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Effect of magnetic fields on spin fluctuations in metals: Electronic specific heat and electrical resistivity

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The spin-fluctuation contribution to the electronic specific-heat enhancement and to the resistivity is calculated in the presence of a magnetic field using the random-phase approximation. The result is a depression of these contributions in the presence of the field thus providing a test of the paramagnon theory. A comparison with experiments on UAl_2 is presented and a measurable effect of several percent is predicted in very high magnetic fields of the order of 100 kG.

I. INTRODUCTION

The theory of persistent spin fluctuations, or paramagnons, in nearly ferromagnetic metals is first formulated by Doniach and Engelsberg[1] and by Berk and Schrieffer.[2] These authors show that the absorption and reemission of paramagnons renormalize the electron self-energy leading to an enhanced effective mass at low temperatures. This effect manifests itself as a low-temperature enhancement of the electronic specific-heat coefficient. The paramagnons also influence the electron transport properties. In analogy to the conventional scattering of electrons by phonons the particles will be scattered by the paramagnons. The temperature dependence of the electrical resistivity predicted by the theory[3,4] displays a T^2 dependence at low temperatures and a linear region at high temperatures.

Experimentally, the best-suited systems in which the paramagnon effects are expected to be seen are the uniform nearly ferromagnetic metals Pd, Rh, and Pt and their alloys with one another. In these metals, there exists a strong intra-atomic Coulomb interaction between d-band electrons of opposite spins. Recent measurements on the narrow-band intermetallic compound UAl_2 (Ref. 5) indicate the presence of spin fluctuations associated with a narrow f band. Although it is generally believed that, for example, Pd exhibits an effective-mass enhancement due to paramagnons, an unambiguous experimental verification for their existence in such a uniformly exchange-enhanced metal has been unsuccessful for the most part.[6,7] The reason is that it is difficult to separate the spin-fluctuation contribution to the specific heat from that due to the phonons. Resistivity measurements, for example, in Pd,[8] display the expected features predicted by the spin-fluctuation theory, namely, the T^2 dependence at low temperatures. However, these experiments may be explained equally well by Baber scattering.[9] Baber scattering may be regarded as a scattering process in which conduction electrons of the s band scatter off d electrons via the screened Coulomb interaction without spin flip. The screened Coulomb interaction is mediated by the exchange of charge density and longitudinal spin fluctuations. Paramagnon scattering, on the other hand, is defined as the electron scattering by transverse spin fluctuations whereby the electrons flip their spins.

In this brief paper we consider the magnetic field dependence of the electron—paramagnon interaction[10,11] and we discuss in particular the field dependence of the mass enhancement and of the resistivity. It was pointed out before by Brinkman and Engelsberg[10] that the application of large magnetic fields offers at least one way of testing the paramagnon theory. If the magnetic field is sufficiently large so that the Zeeman splitting energy of opposite spin states is comparable to or larger than the characteristic spin-fluctuation energy, then the paramagnons no longer have enough energy to flip the spins and, therefore, the inelastic spin-flip scattering is quenched. Hence a decrease of the specific-heat enhancement and of the resistivity with increasing magnetic field is to be expected. However, to see such an effect one needs a metallic substance with a low Fermi energy so that the paramagnon energy becomes comparable to the Zeeman energy. Such substances are more likely to be found among the narrow-band and the f-band metals where the density of states is large, corresponding to "flat" bands. So far there exists only one narrow-band intermetallic compound, namely, UAl_2,[5] where an experimental indication is given for a small magnetic field dependence of the electronic mass enhancement, whereas recent measurements of the field dependence of the electrical resistivity of Pd containing 1.7 ppm of Fe do not support a spin-fluctuation contribution to the magnetoresistance.[12]

The model used in this paper to study the magnetic field effects on the electron-paramagnon interaction is based on the random-phase approximation (RPA) with an effective interaction describing the particle-hole correlation function. Since we are not interested in ferromagnetic phase transitions we believe the RPA to be valid, although it breaks down near the Curie temperature of a ferromagnet. It is also true that there is currently no other approach in which the particle-hole correlation function may be readily obtained. For the effective-exchange interaction it is known that a zero-range model results in a gross overestimation of the mass enhancement. The inclusion of a momentum-dependent exchange interaction reduces this enhancement.[13] For simplicity, that is, to avoid a large numerical effort the results of which cannot be easily assessed, we adopt the simplest generalization of the original one-parameter interaction model. As proposed by Fay and Appel,[14] we replace the effective interaction constant I by $I(q)=Ia^2/(a^2+q^2)$ where the parameter a is of the order of the Fermi wave vector. This two-parameter model must of course be considered as a phenomenological model allowing for a reasonable fit of the paramagnon mass enhancement.

II. SPECIFIC-HEAT ENHANCEMENT

The shift in the electron entropy due to the electron-phonon and the electron-spin-fluctuation interaction is given by

$$\Delta S=-\sum_\sigma \int \frac{d\vec{p}}{(2\pi)^3}\frac{\partial f(\beta\epsilon_{p\sigma})}{\partial T}\,\mathrm{Re}\Sigma_\sigma(\vec{p},\epsilon_{p\sigma},\bar{H})$$
$$\equiv \sum_\sigma k_B N(p_F)\int d\epsilon\,\frac{\partial f(\epsilon)}{\partial\epsilon}\,\epsilon\,\mathrm{Re}\Sigma_\sigma(p_F,\epsilon_{p\sigma}\beta^{-1},\bar{H})\,. \tag{1}$$

This is a generalization of the equation given by Abrikosov *et al.*[15] in the absence of an external field. In Eq. (1), $\epsilon_{p\sigma}=\epsilon_p-\sigma\mu_B\bar{H}$ ($\sigma=+1$ for spin up, $\sigma=-1$ for spin down) is the quasiparticle energy and $\bar{H}$ is the effective magnetic field. The quantity Σ_σ is the electron self-energy including the contributions from phonons and longitudinal and transverse spin fluctuations. Since the self-energy depends only weakly on the momentum p, in the self-energy we set p equal to the appropriate Fermi momentum

$$p_{F\sigma}=[2m(\epsilon_F-\sigma\mu_B\bar{H})]^{1/2}\,.$$

The main contribution to the integral comes from the region near $\epsilon_{p\sigma}=0$. The shift in the specific heat in the zero-temperature limit is then given by

$$\Delta C_v=T\frac{\partial\Delta S}{\partial T}=T\sum_\sigma k_B^2 N(p_{F\sigma})\left(\frac{\partial}{\partial\omega}\Sigma(p_{F\sigma},\omega,\bar{H})\right)_{\omega=0}\times\int d\epsilon\,\frac{\partial f}{\partial\epsilon}\epsilon^2$$
$$=-Tk_B^2\frac{\pi^2}{3}\sum_\sigma N(p_{F\sigma})\left(\frac{\partial}{\partial\omega}\Sigma(p_{F\sigma},\omega,\bar{H})\right)_{\omega=0}\,. \tag{2}$$

It should be remarked that a bosonlike term in Eq. (1) has been neglected which has, as its leading contribution, a $T^3\ln T$ temperature dependence.[10,16]

For the calculation of the self-energy we use the Green's function

$$G_\sigma(\vec{p},\omega)^{-1}=\omega-\epsilon_{\vec{p}}+\sigma\mu_B H-\Sigma^{\mathrm{HS}}(\vec{p},\omega,\bar{H})$$
$$-\sigma\Sigma^{\mathrm{HA}}(\vec{p},\omega,\bar{H})-\Sigma_\sigma(\vec{p},\omega,\bar{H})\,. \tag{3}$$

Here $\Sigma^{\mathrm{HS}}=\frac{1}{2}(\Sigma_\uparrow^{\mathrm{H}}+\Sigma_\downarrow^{\mathrm{H}})$ and $\Sigma^{\mathrm{HA}}=\frac{1}{2}(\Sigma_\uparrow^{\mathrm{H}}-\Sigma_\downarrow^{\mathrm{H}})$ are the symmetric and antisymmetric Hartree self-energy contributions; Σ_σ is the sum of the electron-phonon and the electron-paramagnon self-energies. We define an effective magnetic field

$$\bar{H}=H-\left(\frac{\partial}{\partial\bar{H}}\Sigma^{\mathrm{HA}}(\bar{H})\right)\bar{H}$$

and retain in Σ^{HA}, which is antisymmetric with respect to $\bar{H}$, only the term linear in $\mu_B\bar{H}$. The symmetric Hartree term can be incorporated into the chemical potential and will be neglected. The self-energy Σ_σ does not lead to a significant contribution to the effective magnetic field as is shown by Doniach and Engelsberg.[1] The self-energy Σ^{HA} is readily calculated; we get

$$\Sigma^{\mathrm{HA}}=\frac{i}{2}\int\frac{d\vec{p}d\omega}{(2\pi)^4}[G_\uparrow(\vec{p},\omega,\bar{H})-G_\downarrow(\vec{p},\omega,\bar{H})]I(0)$$
$$=-\frac{I(0)}{2}\int d\epsilon\,N(\epsilon)[f(\epsilon_{p\uparrow})-f(\epsilon_{p\downarrow})]\,, \tag{4}$$

where f_σ is the quasiparticle distribution function of the interacting system. From Eq. (4) we have

$$\frac{\partial}{\partial\bar{H}}\Sigma^{\mathrm{HA}}=+I(0)\int d\epsilon\,N(\epsilon)\frac{\partial}{\partial\epsilon}f(\epsilon)\approx-N(0)I(0)\,. \tag{5}$$

The effective field is then calculated as

$$\bar{H}=[1-N(0)I(0)]^{-1}H=SH\,, \tag{6}$$

where S is the Stoner factor. This result indicates that S should be calculated using the bare band mass. Hence the Green's function is given by.

$$G^{-1}(\vec{p}, \omega, \bar{H}) = \omega - \epsilon_{\vec{p}} - \sigma\mu_B \bar{H} - \Sigma_\sigma(\vec{p}, \omega, \bar{H}) . \quad (7)$$

The electron self-energy calculated in the zero-temperature limit is evaluated using the standard procedure and the approximations analogous to the calculation of the electron-phonon self-energy.[17] Although we do not calculate here the phonon contribution we retain it in the following to exhibit the analogy between phonons and paramagnons. The result is

$$\Sigma_\uparrow(p_{F\uparrow}, \omega, \bar{H}) = \int_0^{2p_{F\uparrow}} \frac{dq\, q}{(2\pi)^2} \frac{m}{p_{F\uparrow}} \int_0^\infty \frac{d\Omega}{\pi} [\mathrm{Im}\, g^2 D(q,\Omega) + \mathrm{Im}T^-(q,\Omega,\bar{H})] \ln\left|\frac{\Omega-\omega}{\Omega+\omega}\right| + \int_{|p_{F\uparrow}-p_{F\downarrow}|}^{p_{F\uparrow}+p_{F\downarrow}} \frac{dq\, q}{(2\pi)^2} \frac{m}{p_{F\uparrow}} \int_0^\infty \frac{d\Omega}{\pi} [\mathrm{Im}T^{-+}(q,\Omega,\bar{H}) \ln|\Omega-\omega| - \mathrm{Im}T^{+-}(q,\Omega,H)\ln|\Omega+\omega|] , \quad (8)$$

where ImD is the phonon-spectral function. $T^\pm$ and $T^{\pm\mp}$ describe the longitudinal and transverse spin fluctuations:

$$T^\mp = -\frac{I^2(q)\chi^\mp(q,\omega,\bar{H})}{1 - I^2(q)\chi^-(q,\omega,\bar{H})\chi^+(q,\omega,\bar{H})} , \quad (9)$$

$$T^{-+} = -\frac{I^2(q)\chi^{-+}(q,\omega,\bar{H})}{1 - I(q)\chi^{-+}(q,\omega,\bar{H})} , \quad (10)$$

with

$$\chi^{-+}(q,\omega,\bar{H}) = i \int \frac{d\vec{p}\, d\omega'}{(2\pi)^4} G_\downarrow(\vec{p},\omega',\bar{H}) G_\uparrow(\vec{q}+\vec{p},\omega+\omega',\bar{H}) , \quad (11)$$

and

$$\chi^\mp(q,\omega,\bar{H}) = i \int \frac{d\vec{p}\, d\omega'}{(2\pi)^4} G_\downarrow(\vec{p},\omega',\bar{H}) G_\uparrow(\vec{q}+\vec{p},\omega+\omega',\bar{H}) . \quad (12)$$

An expression analogous to Eq. (8) is obtained for the spin-down self-energy by reversing the spins (and exchanging + and −). Note that for the calculation of the susceptibilities, Eqs. (11) and (12), the Green's function, Eq. (7), is used neglecting Σ_σ. This neglect is justified as shown elsewhere.[18]

The specific-heat shift, Eq. (2), is calculated from Eq. (8). The result is

$$\Delta C_V = T k_B^2 \frac{\pi^2}{3} N(0) \left(\int_0^{2(1+h)^{1/2}} dq\, q[g^2 D(q,\Omega=0) + T^-(q,\Omega=0,h)] + \int_0^{2(1-h)^{1/2}} dq\, q[g^2 D(q,\Omega=0) + T^+(q,\Omega=0,h)] + \int_{|(1+h)^{1/2}-(1-h)^{1/2}|}^{(1+h)^{1/2}+(1-h)^{1/2}} dq\, q[T^{-+}(q,\Omega=0,h) + T^{+-}(q,\Omega=0,h)] \right) \frac{mp_F}{4\pi^2} , \quad (13)$$

where q corresponds to q/p_F and h corresponds to $\mu_B \bar{H}/\epsilon_F$.

III. TRANSPORT PROPERTIES

In this section we formulate the Boltzmann equation to describe the electron scattering from phonons and from paramagnons in the presence of a magnetic field. We consider the approach to equilibrium of the one-particle distribution function defined for the Hartree one-particle states. Following Baym and Kadanoff[19] the collision term in the Boltzmann equation is formulated by using the retarded and advanced self-energy functions $\Sigma^{\gtrless}(\vec{k},\omega)$ for the emission and readsorption of a phonon and a paramagnon taken on the bare-particle energy shell,

$$\left.\frac{\partial f_{\vec{k}\sigma}}{\partial t}\right|_{\mathrm{coll}} = -f(\epsilon_{\vec{k}\sigma})\Sigma_\sigma^>(\vec{k},\epsilon_{\vec{k}\sigma},H) + [1 - f(\epsilon_{\vec{k}\sigma})]\Sigma_\sigma^<(\vec{k},\epsilon_{\vec{k}\sigma},H) , \quad (14)$$

where $\epsilon_{\vec{k}\sigma} = \epsilon_{\vec{k}} - \sigma\mu_B H$; here and in the following we write H instead of $\bar{H}$. The retarded and advanced self-energies are written explicitly in Appendix A. The spin-down distribution function is given by

$$\left.\frac{\partial f_{\vec{k}\downarrow}}{\partial t}\right|_{\text{coll}} = \int \frac{d\vec{q}}{(2\pi)^3} \int_{-\infty}^{+\infty} \frac{d\Omega}{\pi} [\operatorname{Im} g^2 D(\vec{q},\Omega) + \operatorname{Im} T^{+}(\vec{q},\Omega,H)] \pi\delta(\epsilon_{\vec{k}-\vec{q}\downarrow} - \epsilon_{\vec{k}\downarrow} + \Omega)$$
$$\times\{ f(\epsilon_{\vec{k}\downarrow})[1-f(\epsilon_{\vec{k}-\vec{q}\downarrow})][1+n(\Omega)] - [1-f(\epsilon_{\vec{k}\downarrow})] f(\epsilon_{\vec{k}\downarrow}) n(\Omega)\}$$
$$+ \int \frac{d\vec{q}}{(2\pi)^3} \int_{-\infty}^{+\infty} \frac{d\Omega}{\pi} \operatorname{Im} T^{+-}(\vec{q},\Omega,H) \pi\delta(\epsilon_{\vec{k}-\vec{q}\uparrow} - \epsilon_{\vec{k}\downarrow} + \Omega)$$
$$\times\{ f(\epsilon_{\vec{k}\downarrow})[1-f(\epsilon_{\vec{k}-\vec{q}\uparrow})][1+n(\Omega)] - [1-f(\epsilon_{\vec{k}\downarrow})] f(\epsilon_{\vec{k}-\vec{q}\uparrow}) n(\Omega)\}. \tag{15}$$

The spin-up distribution function is given by an analogous equation. The condition for detailed balance is satisfied for the equilibrium distribution

$$f(\epsilon)[1-f(\epsilon')][1+n(\Omega)] = [1-f(\epsilon)] f(\epsilon') n(\Omega) \quad \text{if} \quad \epsilon' - \epsilon + \Omega = 0. \tag{16}$$

The Boltzmann transport equation is given by

$$-e(\vec{E} + \vec{v}_{\vec{k}\sigma} \times \vec{H}) \frac{\partial f(\epsilon_{\vec{k}\sigma})}{\partial \vec{k}} = \left.\dot{f}_{\vec{k}\sigma}\right|_{\text{coll}} . \tag{17}$$

In the following it is assumed that the phonon and paramagnon relaxation times are large in comparison with the relaxation time of the electrons. Writing the deviation of $f_{\vec{k}\sigma}$ from its equilibrium value $f^0_{\vec{k}\sigma}$ as

$$f(\epsilon_{\vec{k}\sigma}) = f^0(\epsilon_{\vec{k}\sigma}) - \frac{\partial f(\epsilon_{\vec{k}\sigma})}{\delta \epsilon_{\vec{k}\sigma}} \phi_{\vec{k}\sigma} \tag{18}$$

we obtain the linearized Boltzmann equation in the form

$$e\vec{E}\cdot\vec{v}_{\vec{k}\sigma} \frac{\partial f^0(\epsilon_{\vec{k}\sigma})}{\partial \epsilon_{\vec{k}\sigma}} = e(\vec{v}_{k\sigma} \times \vec{H}) \frac{\partial f^0(\epsilon_{\vec{k}\sigma})}{\partial \epsilon_{\vec{k}\sigma}} \frac{\partial}{\partial \vec{k}} \phi_{\vec{k}\sigma} + \frac{1}{k_B T} \sum_{\sigma'} \int \frac{d\vec{k}'}{(2\pi)^3} V(\vec{k}'\sigma', \vec{k}\sigma, H)(\phi_{\vec{k}'\sigma'} - \phi_{\vec{k}\sigma}), \tag{19}$$

where the transition rates are given by

$$V(\vec{k}'\downarrow, \vec{k}\downarrow, H) = \int d\vec{q}\,\delta(\vec{k} - \vec{k}' - \vec{q}) \int_{-\infty}^{+\infty} \frac{d\Omega}{\pi} [\operatorname{Im} g^2 D(\vec{q},\Omega) + \operatorname{Im} T^{+}(\vec{q},\Omega,H)]$$
$$\times \pi\delta(\epsilon_{\vec{k}'\downarrow} - \epsilon_{\vec{k}\downarrow} + \Omega) n(\Omega)[1 - f^0(\epsilon_{\vec{k}\downarrow})] f^0(\epsilon_{\vec{k}'\downarrow}) \tag{20}$$

and

$$V(\vec{k}'\uparrow, \vec{k}\downarrow, H) = \int d\vec{q}\,\delta(\vec{k} - \vec{k}' - \vec{q}) \int_{-\infty}^{+\infty} \frac{d\Omega}{\pi} \operatorname{Im} T^{+-}(\vec{q},\Omega,H) \pi\delta(\epsilon_{k'\uparrow} - \epsilon_{k\downarrow} + \Omega) n(\Omega)[1 - f^0(\epsilon_{k\downarrow})] f^0(\epsilon_{k'\uparrow}). \tag{21}$$

The transition rate is symmetric:

$$V(\vec{k}'\sigma', \vec{k}\sigma) = V(\vec{k}\sigma, \vec{k}'\sigma'). \tag{22}$$

We solve the Boltzmann transport equation with a variational procedure. The usual approach of solving the Boltzmann transport equation by minimizing the dissipation function breaks down in the presence of a magnetic field. The reason is that the magnetic scattering operator is not self-adjoint.[20] The variational method can be extended in the presence of a magnetic field[21] if the following dissipation function is used:

$$D = -\frac{1}{k_B T} \sum_{\sigma} \int \frac{d\vec{k}}{(2\pi)^3} \phi^*_{\vec{k}\sigma} \dot{f}(\epsilon_{\vec{k}\sigma}), \tag{23}$$

where $\phi^*_{\vec{k}\sigma}$ stands for $\phi_{\vec{k}\sigma}(-H)$, and $\phi_{\vec{k}\sigma}$ for $\phi_{\vec{k}\sigma}(H)$.

Although in the case without magnetic field one knows whether D in Eq. (23) is maximal or minimal, in the present case we can conclude only that D is stationary.

The variational problem is then solved by expanding the unknown function $\phi_{\vec{k}\sigma}$ in a set of trial functions:

$$\phi_{\vec{k}\sigma} = \sum_i \phi_i(\vec{k}\sigma) u_i, \qquad \phi^*_{\sigma} = \sum_i u^*_i \phi_i(\vec{k}\sigma). \tag{24}$$

For the calculation of the electrical conductivity the following trial function is used:

$$\phi_i(\vec{k}\sigma) = v_i(\epsilon_{\vec{k}\sigma}), \quad i = x, y, z. \tag{25}$$

The velocities v_i are independent of the spin indices assuming that the effective band masses in the two-spin states are the same. Furthermore, we assume an isotropic system.

With this trial function Eq. (25), the variational solution for the electrical resistivity tensor is given by

$$\rho(H)=\frac{1}{j_0^2(H)}\begin{bmatrix}\rho_0(H) & -h_0(H)H_z & h_0(H)H_y\\ h_0(H)H_z & \rho_0(H) & -h_0(H)H_x\\ -h_0(H)H_y & h_0(H)H_x & \rho_0(H)\end{bmatrix}, \tag{26}$$

where

$$\rho_0(H)=\frac{1}{6}\frac{1}{k_BT}\sum_{\sigma,\sigma'}\int\frac{d\vec{k}}{(2\pi)^3}\times\int\frac{d\vec{k}'}{(2\pi)^3}(\vec{v}(\vec{k}')-\vec{v}(\vec{k}))^2V(\vec{k}'\sigma',\vec{k}\sigma), \tag{27}$$

$$h_0(H)=-\frac{1}{3}\frac{e}{m}\sum_\sigma\int\frac{d\vec{k}}{(2\pi)^3}\vec{v}^2\frac{\partial f^0(\epsilon_{\vec{k}\sigma})}{\partial\epsilon_{\vec{k}\sigma}}, \tag{28}$$

$$j_0(H)=mh_0\,. \tag{29}$$

From the symmetry properties

$$\rho_0(H)=\rho_0(-H),\quad h_0(H)=h_0(-H), \tag{30}$$

it is readily seen that Onsager's reciprocity relations are satisfied: $\rho_{\alpha\beta}(H)=\rho_{\beta\alpha}(-H)$. The Hall resistivity coefficient $h_0(H)$ is calculated up to order h^2, $h=\mu_BH/\epsilon_F$, for low temperatures:

$$h_0(H)=\frac{2\sqrt{2m}}{3\pi^2}\epsilon_F^{3/2}(1+\tfrac{3}{8}h^2)\left(1+\frac{\pi^2}{8}\frac{(k_BT)^2}{\epsilon_F^2}\right). \tag{31}$$

The evaluation of the magnetoresistance coefficient $\rho_0(H)$ may be performed in the case of high and low temperatures (for details see Appendix B). For the paramagnon contribution to $\rho_0(H)$ one obtains for high temperatures

$$\rho_0(H)=f(H)k_BT\,, \tag{32}$$

where

$$f(H)=\frac{1}{6(2\pi)^3}\left(\int_0^{2p_F(1-h)^{1/2}}dq\,q^3T^+(q,0,H)+\int_0^{2p_F(1+h)^{1/2}}dq\,q^3T^-(q,0,H)+2\int_{p_F|(1+h)^{1/2}-(1-h)^{1/2}|}^{p_F[(1+h)^{1/2}+(1-h)^{1/2}]}dq\,q^3T^{-+}(q,0,H)\right). \tag{33}$$

For very low temperatures one obtains

$$\rho_0(H)=g(H)(k_BT)^2\,, \tag{34}$$

where

$$g(H)=\frac{1}{6(2\pi)^2}\frac{\pi}{3}\left[\int_0^{2p_F(1-h)^{1/2}}dq\,q^3\lim_{\Omega\to0}\left(\frac{\mathrm{Im}T^+(q,\Omega,H)}{\Omega}\right)+\int_0^{2p_F(1+h)^{1/2}}dq\,q^3\lim_{\Omega\to0}\left(\frac{\mathrm{Im}T^-(q,\Omega,H)}{\Omega}\right)+2\int_{p_F[(1+h)^{1/2}-(1-h)^{1/2}]}^{p_F[(1+h)^{1/2}+(1-h)^{1/2}]}dq\,q^3\lim_{\Omega\to0}\left(\frac{\mathrm{Im}T^{-+}(q,\Omega.H)}{\Omega}\right)\right]. \tag{35}$$

These equations for the magnetoresistance are applied to Pd and UAl_2 in the following section.

IV. RESULTS

In calculating the longitudinal and transverse susceptibilities in the presence of a magnetic field we have used the RPA expressions and assumed a parabolic band. Furthermore, we have neglected effects due to the quantization of the electron states in the presence of a magnetic field. The effect of orbital quantization is difficult to assess and has not been considered in this paper.

In Fig. 1 we show for a typical paramagnetic metal, such as Pd, the results for the effective-mass enhancement and the high- and low-temperature resistivity coefficients $\rho_0(H)$ as functions of the magnetic field $h=S\mu_BH/\epsilon_F$. The Stoner factor is assumed to be 10 and the effective-mass enhancement due to paramagnons is assumed to be 0.37. The fit of the potential parameter gives then $a^2=0.5k_F^2$. For increasing values of h the effective-mass enhancement and the low-temperature resistivity coefficient are decreasing. However, the effects in general are small, the reason being that the spin-fluctuation temperature $T_{SF}=\epsilon_F/k_BS$ is large (250 K) and, therefore, the value of h is small, of order of 0.026, for an external field of 100 kG ($\mu_BH\sim6.7\times10^{-7}k_BT$, H measured in G). For this field, the depression of the mass enhancement is smaller than 1%. However,

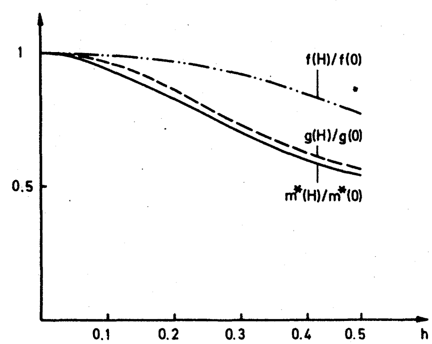

FIG. 1. The effective-mass enhancement, the high-temperature coefficient $\rho_0(H)=f(H)T$, and the low temperature coefficient $\rho_0(H)=g(H)T^2$ are shown as functions of the magnetic field $h=S\mu_B H/\epsilon_F$ for a metal with $S=10$, $m^*(H=0)=0.37$.

the situation is much improved if we consider UAl_2. The specific-heat, susceptibility, and resistivity measurements provide a unified picture of paramagnon effects in UAl_2.[5] The value of the Stoner factor S and of the mass enhancement due to spin fluctuations is found to be 4 and 0.3–0.6, respectively. The spin-fluctuation temperature T_{SF} is small, about 16 K. Hence the parameter h may be as large as 0.4 for 100 kG. So far measurements have been performed up to 43 kG corresponding to $h=0.18$, and a depression of the mass enhancement of about 1.5% has been observed. From Fig. 2 we predict a decrease of the electron-mass enhancement of about 5%. This prediction, however, depends sensitively on the value of T_{SF}. The authors of Ref. 5 claim that the low-temperature variation of χ satisfies a temperature dependence consistent with a characteristic temperature $T_{SF}=40$ K. With this value of T_{SF} we obtain a decrease of the effective mass of about 1%, consistent with the experiments. Furthermore, note that we have assumed a parabolic band. In f-band metals, such as UAl_2, band-structure effects can also be important. In order to distinguish between band-structure effects and paramagnon effects it is of some interest to perform resistivity measurements in very high fields by comparing the Hall resistivity with the normal resistivity. In the Hall resistivity, which is independent of the spin-fluctuation scattering, band-structure effects should be seen. Furthermore, we would like to remind the reader that we have used a phenomenological q-dependent exchange interaction potential. In Fig. 3 it is shown how the choice of the potential parameters influences the magnetic field dependence of the mass enhancement.

V. CONCLUSIONS

The experimental situation regarding proof of the existence of persistent spin fluctuations in uniform nearly ferromagnetic metals has much improved with the discovery that in UAl_2 paramagnons play a pertinent role for the mass enhancement of the conduction electrons. In particular, measurements in the presence of a magnetic field of 43 kG indicate a small field dependence of the electron-mass enhancement. It would be of interest to perform the experiments at larger fields, such as 100 kG or more, since, as our semiquantitative calculations show, an effect of at least several percent should be expected in UAl_2. Such measurements would provide a test of the extent to which spin fluctuations contribute to the electron-mass enhancement and to the resistivity. If band-structure effects are not the dominant feature, it should be possible to separate the phonon con-

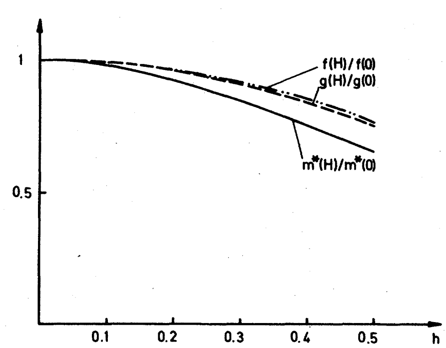

FIG. 2. The effective-mass enhancement and the high- and the low-temperature resistivity coefficients (cf. Fig. 1) are shown as functions of h for UAl_2 [$S=4$; $m^*(H=0)=0.345$].

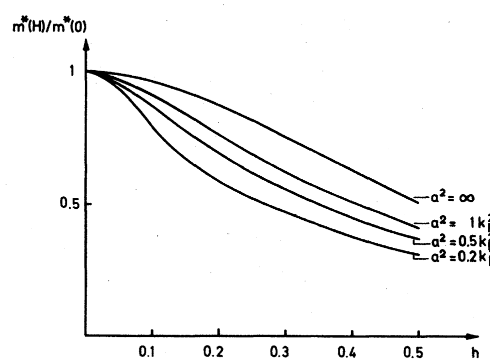

FIG. 3. The effective-mass enhancement is shown as a function of h for different values of the potential parameter a^2 for a metal with $S=50$.

tribution from the spin-fluctuation contribution to the mass enhancement since the phonon-mass enhancement is independent of the magnetic field. Similary the resistivity measurement at low temperatures in high magnetic fields should decide whether the resistivity is due to spin-fluctuation scattering or due to Baber scattering (or due to some other mechanism such as the anisotropy of the Fermi surface[22,23]). Since Baber scattering is a spin-independent scattering due to the screened Coulomb interaction whereby the spin states do not change, an appreciable dependence on the magnetic field is not to be expected, in contrast to the spin-flip scattering due to paramagnons.

APPENDIX A: THE RETARDED AND ADVANCED SELF-ENERGIES

The retarded and advanced self-energies are given by

$$\begin{aligned}\Sigma^{>}(\vec{k},\epsilon_{\vec{k}\downarrow},H) = &-\int\frac{d\vec{q}}{(2\pi)^3}\int_{-\infty}^{+\infty}\frac{d\Omega}{\pi}\,[\mathrm{Im}\,g^2D(\vec{q},\Omega)+\mathrm{Im}\,T^{+}(\vec{q},\Omega,H)]\\ &\times\pi\delta(\epsilon_{\vec{k}-\vec{q}\downarrow}-\epsilon_{\vec{k}\downarrow}+\Omega)[1+n(\Omega)][1-f(\epsilon_{\vec{k}-\vec{q}\downarrow})]\\ &-\int\frac{d\vec{q}}{(2\pi)^3}\int_{-\infty}^{+\infty}\frac{d\Omega}{\pi}\,\mathrm{Im}\,T^{+-}(\vec{q},\Omega,H)\pi\delta(\epsilon_{\vec{k}-\vec{q}\uparrow}-\epsilon_{\vec{k}\downarrow}+\Omega)[1+n(\Omega)][1-f(\epsilon_{\vec{k}-\vec{q}\uparrow})]\end{aligned} \tag{A1}$$

and

$$\begin{aligned}\Sigma^{<}(\vec{k},\epsilon_{\vec{k}\downarrow},H) = &-\int\frac{d\vec{q}}{(2\pi)^3}\int_{-\infty}^{+\infty}\frac{d\Omega}{\pi}\,[\mathrm{Im}\,g^2D(\vec{q},\Omega)+T^{+}(\vec{q},\Omega,H)]\pi\cdot\delta(\epsilon_{\vec{k}-\vec{q}\downarrow}-\epsilon_{\vec{k}\downarrow}+\Omega)\,f(\epsilon_{\vec{k}-\vec{q}\downarrow})n(\Omega)\\ &-\int\frac{d\vec{q}}{(2\pi)^3}\int_{-\infty}^{+\infty}\frac{d\Omega}{\pi}\,\mathrm{Im}\,T^{+-}(\vec{q},\Omega,H)\pi\delta(\epsilon_{\vec{k}-\vec{q}\uparrow}-\epsilon_{\vec{k}\downarrow}+\Omega)\,f(\epsilon_{\vec{k}-\vec{q}\uparrow})n(\Omega)\,.\end{aligned} \tag{A2}$$

APPENDIX B: EVALUATION OF THE RESISTIVITY COEFFICIENT $\rho_0(H)$

It is sufficient to consider the contribution from the transversal spin fluctuations, the other contributions being evaluated analogously. From Eqs. (27) and (21) one finds

$$\begin{aligned}\rho_0^{-+}(H) &= \frac{1}{6m^2k_BT}\int\frac{d\vec{k}}{(2\pi)^3}\int\frac{d\vec{k}'}{(2\pi)^3}\,(\vec{k}'-\vec{k})^2V(\vec{k}'\downarrow,\vec{k}\uparrow)\\ &= \frac{\pi}{6m^2k_BT}\int\frac{dq}{(2\pi)^3}\,q^2\int_{-\infty}^{+\infty}\frac{d\Omega}{\pi}\,\mathrm{Im}\,T^{-+}(q,\Omega,H)B(q,\Omega,H)n(\Omega)\,,\end{aligned} \tag{B1}$$

where

$$\begin{aligned}B(q,\Omega,H) &= \int\frac{d\vec{k}}{(2\pi)^3}\,\delta\left(\frac{q^2}{2m}-\frac{2\vec{k}\cdot\vec{q}}{2m}+\Omega+2H\mu_B\right)\\ &\quad\times\left(\left\{1+\exp\left[-\beta\left(\frac{k^2}{2m}-\mu_BH-\mu\right)\right]\right\}\left\{1+\exp\left[\beta\left(\frac{k^2}{2m}-\frac{2\vec{k}\cdot\vec{q}}{2m}+\mu_BH-\mu\right)\right]\right\}\right)^{-1}\\ &= \frac{m^2}{q(2\pi)^2}\int_{-\mu-\mu_BH+[q/2+m/q(\Omega+2\mu_BH)]^2/2m} d\epsilon\,[1+\exp(-\beta\epsilon)]^{-1}\{1+\exp[\beta(\epsilon-\Omega)]\}^{-1}\,.\end{aligned} \tag{B2}$$

Because of the Fermi functions the main contributions to the integral come from the region

$$|q/2+m(\Omega+2\mu_BH)/q|^2\leqslant 2m(\epsilon_F+\mu_BH)\,.$$

Since the characteristic spin-fluctuation energy is small in comparison with the Fermi energy the lower limit in Eq. (B2) may be effectively set equal to $-\infty$ if we restrict the allowed q values to the region

$$|q/2+2m\mu_BH/q|\leqslant 2m(\epsilon_F+\mu_BH)\,.$$

One then obtains

$$B(q,\Omega,H)=\frac{m^2}{q(2\pi)^2}\,\frac{\Omega}{1-\exp(-\beta\Omega)}\,. \tag{B3}$$

Inserting Eq. (B3) into Eq. (B1) yields

$$\rho_0^{-+}(H)=\frac{1}{6(2\pi)^3k_BT}\int_{q\min}^{q\max}dq\,q^3\int_{-\infty}^{+\infty}\frac{d\Omega}{\pi}\,\frac{\Omega}{[1-\exp(-\beta\Omega)][\exp(\beta\Omega)-1]}\,\mathrm{Im}T^{-+}(q,\Omega,H)\,, \tag{B4}$$

with $q_{\min}=p_F[(1+h)^{1/2}-(1-h)^{1/2}]$, $q_{\max}=p_F[(1+h)^{1/2}+(1-h)^{1/2}]$. In the case of high temperatures (but small in comparison with the Fermi energy) $\exp(\pm\beta\Omega)$ can be expanded:

$$\begin{aligned}\rho_0^{-+}(H)&=\frac{1}{(2\pi)^3}\,\frac{1}{6k_BT}\int_{q\min}^{q\max}dq\,q^3\int_{-\infty}^{+\infty}\frac{d\Omega}{\pi}\,(k_BT)^2\,\mathrm{Im}T^{-+}(q,\Omega,H)\,\frac{1}{\Omega}\\&=\frac{k_BT}{(2\pi)^3}\,\frac{1}{6}\int_{q\min}^{q\max}dq\,q^3T^{-+}(q,0,H)\,.\end{aligned} \tag{B5}$$

In the case of low temperatures (that is, small in comparison with the spin-fluctuation energy) one finds

$$\begin{aligned}\rho_0^{-+}(H)&=\frac{1}{(2\pi)^3}\,\frac{1}{6k_BT}\int_{q\min}^{q\max}dq\,q^3\int_{-\infty}^{+\infty}\frac{dx}{\pi}\,\frac{x^2(k_BT)^3}{[1-\exp(-x)][\exp(x)-1]}\,\frac{\mathrm{Im}T^{-+}(q,xk_BT,H)}{xk_BT}\\&=\frac{1}{(2\pi)^3}\,\frac{(k_BT)^2}{6}\,\frac{\pi}{3}\int_{q\min}^{q\max}dq\,q^3\lim_{y\to 0}\left(\frac{\mathrm{Im}T^{-+}(q,y,H)}{y}\right).\end{aligned} \tag{B6}$$

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