

Experimental study of localization in thin wires

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The electrical properties of wires with cross-sectional areas in the range 1×10^{-11} to 3×10^{-10} cm² have been studied. At temperatures below about 10 K, the resistance of the wires increases as the temperature is lowered. The magnitude of the resistance rise increases both as the cross-sectional area of the wire is made smaller and as the amount of randomness is increased. This behavior is in qualitative agreement with the recent predictions by Thouless concerning localization in thin wires. However, the magnitude of the resistance rise is much smaller than originally predicted, and to obtain quantitative agreement with the experimental results it is necessary to assume that the rate at which electrons are inelastically scattered is proportional to the temperature and independent of the size of the wire. This is not consistent with the usual electron-phonon or electron-electron scattering process, and suggests that either the sample geometry affects these processes or that scattering from some other source, such as the tunneling levels believed to be present in all highly disordered systems, is the dominant inelastic scattering process. Experiments on thin films with resistances per square in the range 15 to 100 Ω are also reported. The behavior of the films is similar to that of the wires; they exhibit a resistance rise at low temperatures, and the magnitude of the rise becomes larger as the resistance per square is increased. These results are in good agreement with the predictions of Abrahams *et al.* for localization in two dimensions and correlate well with recent experimental results of Dolan and Osheroff. The effect of a magnetic field has also been investigated. It is found that a field of 75 kOe has no significant effect on the behavior of the wires. However, a field of this size appears to change the behavior of the films markedly. While our results are all consistent with theoretical predictions based on localization, they are also consistent with recent theories based on electron-electron interaction effects in disordered systems.

I. INTRODUCTION

In recent years there has been a great deal of interest in the properties of random systems. One of the very interesting developments in this field was the prediction by Anderson¹ that when the amount of randomness exceeds a critical amount, all of the electronic states will be spatially localized. This is in sharp contrast to the extended (i.e., nonlocalized) states which are characteristic of a perfect, periodic system. Since transport properties, such as the electrical conductivity, are strongly dependent on the nature of the electronic states, the concept of localization plays a key role in present theories of transport in random systems. In particular, in a system in which all of the states are localized, electrical conduction at low temperatures will proceed via hopping from one localized state to another.² Since this hopping will be a thermally activated process, the hopping probability and hence the electrical conductivity will decrease as the temperature is lowered, and both will go to zero as $T \rightarrow 0$. This is in contrast to the behavior of a system with extended states; in this case the conductivity at low temperatures is independent of temperature and has a nonzero value as $T \rightarrow 0$.

Localization has been studied both theoretically and experimentally in a number of two- and three-

dimensional systems.³ While the theory also predicts very interesting behavior in one dimension, this case did not appear to be experimentally accessible until very recently when Thouless⁴⁻⁶ argued that all systems, regardless of their cross-sectional area will behave one-dimensionally insofar as localization is concerned, provided only that they are sufficiently long. Specifically, Thouless predicted that localization will cause all one-dimensional systems (i.e., wires) with impurity resistances greater than about 8 k Ω to exhibit thermally activated conduction at low temperatures and to be insulators at absolute zero. The temperature at which this effect becomes observable, T_L , is predicted to increase both as the cross-sectional area, A , of the wire is decreased and as the amount of randomness (i.e., the impurity resistance) is increased. For macroscopic wires, T_L is predicted⁴ to be unattainably low, but for very random wires with $A = 2.5 \times 10^{-11}$ cm² (a radius of order 250 Å), T_L is predicted to be of order 1 K.

We have searched for the effect of localization in thin wires, and in this paper we report the initial results of this work. We find strong evidence that the predicted effects do occur, and the results are in good general agreement with the theory.

The outline of this paper is as follows. In Sec.

II we review the pertinent theoretical work and describe the predictions relevant to our experiments. The experimental techniques used to fabricate the samples studied in this work along with the measurement techniques are described in Sec. III. In Section IV we present our results and compare them with the theoretical predictions. Our results are compared with those of other workers in Sec. V, and Sec. VI contains our conclusions. We note that a preliminary account of some of the results described in this paper was given in Ref. 7.

II. THEORY

A. Localization in one dimension

In a three-dimensional system, localization of all of the electronic states does not occur until the randomness exceeds a certain nonzero amount.¹⁻³ That is, in the presence of weak disorder some of the states in a three-dimensional system will remain extended. However, the behavior in one dimension is believed to be quite different. It was first shown by Mott and Twose⁸ and by Landauer⁹ that any amount of disorder, no matter how small, would cause all of the states in a one-dimensional system to be localized, and that this would lead to a vanishing of the conductivity at absolute zero. Since this work, a number of other authors¹⁰ have proven this result with greater rigor, and it seems fairly well established theoretically that the electrical conductivity of a random one-dimensional system will be zero at absolute zero. At finite temperatures the electrons will be able to hop from one localized state to another, and thus one would expect the electrical conductivity to be thermally activated at low temperatures.

Although the results described above were all obtained from models in which electrons move in a strictly one-dimensional potential, it has been suggested¹¹ that they should apply to all systems, provided that the ratio of the length to the cross-sectional area is sufficiently large. This problem has been considered in detail by Thouless,^{4,5} who has discussed the conditions under which this effect should be observable. Using scaling arguments similar to those developed previously,^{3,12} Thouless showed that for a one-dimensional system, i.e., a wire, the electronic states will be localized with a localization length equal to the length of wire corresponding to an impurity resistance of order $2\hbar/e^2 \approx 8 \text{ k}\Omega$. Thus any wire with a resistance greater than this amount (that is, longer than the localization length) will have a thermally activated conductance at low temperatures and be an insulator at absolute zero. Thou-

less argued further that the effect—the thermally activated conductance—will only be observed at very low temperatures and with very thin wires for the following reason. At “high” temperatures an electron will experience frequent inelastic collisions with phonons, electrons, and any other excitations which are thermally excited. Each of these collisions will cause the electron to jump from one localized state to another.¹³ If the inelastic scattering rate is very high, then the electron will change states many times before it can diffuse a distance of the order of the localization length. As a result, the electron will never “know” that it was in a localized state, and the localization will have no effect on the conductance. The effect of localization will only be felt when the mean free path for inelastic scattering (that is, the distance the electron travels between inelastic scattering events), l_i , is comparable to the localization length L . This can be accomplished either by reducing the cross-sectional area of the wire and/or increasing the amount of randomness which will decrease L , or by reducing the temperature and hence increasing l_i . No matter how large the cross section is, there will always be a temperature T_L at which l_i is equal to L , and this will be approximately the temperature at which localization becomes observable. For a very random wire with a diameter of 500 \AA , Thouless⁴ has shown that $T_L \approx 1 \text{ K}$, where he considered inelastic scattering due to both phonons and electrons. In addition, the above argument implies that T_L will scale inversely with the diameter of the wire, and this explains why the effect is not ordinarily observed.

The quantitative behavior of the resistance has been considered by several authors. Abrahams *et al.*¹⁴ have utilized scaling arguments similar to those developed previously, Thouless¹⁵ has considered the diffusive motion of the electrons directly, and Anderson *et al.*¹⁶ have also employed the scaling equations. The results of all three theories are essentially the same and can be described as follows. The resistance of a wire at zero temperature can be written as

$$R = R_0 \left(1 + \frac{l}{L} + \dots \right), \quad (1)$$

where R_0 is the resistance due to impurity (elastic) scattering, l is the length of the wire, and L is again the localization length. Here we give only the leading terms in the expansion of R . This will be a good approximation when the effect of localization is small, as will be seen to be the case in the present experiments. At finite temperatures l must be replaced by the distance the electron diffuses between inelastic col-

lisions,^{15,17,18} which is just the inelastic mean free path, l_i . Since the electronic motion will be diffusive, l_i will be given by

$$l_i = \sqrt{D\tau_i}, \quad (2)$$

where τ_i is the inelastic scattering time and D is the appropriate diffusion constant. Using the Einstein relation together with the usual relations for an electron gas, we have^{19,20}

$$D = \frac{v_F^2 \tau_e}{3}, \quad (3)$$

where v_F is the Fermi velocity and τ_e is the elastic scattering time. The localization length is given through the relation

$$R_L = \frac{\rho_e L}{A}. \quad (4)$$

Here ρ_e is the resistivity due to impurity scattering, A is the cross-sectional area, and R_L is the maximum metallic resistance which Anderson *et al.*^{15,16} have predicted to be 36.5 k Ω . Combining all of the above results we find²¹

$$\begin{aligned} R &= R_0 \left(1 + \frac{\rho_e \sqrt{2D\tau_i}}{R_T A} \right) \\ &= R_0 \left(1 + \frac{\rho_e \sqrt{2v_F^2 \tau_e \tau_i}}{\sqrt{3} R_T A} \right). \end{aligned} \quad (5)$$

From (5) we see that the fractional resistance rise (which is just the second term in the parentheses) should exhibit the following behavior: (1) it should scale as A^{-1} , (2) it should scale with the impurity content, e.g., the amount of randomness, as $\sqrt{\rho_e}$ [this follows from Eq. (5) since ρ_e should be proportional to τ_e^{-1}], and (3) all of the temperature dependence of the resistance arises from the temperature dependence of τ_i .

B. Inelastic scattering time

From the above discussion, it is clear that the inelastic scattering time, τ_i , plays a key role in the prediction for R . Thouless⁴ originally considered inelastic scattering by phonons and by electrons. For the latter, the scattering time goes as²⁰

$$\tau_i \sim T^{-\alpha}, \quad (6)$$

with $\alpha = 2$. For phonons the scattering time is still given by (6) but the value of α depends on whether the phonons are three dimensional or one dimensional, and on whether the material is clean or dirty.⁴ Determining the effective dimensionality of the phonons is not a simple problem since in the present experiments the wires are attached to a substrate. For three-dimensional phonons in a clean material (by "clean" we

mean that the wavelength of a thermal phonon is much less than the electronic mean free path for elastic scattering), one expects⁴

$$\tau_i \sim T^{-3}, \quad (7)$$

where the coefficient of proportionality depends on material parameters but not on the size of the wire. For one-dimensional phonons in a clean material one expects

$$\tau_i \sim AT^{-1}. \quad (8)$$

The factor of A from the coefficient of proportionality is displayed explicitly here to emphasize that in this case τ_i also depends upon the area of the wire. In a dirty material (i.e., one in which the wavelength of a thermal phonon is much greater than the electronic mean free path) the results for τ_i are somewhat different. In this case Thouless found⁴

$$\tau_i \sim T^{-4} \quad (9)$$

for three-dimensional phonons and

$$\tau_i \sim AT^{-2} \quad (10)$$

for one-dimensional phonons.

Since the original work of Thouless,⁴ it has been suggested²² that scattering from the two level-tunneling modes believed to be present in all amorphous materials²³ may be important in the present experiments. Recently Black *et al.*²⁴ have considered inelastic scattering from tunneling modes in detail and have used experimental data to estimate the value

$$\tau_i = (2.3 \times 10^{-9}) T^{-1} \text{ sec-K} \quad (11)$$

for the metallic glass PdSiCu. The temperature dependence of τ_i for electron-tunneling-mode scattering should be the same for all materials.²⁴ While the numerical factor in (11) will in general be different for different materials, it does indicate the order of magnitude which might be expected.

We should also note that it has been shown very recently that the presence of impurities will modify the electron-electron scattering rate discussed above. For a dirty one-dimensional system it has been predicted that^{22,25}

$$\tau_i \sim AT^{-1/2}. \quad (12)$$

The inelastic mean free path will be determined primarily by the scattering mechanism which yields the smallest value for τ_i . Since it is very difficult to make quantitative estimates of the magnitudes of the inelastic scattering times discussed above [i.e., the coefficients of proportionality in (6)–(12)], it is not possible to determine ahead of time which mechanism will be

dominant in a given experiment. We will, therefore, use the experimental results to infer which mechanism(s) are dominant. This is possible since the different mechanisms yield different dependences on temperature and cross-sectional area.

C. Many-body effects

Altshuler *et al.*²⁵ have predicted that in a disordered one-dimensional system electron-electron interactions will lead to a divergence in the resistivity which is given by

$$R = R_0 \left[1 + \frac{e^2 \rho e (2 - F)}{\pi \hbar A} \left(\frac{D \hbar}{2 k_B T} \right)^{+1/2} \right], \quad (13)$$

where k_B is Boltzmann's constant and F is a screening parameter whose value lies between 0 and 1. Since \hbar/e^2 is approximately equal to R_L , this prediction is identical in form to (5) provided that the dominant inelastic scattering mechanism is electron-tunneling-level scattering. We will see in Sec. IV that our experimental results are consistent with (13). Since, as discussed above, the absolute magnitude of the electron-tunneling-level scattering time in our samples cannot be determined independently at present, our experimental results are not able to distinguish between the two theories. We will return to this problem in Sec. V. In discussing the experimental results for our thin wires we will for convenience often refer to "the theory" or "localization effects." It is to be understood that these statements refer to the predictions of *both* localization theory and the many-body theory.

D. Localization in two dimensions

While the qualitative behavior which is expected to arise from localization in one dimension is fairly well established, this is not the case in two dimensions. A considerable amount of theoretical and experimental work²⁶ seems to provide strong support for the existence of extended states when the disorder is weak and a mobility edge together with a minimum metallic conductivity in the neighborhood of $(30 \text{ k}\Omega/\square)^{-1}$. However, some recent work¹⁴ has argued against this point of view, and it has been suggested that even when the disorder is weak, all of the states will be localized, just as is the case in one dimension. These predictions are of interest for the present work since, as we will see, it is very important to compare the behavior of thin wires with that of thin films of the same material.

If there are extended states when the resistance is below $30 \text{ k}\Omega/\square$, then films with resistances below this value should exhibit ordinary metallic behavior at low temperatures. That is, the re-

sistance should be independent of temperature²⁷ and approach a finite nonzero value as $T \rightarrow 0$. If, on the other hand, there are no extended states even with weak disorder, then all metal films, regardless of their resistance, should exhibit a resistance which rises as the temperature is lowered, and becomes infinite as T approaches zero. In this case the resistance at high temperatures is predicted to be given by¹⁴

$$R = R[1 - B \ln(l)], \quad (14)$$

where $B = 2R_\square e^2 b / \pi^2 \hbar$. Here b is a constant of order unity and R_\square is the resistance per square. If at finite temperatures l is equal to the inelastic mean free path, then R will rise logarithmically at low temperatures. As will be discussed below, our experiments as well as those of previous workers are consistent with (14) and not with theories which predict ordinary metallic behavior for films with resistances less than $30 \text{ k}\Omega/\square$. However, it has recently been proposed²⁵ that in a disordered two-dimensional system the electron-electron interactions will lead to a singularity in the conductivity which, to within a constant of order unity, is identical to that given in (14). Since the two theories, one based on localization and the other based on many-body effects, make virtually identical predictions, it does not appear that resistance measurements alone can distinguish between the two theories. This problem will be discussed further in Sec. V.

E. Conditions for one- and two-dimensional behavior

It is important to consider the length scales which determine the effective dimensionality of a particular sample. For a thin film this amounts to asking how small the thickness of the film must be in order for it to behave two-dimensionally. For a thin wire, the dimension in question is the diameter of the wire. Thouless¹⁵ has argued that insofar as localization is concerned, the inelastic mean free path l_i is the relevant length (we will see that our experiments are consistent with this prediction). Thus to obtain two-dimensional behavior, the film thickness must be less than l_i , and to obtain one-dimensional behavior the lateral dimensions of the wire must both be less than l_i . Since the temperature dependence of the resistance can be used in conjunction with (5) to estimate l_i , these conditions can be used as a consistency check on the overall interpretation. We should also note that the many-body theory²⁵ predicts that the relevant length scale will be $\sqrt{D \hbar / k_B T}$. Since this length can be estimated from the measured resistance, it is again possible to make a consistency check as to the effective dimensionality of a sample.

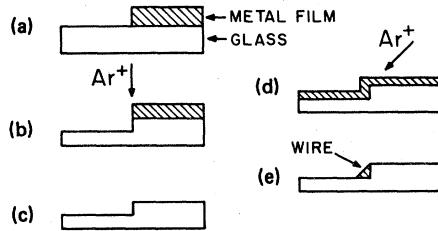


FIG. 1. Description of the lithographic process used to make the wires studied in this work.

III. EXPERIMENTAL TECHNIQUE

A. Fabrication

The wires studied in the present work were fabricated using a novel lithographic technique which we shall refer to as "step lithography." It is based on work by Feuer and Prober²⁸ who have shown how to make well defined yet extremely shallow steps in materials such as glass. The fabrication process is illustrated schematically in Fig. 1. Conventional conformal lithographic techniques are used to coat half of a glass substrate with a metal film. This structure [Fig. 1(a)] is then bombarded at normal incidence with Ar^+ ions which "mill" into the surface [Fig. 1(b)]. The metal film is then removed chemically leaving a step in the substrate [Fig. 1(c)]. A second metal film is next deposited in such a way that the step is coated. The surface is then milled again with Ar^+ ions, but this time at an angle such that the metal on the side of the step is in the shadow of the step [Fig. 1(d)]. The result is a wire which runs along the step [Fig. 1(e)].

We have used Cr for the first metal film [Fig. 1(a)] although any metal should be suitable. However, metals such as Cr which have lower milling rates than glass²⁹ are desirable since this will minimize rounding at the bottom of the step due to nonidealities in the edge profile of the metal film [Fig. 1(a)].

In our experiments the Ar^+ beam was produced by a Commonwealth Scientific model 2-30 ion gun. The initial step was milled at high energies, typically 5 keV, while a somewhat lower energy, typically 500 eV, was found to be best for the final milling [Fig. 1(d)]. Our attempts to make wires using high energies for the final milling were unsuccessful, presumably because of multiple scattering and ion penetration effects.

Figure 2(a) shows a picture of a rather large wire whose diameter (which we take here to be the distance across the wire when viewed at normal incidence) is approximately 1100 Å. The wire is seen to be continuous and to have a very uniform cross section. The effects of finite grain size in

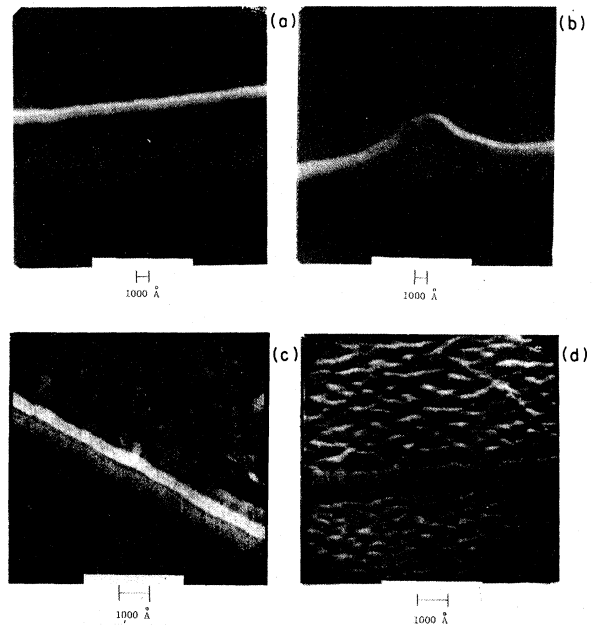


FIG. 2. Scanning electron micrographs of several wires. (a) A relatively large "dirty" (see Sec. III. C for an explanation of this term) wire viewed at normal incidence. The high side of the step is at the top of the picture. (b) Picture of the same wire as in (a) but taken at a different location along the wire. The high side of the step is again at the top of the picture. (c) A small "clean" (see Sec. III. C) wire viewed at normal incidence. The high side of the step is at the upper right. (d) Picture of the same wire as in (c), but viewed at 45° from the normal, i.e., looking into the step. The high side of the step is at the top of the picture.

the original metal film are barely visible at this scale and will be discussed further below. We also note that a small amount of "lumpiness" is visible in the glass both above and below the step. This is not inherent to the glass substrates which were used,³⁰ but is due to the ion milling. These regions were covered with a metal film which was then milled away. Because of grain size effects, the film milled nonuniformly so that some regions were milled away faster than others.³¹ This resulted in the lumpiness in the glass after all of the metal was removed.

Figure 2(b) shows a picture of the same wire in a region in which the step was not perfectly straight. We were originally concerned that defects such as these would cause discontinuities or other problems in the wires. However, as can be seen from Fig. 2(b), the wire conforms to the step very well and the cross-sectional area is not altered significantly in this region. We should also note that this was by far the worst defect in this particular wire. Our steps were thus of very high "quality" even over distances of

order 200 μm , which was the length of a typical wire.

Figure 2(c) shows a picture of one of our smallest wires. Here the diameter is approximately 350 \AA . The wire is seen to have a very uniform cross section, and it thus appears that it should be possible to make even smaller wires with the step lithography technique.

In Fig. 2(c) the lumpiness in the glass is clearly evident, as is a "hazy" region on the low side of the wire. This region arises in the following way. The final cross section of the wire shown in Fig. 1(e) will be obtained only if the ratio of the milling rate of the glass to that of the metal film is infinite. A finite ratio will lead to a somewhat different cross section. In general, the cross section will depend in a complicated way on the ratio of the milling rates, the milling angle, the milling time, the thickness of the metal film relative to the height of the step, and the profile of the film in the vicinity of the step. If we assume simple line-of-sight milling, ignore redeposition effects,³² and assume that the profiles of the metal film and the underlying step are both perfectly "sharp" [as shown in Fig. 1(d)], then cross sections like those shown in Fig. 3 will be obtained. If the thickness of the initial film is comparable to the height of the step, then the cross section just after the upper and lower half planes have been milled away will appear as shown in Fig. 3(a). If the milling is continued, the cross section will change as shown in Fig. 3(b) and 3(c). If the thickness of the initial film is much less than the height of the step, then a cross section like that shown in Fig. 3(d) will result.

As discussed above, a number of simplifying assumptions were made in deriving the results in Fig. 3, and it seems likely that the actual cross sections will be more complicated. Nevertheless, cross sections of the four general types shown in Fig. 3 have all been observed. The wire shown in Fig. 2(a) corresponds to the cross section in Fig. 3(a), while the wire in Fig. 2(c) corresponds to Fig. 3(b). The hazy region seen on the low side

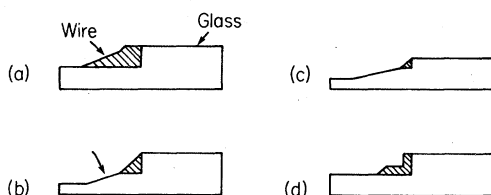


FIG. 3. Realistic wire cross sections derived using the assumptions and conditions discussed in the text. The arrow in (b) indicates the "hazy" region noted in connection with Fig. 2(c) (see text for discussion).

of the wire in Fig. 2(c) is the region in which the substrate makes a finite angle with the original glass surface. This can be seen more clearly from Fig. 2(d) which shows the same wire viewed at an angle of 45° from the normal to better expose the wire. The contrast between the wire and the glass has now decreased, but a dark strip is clearly visible below the wire. This strip is the region of the substrate indicated by the arrow in Fig. 3(c). We note that the types of cross sections obtained in the wires shown in Fig. 2 and in all of our other wires were always consistent with the measured step heights, film thickness, and milling times of each sample.

Leads were attached to the wires in the following way. Each slide contained one or more long (several mm) steps. After coating the steps with the metal film [Fig. 1(d)], strips of silver paint were laid across the steps. The strips were separated by typically 100 to 400 μm and were positioned so that only the straightest and most defect-free regions of the step (as viewed with an optical microscope) were exposed. When the final milling was performed, the strips of silver paint (which were much thicker than the underlying film) protected the film, and thus wires were formed only in the regions between the strips. The strips then served as contact pads since the silver paint was in good electrical contact with the underlying film which was itself continuous with the neighboring wires. Contacts to the outside world were then made via the silver paint pads. This method made it possible to avoid imperfect regions of the step and to put several potential wires on a single slide. The yield of good (i.e., electrically continuous) wires varied from slide to slide, and was typically 30–70%.

The cross-sectional area was determined using the relation

$$A = \frac{\rho l}{R}, \quad (15)$$

where ρ is the resistivity, and l and R are the measured length and resistance of a wire. The values of A determined in this way were in good agreement with, but more accurate than values obtained from examination with a scanning electron microscope, and from the known step heights, milling times, etc. For example, the wire in Fig. 2(a) had an area of $A = 1.19 \times 10^{-10} \text{ cm}^2$ ($\sqrt{A} = 1090 \text{ \AA}$) as determined from (15), in good agreement with the value of the diameter noted above.

B. Film preparation

All of the wires studied in the present work were composed of $\text{Au}_{80}\text{Pd}_{20}$. This material was

chosen for several reasons. (1) Neither Au nor Pd has a local magnetic moment, thus avoiding the Kondo effect. (2) Thin films can be readily produced either by sputtering or thermal evaporation. (3) AuPd films are known to have a very small grain size,³³ thus reducing problems with granularity of the wires. (4) AuPd does not readily oxidize, so that films and wires can be stored without any special precautions.

The AuPd films used in this work were deposited in two ways: dc sputtering and thermal evaporation. The sputtered films had a very high resistivity, presumably due to trapped Ar and O₂ and other defects. The resistivity was $370 \pm 90 \mu\Omega \text{ cm}$ where the uncertainty represents sample-to-sample variations as well as measurement uncertainties (due largely to the uncertainties in the film thickness). The resistance ratios of the films were in the range 1.028–1.042, where this represents (the maximum) sample-to-sample variations. Here we “define” the resistance ratio to be the ratio of the resistance at room temperature to that at 12 K. (This definition was chosen to avoid complications from the resistance rise due to localization which in our samples is small above 12 K.) The evaporated films were prepared by thermal evaporation of measured amounts of Au₆₀Pd₄₀ wire from Al₂O₃-coated boats. The pressure in the belljar during the evaporation was kept fairly high, typically 3×10^{-5} torr. The evaporated films had resistivities of $100 \pm 25 \mu\Omega \text{ cm}$ and resistance ratios in the range 1.060–1.080. In the following we will refer to the sputtered films and wires made from them as “dirty,” and the evaporated films and wires made from them as “clean.”

If we use the usual free-electron model for the resistivity, we expect

$$\rho = \frac{m}{ne^2} \left(\frac{1}{\tau_i} + \frac{1}{\tau_e} \right). \quad (16)$$

The resistance ratio Γ is then given by

$$\Gamma = \frac{\rho(300 \text{ K})}{\rho(0)} = 1 + \frac{\tau_e}{\tau_i}. \quad (17)$$

If we assume τ_i at room temperature to have the same value²⁰ as in Au, 3×10^{-14} sec, then

$$\tau_e = (\Gamma - 1)\tau_i \sim 1.1 \pm 0.3 \times 10^{-15} \quad (18)$$

expressed in sec, using the measured value of Γ for our dirty films [note that the difference between $\rho(0)$ and $\rho(12 \text{ K})$ in (16) is negligible for our purposes]. Inserting this into (16) gives, in $\mu\Omega \text{ cm}$,

$$\rho(300 \text{ K}) = 58 \pm 15, \quad (19)$$

which is far below the measured value. While

there is no reason to expect the free-electron model to work terribly well, we find the magnitude of the discrepancy to be surprising and may indicate that the inelastic scattering processes in our dirty films are different from those found in a clean system. A similar discrepancy is found for the clean films.³⁴

Examination with a scanning electron microscope (SEM) showed that both the dirty and clean films were smooth to within the resolution of the microscope, which was about 100 Å. Transmission electron microscope studies of films lifted off from their substrates showed that both types of films were polycrystalline with a grain size of approximately 50 Å. After a “light” ion milling of the films, lumpiness could be readily observed with the SEM, as discussed above in connection with Fig. 2.

Since the final fabrication step involved ion milling the AuPd films, it is possible that the wires could have been “damaged” by this process and that they could contain implanted Ar.³⁵ Fortunately such effects should not affect the properties of interest in the present work. We note, however, that the wires always exhibited the same resistance ratio as films deposited at the same time.³⁶ This implies that any damage, etc., due to the milling is not large. Moreover, it also means that the resistivity of the wires is the same as that of the films and thus justifies the use of (15) to determine the cross-sectional area.

C. Measurement techniques and apparatus

The measurements presented in this paper were begun at Yale and completed at Purdue. This led to a time “gap” in this work which, as will be seen below, led to some interesting results.

Two different cryostats were used in this work. A ⁴He cryostat was used for measurements in the range 1.3 to 30 K. The sample was mounted on a copper block which was enclosed in a vacuum can, which in turn was immersed in liquid ⁴He. Temperatures above 4.2 K were obtained by evacuating the can and employing a heater, while below 4.2 K a small amount of exchange gas was admitted into the can. The temperature was measured with a calibrated germanium thermometer.³⁷ The outside of the vacuum can was lined with Pb and μ metal and the magnetic field at the sample was less than 0.05 Oe.

The same cryostat was used for measurements in large magnetic fields. For these measurements, the vacuum can (which in this case was unlined) was inserted into the bore of a superconducting solenoid. The thermometer was positioned well outside the high-field region, so

that the magnetoresistance was negligible.

In later measurements a ^3He cryostat was employed. This cryostat has been described elsewhere.³⁸ The samples were located in the inner vacuum can along with two calibrated germanium thermometers.³⁹ A heater, which was approximately 1 m of 75- μm diameter evanohm wire, was wound around the outside of the inner can. A small amount of ^3He gas was admitted into the inner vacuum can to ensure good thermal contact between the sample, the thermometers, the heater, and the ^3He pot.

In our initial experiments with the wires the resistance was measured using a quasifour-terminal dc method.⁴⁰ In this setup two separate leads were attached to each of the silver-paint contact pads of the sample. In order to check for possible spurious (i.e., temperature-dependent) contact effects, a few measurements were made using a true four-terminal setup in which adjacent wires on the same substrate were used as current leads. These measurements indicated that contact effects were negligible. In later experiments, a two-terminal ac Wheatstone-bridge arrangement⁴¹ was used. ac and dc measurements on the sample were always in good agreement. The ac setup was preferable, however, as it had a higher voltage sensitivity than could be obtained with the dc method. This allowed the use of smaller sample currents, which was particularly important with the thinnest wires at the lowest temperatures. The ac bridge was operated at frequencies in the range 30–100 Hz, and it was possible to resolve voltage changes of approximately 1×10^{-8} V with a one-second time constant.

Measurements on the films were all made with a four-terminal dc arrangement. The films were sliced into strips typically 0.5 mm wide and 1 cm long, and four separate leads were attached, either with silver paint or by slicing the original film in the appropriate way. We note, however, that even though four separate leads were attached, contact effects are still possible since the resistance of the contact regions was a significant fraction of the total resistance of the sample (typically 10%). (Note that this was not the case for the wires since they had much larger resistances.) Fortunately, by comparing measurements made on both the clean and dirty samples and on samples of various lengths, it was possible to show that contact effects were in fact negligible for measurements made in zero magnetic field. Insufficient data are available for the films to establish this fact in finite magnetic fields, although we believe that contact effects were also negligible in this case.

IV. EXPERIMENTAL RESULTS

A. Thin films

During the course of our studies of thin wires, it became clear that a careful study of the AuPd films was needed to enable us to distinguish which effects were due to the "one-dimensionality" of the wires and which were intrinsic to the AuPd films. The results of this study are described in this section.

Figure 4(a) shows the resistance of a typical dirty AuPd film as a function of temperature. As discussed in Sec. III, the resistance rise observed at low temperatures is not a contact effect, but appears to be an intrinsic property of the AuPd film. Most of our measurements were made on films approximately 400 Å thick. For these films, the resistance rise at 1.5 K was $(10 \times 2) \times 10^{-4}$ of the total resistance,⁴² where the uncertainty is the sample-to-sample variation observed in the eight samples which were studied. We note that this variation is of the same magnitude as variations in the resistances per square of the samples. Thicker films exhibited somewhat smaller resistance rises, approximately 7×10^{-4}

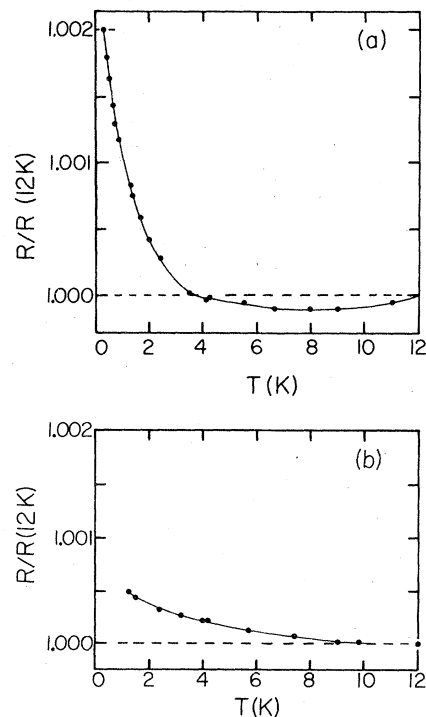


FIG. 4. Resistance as a function of temperature for (a) a dirty film with $R_0 = 100 \Omega$, and (b) a clean film with $R_0 = 13 \Omega$. For purposes of comparison, the values are normalized by the value of the resistance at 12 K; this was 1240 Ω for the dirty sample and 1140 Ω for the clean sample.

for 800-Å films, and 5×10^{-4} for 1600-Å films. Films which were "lightly" ion milled had a resistance rise which was equal to or slightly (~20%) greater than that found prior to ion milling. Similar behavior was observed in the clean AuPd films as shown in Fig. 4(b). The resistance rise was smaller in the clean samples by approximately a factor of two at 1.5 K for samples of the same thickness. The resistance rise in the clean films was also observed to increase as the film thickness was made smaller.

Edwards *et al.*⁴³ have reported that bulk AuPd alloys exhibit a resistivity rise at low temperatures. For $\text{Au}_{60}\text{Pd}_{40}$ they find a rise in the resistivity at 1.5 K of approximately $3 \times 10^{-3} - \mu\Omega$ cm, which is much smaller than the $0.4 - \mu\Omega$ cm rise which we observe in our 400-Å thick dirty films. This difference suggests that the resistance rise observed in our thin films is not intrinsic to bulk AuPd, but is due either to the geometry of the films or possibly the very high concentration of impurities present in our films. It is interesting to note that while several different mechanisms have been proposed to explain the behavior observed by Edwards *et al.*,⁴³ they all predict a variation of the form

$$\rho = \rho_0(1 - aT^2), \quad (20)$$

at low temperatures. This functional form is in good agreement with the results for AuPd and for a number of other Pd alloys.^{43,44} However, the resistance rise of our thin films is not consistent with the functional form (20) but, as shown in Fig. 5, our results are quite consistent with a logarithmic temperature dependence. Since the latter is the form predicted for localization in two dimensions,¹⁴ we tentatively conclude that the effect seen in our film is due to localization. How-

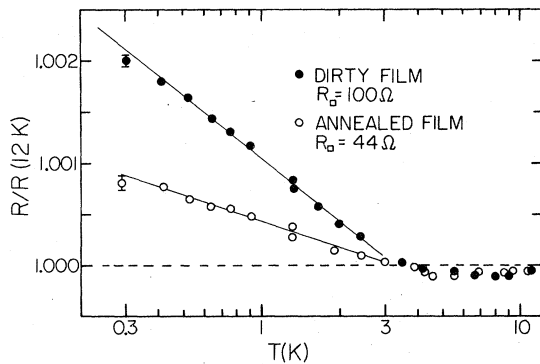


FIG. 5. Resistance as a function of the logarithm of the temperature for (a) a dirty film with $R_{\square} = 100 \Omega$, and (b) a dirty film which had been annealed as described in the text and which had $R_{\square} = 44 \Omega$. The values of $R(12 \text{ K})$ were 1230 Ω and 880 Ω , respectively.

ever, since a logarithmic variation is also consistent with the Kondo effect, we performed the following experiment. A dirty sample was cut into two pieces. One piece was annealed at 300°C for 24 h in an Ar atmosphere, while the other piece was stored as usual at room temperature. The resistance of the annealed sample after this procedure was 44% of its value prior to annealing. The resistances of the two samples were then measured at low temperatures and the results are shown in Fig. 5. The fractional resistance rise is seen to be much smaller in the annealed sample than in the unannealed sample. If this resistance rise was due to Kondo effect, i.e., magnetic impurities, then we would expect that the fractional rise would be larger in the annealed sample since annealing would not be expected to change the concentration of magnetic impurities. This is clearly not the case, and we therefore conclude that the Kondo effect is not important in our experiments.

The theory^{14,18} predicts that the resistance rise due to localization should scale as R_{\square}^{-1} , where R_{\square} is the resistance per square [Eq. (14)]. Our results for the clean and dirty films are qualitatively consistent with this prediction since as noted above, the magnitude of the resistance rise decreases as the films are made thicker and/or cleaner. However, the results for the dirty films as a function of thickness do not appear to scale quite as strongly with R_{\square} as the theory predicts.⁴⁵ Part of this discrepancy could be due to uncertainties in the values of R_{\square} for our samples, but further work is needed to clarify this point. The results of the annealing experiment are in good agreement with the theory, since the annealing reduced R_{\square} by 44% and the magnitude of the resistance rise decreased by approximately 40% (see Fig. 5). We thus find semiquantitative agreement between localization theory and experiment for our thin-film results.⁴⁶ However, since the theory based on electron-electron interaction effects makes predictions which are identical to localization theory, we are unable to determine which one is responsible for the behavior we have observed.

We also attempted to study the inelastic-scattering time through measurements of the resistance at high temperatures, and some typical results are shown in Fig. 6. The behavior is consistent with a T^2 dependence (but not a T or T^4 dependence) in the range 10–30 K. An interesting feature of these results is that the relative rise in the resistivity is much larger in the dirty samples than in the clean samples. This is opposite to what one would ordinarily expect if the inelastic-scattering time is approximately the same in

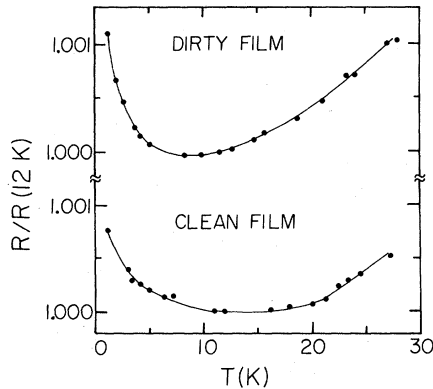


FIG. 6. Resistance as a function of temperature for a dirty film and a clean film at high temperatures.

the two samples. A possible explanation of this result is that increasing the randomness causes an increasing breakdown of the conservation of wave-vector requirement in electron-phonon scattering.⁴⁷ Because of this complication, we have not attempted to extract a value of the inelastic-scattering times from these results.³⁴

B. Thin wires

The current voltage (I - V) characteristics of several wires were studied at various temperatures. In all cases the I - V curves were linear at low currents while at high currents Joule heating was clearly evident. The wires thus behaved as ordinary Ohmic resistors. All of the results given below were, unless explicitly noted otherwise, obtained at currents sufficiently small that Joule heating was negligible.

We will first consider the results for the dirty wires. Typical results for the resistance over a

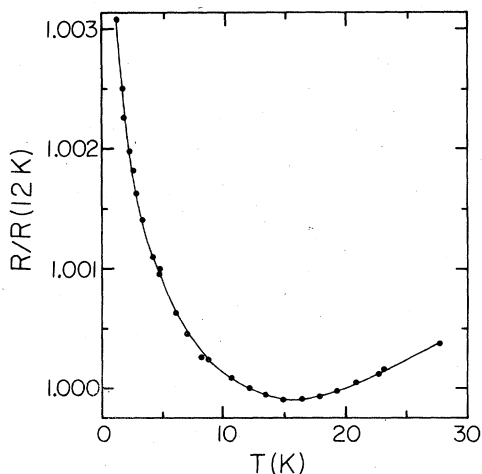


FIG. 7. Resistance as a function of temperature for a dirty wire with $\sqrt{A} = 890 \text{ \AA}$. $R(12 \text{ K}) = 58 \text{ k}\Omega$.

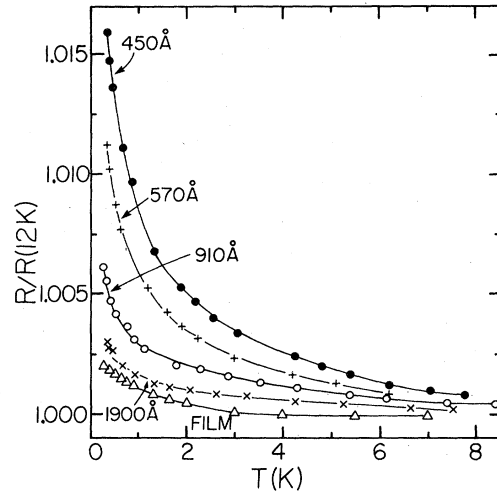


FIG. 8. Resistance as a function of temperature for several dirty wires. The \sqrt{A} of each wire is indicated in the figure. For purposes of comparison, the results for each sample are normalized by the value of R at 12 K. $R(12 \text{ K}) = 360 \text{ k}\Omega$ for the 450- \AA wire, 200 k Ω for the 570- \AA wire, 78 k Ω for the 910- \AA wire, and 32 k Ω for the 1900- \AA sample. The results for a dirty film are also shown for comparison.

wide temperature range are shown in Fig. 7. Above about 15 K the behavior is quantitatively similar to that found in the dirty films (Fig. 6). At lower temperatures the resistance of the wire increases much more rapidly than that of the film,

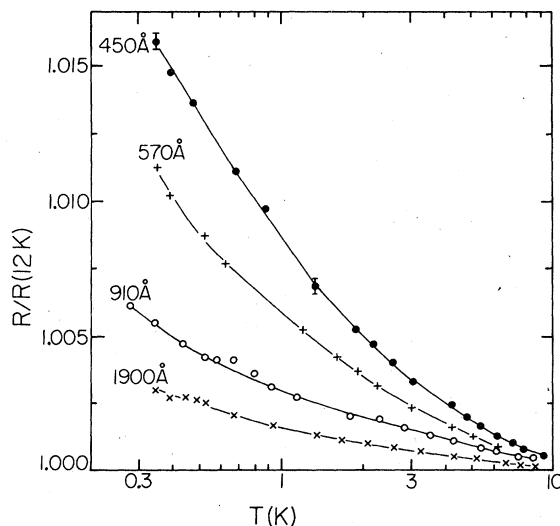


FIG. 9. Resistance as a function of the logarithm of the temperature for several dirty wires. The data are the same as that shown in Fig. 8. Typical error bars are shown at selected points. Note that the uncertainty increases as the temperature decreases since lower currents must then be used to avoid significant Joule heating.

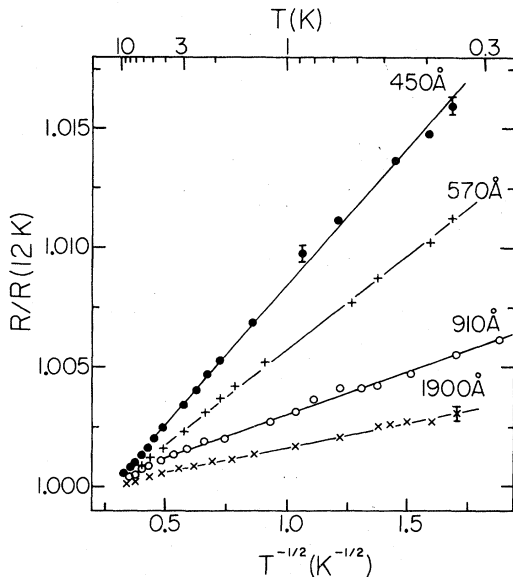


FIG. 10. Resistance as a function of $T^{-1/2}$ for several dirty wires. The data are the same as that shown in Fig. 8. For the sake of clarity some overlapping points have been omitted.

and we attribute this to the effect of localization in one dimension.

The results for several other wires are shown in Fig. 8, where for comparison we also show the behavior of a thin film. The resistance rise is seen to increase as the cross-sectional area of the wires is decreased, and for the wire with $\sqrt{A} = 450 \text{ \AA}$ the rise at 0.3 K is nearly ten times larger than that observed in the film. In Fig. 9 we show the same results plotted against a logarithmic temperature scale. It is clear that for the wires the temperature dependence is definitely faster than logarithmic and is thus different from that observed in the thin films. Figure 10 shows the same results plotted against $T^{-1/2}$. Below about 5 K the results are consistent with a $T^{-1/2}$ dependence, while at higher temperatures there is a small deviation towards lower values of the resistance.⁴⁸ Figure 11 shows a plot of the fractional resistance rise at a constant temperature as a function of A^{-1} . The data at $A^{-1} = 0$ are the thin-film results and we see that the results for the wires extrapolate smoothly to the thin-film behavior as $A^{-1} \rightarrow 0$. This is to be expected for the following reason. The dirty wires were all made from AuPd films which were between 400 and 800 \AA thick. The cross sections of wires with \sqrt{A} greater than about 1000 \AA were, therefore, similar to Fig. 3(d), i.e., the effective width was much greater than the thickness and hence as $A \rightarrow \infty$ the "wires" essentially become thin films.⁴⁹ We interpret the amount by which the

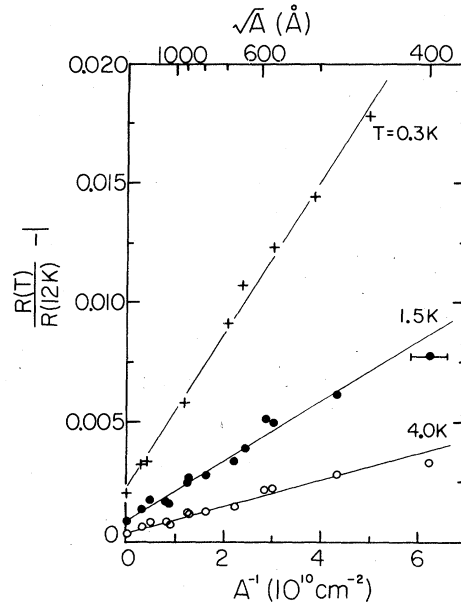


FIG. 11. The fractional resistance rise at 4.0, 1.5, and 0.3 K as a function of A^{-1} for the dirty wires.

resistance rise of the wires exceeds that of the films as being due to one-dimensional localization. From Fig. 11, we see that this "one-dimensional" contribution to the resistance rise varies as A^{-1} .

We should note that the lengths of our wires varied from approximately 90 to 300 μm . Since there was no correlation between the cross-sectional area and the length, the results in Fig. 11 indicate that the resistance rise is independent of the length of the wire. In particular, two wires with nearly identical areas (3.5×10^{-11} and $3.3 \times 10^{-11} \text{ cm}^2$) but lengths differing by nearly a factor of two (90 and 160 μm , respectively), exhibited the same resistance rise to within 2%, well within the experimental error.⁵⁰ While the theory does predict that the resistance will, at low temperatures, vary exponentially with the length of the wire, this behavior should be observable only when the length is comparable to the inelastic mean free path. We will see below that this mean free path is much less than 1 μm in our wires, so the absence of a length dependence is consistent with the theory.

The principal results from the experiments on the dirty wires are the $T^{-1/2}$ variation of the resistance at low temperatures and the A^{-1} dependence of the overall effect. These results are consistent with the theory of localization only if the inelastic scattering time is proportional to T^{-1} and is independent of the size of the wire. The only known scattering mechanism which is consistent with this behavior is electron-tunneling-level scattering. If we use the electron-

tunneling-mode scattering time for PdSiCu given by Black *et al.*²⁴ [Eq. (11)] in conjunction with the prediction [Eq. (5)], we find a resistance rise of

$$\left(\frac{\Delta R}{R}\right)_{\text{theory}} = (0.088)T^{-1/2}, \quad (21)$$

where we have used the parameters appropriate for one of our 450-Å diameter dirty wires.⁵¹ From Fig. 10 we find at low temperatures

$$\left(\frac{\Delta R}{R}\right)_{\text{expt}} = (0.012)T^{-1/2}. \quad (22)$$

The theory thus overestimates the resistance rise by approximately a factor of 7. This discrepancy is probably due to the uncertainty in the inelastic scattering rate. It would be necessary to increase this scattering rate by a factor of approximately 50 to remove this discrepancy. It is also possible that the value of τ_e and the theoretical prediction [Eq. (5)] could both be in error by factors of order unity, but it seems unlikely that this could account for all of the discrepancy. Further work to refine the estimate for τ_i would clearly be of interest. With regard to the inelastic-scattering mechanism, it is interesting to recall that our films (and almost certainly also the wires) are polycrystalline rather than amorphous; however, it seems plausible that the high degree of disorder in our films could give rise to the existence of tunneling modes. We also note that the electron-phonon scattering rate [Eq. (9)] will, by virtue of its stronger temperature dependence, dominate over the scattering from tunneling modes at high temperatures. This could account for the change in temperature dependence above about 5 K which is seen in Fig. 10.

Our results are also quite consistent with the theory of Altshuler *et al.*²⁵ This is not surprising since as discussed in Sec. II, the prediction of this theory is identical in form to that of localization theory when the inelastic scattering is due to tunneling levels. We are therefore unable to distinguish between these two theories. We should note, however, that the interaction effect theory²⁵ also correctly predicts the magnitude of the resistance rise to within a factor of order unity. Unfortunately a more precise comparison is not possible because of uncertainties in the various material-dependent parameters which enter (13).

It should be emphasized that the above conclusions are based on the $T^{-1/2}$ and the A^{-1} dependences which have been observed. However, it is possible that this is not the true "asymptotic" behavior. For example, our large wires have been shown to behave somewhat "two-dimensionally." As the area of the wire is gradually decreased, the behavior should cross over and be-

come one dimensional. Since there is no reliable information about either the size of the crossover region or the behavior in this region, it is possible that we are not observing the true one-dimensional behavior. All that we can do is note that a $T^{-1/2}$ dependence is consistent with the data over the temperature range $5 \text{ K} \gtrsim T \gtrsim 0.3 \text{ K}$, whereas $\log(T)$ and T^{-1} dependences are not. As for the dependence on area, our results are consistent with an A^{-1} behavior over the entire range investigated. Our results are not consistent with an A^{-2} , $A^{-3/2}$, or $A^{-1/2}$ dependence unless the data are restricted to a fairly narrow range of areas.

We next consider the behavior of the clean wires. Figure 12 shows the resistance as a function of temperature for several clean wires. The resistance rise is seen to be larger than in the clean films (Fig. 4) and to increase as the cross-sectional area is made smaller. This rise, however, is much smaller than that found in dirty wires of the same cross sections, consistent with the theoretical prediction (5). Figure 13 shows the results for one wire plotted against the logarithm of the temperature and also against $T^{-1/2}$. As was the case with the dirty wires, the behavior does not appear to be logarithmic, but is more consistent with $T^{-1/2}$ dependence. Figure 14 shows results for the relative resistance rise at 1.5 K with the rise found in the thin films subtracted.⁵² The variation is consistent with the A^{-1} dependence observed in the dirty wires.

The results for the clean wires are thus quite similar to those obtained with the dirty wires. They also indicate that damage and/or implantation occurring during the ion milling are not responsible for the resistance rise. If this were the case, the relative rise would be larger in the

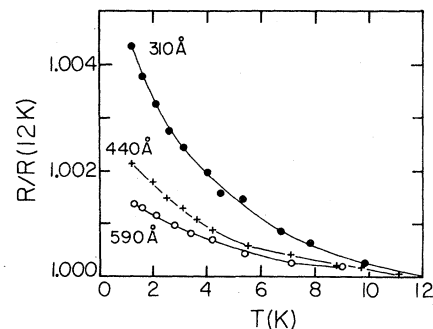


FIG. 12. Resistance as a function of temperature for several clean wires. The \sqrt{A} of each wire is indicated in the figure. For purposes of comparison, the results for each sample are normalized by the value of R at 12 K. $R(12 \text{ K}) = 122 \text{ k}\Omega$ for 310-Å wire, 90 kΩ for the 440-Å wire, and 61 kΩ for the 590-Å wire.

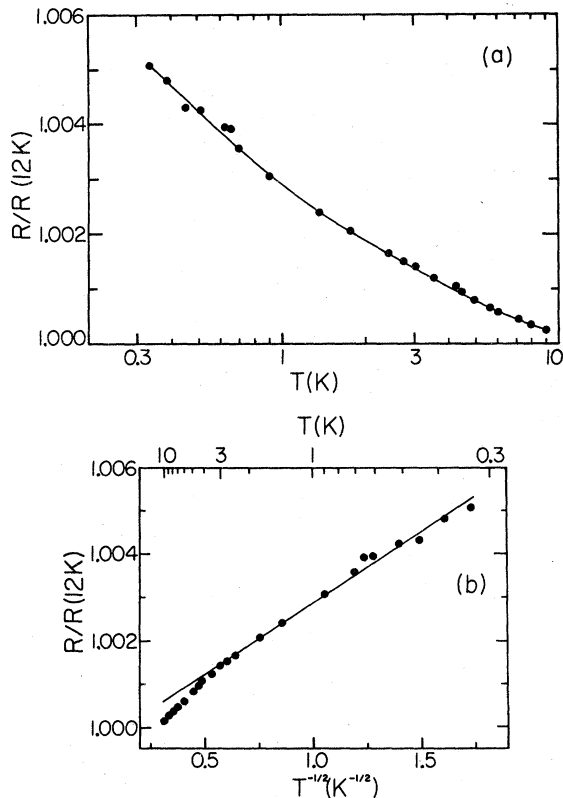


FIG. 13. Resistance as a function of (a) the logarithm of the temperature and (b) $T^{-1/2}$ for a clean wire with $\sqrt{A} = 420 \text{ \AA}$. $R(12 \text{ K}) = 80 \text{ k}\Omega$.

clean wires than in the dirty ones, and this is not observed.

The resistance rise observed in the clean wires is smaller than that observed in the dirty wires by a factor of 6 ± 1 , which is approximately equal to the ratio of the resistivities of the dirty and clean materials. This is not consistent with the theory [Eq. (5)] which predicts that the effect will be smaller by a factor of

$$\left[\frac{(\sqrt{\tau_e} \rho_e)_{\text{dirty}}}{(\sqrt{\tau_e} \rho_e)_{\text{clean}}} \right] = (\rho_{e,\text{dirty}} / \rho_{e,\text{clean}})^{1/2},$$

which is equal to 2.0 ± 0.5 in the present case. In making this comparison we are implicitly as-

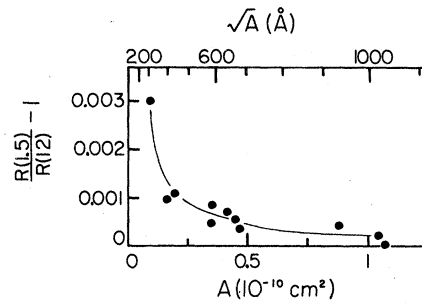


FIG. 14. The fractional resistance rise at 1.5 K as a function of A for the clean wires. The resistance rise of a thin film deposited at the same time as the wire has been subtracted, so that here zero resistance rise corresponds to the thin-film behavior.

suming that τ_i , and hence the density of tunneling modes, is the same in the clean and dirty materials. Since the materials were prepared in different ways, there is no *a priori* reason to expect this to be the case, and this could account for the discrepancy between theory and experiment.

As noted above, all of our samples were transported from Yale to Purdue midway through the present work. All of the wires and films studied after this time exhibited a room-temperature resistance which was typically 20% less than the value prior to being transported. Moreover, the resistance rise observed at low temperatures decreased by approximately the same amount. We believe that when the wires were transported, environmental effects (such as increased temperature or humidity) resulted in partial annealing and hence a decrease in resistance. Since the resistance rise should (according to the theory) decrease when the amount of randomness (i.e., the impurity resistance) is decreased, the resistance rise at low temperatures should therefore also be reduced. Table I shows the results for several wires and we see that they are consistent with the behavior inferred from the comparison of the results for the clean and dirty wires, namely, that the resistance rise scales approximately as the ratio of the resistivities.

This result explains why the sample-to-sample

TABLE I. Values of the room-temperature resistance R and fractional resistance rise at 1.5 K, $\Delta R/R$, for several dirty wires before (subscript i) and after (subscript f) being transported.

R_i (k Ω)	$(\Delta R/R)_i (\times 10^{-4})$	R_f (k Ω)	$(\Delta R/R)_f (\times 10^{-4})$	R_i/R_f	$(\Delta R/R)_i (\Delta R/R)_f$
460	77	360	65	1.21	1.18
122	39	104	34	1.17	1.15
290	61	205	44	1.41	1.39

variations in the resistivity discussed in Sec. III. B did not affect comparisons made between different wires (i.e., Fig. 11). By using (15) to determine the value of A for each wire, we effectively remove errors due to variations in the resistivity, *provided* that the resistance rise does indeed scale with the resistivity.⁵³ Note, however, that these variations should affect the value of A by no more than about 20%, and hence that the values of A quoted in this paper should be accurate on an absolute basis to within this accuracy.

C. Conditions for one-dimensional behavior

From our results for the resistance rise we can use (5) to estimate the value of l_i , the elastic mean free path. From the data for a dirty wire with $A = 450 \text{ \AA}$ (see Fig. 8), we find $l_i = 880 \text{ \AA}$ at 4 K and 3200 \AA at 0.3 K. The lateral dimensions of our small wires are thus comparable to or smaller than l_i at the temperatures studied, and one would therefore expect them to behave one-dimensionally. Our larger wires do not satisfy these criteria quite as well, and this is consistent with the finding (see Fig. 11) that the "two-dimensional" contribution is a significant fraction of the total resistance rise for the large wires. For our thin films we see that l_i is comparable to or larger than the film thickness, and thus it is reasonable to interpret our results in terms of two-dimensional behavior. Comparisons with the theory of Altshuler *et al.*²⁵ yield a value of the length scale similar to that of l_i quoted above.

D. Heating effects

All of the results described above were obtained with sample currents sufficiently small that Joule heating was negligible. For our smallest wires at the lowest temperatures, the maximum allowable current was approximately 5×10^{-10} ampere, corresponding to a field strength of 9×10^{-3} V/cm and a current density of 25 amperes/cm². At higher temperatures and with larger wires it was possible to use much larger currents, but the sample current was usually kept below 4×10^{-9} ampere. Some results for the resistance as a function of sample current are shown in Fig. 15. We see that the maximum allowable current decreases very rapidly as the temperature is lowered. We also see that at low currents R varies approximately quadratically with I , as would be expected from simple Joule heating. This dependence breaks down at large I (larger than those shown in Fig. 15), presumably because the temperature of the "heat bath" becomes significantly altered. We can use the data at low currents to estimate the effective thermal resistance between the sample and the heat bath in the following way.

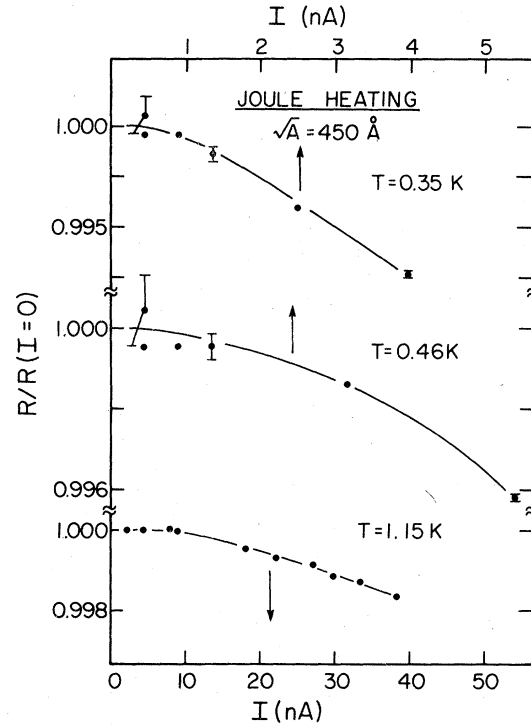


FIG. 15. Resistance ($=dV/dI$) as a function of the measuring current (rms) at several temperatures for a dirty wire with $\sqrt{A} = 450 \text{ \AA}$. The resistance of this wire as a function of temperature is shown in Fig. 8.

Let us assume that the sample is connected to a thermal reservoir by a thermal resistance R_{th} . Then

$$\Delta T = QR_{th}, \quad (23)$$

where ΔT is the temperature rise. Q is the heat "current" which is given by

$$Q = I^2 R_s, \quad (24)$$

where R_s is the electrical resistance of the sample. Thus we have

$$\Delta T = (R_s R_{th}) I^2. \quad (25)$$

From the behavior at low currents shown in Fig. 15 and the data for R_s as a function of temperature shown in Fig. 8, we have estimated R_{th} at various temperatures. The results are shown in Fig. 16. We see that R_{th} varies approximately as T^{-5} or T^{-4} .

To interpret this result, we need to consider the nature of the thermal reservoir discussed above. In general we expect¹⁸

$$R_{th} \sim \frac{\tau_i}{C_s}, \quad (26)$$

where τ_i is the scattering time for the process through which the sample and the reservoir ex-

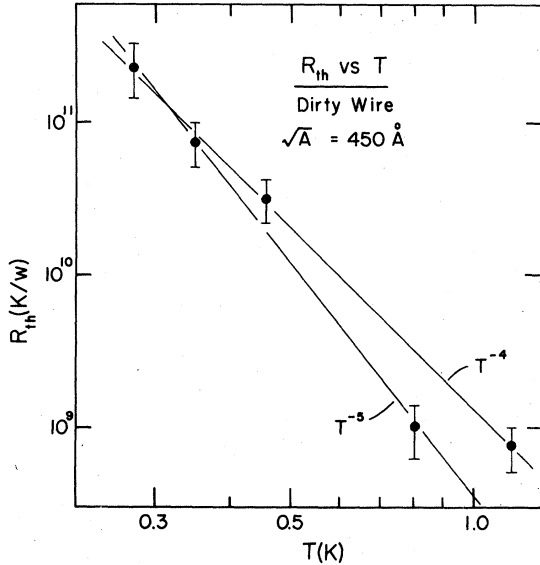


FIG. 16. Log-log plot of R_{th} as a function of temperature for a dirty wire with $\sqrt{A} = 450 \text{ \AA}$. The solid lines are drawn proportional to $T^{-\alpha}$ with $\alpha = 4$ and 5 as indicated.

change energy, and C_s is the heat capacity of the sample. In our case (if we assume that the effects we have observed are due to localization) we need to consider heat flow between the electrons and the tunneling modes, between the tunneling modes and the phonons of the metal film, and between the phonons of the metal film and the phonons of the substrate. There will be a thermal resistance given by an expression like (26) between each of these thermal reservoirs. The temperature rise of the electrons will be determined predominantly by the largest thermal resistance. If this is the thermal resistance between the electrons and the tunneling modes, then from (26) we would expect

$$R_{th} \sim T^{-2}, \quad (27)$$

since $C_s \sim T$ (the ordinary electronic heat capacity)⁵⁴ and τ_i is in this case proportional to T^{-1} [Eq. (11)]. This does not appear to be consistent with the behavior observed in Fig. 16. If the dominant thermal resistance is between the tunneling modes and the phonons, then we would expect a much stronger temperature dependence since in this case $\tau_i \sim T^{-\alpha}$ with $\alpha = 4$ for three-dimensional phonons, and $\alpha = 2$ for one-dimensional phonons.⁵⁵ C_s is still⁵⁴ proportional to T , so we find

$$R_{th} \sim T^{-5} \quad (28)$$

from three-dimensional phonons, and

$$R_{th} \sim T^{-3} \quad (29)$$

for one-dimensional phonons and (28) is quite consistent with the observed temperature dependence. The absolute magnitude of τ_i for this process is very difficult to estimate. We should note, however, that in glasses τ_i [in (28)] has been found to be relatively large. In fact, if τ_i is sufficiently large, then it is possible that energy transfer from the electrons directly to the phonons by means of electron-phonon scattering will be important. In this case (see Sec. II.B), $\tau_i \sim T^{-\alpha}$ with $\alpha = 4$ for three-dimensional phonons in a dirty material and $\alpha = 2$ for one-dimensional phonons in a dirty material, thus leading to $R_{th} \sim T^{-5}$ and T^{-3} , respectively. Finally, the dominant thermal resistance could be between the phonons in the metal film and those in the substrate. This is just the usual thermal boundary resistance which is generally found⁵⁶ to vary as T^{-3} . In this case, it is possible to make a quantitative estimate of the magnitude of R_{th} . Typical boundary resistances⁵⁶ are in the range $1-10 \text{ K cm}^2/\text{W}$ at 1 K . The data in Fig. 16 were for a wire with an approximately triangular cross section (Fig. 3), an area $A = 2.0 \times 10^{-11} \text{ cm}^2$, and a length of $200 \text{ }\mu\text{m}$. Using the above value for the boundary resistance we find $R_{th} = 4 \times 10^9 - 4 \times 10^7 \text{ K/W}$ at 1 K , which is several orders of magnitude smaller than the measured value. This could mean that the thermal resistance between the substrate and the metal film is not the dominant thermal resistance, but it is also possible that the boundary resistance in our samples is much larger than the typical value quoted above.⁵⁷

On the basis of our results, we conclude that the Joule heating in our samples is not controlled by the thermal resistance between the electrons and the tunneling modes. That is, the inelastic scattering time which determines the temperature dependence of the resistance does *not* determine the magnitude of the Joule heating. Further experiments should make it possible to determine which of the thermal resistances discussed above controls the Joule heating of the electrons.⁵⁸

E. Magnetic field effects

There is at present no detailed theory of how a magnetic field will affect localization in thin wires. However, it has been suggested⁵⁹ that if μH (where μ is the nuclear magnetic moment and H is the magnetic field) is comparable to or greater than the spacing between the localized energy levels, scattering processes in which a nuclear spin is flipped may help to delocalize electrons and hence reduce the resistance rise observed at low temperatures. Unfortunately, ¹⁹⁷Au which is 100% abundant has a nuclear moment of only $0.14 \mu_N$ while the only naturally

abundant (22%) Pd isotope which has a magnetic moment is ^{105}Pd with a moment of $0.64 \mu_N$ (Ref. 60). If $H=75 \text{ kOe}$, then for the Au nuclei we have $\mu H=5.3 \times 10^{-20} \text{ erg}$ while for the Pd nuclei we have $\mu H=2.4 \times 10^{-19} \text{ erg}$. This is to be compared with the spacing between localized levels which in our smallest dirty wires is approximately $7 \times 10^{-21} \text{ erg}$, so that we might expect to see some sort of an effect in magnetic fields of order 75 kOe . Some typical experimental results are shown in Fig. 17. Other than a temperature-independent magnetoresistance, the magnetic field is seen to have no significant effect on the resistance rise except possibly at the highest temperatures studied. Further theoretical work on this problem would clearly be of interest.

It is interesting to note that a magnetic field did have a large effect on the behavior of our thin films. Some results are given in Fig. 18 which shows the behavior with the field parallel and perpendicular to the direction of the current. For experimental reasons it was not possible to make both of these measurements on the same sample, and this could account for the difference in the behavior in the two cases. In any case, the magnetic field is seen to have a significant effect on

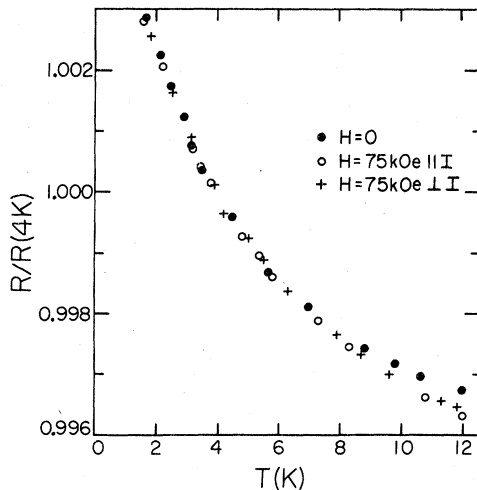


FIG. 17. Resistance of a dirty wire with $\sqrt{A}=400 \text{ \AA}$ in zero magnetic field (\bullet), and in a magnetic field of 75 kOe with the field parallel (\circ), and perpendicular ($+$) to the axis of the wire. Each curve has been normalized by the value of the resistance at 4.0 K to demonstrate that below about 8 K , the temperature dependence in the different cases is the same to within experimental error. For $H \parallel I$

$$[R(H=75 \text{ kOe}, T=4 \text{ K}) - R(H=0, T=4 \text{ K})] = 0.79 \text{ k}\Omega,$$

while for $H \perp I$,

$$[R(H=75 \text{ kOe}, T=4 \text{ K}) - R(H=0, T=4 \text{ K})] = 0.72 \text{ k}\Omega.$$

$$R(H=0, T=4 \text{ K}) = 410 \text{ k}\Omega.$$

the resistance rise. Since the effect of the field is much larger in the films than in the wires, we conclude that the effect is not intrinsic to bulk AuPd. It could conceivably be a contact effect (see Sec. III above), but we see no reason why a contact effect should be temperature and field dependent in this way. We therefore tentatively conclude that a magnetic field has a significant effect on localization in two-dimensional systems and that the field acts to further localize the electrons. Further experimental work on this problem is clearly needed, and is currently underway.⁶¹

V. COMPARISON WITH PREVIOUS WORK

A. One dimension

The first attempts to observe localization in one dimension were unsuccessful. Dolan *et al.*⁶² reported null results in thin film "wires" which were very thin and relatively wide (typically $30 \text{ \AA} \times 10000 \text{ \AA}$) strips of granular AuPd films. These workers also reported null results in MOSFET devices which had narrow gates. At the same time, Garland *et al.*⁶³ reported null results in a percolative mixture of small ($\sim 100 \text{ \AA}$) Ag particles in a KCl matrix. The samples studied by these workers were, according to the original theoretical estimates,⁴ expected to exhibit a significant resistance rise at the temperatures investigated. However, we have seen that the original estimates for τ_i are much too large to be consistent

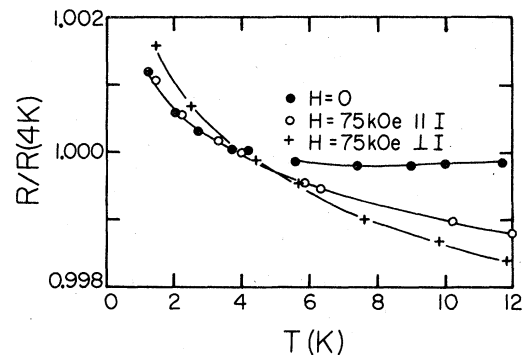


FIG. 18. Resistance of dirty films with $R_0=100 \Omega$ in zero magnetic field (\bullet), and in a magnetic field of 75 kOe with the field parallel (\circ) and perpendicular ($+$) to the direction of the current. In both cases the field was in the plane of the film. Note that the measurements in a field were made on two different samples, but that in zero field these samples exhibited the same temperature dependence to within $1/10^4$. For $H \parallel I$,

$$[R(H=75 \text{ kOe}, T=4 \text{ K}) - R(H=0, T=4 \text{ K})] = 2.3 \Omega$$

with $R(H=0, T=4 \text{ K}) = 1040 \Omega$. For $H \perp I$,

$$[R(H=75 \text{ kOe}, T=4 \text{ K}) - R(H=0, T=4 \text{ K})] = 3.3 \Omega,$$

with $R(H=0, T=4 \text{ K}) = 1400 \Omega$.

with our results. If we use our observed dependences on impurity resistance, area, and temperature to extrapolate to the parameter values appropriate to the experiment of Garland *et al.*,⁶³ we find a resistance rise which is smaller than their experimental resolution. Dolan *et al.*⁶² do not give sufficient data to make such a comparison.

More recently, Dolan and Osheroff⁶⁴ have reported possible one-dimensional behavior in their narrowest (1000 Å wide) thin-film wires. They noted, however, that there may have been significant nonuniformities in the widths of these samples, as the behavior was not reproducible from sample to sample. Some of their samples exhibited an $\exp(T^{-1/2})$ temperature dependence. At high temperatures this reduces to the $T^{-1/2}$ dependence which we have observed in our wires, so it appears that the results of Dolan and Osheroff are consistent with our results. The samples of Dolan and Osheroff also exhibited highly nonlinear current-voltage characteristics. We have seen no evidence for behavior of this kind in our samples.⁶⁵

Chaudhari and Habermeier⁶⁶ have reported results on very thin and relatively wide (typically $50 \text{ Å} \times 5000 \text{ Å}$) strips of continuous W-Re films. These wires have areas and resistivities quite comparable to our samples. They exhibit a resistance rise which scales as A^{-1} and varies approximately as $T^{-1/2}$, just as we observe in our samples. Moreover, the absolute magnitude of the resistance rise is in good agreement with our results. Chaudhari and Habermeier⁶⁶ have also reported a length dependence of the resistance which does not appear to be consistent with the present theory.

The only experiments which have studied wires which were not attached to substrates (or embedded in a matrix) are those of Overcash *et al.*⁶⁷ These authors studied free-standing whiskers of Bi and Bi doped with Sb, with diameters as small as 1400 Å. No evidence was found for localization, as the resistance was always found to decrease as the temperature was lowered. Overcash *et al.*⁶⁷ have interpreted these results as being in conflict with localization theory. However, if one extrapolates our results for AuPd wires using the dependences on A , T , and ρ_e which we have observed, one finds a resistance rise which is smaller than the experimental resolution in Ref. 67.

B. Two dimensions

Dolan and Osheroff⁶⁴ have performed extensive measurements on granular AuPd films. They find a resistance rise at low temperatures which var-

ies logarithmically with temperature, as predicted by the theory of Abrahams *et al.*¹⁴ Since the theory also predicts that the resistance rise will scale with the resistance per square, R_{\square} , it is possible to compare the results for our uniform (i.e., nongranular) films directly with those of Dolan and Osheroff. From Fig. 5, we see that

$$\frac{\Delta R}{R} \cong - (0.0020) \log_{10}(T) + \text{constant}, \quad (30)$$

for a film with $R_{\square} = 100 \text{ } \Omega$. Dolan and Osheroff find⁶⁴

$$\frac{\Delta R}{R} \cong - (0.093) \log_{10}(T) + \text{constant} \quad (31)$$

for a film with $R_{\square} = 4600 \text{ } \Omega$. The reduced resistance rise $(\Delta R/R)/R_{\square}$ is thus $2.0 \times 10^{-5} \text{ } \Omega^{-1}$ for our sample and $2.0 \times 10^{-5} \text{ } \Omega^{-1}$ for the sample of Dolan and Osheroff. This agreement suggests that the detailed nature of the sample, i.e., uniform versus granular, is not important in this case.

Our thin film results are thus in good agreement both with the results of Dolan and Osheroff⁶⁴ and with the theory of Abrahams *et al.*¹⁴ We wish to emphasize, however, that all of these results are also consistent with the theory of Altshuler *et al.*²⁵

VI. CONCLUSIONS

We have presented the results of a systematic study of localization in thin wires. Our principal results—the dependence of the effect on cross-sectional area, temperature, and the amount of disorder—are all in good general agreement with the theory.^{4,14-16} Localization theory can account for our results quantitatively only if the inelastic scattering rate is proportional to the temperature and independent of the cross-sectional area. A mechanism which satisfies these requirements is scattering from the tunneling modes which are believed to exist in all highly disordered systems.⁶⁸ Our results are also consistent with the theory of Altshuler *et al.*²⁵ which attributes this behavior not to localization but to electron-electron interaction effects. Further work is needed to determine which of these theories is applicable to the present experiments. Independent measurements of τ_i should make it possible to distinguish between the two theories.⁶⁹ A unified theory which includes both localization and interaction effects would certainly also be of interest.

ACKNOWLEDGMENTS

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*Present address.

¹P. W. Anderson, Phys. Rev. **109**, 1492 (1958).

²See for example, N. F. Mott and E. A. Davis, *Electronic Processes in Non-Crystalline Materials* (Clarendon, Oxford, 1971).

³See for example, D. J. Thouless, Phys. Rep. **13C**, 93 (1974), and in *The Metal-Nonmetal Transition in Disordered Systems*, Proceedings of the Nineteenth Scottish Universities Summer School in Physics, 1978, edited by L. R. Friedman and D. P. Tunstall (Scottish Universities Summer School in Physics, 1979); N. Mott, M. Pepper, S. Pollitt, R. H. Wallis, and C. J. Adkins, Proc. R. Soc. London Ser. A **345**, 169 (1975). These papers also contain extensive references to earlier work.

⁴D. J. Thouless, Phys. Rev. Lett. **39**, 1167 (1977).

⁵C. J. Adkins, Philos. Mag. **36**, 1285 (1977), has advanced arguments similar to those of Thouless (Ref. 4), although he finds somewhat different results for the conditions under which localization should be observable.

⁶For a review of recent work, see D. J. Thouless, J. Non-Cryst. Solids **35** and **36**, 3 (1980).

⁷N. Giordano, W. Gilson, and D. E. Prober, Phys. Rev. Lett. **43**, 725 (1979).

⁸N. F. Mott and W. D. Twose, Adv. Phys. **10**, 107 (1961).

⁹R. Landauer, Philos. Mag. **21**, 863 (1970) and references contained therein, including IBM J. Res. Dev. **1**, 223 (1957). See also the discussion in R. Landauer, in *Electrical Transport and Optical Properties of Inhomogeneous Media*, edited by J. C. Garland and D. B. Tanner (AIP, New York, 1978), p. 2.

¹⁰See for example, B. I. Halperin, Adv. Chem. Phys. **13**, 123 (1967); K. Ishii, Prog. Theor. Phys. Suppl. **53**, 77 (1973); A. A. Abrikosov and I. A. Ryzhkin, Adv. Phys. **27**, 147 (1978); D. J. Thouless, J. Phys. C **6**, L49 (1973), and references contained therein.

¹¹G. Yuval, Phys. Lett. **53A**, 136 (1975).

¹²J. T. Edwards and D. J. Thouless, J. Phys. C **5**, 807 (1972); D. C. Licciardello and D. J. Thouless, Phys. Rev. Lett. **35**, 1475 (1975); J. Phys. C **8**, 4157 (1975).

¹³Strictly speaking, this argument should be formulated in terms of wave packets (see Ref. 6), but this does not change the physical ideas which are involved.

¹⁴E. Abrahams, P. W. Anderson, D. C. Licciardello, and T. V. Ramakrishnan, Phys. Rev. Lett. **42**, 673 (1979).

¹⁵D. J. Thouless, Solid State Commun. **34**, 683 (1980).

¹⁶P. W. Anderson, D. J. Thouless, E. Abrahams, and D. S. Fisher, Phys. Rev. B **22**, 3519 (1980).

¹⁷It has been suggested that the appropriate length scale at finite temperatures might be a quantity called the "incoherence length" which is proportional to τ_i (see Ref. 18). However, it now appears (see Ref. 15) that this is not the relevant length scale.

¹⁸P. W. Anderson, E. Abrahams, and T. V. Ramakrishnan, Phys. Rev. Lett. **43**, 718 (1979).

¹⁹More precisely, we use the Einstein relation $\mu = (eD/n)(dn/d\mathcal{E})$ given by Ashcroft and Mermin (Ref. 20) together with the relation $\sigma = ne\mu$. Here μ is the mobility, e is the charge, n is the density, and \mathcal{E} is the chemical potential of the electrons. When combined with the usual relations for an electron gas (Ref. 20), one then finds $D = v_F^2 \tau_e / 3$. We note that the Einstein relation quoted above differs (by a factor of $\frac{1}{2}$ on the right-hand side) from that used by Thouless (Ref. 4). Presumably this is because Thouless allowed for an exponential distribution of relaxation times with a mean of τ_e whereas we have implicitly assumed a delta-function distribution for τ_e . This difference leads to a factor-of-two difference in the diffusion constant and hence a $\sqrt{2}$ difference in the value of l_i . This is not of any quantitative significance in interpreting the experiments since the uncertainty in τ_i is much larger than this amount.

²⁰N. W. Ashcroft and N. D. Mermin, *Solid State Physics* (Holt, Rinehart and Winston, New York, 1976).

²¹Note that we have added an extra factor of $\sqrt{2}$ to the second term in the brackets in (5). This factor is necessary because of the difference in the definitions of the diffusion constants used by Thouless (Ref. 15) and that quoted in (3) above. See Ref. 19 and also the discussion concerning Eq. (6) in Ref. (15) for a discussion of this point.

²²P. A. Lee, private communication.

²³See for example, P. W. Anderson, B. I. Halperin, and C. M. Varma, Philos. Mag. **25**, 1 (1972); W. A. Phillips, J. Low Temp. Phys. **7**, 351 (1972).

²⁴J. L. Black, B. L. Gyorffy, and J. Jäckle, Philos. Mag. **B40**, 331 (1979).

²⁵B. L. Altshuler, A. G. Aronov, and P. A. Lee, Phys. Rev. Lett. **44**, 1288 (1980); B. L. Altshuler, D. Khmel'nitzkii, A. I. Larkin, and P. A. Lee (to be published).

²⁶P. A. Lee, Phys. Rev. Lett. **42**, 1492 (1979); see also

Mott *et al.* in Ref. 3.

- ²⁷We are assuming here that the contribution from electron-phonon and electron-electron scattering is negligible, as will be the case at low temperatures.
- ²⁸M. D. Feuer and D. E. Prober, *Appl. Phys. Lett.* **36**, 226 (1980); see also D. E. Prober, M. D. Feuer, and N. Giordano, *Appl. Phys. Lett.* **37**, 94 (1980).
- ²⁹See for example, C. M. Melliar-Smith, *J. Vac. Sci. Technol.* **13**, 1008 (1976).
- ³⁰We used No. 1-1/2 cover glasses made by Corning Glass Works, Corning, N.Y.
- ³¹More precisely, different grains in the film have different crystal faces exposed to the Ar⁺ beam. Since the different crystal faces will, in general, mill at slightly different rates, a film which is initially "smooth" will become "rough" after it is milled for a period of time.
- ³²See for example, H. W. Lehmann, L. Krausbauer, and R. Widmer, *J. Vac. Sci. Technol.* **14**, 281 (1977).
- ³³See, for example, P. Echlin, *Scanning Electron Microscope Symposium—Scanning Electron Microscopy, Proceedings, 1978* (AMF Ohare, Chicago, 1978), Vol. 1, p. 109.
- ³⁴It is also possible that deviations from Matthiessen's rule [see for example, J. Bass, *Adv. Phys.* **21**, 431 (1972)] are responsible for this discrepancy.
- ³⁵At the energies employed, the Ar⁺ should penetrate less than 10 Å. See for example, J. W. Mayer, L. Eriksson, and J. A. Davies, *Ion Implantation in Semiconductors* (Academic, New York, 1970).
- ³⁶The resistance ratios of wires and films prepared at the same time agreed to typically better than 1/10³. In addition the resistance ratio of the wires was not affected (typically to within a few parts per thousand) by thermal cycling between room temperature and low temperature, or by "aging" at room temperature (except for the changes which occurred while the samples were transported as described in Sec. III).
- ³⁷This thermometer was a Model CR-2500H from Cryocal, Inc., St. Paul, Minn. and was calibrated against a second germanium thermometer which had previously been calibrated by Cryocal.
- ³⁸G. Seidel and P. H. Keesom, *Rev. Sci. Instrum.* **29**, 606 (1958).
- ³⁹These thermometers were Cryocal Models CR-2500H and CR-100. The former was calibrated by the manufacturer while the latter was calibrated against the vapor pressures of ³He and ⁴He, and the susceptibility of a paramagnetic salt, in a previous experiment.
- ⁴⁰A battery-powered current source was used in conjunction with Data Precision 3500 and Hewlett-Packard 3465A digital voltmeters.
- ⁴¹A Princeton Applied Research Model 126 Lock-in Amplifier was used as the null detector.
- ⁴²Here and in the following we define the fractional resistance rise at temperature *T* as $[R(T) - R(12\text{ K})]/R(12\text{ K})$. This definition was chosen because the effects of both localization (which causes the resistance rise at low temperatures) and electron-phonon scattering (which causes the resistance rise at high temperatures) are both small at 12 K.
- ⁴³L. R. Edwards, C. W. Chen, and S. Legvold, *Solid State Commun.* **8**, 1403 (1970).
- ⁴⁴See for example, A. P. Murani, *Phys. Rev. Lett.* **33**, 91 (1974); D. Greig and J. A. Rowlands, *J. Phys. F* **4**, 536 (1974).
- ⁴⁵For this reason we tentatively ruled out localization in our earlier report (Ref. 7). However, the new measurements at lower temperatures (Fig. 5) together with the results of the annealing experiment are strong evidence that localization is in fact responsible.
- ⁴⁶We should note that a resistance rise at low temperature has been observed in thin Pd films by S. Schmid-Marčić and J. A. Mydosh [*Solid State Commun.* **17**, 795 (1975)]. These authors also observed that the effect became larger as the resistivity (and hence presumably also the sheet resistance) was increased. They attributed this resistance rise to the same mechanisms discussed in connection with Pd alloys (see Refs. 43 and 44). However, their results do not appear to be consistent with the functional form (20) and it seems entirely possible that the resistance rise was due instead to localization.
- ⁴⁷I. A. Campbell, A. D. Caplin, and C. Rizzuto, *Phys. Rev. Lett.* **26**, 239 (1971).
- ⁴⁸In our earlier report (Ref. 7) we were unable to distinguish between a log(*T*) and a $T^{-1/2}$ dependence. The new measurements are over a wider temperature range, and as can be seen from Figs. 9 and 10, it is now possible to make this distinction.
- ⁴⁹As noted earlier, the 800-Å-thick films exhibit a slightly (3×10^4) smaller resistance rise than the 400-Å films. A large wire made from the thicker films would have a smaller effective value of R_{\square} , and the "two-dimensional" contribution to its overall resistance rise should, therefore, be smaller by this amount. The magnitude of this effect is comparable to the experimental error and is thus not apparent in Fig. 11. However, this effect is important for the clean wires (see below).
- ⁵⁰In making this comparison we have taken the small difference in the cross-sectional areas into account.
- ⁵¹We have taken $v_F = 1.4 \times 10^8$ cm/sec and $\tau_p = 1.1 \times 10^{-15}$ sec (see Sec. III. B) and assumed that the localization length is the length of the wire which gives a total impurity resistance of 36 500 Ω.
- ⁵²Here we have subtracted off the resistance rise found in the thin films so as to extract (albeit in an approximate manner) the contribution arising from one-dimensional localization. Such a subtraction was not necessary for the dirty wires since the resistance rise of the dirty films was much smaller than that of the wires. For the clean wires this is not the case, and wires made from 200-Å-thick films exhibited significantly larger resistance rises than wires of the same area made from 400-Å-thick films. However, after the resistance rise of the films (which is larger for the thinner films) is subtracted, the results are to within the errors dependent only on the cross-sectional area and not on the initial film thickness.
- ⁵³We note that for the samples measured after being transported, new "effective" values of *A* were determined using the new values of the room-temperature resistance. These effective values were, of course, slightly larger than the old values. Throughout this paper the values of *A* quoted are always the effective values determined from the room-temperature resistance of the wire just prior to the measurements in question. This has the effect of scaling the results to values appropriate for a wire with a resistivity of the

"unannealed" samples.

⁵⁴In estimating the heat capacity of the electronic "system" it is also important to consider nuclear contributions to the heat capacity. In zero magnetic field it is necessary to take into account only the nuclear quadrupole moment. Since AuPd is cubic, one might expect this contribution to vanish. However, AuPd is a disordered alloy and the site symmetry will not in general be cubic. As a result, estimating this contribution to the heat capacity is not completely straightforward, but fortunately measurements of this effect in Au-Ag and Ag-Cu alloys have been reported [D. L. Martin, *Phys. Rev.* **176**, 790 (1968); *Can. J. Phys.* **46**, 923 (1968)]. If we use the value for Au derived by Martin, and also scale it according to the nuclear spin and quadrupole moment to get a corresponding estimate for Pd, then we find a total nuclear contribution to the heat capacity of $4.5/T^2$ ($\mu\text{JK}/\text{mole}$). This is to be compared with the electronic contribution which is approximately $4.0T$ ($\text{mJ}/\text{mole K}$) [see, for example, N. E. Phillips, *Crit. Rev. Solid State Sci.* **2**, 467 (1971)]. Thus at 0.3 K the nuclear contribution is only 4% of the electronic contribution, and our use of the approximation $C \sim T$ is easily justified. In this regard we should also note that the lattice specific heat may be estimated as approximately $0.32 T^3$ ($\text{mJ}/\text{mole K}^4$) (see Phillips, referred above), and thus it is also negligible compared to the electronic contribution in the temperature range of interest.

⁵⁵This result for τ_i follows directly from Ref. 23 [see also J. Jäckle, *Z. Phys.* **257**, 212 (1972)]. It was derived by integrating τ_i^{-1} for tunneling levels with energy E [see Jäckle's Eq. (7)] from $E=0$ to $E=\omega_D$, assuming a Debye model for the phonons with ω_D being the Debye frequency. I thank J. L. Black for a very helpful discussion on this point.

⁵⁶See, for example, V. E. Holt, *J. Appl. Phys.* **37**, 798 (1966).

⁵⁷It is also possible for example, that the phonons in the wire are one dimensional and that this could change the value of the thermal boundary resistance.

⁵⁸Our results are also consistent with the following model. It is possible that the tunneling modes are not important and that electron-electron scattering or electron-electron interaction effects determine the temperature dependence of the resistance. However, since these processes cannot remove energy from the

electronic system, some other process such as electron-phonon scattering will determine R_{th} . It is not possible to determine at this time which (if either) of these models (i.e., the one discussed here or the one discussed in the text) is relevant to our samples. In any case it seems clear that two different inelastic scattering times must be important in our samples.

⁵⁹D. J. Thouless, private communication.

⁶⁰See, for example, *American Institute of Physics Handbook*, 3rd ed., edited by D. E. Gray (McGraw-Hill, New York, 1972).

⁶¹G. Bergmann [*Phys. Rev. Lett.* **43**, 1357 (1979)] has reported that in the presence of a magnetic field, very thin Pd films exhibit a resistance increase very similar to what we observe in our films. The effects may have a common origin.

⁶²G. J. Dolan, D. D. Osheroff, and D. C. Tsui, *Bull. Am. Phys. Soc.* **24**, 233 (1979).

⁶³J. C. Garland, W. J. Gully, and D. B. Tanner, *Bull. Am. Phys. Soc.* **24**, 280 (1979); *Phys. Rev. B* **22**, 507 (1980).

⁶⁴G. J. Dolan and D. D. Osheroff, *Phys. Rev. Lett.* **43**, 721 (1979); G. J. Dolan, in *Inhomogeneous Superconductors—1979*, edited by D. U. Gubser, T. L. Franca-villa, S. A. Wolf, and J. R. Leibowitz (AIP, New York, 1979), p. 118.

⁶⁵Very recently, G. J. Dolan [*Bull. Am. Phys. Soc.* **25**, 355 (1980)] has studied narrower, continuous AuPd wires, and finds behavior similar to what we observe.

⁶⁶P. Chaudhari and H.-U. Habermeier, *Phys. Rev. Lett.* **44**, 40 (1980); *Solid State Commun.* **34**, 687 (1980).

⁶⁷D. R. Overcash, B. A. Ratnam, M. J. Skove, and E. P. Stillwell, *Phys. Rev. Lett.* **44**, 1348 (1980).

⁶⁸S. T. Chui [*Bull. Am. Phys. Soc.* **25**, 370 (1980), and unpublished] has predicted that in a thin wire the electron-electron scattering process will be modified so as to lead to an inelastic scattering time of form $\tau_i \sim T^{-(2+\alpha)}$ with $\alpha < 0$. If $\alpha \sim -1$, this scattering process could conceivably account for our results. However, the theory of Chui appears to depend crucially upon conservation of wave vector, which is certainly not the case in our experiments since in our samples the elastic mean free path is of order an interatomic spacing.

⁶⁹Such measurements are currently underway, P. Chaudhari, A. N. Broers, C. C. Chi, R. Laibowitz, E. Spiller, and J. Viggiano, *Phys. Rev. Lett.* **45**, 930 (1980).

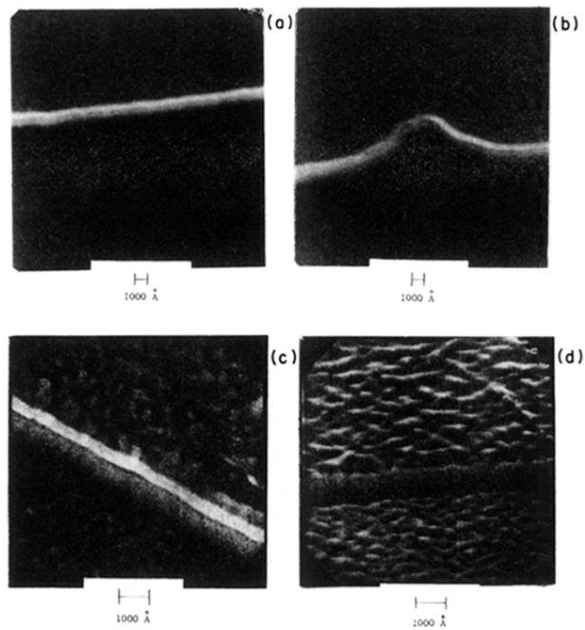


FIG. 2. Scanning electron micrographs of several wires. (a) A relatively large "dirty" (see Sec. III. C for an explanation of this term) wire viewed at normal incidence. The high side of the step is at the top of the picture. (b) Picture of the same wire as in (a) but taken at a different location along the wire. The high side of the step is again at the top of the picture. (c) A small "clean" (see Sec. III. C) wire viewed at normal incidence. The high side of the step is at the upper right. (d) Picture of the same wire as in (c), but viewed at 45° from the normal, i.e., looking into the step. The high side of the step is at the top of the picture.