^2 \gg \omega_0\omega_r$. We conclude that the Raman instability is possible in the isotropic n-type piezoelectric semiconductors only when the frequency of the molecular vibra-

tions (ω_r) is greater than the phenomenological electron-collision frequency (ν).

IV. MAGNETOACTIVE SEMICONDUCTOR PLASMA

In this section we investigate the SRS process and analyze the possibility of Raman instability in a piezoelectric magnetoactive (i.e., $B_0 \neq 0$) n-type semiconductor plasma. For this purpose, we solve the dispersion relation (2.20) as

$$(\omega_T^2 - \omega^2 + i\omega\gamma)\left[\left(\frac{\omega_R^2(\bar{\omega} - i\nu)}{\omega_1[(\bar{\omega} - i\nu)^2 - \omega_c^2]} - 1\right)\left(k^2c_l^2 - \omega_1^2 + \frac{\omega_p^2\omega_L^2\omega_1(\bar{\omega} - i\nu)}{\omega_T^2[(\bar{\omega} - i\nu)^2 - \omega_c^2]}\right) - \frac{\omega_p^4\omega_L^4\omega_c^2}{\omega_T^4[(\bar{\omega} - i\nu)^2 - \omega_c^2]^2}\right]$$

$$= \frac{(1-Q)\epsilon N\left(\frac{\partial\alpha}{\partial u}\right)_0 |E_0|^2}{2M}\left(k^2c_l^2 - \omega_1^2 + \frac{\omega_p^2\omega_L^2\omega_1(\bar{\omega} - i\nu)}{\omega_T^2[(\bar{\omega} - i\nu)^2 - \omega_c^2]}\right). \quad (4.1)$$

Assuming $\bar{\omega} > \nu$, $\omega_c > \nu$, $\omega_1 > \omega_p$, and $k^2c_l^2 \approx \omega_1^2$, one obtains Eq. (4.1) in the simplified form as

$$(\omega_T^2 - \omega^2 + i\omega\gamma)\left[\left(\frac{\omega_R^2\bar{\omega}}{\omega_1(\bar{\omega}^2 - \omega_c^2)^{-1}}\right)\frac{\omega_p^2\omega_L^2\omega_1\bar{\omega}}{\omega_T^2(\bar{\omega}^2 - \omega_c^2)} - \frac{\omega_p^4\omega_L^4\omega_c^2}{\omega_T^4(\omega^2 - \omega_c^2)^2}\right] = \frac{(1-Q)D_1\omega_p^2\omega_L^2\omega_1\bar{\omega}}{\omega_T^2(\bar{\omega}^2 - \omega_c^2)}, \quad (4.2)$$

where $D_1 = N\epsilon(\partial\alpha/\partial u)_0 E_0^2/2M$. Using the method followed in Sec. III, the dispersion relation (4.2) is solved for complex $\omega(=\omega_r + i\omega_i)$, which gives

$$\omega_i = -\frac{D|Q|(\omega_r^2 - \omega_c^2)\omega_1}{2[\omega_r(\omega_R^2\omega_r - \omega_1\omega_r^2 + \omega_1\omega_c^2) - \omega_p^2\omega_c^2]} + \frac{\gamma}{2}, \quad (4.3)$$

where

$$D|Q| = \epsilon Ne\gamma\left(\frac{\partial\alpha}{\partial u}\right)_0 \omega_0 E_0^2 / m\beta(\omega_0^2 - \omega_c^2).$$

Using Eq. (4.3), the threshold value of the electric-field amplitude of the pump at $B_0 \neq 0$ is obtained by making $\omega_i = 0$ as

$$(E_{0\text{th}})_{B_0\neq 0} = \left[\frac{m\beta}{\epsilon Ne(\partial\alpha/\partial u)_0}\frac{(\omega_0^2 - \omega_c^2)}{\omega_0\omega_1}\frac{[\omega_r(\omega_R^2\omega_r - \omega_1\omega_r^2 + \omega_1\omega_c^2) - \omega_p^2\omega_c^2]}{(\omega_r^2 - \omega_c^2)}\right]^{1/2}. \quad (4.4)$$

The threshold value obtained in this case reduces to that for isotropic plasma if $\omega_c = 0$ and $\omega_1 = \omega_0$ in Eq. (4.4) whence it becomes identical with Eq. (3.5). From Eq. (4.4) the real value of $(E_{0\text{th}})_{B_0\neq 0}$ is obtained only when $\omega_0^2 < \omega_c^2$, $\omega_c^2 > \omega_r^2$, and $\omega_p^2\omega_c^2 > \omega_r(\omega_R^2\omega_r - \omega_1\omega_r^2 + \omega_1\omega_c^2)$. These inequalities can be satisfied in the moderately piezoelectric n-type semiconductors with low values of the electron effective mass.

The growth rate of the unstable Raman mode above the threshold in a magnetoactive n-type piezoelectric semiconductor plasma is found to be negative. From Eq. (4.3) at $\gamma \ll$ (first factor in the rhs), we obtain

$$|\omega_i|_{B_0\neq 0} = \frac{\gamma}{2}\left(\frac{\epsilon Ne(\partial\alpha/\partial u)_0\omega_0 E_0^2(\omega_r^2 - \omega_c^2)\omega_1}{\{M\beta(\omega_0^2 - \omega_c^2)[\omega_r(\omega_R^2\omega_r - \omega_1\omega_r^2) + \omega_1\omega_c^2) - \omega_p^2\omega_c^2]\}}\right). \quad (4.5)$$

The dependence of growth rate on the pump amplitude is found similar to that in the isotropic plasma as can be seen by comparing Eq. (4.5) for $\omega_c = 0$ with Eq. (3.6).

V. RESULTS AND DISCUSSIONS

The results obtained in Sec. III and Sec. IV can be used to make a comparative study of the Raman instability in isotropic and magnetoactive semiconductors. Comparing Eqs. (3.5) and (4.4), one obtains

$$\frac{(E_{0\text{th}})_{B_0\neq 0}}{(E_{0\text{th}})_{B_0=0}} = \left(\frac{(\omega_0^2 - \omega_c^2)\{[\omega_r(\omega_R^2\omega_r - \omega_1\omega_r^2 + \omega_1\omega_c^2) - \omega_p^2\omega_c^2]\}}{\omega_0\omega_1(\omega_r^2 - \omega_0^2)(\omega_R^2 - \omega_1\omega_r)}\right)^{1/2}. \quad (5.1)$$

Similarly, the comparison of Eqs. (3.6) and (4.5) yields

$$\frac{|\omega_i|_{B_0\neq 0}}{|\omega_i|_{B_0=0}} = \frac{\omega_0\omega_1(\omega_R^2 - \omega_1\omega_r)(\omega_r^2 - \omega_c^2)}{(\omega_0^2 - \omega_c^2)[\omega_r(\omega_R^2\omega_r - \omega_1\omega_r^2 + \omega_1\omega_c^2) - \omega_p^2\omega_c^2]}. \quad (5.2)$$

Equations (5.1) and (5.2) can be studied over a wide range of different system parameters like ω_c, ω_p, and ω_r to see the effect of the transverse magnetostatic field on the threshold pump amplitude and the growth rate of the unstable mode well above the threshold. It can further be seen that

at $B_0=0$, one can obtain

$$(E_{0th})_{B_0\neq0}/(E_{0th})_{B_0=0}=|\omega_i|_{B_0\neq0}/|\omega_i|_{B_0=0}=1\,.$$

The principal point of the discussion is to examine the advantage of the application of the large transverse magnetostatic field. We consider that B_0 is so large that $\omega_c^2\gg\omega_r^2$ where $\omega_r^2=(\omega_T^2+\omega_p^2)/2$, $\omega_c^2\approx\omega_0^2$, $\omega_1\approx\omega_0$, and the condition $\omega_p^2\omega_c^2>\omega_r(\omega_R^2\omega_r-\omega_1\omega_r^2+\omega_1\omega_c^2)$ is satisfied. Then from Eq. (4.4), the threshold value of the pump amplitude diminishes rapidly with the increase of the magnetostatic field, and at $\omega_c^2=\omega_0^2$, the threshold becomes quite small but not zero because in that case the electron collision frequency ν would have to be considered instead of neglected, as in the present analysis and retained as in an earlier paper (Ref. 19). At the same time, under the same physical condition, the growth rate of the unstable Raman mode increases with the increase in the value of ω_c [Eq. (4.5)]. These results are in qualitative agreement with the work of Willett and Maraghechi.[13] For large transverse magnetostatic field [such that $\omega_c(\approx\omega_0)>\omega_r$ and $\omega_c>\omega_R$], from Eq. (5.1), one obtains

$$\frac{(E_{0th})_{B_0\neq0}}{(E_{0th})_{B_0=0}}=\left(\frac{\omega_0^2-\omega_c^2}{\omega_0^2}\right)^{1/2}. \tag{5.3}$$

Similarly, Eq. (5.2) reduces to

$$\frac{|\omega_i|_{B_0\neq0}}{|\omega_i|_{B_0=0}}=\frac{\omega_0^2}{\omega_0^2-\omega_c^2}. \tag{5.4}$$

From Eq. (5.3) it can be observed that the threshold value of the electric-field amplitude of the pump necessary for the onset of Raman instability can significantly be diminished by applying a very large transverse magnetostatic field such that $\omega_c\sim\omega_0$. Equation (5.4) shows that the growth rate of the unstable Raman mode is considerably increased in that case. It should be noted here that the threshold does not reduce to zero at $\omega_c=\omega_0$, because in that case the term ν which has been neglected in the analysis in comparison with ω_c and ω_0, must be present there. The same term should also be retained in Eq. (5.4) if one wants to study the phenomenon at $\omega_c=\omega_0$. Similar results have been obtained earlier.[19,22,23] It is observed from Eqs. (5.3) and (5.4) that at $B_0=11.3$ T when $\omega_c=0.9\,\omega_0$, the ratios are about 0.436 and 5.3, respectively, whereas at $B_0=13.6$ T, $\omega_c=0.96\,\omega_0$; the ratios become 0.2 and 25, respectively. The analytical results are applied to a semiconductor like n-InSb crystal at 77 K. The physical constants are $m=0.014m_0$, $\beta=0.054$ C m^{-2}, $\rho=5.8\times10^3$ kg m^{-3}, $\epsilon_1=17.8$, $\epsilon_\infty=15.68$, $\nu=3.5\times10^{11}$ sec^{-1}, $c_s=4\times10^3$ msec^{-1} and the differential polarizability $(\partial\alpha/\partial u)_0=1.04\times10^{-16}$ (mks units). The later values have been calculated from Debye temperature of n-InSb crystal (278 K), molecular weight of InSb (236.47), and the fact that the crystal is irradiated with a pulsed 10.6-μm CO_2 laser.

Using the above parameters in Eq. (3.5) the threshold value of the electric-field amplitude of the pump required for the onset of instability in the isotropic n-type InSb crystal is obtained as 3.13×10^5 V m^{-1} at $k=10^7$ m^{-1}. In order to obtain the power density (also known as irradiance) I corresponding to the threshold value of the electric-field amplitude of the pump (E_0), we have employed the relation given by[26]

$$E_0=19.41\,I^{1/2}\text{ or }I=(E_0/19.41)^2,$$

where E_0 (rms value) is in V m^{-1} and I is in W m^{-2}. Calculations yield a value of the threshold power density $I\sim2.6\times10^8$ W m^{-2} at $E_0=3.13\times10^5$ V m^{-1}. Using Eq. (3.6), we obtain the growth rate $|\omega_i|_{B_0=0}$ of the unstable Raman mode at an electric-field amplitude, $E_0\sim10^6$ V m^{-1} (corresponding power density $I\sim2.65\times10^9$ W m^{-2}) as 4.8×10^{11} sec^{-1}. For semiconductors, parametric gains are limited by 2×10^{11} to 6×10^{11} W m^{-2} damage thresholds for long pulses.[27] We have assumed that the onset of Raman instability takes place well below the damage threshold. On the other hand, the damage threshold can be increased by reducing the pulse duration.[27] In the present simplified treatment, we have neglected the presence of other nonlinear processes in the crystal at power densities up to 2.65×10^9 W m^{-2}.

The value of the threshold electric-field amplitude $(E_{0th})_{B_0=0}$ in the present investigation is found to be of the same order as that necessary for the onset of Brillouin instability but is nearly twice that in the later case which was found to be 1.7×10^5 V m^{-1} (Sec. V of Ref. 19), the physical condition being the same in both cases of Raman instability and Brillouin instability. This result is in fair agreement with the result of Lashmore-Davies[28] who has shown that the Brillouin process needs a lower threshold than that needed for the Raman process, and at high densities, they are comparable to each other. On the other hand, the growth rate, which is found to be 1.8×10^{12} sec^{-1} in the case of Raman instability is very much larger than that in the case of Brillouin instability at the same value of $E_0=10^6$ V m^{-1} because in the later phenomenon, it has been found to be only 8.3×10^3 sec^{-1} (see Sec. V of Ref. 19).

One may conclude from the above discussion that the growth rate of the unstable Raman mode far exceeds that of the Brillouin mode at the same value of the electric-field amplitude of the pump well above threshold in the case of an isotropic plasma; whereas the comparative increase in the

growth rate of the Brillouin mode due to the application of the transverse magnetostatic field to the semiconductor is much larger than that for the Raman mode under the same condition which is evident when comparing the value of $|\omega_i|_{B_0\neq 0}/|\omega_i|_{B_0=0} = 1.85\times 10^5$ (Ref. 19) with Eq. (5.4) of the present investigation which yields a ratio equal merely to 5.3 at $\omega_c = 0.9\,\omega_0$ (at $B_0 = 11.3$ T).

The present analysis has a marked similarity with that of Brillouin instability[19] so far as the behavior with regard to the application of a large transverse magnetostatic field is concerned, and the instability disappears at a value of B_0 when ω_c becomes larger than ω_0.

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