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p-wave superconductors in magnetic fields

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The possibility of vortex solutions existing for a bulk superconductor with p-wave pairing and isotropic normal-state properties is investigated on the basis of Gorkov's theory of weakly coupled superconductors. Assuming translational symmetry in the direction of the magnetic field only anisotropic states are found. Among these the polar state gives the highest upper critical field. States of the Anderson-Brinkman-Morel type are also found. Apart from the absence of Pauli paramagnetic limiting, the temperature dependence of the corresponding upper critical fields does not differ dramatically from that obtained for s-wave superconductors. Contrary to the predictions for the case of fast rotating superfluid ^{3}He, which is closely related to the problem considered here, it appears to be impossible to construct vortex lattices without singular vortex cores.

I. INTRODUCTION

Generalizations of the BCS theory of superconductivity including other than s-wave pairing have been developed and applied to ^{3}He at a very early stage.[1,2] The discovery[3] of superfluidity in ^{3}He presented an incentive to reexamine the possibility of p-wave pairing in metals. Since it is generally accepted that spin fluctuations play an important role in providing an attractive interaction between ^{3}He atoms,[4] attention was first turned to nearly ferromagnetic materials like palladium. The predicted transition temperature due to spin fluctuations alone, however, turned out to be discouragingly small,[5] and including the electron-phonon interaction did not improve the situation much,[6,7] unless some anomalous phonon properties are assumed.[8,9] Thus it is not surprising that the search for p-wave superconductivity in several transition metals has so far been unsuccessful.[10] A more promising approach might be to investigate materials like $ZrZn_2$ (Ref. 9) in which the electron-electron interaction can be changed by varying external parameters like pressure.[11]

We shall not pursue the question of which material is most likely to exhibit p-wave superconductivity any further but instead address the problem of how a p-wave superconductor differs from an s-wave superconductor, and in fact, how we could recognize a material as being a p-wave superconductor.

The most obvious difference is the presence of equal spin pairs in p-wave superconductors, which leads to different predictions for the Knight shift.[1,2] But since the Knight shift in s-wave superconductors does not always agree with the predictions of BCS theory due to a number of complicating features present in most materials,[12] measurements of the Knight shift are hardly suitable to identify a p-wave superconductor.

It has been pointed out by Balian and Werthamer[2] (BW) that their energetically most favorable isotropic state has the same thermodynamic and transport properties as an s-wave state and, in particular, shows a Meissner effect.

Some doubt has been cast on this last conclusion by Machida and Klemm[13] based on the idea that the equal spin pairs present in the p-wave state would benefit energetically from the magnetic field. However, the solution they suggest which has a finite homogeneous magnetic field inside the superconductor and vanishing currents [Ref. 13, Eq. (8)] contradicts the current equation [Ref. 13, Eq. (3)] from which it was derived. We conclude, therefore, that a p-wave superconductor in the BW state does indeed show a complete Meissner effect.

With the magnetic field excluded from the bulk of the sample the wealth of phenomena observed in superfluid ^{3}He would not be present in a p-wave superconductor. Hence, in order to detect any effects related to the presence of equal spin pairs the magnetic field must enter the sample without destroying superconductivity.

One way to achieve this is the application of the magnetic field parallel to a thin film. Thin films, however, tend to be dirty, and nonmagnetic[2,6] as well as magnetic[2,14] impurity scattering reduces the transition temperature for the onset of p-wave superconductivity.

This dependence of T_c on the presence of nonmagnetic impurities is, in fact, the only property predicted so far in which the BW state differs drastically from the s-wave state.

In addition to impurity scattering, diffuse surface scattering reduces the transition temperature for any kind of p-wave state.[15,16] Because of these restrictions it appears to be unlikely that p-wave pairs would form in a thin film.

We, therefore, decided to investigate the possibility of a type-II state existing in a p-wave superconductor, which would allow the magnetic field to penetrate the bulk of the sample in the form of vortex lines. Since for equal spin pairing the upper critical field is not limited by Pauli paramagnetism[17] we could expect very high critical fields provided the effect of the orbital diamagnetism can be reduced.

The problem considered here is very similar to the rotation[18] of superfluid ^{3}He and a few comments will be included.

In the following section the gap equation is derived from which the possible vortex solutions are obtained in Sec. III. The corresponding upper critical fields are presented in Sec. IV.

II. GAP EQUATION

In this section we shall derive an equation for the superconducting order parameter defined through

$$\Delta_{\sigma\sigma'}(\vec r,\vec r\,')=V(\vec r-\vec r\,')F_{\sigma\sigma'}(\vec r,\vec r\,',0^+) \ , \tag{1}$$

which allows for more general than s-wave pairing. $V(\vec r-\vec r\,')$ is the attractive two-body interaction of the weak-coupling theory. Using Gorkov's[19] description of weakly coupled superconductors it is straightforward to derive the equations of motion for the normal and anomalous Green's functions

$$\left\{i\omega_n-\frac{1}{2m}[\vec p+\epsilon\vec A(\vec r)]^2+E_F-\sigma\mu_B H(\vec r)\right\}G_{\sigma\sigma'}(\vec r,\vec r\,',\omega_n)+\sum_\rho\int d^3\xi\,\Delta_{\sigma\rho}(\vec r,\vec\xi)F^+_{\rho\sigma'}(\vec\xi,\vec r\,',\omega_n)=\delta(\vec r-\vec r\,')\delta_{\sigma\sigma'} \ ,$$

$$\left\{-i\omega_n-\frac{1}{2m}[\vec p-\epsilon\vec A(\vec r)]^2+E_F-\sigma\mu_B H(\vec r)\right\}F^+_{\sigma\sigma'}(\vec r,\vec r\,',\omega_n)+\sum_\rho\int d^3\xi\,\Delta^*_{\sigma\rho}(\vec r,\vec\xi)G_{\rho\sigma'}(\vec\xi,\vec r\,',\omega_n)=0 \ , \tag{2}$$

appropriate for a charged superfluid in a magnetic field $\vec H(\vec r)$, which is chosen to be parallel to the z axis. The spin variable takes on the value $+1$ (or $\uparrow$) and -1 (or $\downarrow$) and ϵ is the absolute value of the electron charge.

Using the normal-state Green's function obtainable from

$$\left\{i\omega_n-\frac{1}{2m}[\vec p+\epsilon\vec A(\vec r)]^2+E_F-\sigma\mu_B H(\vec r)\right\}G^0_{\sigma\sigma'}(\vec r,\vec r\,',\omega_n)=\delta(\vec r-\vec r\,')\delta_{\sigma\sigma'} \ , \tag{3}$$

which is diagonal with respect to spin indices, we can rewrite Eq. (2) in the form

$$\begin{aligned}G_{\sigma\sigma'}(\vec r,\vec r\,',\omega_n)&=G^0_{\sigma\sigma'}(\vec r,\vec r\,',\omega_n)\delta_{\sigma\sigma'}-\int d^3\xi'\,d^3\xi\sum_\rho G^0_{\sigma\sigma}(\vec r,\vec\xi\,',\omega_n)\Delta_{\rho\sigma}(\vec\xi\,',\vec\xi)F^+_{\rho\sigma'}(\vec\xi,\vec r\,',\omega_n) \ ,\\ F_{\sigma\sigma'}(\vec r,\vec r\,',\omega_n)&=\int d^3\xi'\,d^3\xi\sum_\rho G^0_{\sigma'\sigma'}(\vec r\,',\vec\xi\,',-\omega_n)\Delta_{\rho\sigma'}(\vec\xi,\vec\xi\,')G_{\sigma\rho}(\vec r,\vec\xi,\omega_n) \ .\end{aligned} \tag{4}$$

From Eqs. (1) and (4) we obtain the gap equation

$$\Delta_{\sigma\sigma'}(\vec r,\vec r\,')=V(\vec r-\vec r\,')T\sum_{\omega_n}\sum_\rho\int d^3\xi'\,d^3\xi\,G^0_{\sigma'\sigma'}(\vec r\,',\vec\xi\,',-\omega_n)\Delta_{\rho\sigma'}(\vec\xi,\vec\xi\,')G_{\sigma\rho}(\vec r,\vec\xi,\omega_n) \ . \tag{5}$$

With $V(\vec r-\vec r\,')=g\delta(\vec r-\vec r\,')$ and $\Delta_{\sigma\sigma'}(\vec r,\vec r\,')=\sigma\Delta(\vec r)\delta(\vec r-\vec r\,')\delta_{\sigma,-\sigma'}$ this reduces to the well-known expression for s-wave superconductors.[20]

Even with the simplifying assumption that the magnetic field is homogeneous, which limits our discussion to the vicinity of the upper critical field, the normal-state Green's function still depends on both its position variables separately. We, therefore, define new Green's functions differing from the old ones by the usual phase factor[21]:

$$\bar G_{\sigma\sigma'}(\vec r,\vec r\,',\omega_n)=e^{i\epsilon\phi(\vec r,\vec r\,')}G_{\sigma\sigma'}(\vec r,\vec r\,',\omega_n) \ , \tag{6}$$

$$\phi(\vec r,\vec r\,')=\int_{\vec r\,'}^{\vec r}d\vec l\cdot\vec A(\vec l) \ . \tag{7}$$

Instead of Eq. (3) we now have

$$\left\{ i\omega_n - \frac{1}{2m}[\vec{p} - \epsilon(\vec{r}-\vec{r}\,')\times\vec{H}]^2 + E_F - \sigma\mu_B H \right\} \times \bar{G}^0_{\sigma\sigma'}(\vec{r}-\vec{r}\,',\omega_n) = \delta(\vec{r}-\vec{r}\,')\delta_{\sigma\sigma'} \ . \quad (8)$$

The semiclassical approximation, which we shall adopt, consists in neglecting the term $(\vec{r}-\vec{r}\,')\times\vec{H}$ in Eq. (8) and thereby omitting Landau levels from our consideration. As discussed by Werthamer[20] we expect this approximation to be valid if either impurity or thermal smearing of the Landau levels is greater than the separation of neighboring levels. Near the zero-field transition temperature, thermal smearing would certainly justify the semiclassical approximation, but since we want to consider pure p-wave superconductors, the smearing argument does not hold for the whole temperature range. However, the oscillations of the upper critical field as calculated by Gruenberg and Gunther[22] turn out to be very small indeed provided the cyclotron frequency is much smaller than the Fermi energy and a finite electron lifetime is introduced, which can be assumed, though, to be much too large to noticeably affect the transition temperature of the p-wave superconductor.[6]

The approximate solution of Eq. (8) with which we shall therefore work is[20]:

$$\bar{G}^0_{\sigma\sigma}(\vec{r}-\vec{r}\,',\omega_n) = -\frac{m}{2\pi|\vec{r}-\vec{r}\,'|}\exp\{ip_F|\vec{r}-\vec{r}\,'|\,[1+(i\omega_n-\sigma\mu_B H)/E_F]^{1/2}\,\mathrm{sgn}\,\omega_n\} \ , \quad (9)$$

or in Fourier space

$$\bar{G}^0_{\sigma\sigma}(\vec{k},\omega_n) = (i\omega_n - \vec{k}^2/2m + E_F - \sigma\mu_B H)^{-1} \ . \quad (10)$$

In view of the short range of the interaction $V(\vec{r}-\vec{r}\,')$ we shall regard the order parameter Eq. (1) as being proportional to $\delta(\vec{r}-\vec{r}\,')$ whenever it is multiplied by a slowly varying function. This is clearly equivalent to the assumption used by Tewordt[23] in his derivation of generalized Ginzburg-Landau equations for superfluid ^{3}He, that the derivatives of

$$\Delta_{\sigma\sigma'}(\vec{r}\,',\vec{k}) = \int d^3r\, e^{-i\vec{k}\cdot(\vec{r}-\vec{r}\,')}\,\Delta_{\sigma\sigma'}(\vec{r}\,',\vec{r}-\vec{r}\,') \ , \quad (11)$$

with respect to $\vec{k}$ are negligible. Because the order parameter itself is a slowly varying function of the center-of-mass coordinate $\frac{1}{2}(\vec{r}+\vec{r}\,')$ we took the liberty to replace this variable by $\vec{r}\,'$ in Eq. (11). In fact, the only dependence on position variables that cannot be considered a slow variation is the dependence of the Green's function on $\vec{r}-\vec{r}\,'$. We can therefore simplify Eq. (5) to

$$\Delta_{\sigma\sigma'}(\vec{r}\,',\vec{r}-\vec{r}\,') = V(\vec{r}-\vec{r}\,')T\sum_{\omega_n}\sum_{\rho}\int d^3\xi'\,d^3\xi\,\bar{G}^0_{\sigma'\sigma'}(\vec{r}\,'-\vec{\xi}\,',-\omega_n) \times e^{2i\epsilon\phi(\vec{\xi}\,',\vec{r}\,')}\Delta_{\rho\sigma'}(\vec{\xi}\,',\vec{\xi}-\vec{\xi}\,')\bar{G}_{\sigma\rho}[\tfrac{1}{2}(\vec{r}\,'+\vec{\xi}\,'),\vec{r}-\vec{\xi},\omega_n] \ . \quad (12)$$

We note that

$$\phi(\xi',r') = \phi[\vec{\xi}\,',\tfrac{1}{2}(\vec{r}\,'+\vec{\xi}\,')] + \phi[\tfrac{1}{2}(\vec{r}\,'+\vec{\xi}\,'),\vec{r}\,']$$

and exploit an identity first derived by Helfand and Werthamer[24] to write this as

$$\Delta_{\sigma\sigma'}(\vec{r}\,',\vec{r}-\vec{r}\,') = V(\vec{r}-\vec{r}\,')T\sum_{\omega_n}\sum_{\rho}\int d^3\xi'\,d^3\xi\,\bar{G}^0_{\sigma'\sigma'}(\vec{r}\,'-\vec{\xi}\,',-\omega_n)\exp[\tfrac{1}{2}i(\vec{\xi}\,'-\vec{r}\,')\cdot\vec{\Pi}(\vec{R})] \times\left(\{\exp[\tfrac{1}{2}i(\vec{\xi}\,'-\vec{r}\,')\cdot\vec{\Pi}(\vec{R})]\Delta_{\rho\sigma'}(\vec{R},\vec{\xi}-\vec{\xi}\,')\}\bar{G}_{\sigma\rho}(\vec{R},\vec{r}-\vec{\xi},\omega_n)\right)_{R=r'} \ , \quad (13)$$

with

$$\vec{\Pi}\cdot(\vec{R}) = \frac{1}{i}\vec{\nabla} + 2\epsilon\vec{A}\,(\vec{R}) \ . \quad (14)$$

There are (at least) three routes along which Eq. (13) can be further simplified. Two of them involve explicit assumptions on the p-wave symmetry of the interaction and the order parameter and the third makes use of the explicit form Eqs. (9) and (10) of the normal-state Green's function. This third approach is presented in Appendix B. Here we want to briefly discuss and compare the first two methods; further details are given in Appendix A.

From Eq. (13) we obtain the following equation for the Fourier transform Eq. (11) of the order parameter:

$$\Delta_{\sigma\sigma'}(\vec{R},\vec{k})=\int\frac{d^3k'}{(2\pi)^3}V(\vec{k}-\vec{k}')T \times\sum_{\omega_n}\sum_{\rho}\int d^3\xi'\,d^3\xi\,e^{i\vec{k}'\cdot(\vec{\xi}'-\vec{\xi})}\bar{G}^{0}_{\sigma'\sigma'}(\xi,-\omega_n)\exp[\tfrac{1}{2}i\vec{\xi}\cdot\vec{\Pi}(\vec{R})] \times\left(\{\exp[\tfrac{1}{2}i\vec{\xi}\cdot\vec{\Pi}(\vec{R})]\Delta_{\rho\sigma'}(\vec{R},\vec{k}')\}\bar{G}_{\sigma\rho}(\vec{R},\vec{\xi}',\omega_n)\right)\ . \tag{15}$$

Approximating $V(\vec{k}-\vec{k}')$ by a point interaction of p symmetry[2]

$$V(\vec{k}-\vec{k}')=3g\hat{k}\cdot\hat{k}' \tag{16}$$

and expanding the order parameter as

$$\Delta_{\sigma\sigma'}(\vec{R},\vec{k})=\sum_{i=1}^{3}\Delta^{(i)}_{\sigma\sigma'}(\vec{R})\,\hat{k}_i\ , \tag{17}$$

where $\hat{k}_i$ is the ith component of the unit vector $\hat{k}$, Eq. (15) can be reduced to

$$\Delta^{(i)}_{\sigma\sigma'}(\vec{R})=3gT\sum_{\omega_n}\sum_{\rho}\sum_{j=1}^{3}\int d^3\xi'\,d^3\xi\int\frac{d^3k}{(2\pi)^3}\hat{k}_i\,\hat{k}_j e^{i\vec{k}\cdot(\vec{\xi}'-\vec{\xi})}\bar{G}^{0}_{\sigma'\sigma'}(\xi,-\omega_n)\exp[\tfrac{1}{2}i\vec{\xi}\cdot\vec{\Pi}(\vec{R})] \times\left(\{\exp[\tfrac{1}{2}i\vec{\xi}\cdot\vec{\Pi}(\vec{R})]\Delta^{(j)}_{\rho\sigma'}(\vec{R})\}\bar{G}_{\sigma\rho}(\vec{R},\vec{\xi}',\omega_n)\right)\ . \tag{18}$$

Ambegaokar *et al.*[15] who considered the linearized version of Eq. (18) for a neutral superfluid, replaced $\hat{k}_j$ by k_j/k_F. This permits the introduction of $(1/i)\nabla_j$ in place of k_j. It is then possible to integrate by parts and their Eqs. (21) and (22) are obtained without difficulty. Their expression (22) for the kernel of the gap equation has, however, a rather unsatisfactory feature: if one simply inserts the normal-state Green's function Eq. (9) one does not obtain their final and correct result Eq. (29) for a bulk system. The origin of this difficulty is the replacement of $\hat{k}_j$ by k_j/k_F, which introduces additional singularities.

To avoid these problems we tried to evaluate the k integral in Eq. (18) directly. In Appendix A we derive the approximate result

$$\int d^3k\,\hat{k}_i\,\hat{k}_j e^{i\vec{k}\cdot(\vec{\xi}'-\vec{\xi})}=(2\pi)^3\hat{\xi}_i\,\hat{\xi}_j\delta(\vec{\xi}-\vec{\xi}')\ . \tag{19}$$

With this result we can drastically simplify Eq. (18):

$$\Delta^{(i)}_{\sigma\sigma'}(\vec{R})=3gT\sum_{\omega_n}\sum_{\rho}\sum_{j=1}^{3}\int d^3\xi\,\hat{\xi}_i\hat{\xi}_j\bar{G}^{0}_{\sigma'\sigma'}(\vec{\xi},-\omega_n) \times\exp[\tfrac{1}{2}i\vec{\xi}\cdot\vec{\Pi}(\vec{R})]\left(\{\exp[\tfrac{1}{2}i\vec{\xi}\cdot\vec{\Pi}(\vec{R})]\Delta^{(j)}_{\rho\sigma'}(\vec{R})\}\bar{G}_{\sigma\rho}(\vec{R},\vec{\xi},\omega_n)\right)\ . \tag{20}$$

This differs from the corresponding equation of an s-wave superconductor only by the presence of the direction cosines $\hat{\xi}_i$ and $\hat{\xi}_j$ and, of course, by the fact that we now have three coupled equations for the three orbital components of the order parameter. From Eqs. (20) and (4) Ginzburg-Landau equations and their extensions can be derived.[15,23]

III. VORTEX SOLUTIONS

In this section we shall show that solutions of Eq. (20) can be obtained which correspond to the Abrikosov solution[25] for type-II superconductors with s-wave pairing. As in the s-wave case we linearize Eq. (20) and insert the normal-state Green's function Eq. (9) to obtain[15,20]

$$\Delta^{(i)}_{\sigma\sigma'}(\vec{R}) = \frac{3gm^2}{4\pi^2} T \sum_{\omega_n} \sum_j \int d^3\xi \, \frac{\xi_i \xi_j}{\xi^4} \exp\{-\xi[2|\omega_n| + i\,\mathrm{sgn}\omega_n \mu_B H(\sigma-\sigma')]/v_F\} e^{i\vec{\xi}\cdot\vec{\Pi}(\vec{R})} \Delta^{(j)}_{\sigma\sigma'}(\vec{R}) \; . \tag{21}$$

Here we have expanded the square root in Eq. (9) to lowest order in ω_n/E_F thus neglecting the splitting of the Fermi level due to the Pauli term. If the magnetic fields are such that this effect becomes important, one probably also needs to worry about the magnetic field dependence of the pairing interaction $V(\vec{r}-\vec{r}\,')$ in Eq. (1). As was to be expected we can see from Eq. (21) that to this order Pauli paramagnetism does not affect equal spin pairing.

The solution of the eigenvalue problem Eq. (21) proceeds along the lines of Helfand and Werthamer[24]: we introduce spherical coordinates with the direction of the magnetic field as polar axis and then use[20]

$$e^{i\vec{\xi}\cdot\vec{\Pi}(\vec{R})}\Delta^{(j)}_{\sigma\sigma'}(\vec{R}) = \exp[(-\epsilon H/2)\xi^2 \sin^2\Theta] \exp(i 2^{-1/2}\xi \sin\Theta e^{-i\phi}\Pi_+)$$
$$\times \exp(i 2^{-1/2}\xi \sin\Theta e^{+i\phi}\Pi_-) \exp(i\xi\cos\Theta \Pi_z)\Delta^{(j)}_{\sigma\sigma'}(\vec{R}) \; , \tag{22}$$

with

$$\Pi_\pm(\vec{R}) = \frac{1}{\sqrt{2}}[\Pi_x(\vec{R}) \pm i\Pi_y(\vec{R})] \; .$$

Because of the symmetry of the problem it seems reasonable to expect that the energetically most favorable solution is independent of $z = R_3$. For such an order parameter the operator $\exp(i\xi\cos\Theta\Pi_z)$ reduces to the identity so that the kernel of Eq. (21) is an odd function of $x = \cos\Theta$ whenever $i = 1, 2$ and $j = 3$ or vice versa. The $i = 3$ component of the order parameter therefore decouples from the $i = 1, 2$ components.

If we, therefore, take $\Delta^{(1)}_{\sigma\sigma'}(\vec{R}) = \Delta^{(2)}_{\sigma\sigma'}(\vec{R}) = 0$ and $\Delta^{(3)}_{\sigma\sigma'}(\vec{R})$ proportional to $f_{p_y,0,N}(\vec{R})$, where[26]

$$f_{p_y,p_z,N}(\vec{R}) = \frac{e^{-ip_z Z} e^{-ip_y Y}}{[(\pi/2\epsilon H)^{1/2} 2^N N!]^{1/2}} \exp[-\epsilon H (X - p_y/2\epsilon H)^2] H_N[(2\epsilon H)^{1/2}(X - p_y/2\epsilon H)] \tag{23}$$

and then use

$$\begin{pmatrix}\Pi_+\\ \Pi_-\end{pmatrix} f_{p_y,p_z,N}(\vec{R}) = \begin{pmatrix}(N+1)^{1/2}\\ \sqrt{N}\end{pmatrix} (2\epsilon H)^{1/2} f_{p_y,p_z,N\pm 1}(\vec{R}) \; , \tag{24}$$

we can reduce Eq. (21) to

$$1 = \frac{gm^2}{2\pi}\int_0^\infty d\xi \int_0^\pi d\Theta \sin\Theta \, 3\cos^2\Theta \, T \sum_{\omega_n} \exp\{-\xi[2|\omega_n| + i\,\mathrm{sgn}\omega_n \mu_B H(\sigma-\sigma')]/v_F\}$$
$$\times \exp[-(\epsilon/2)H\xi^2\sin^2\Theta] L_N(\epsilon H \xi^2 \sin^2\Theta) \; . \tag{25}$$

This equation determines the upper critical field. In Eqs. (23) and (25), H_N and L_N are Hermite and Laguerre polynomials, respectively, and $\vec{R} \equiv (X,Y,Z)$.

Since the functions Eq. (23), which form a complete orthonormal set, are degenerate with respect to p_y, we can choose as a solution for $\Delta^{(3)}_{\sigma\sigma'}(\vec{R})$ linear combinations such as[26]

$$g_{\vec{p},N}(\vec{R}) = \left(\frac{2\pi}{\epsilon H}\right)^{1/4} \sum_{n=-\infty}^{+\infty} e^{-ip_x x_0 n} f_{p_y+nk,p_z,N}(\vec{R}) \; , \tag{26}$$

which again form a complete orthonormal set. Choosing $k = (4\pi\epsilon H)^{1/2}$ and $kx_0 = 2\pi$ these functions transform under a translation through the vector

$$\vec{a} = [n_x(\pi/\epsilon H)^{1/2}, n_y(\pi/\epsilon H)^{1/2}, a_z] \; , \quad n_x, n_y \text{ integer} \tag{27}$$

according to

$$g_{\vec{p},N}(\vec{R} + \vec{a}) = e^{-i\vec{p}\cdot\vec{a}} e^{-i2\epsilon H a_x Y} g_{\vec{p},N}(\vec{R}) \tag{28}$$

and therefore represent a square vortex lattice. For $\vec{p} = 0$ and $N = 0$ this is the Abrikosov solution.[25]

If the Pauli term is included in Eq. (21), the formation of pairs with antiparallel spins is suppressed and we expect the order-parameter matrix to be of the form

$$\tilde{\Delta}(\vec{R},\vec{k}) \equiv \begin{pmatrix} \Delta_{\uparrow\uparrow}(\vec{R},\vec{k}) & \Delta_{\uparrow\downarrow}(\vec{R},\vec{k}) \\ \Delta_{\downarrow\uparrow}(\vec{R},\vec{k}) & \Delta_{\downarrow\downarrow}(\vec{R},\vec{k}) \end{pmatrix} = \Delta^{(3)}(\vec{R})\sqrt{3}\hat{k}_3 \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} . \tag{29}$$

In vector notation[27] this would read

$$d_x(\hat{k}) = \sqrt{3}\hat{k}_3 . \tag{30}$$

This is the so-called "polar" state.[27]

If the Pauli term is neglected, all three spin components of the order parameter can occur simultaneously and the following unitary state can be constructed

$$\tilde{\Delta}(\vec{R},\vec{k}) = \Delta^{(3)}(\vec{R})\hat{k}_3 \begin{pmatrix} -1+i & 1 \\ 1 & 1+i \end{pmatrix} , \tag{31}$$

or in vector notation,

$$d_x(\hat{k}) = d_y(\hat{k}) = d_z(\hat{k}) = \hat{k}_3 . \tag{32}$$

Ignoring the spatial dependence Eq. (26) of $\Delta^{(3)}(\vec{R})$ for the moment we note that this particular p-wave state has no gap in the excitation spectrum for quasiparticles traveling parallel to the z axis. In the remainder of this paper we shall omit the Pauli term from Eq. (21). At the same time we shall consider only states with equal spin pairing.

We now turn to a discussion of Eq. (21) for $i=1$, 2 to see whether some novel type of solution emerges. The functions Eq. (23) are no longer solutions of the eigenvalue problem Eq. (21) because, due to the presence of the azimuthal angle ϕ in the direction cosines, the operator in Eq. (21) generates $f_{p_y,0,N\pm 2}(\vec{R})$ when acting on $f_{p_y,0,N}(\vec{R})$. However, inserting the expansion

$$\Delta^{(i)}_{\sigma\sigma'}(\vec{R}) = \sum_{N=0}^{\infty} a_N^{(i)} g_{p_x,p_y,0,N}(\vec{R}) \tag{33}$$

and using Eqs. (24) and (26) we transform Eq. (21) to an algebraic system of equations which can be simplified to

$$\begin{pmatrix} 1-\alpha_0^{(1,2)} & 0 & 0 & 0 & 0 & \cdots \\ 0 & 1-\alpha_0^{(1,2)} & -\beta_0 & 0 & 0 & \cdots \\ 0 & -\beta_0 & 1-\alpha_2^{(1,2)} & 0 & 0 & \cdots \\ 0 & 0 & 0 & 1-\alpha_2^{(1,2)} & -\beta_2 & \cdots \\ 0 & 0 & 0 & -\beta_2 & 1-\alpha_4^{(1,2)} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \end{pmatrix} \begin{pmatrix} a_0^{(1)}+ia_0^{(2)} \\ a_0^{(1)}-ia_0^{(2)} \\ a_2^{(1)}+ia_2^{(2)} \\ a_2^{(1)}-ia_2^{(2)} \\ a_4^{(1)}+ia_4^{(2)} \\ \vdots \end{pmatrix} = 0 . \tag{34}$$

Another identical system of equations is obtained for odd N. For clarity, spin indices have been suppressed. The coefficients $\alpha_N^{(1,2)}$ and β_N are given by

$$\alpha_N^{(1,2)} = \frac{gm^2}{2\pi}\int_0^\infty d\xi \int_0^\pi d\Theta\, \tfrac{3}{2}\sin^3\Theta\, T\sum_{\omega_n}\exp(-2\xi|\omega_n|/v_F)\exp[(-\epsilon/2)H\xi^2\sin^2\Theta]L_N(\epsilon H\xi^2\sin^2\Theta) , \tag{35}$$

$$\beta_N = \frac{gm^2}{2\pi}\int_0^\infty d\xi \int_0^\pi d\Theta\, \tfrac{3}{2}\sin^3\Theta\, T\sum_{\omega_n}\exp(-2\xi|\omega_n|/v_F)\exp[(-\epsilon/2)H\xi^2\sin^2\Theta] \times \sum_{m=0}^{N}(-\epsilon H\xi^2\sin^2\Theta)^{m+1}\binom{N}{m}\frac{[(N+1)(N+2)]^{1/2}}{(m+2)!} . \tag{36}$$

The sum in Eq. (36) could be expressed as a generalized Laguerre polynomial.

The simplest solution possible, which requires

$$\alpha_0^{(1,2)} = 1 \tag{37}$$

evidently is

$$a_0^{(1)} = ia_0^{(2)}, \quad a_N^{(i)} = 0 \text{ for } N > 0 . \tag{38}$$

This corresponds to an axial or Anderson-Brinkman-Morel (ABM) state[1,27]

$$\tilde{\Delta}(\vec{R},\vec{k}) = \Delta^{(1,2)} g_{p_x,p_y,0,0}(\vec{R}) \times (\tfrac{3}{2})^{1/2} \begin{pmatrix} -\hat{k}_1+i\hat{k}_2 & 0 \\ 0 & -\hat{k}_1+i\hat{k}_2 \end{pmatrix} , \tag{39}$$

where $\Delta^{(1,2)}$ is a constant. In vector notation[27] this would be written as

$$\Delta^{(1,2)}|g_{p_x,p_y,0,0}(\vec{R})|\,(\tfrac{3}{2})^{1/2}\hat{d}(\hat{m}\cdot\hat{k}+i\hat{n}\cdot\hat{k}) \; . \quad (40)$$

The unit vectors $\hat{m}$ and $\hat{n}$ are given by

$$\hat{m}=\begin{pmatrix}-\cos\theta(\vec{R})\\ \sin\theta(\vec{R})\\ 0\end{pmatrix},\quad \hat{n}=\begin{pmatrix}\sin\theta(\vec{R})\\ \cos\theta(\vec{R})\\ 0\end{pmatrix}\; , \quad (41)$$

where $\theta(\vec{R})$ is the phase of $\Delta^{(1,2)}g_{p_x,p_y,0,0}(\vec{R})$, which is an oscillatory function of $\vec{R}$. However, the third vector of the triad, $\hat{l}=\hat{m}\times\hat{n}=(0,0,-1)$ remains fixed and is uniquely determined by Eq. (38). Hence, the freedom in the choice of $\hat{l}$ assumed by Fujita *et al.*,[28] which allowed these authors to construct singularity-free vortex lattices for the case of fast rotating superfluid ^{3}He, does not exist.

In addition to this solution, which as far as the spatial dependence of the order-parameter amplitude is concerned, is again identical to the Abrikosov vortex lattice,[25] we can find a different type of solution for any N provided

$$(1-\alpha_N^{(1,2)})(1-\alpha_{N+2}^{(1,2)})=\beta_N^2 \; . \quad (42)$$

These solutions require

$$a_N^{(1)}=-ia_N^{(2)}\; ,\quad a_{N+2}^{(1)}=ia_{N+2}^{(2)}\; ,$$
$$a_M^{(1)}=a_M^{(2)}=0 \text{ if } M<N \text{ or } M>N+2\; . \quad (43)$$

The two equations for $a_N^{(1)}$ and $a_{N+2}^{(1)}$ that remain to be solved yield

$$a_{N+2}^{(1)}=\frac{1-\alpha_N^{(1,2)}}{\beta_N}a_N^{(1)}\; . \quad (44)$$

This gives an order parameter of the ABM type Eq. (39) which, even for $N=0$ has a more complex spatial dependence:

$$\vec{\Delta}(\vec{R},\vec{k})\propto\left[\left(g_{\vec{p},N}(\vec{R})+\frac{1-\alpha_N^{(1,2)}}{\beta_N}g_{\vec{p},N+2}(\vec{R})\right)\hat{k}_1+\left(g_{\vec{p},N}(\vec{R})-\frac{1-\alpha_N^{(1,2)}}{\beta_N}g_{\vec{p},N+2}(\vec{R})\right)i\hat{k}_2\right]\begin{pmatrix}-1&0\\0&1\end{pmatrix}\; , \quad (45)$$

$g_{\vec{p},N}(\vec{R})$ is given by Eq. (26) with $p_z=0$.

In the following section we shall calculate the upper critical field for the various solutions which we have just obtained.

IV. UPPER CRITICAL FIELD

The upper critical field is obtained from Eqs. (25), (37), or (42). Neglecting the Pauli term we define the right-hand side of Eq. (25) as $\alpha_N^{(3)}$ and include for comparison the s-wave case. We then need to calculate

$$\alpha_N^{(\lambda)}=T\sum_{\omega_n}\int d^3\xi\, Q_N^{(\lambda)}(\vec{\xi})\exp[-(\epsilon/2)H(\xi_x^2+\xi_y^2)]\; , \quad (46)$$

where

$$\begin{pmatrix}Q_N^{(s)}\\ Q_N^{(3)}\\ Q_N^{(1,2)}\end{pmatrix}=\frac{gm^2}{4\pi^2}\frac{1}{\xi^4}\begin{pmatrix}\xi^2\\ 3\xi_z^2\\ \frac{3}{2}(\xi_x^2+\xi_y^2)\end{pmatrix}\times\exp(-2\xi|\omega_n|/v_F)L_N[\epsilon H(\xi_x^2+\xi_y^2)]\; . \quad (47)$$

Taking the Fourier transform of $Q_N^{(\lambda)}(\vec{\xi})$, introducing the zero-field transition temperature $T_c(0)$ and defining dimensionless variables[24]

$$t=T/T_c(0)\; , \quad (48)$$

$$h=2\epsilon H[v_F/2\pi T_c(0)]^2\; , \quad (49)$$

we obtain for $\alpha_N^{(\lambda)}$:

$$\begin{pmatrix}\alpha_N^{(s)}\\ \alpha_N^{(3)}\\ \alpha_N^{(1,2)}\end{pmatrix}=1+N(0)g\ln\frac{1}{t}+N(0)g\times\sum_{n=-\infty}^{+\infty}\int_0^\infty dp\,pe^{-p^2}\int_{-1}^{+1}dx\int_0^{2\pi}\frac{d\psi}{2\pi}\begin{pmatrix}1\\ 3(1-x^2)\cos^2\psi\\ \frac{3}{2}-\frac{3}{2}(1-x^2)\cos^2\psi\end{pmatrix}$$
$$\times\left[\sum_{m=0}^{N}\binom{N}{m}\frac{(2m)!}{m!}\left(-\frac{h}{2t^2}\right)^m\frac{[1-(1-x^2)\cos^2\psi]^m}{(|2n+1|+i\sqrt{h}px/t)^{2m+1}}-\frac{1}{|2n+1|}\right]\; . \quad (50)$$

For β_N Eq. (36) an expression similar to the one for $\alpha_N^{(1,2)}$ is obtained. Expanding Eq. (50) in powers of $\sqrt{h}/t$ we find the slopes of the upper critical fields near $T_c(0)$:

$$\begin{pmatrix} -dh_N^{(s)}/dt \\ -dh_N^{(3)}/dt \end{pmatrix} = \frac{1}{2N+1}\frac{4}{7\zeta(3)}\begin{pmatrix} 3 \\ 5 \end{pmatrix} . \tag{51}$$

For both s-wave and polar p-wave state [Eq. (29)] order parameters of the form Eq. (26) with $N=0$ (and $p_z=0$) give the highest slope. For the ABM state Eq. (39) we find

$$-dh_0^{(1,2)}/dt = \frac{4}{7\zeta(3)}\frac{5}{2} . \tag{52}$$

However, for the state Eq. (45) the slope is

$$-dh_N^{(1,2)}/dt = \frac{4}{7\zeta(3)}\frac{5}{2}\frac{2N+3+(N^2+3N+6)^{1/2}}{3(N^2+3N+1)} , \tag{53}$$

which for $N=0$ is only slightly smaller than the value obtained for the polar state.

The critical field at zero temperature is also easily obtained from Eq. (50) by rewriting the sums over n as polygamma functions and then using their asymptotic expansions. The results for $N=0$ are

$$\begin{pmatrix} h_0^{(s)}(0) \\ h_0^{(3)}(0) \\ h_0^{(1,2)}(0) \end{pmatrix} = \frac{e^2}{4\gamma}\begin{pmatrix} 1 \\ e^{2/3} \\ e^{-1/3} \end{pmatrix} = \begin{pmatrix} 1.037 \\ 2.020 \\ 0.743 \end{pmatrix} . \tag{54}$$

The state Eq. (45) again gives for $N=0$ a higher value than $h_0^{(1,2)}(0)$:

$$\tilde{h}_0^{(1,2)}(0) = \frac{e^2}{4\gamma}e^{-1/3}e^{\sqrt{3}-1} = 1.545 . \tag{55}$$

$h_0^{(\lambda)}(t)$ is calculated numerically for the whole temperature range from

$$\ln t = \sum_{n=-\infty}^{+\infty}\left[s_0^{(\lambda)}(\omega_n) - \frac{1}{|2n+1|}\right] , \tag{56}$$

with

$$\begin{pmatrix} s_0^{(s)}(\omega_n) \\ s_0^{(3)}(\omega_n) \\ s_0^{(1,2)}(\omega_n) \end{pmatrix} = 2\frac{t}{\sqrt{h}}\int_0^\infty du\, e^{-u^2}\begin{pmatrix} \tan^{-1}\alpha_\omega u \\ \frac{3}{2}\{[1+(\alpha_\omega u)^{-2}]\tan^{-1}\alpha_\omega u - (\alpha_\omega u)^{-1}\} \\ \frac{3}{4}\{[1-(\alpha_\omega u)^{-2}]\tan^{-1}\alpha_\omega u + (\alpha_\omega u)^{-1}\} \end{pmatrix} , \tag{57}$$

$$\alpha_\omega = \sqrt{h}/t|2n+1| , \tag{58}$$

which is obtained from Eq. (50) by integrating with respect to x.

For the generalized ABM state [Eq. (45)] $\ln t$ is obtained from a quadratic equation with coefficients somewhat more complicated than Eq. (57) which shall not be given here. The results of the numerical calculations are shown in Fig. 1. We note that the polar state Eq. (29) gives the highest critical field with the generalized ABM state Eq. (45) a close second. The simple ABM state Eq. (39) has the lowest upper critical field.

A criterion for the stability of the polar vortex state against formation of a field free configuration is obtained by comparing the slope at $T_c(0)$ with the slope of the thermodynamic critical field[2,20] of the BW state

$$\frac{dH_{c2}/dt}{dH_c/dt} = \frac{5}{3}\sqrt{2}\kappa . \tag{59}$$

The parameter κ introduced here is the same as that used in BCS theory.[20] The criterion for type-II superconductivity in the presence of p-wave pairing, therefore, is

$$\kappa > \frac{3}{5}\frac{1}{\sqrt{2}} .$$

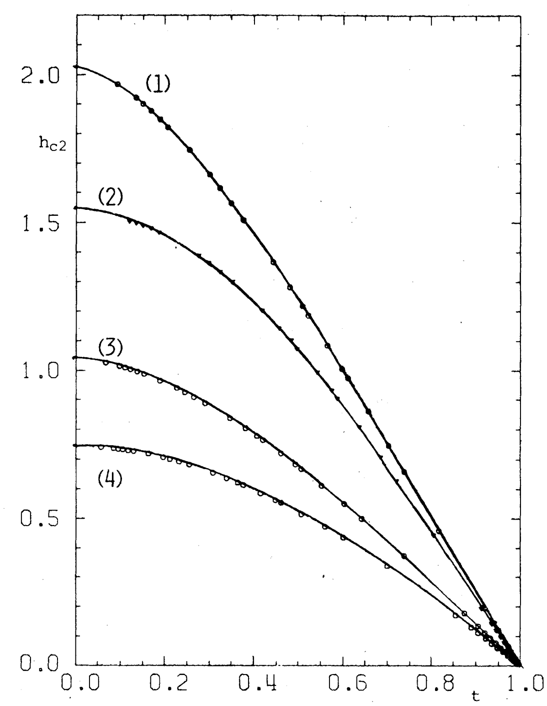

FIG. 1. Upper critical field in reduced units [Eq. (49)] as a function of reduced temperature for (1) the polar state [Eq. (29)], (2) the generalized ABM state [Eq. (45)], (3) the s-wave state, and (4) the ABM state [Eq. (39)].

V. CONCLUSIONS

We have shown that the linearized gap equation permits a variety of solutions for the order parameter of a p-wave superconductor which describes the penetration of the magnetic field in the form of a regular array of vortex lines. However, a state with the symmetries of the famous Balian-Werthamer state[2] is not possible.[29] Among the allowed states the polar state Eq. (29) has the highest upper critical field, which at zero temperature is about twice as large as that of an s-wave superconductor. Solutions of the ABM type have also been found but there is no degeneracy that would allow the construction of a vortex lattice without singular vortex cores.[28]

The magnetic field variation associated with the vortices is very much like that of s-wave superconductors in the vortex state and hence very small near the transition. Also, the paramagnetism of the electron spins in a p-wave superconductor with equal spin pairing is very similar to that of a normal metal so that a p-wave vortex state does not predict the large variation in internal magnetic fields necessary to explain the puzzling experiments of Tse *et al.*[29] A transition to an antiferromagnetic state would provide an explanation, but we then need to understand not only how the p-wave state and an itinerant antiferromagnetic state can coexist, which is not inconceivable, but why they should form simultaneously.

In summary we can say that, contrary to Leggett's[30] anticipation, the behavior of necessarily anisotropic p-wave superconductors in a magnetic field does not differ appreciably from that of ordinary s-wave superconductors.

The only qualitative difference between s-wave and p-wave superconductors in a vortex state is the presence or absence of the Pauli paramagnetic limit of the upper critical field. However, this difference can gain quantitative importance only if the overwhelming effect of the orbital diamagnetism can be eliminated without destroying p-wave superconductivity. This could possibly be achieved in clean layered compounds with the magnetic field parallel to the layers.[31,32] A detailed study of this problem along the lines of the present paper is under way.[33]

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APPENDIX A

In this appendix we want to show that the expression given in Eq. (19) for the integral

$$J_{ij}=\int d^3k\,\hat{k}_i\hat{k}_j e^{i\vec{k}\cdot(\vec{\xi}'-\vec{\xi})} \tag{A1}$$

represents a good approximation, at least within the context of Eq. (18). We start from the following exact relation

$$J_{ij}=\nabla_i'\nabla_j\int d^3k\,k^{-2}e^{i\vec{k}\cdot(\vec{\xi}'-\vec{\xi})}$$
$$=2\pi^2\nabla_i'\nabla_j\frac{1}{|\vec{\xi}-\vec{\xi}'|}\ . \tag{A2}$$

For comparison, Ambegaokar *et al.*[15] use the following approximation:

$$J_{ij}\simeq(2\pi)^3\nabla_i'\nabla_j\delta(\vec{\xi}-\vec{\xi}')\ . \tag{A3}$$

From Eq. (A2) we obtain

$$J_{ij}=(2\pi)^3\nabla_i'\nabla_j\sum_{l=0}^{\infty}\left[\frac{\xi^l}{\xi'^{l+1}}\Theta(\xi'-\xi)+\frac{\xi'^l}{\xi^{l+1}}\Theta(\xi-\xi')\right]\times\frac{1}{2l+1}\sum_{m=-l}^{+l}Y_{lm}(\hat{\xi})Y_{lm}^*(\hat{\xi}')\ . \tag{A4}$$

The gradient operator in polar coordinates can be written as

$$\vec{\nabla}=\hat{\xi}\frac{\partial}{\partial\xi}+\frac{1}{\xi}\vec{\mathrm{T}}(\hat{\xi})\ , \tag{A5}$$

where

$$\vec{\mathrm{T}}(\hat{\xi})=\begin{pmatrix}\frac{1}{2}\cos\theta(L_+-L_-)-\sin\theta\sin\phi\frac{\partial}{\partial\phi}\\ \frac{1}{2i}\cos\theta(L_++L_-)+\sin\theta\cos\phi\frac{\partial}{\partial\phi}\\ -\sin\theta\frac{\partial}{\partial\theta}\end{pmatrix}\ . \tag{A6}$$

As usual, $L_{\pm}=L_x\pm iL_y$ is defined in terms of angular momentum operators.

Differentiating with respect to ξ first, the Θ functions do not contribute. However, differentiating the result with respect to ξ' we obtain a term proportional to $\delta(\xi-\xi')$. The sum over l and m then reduces to the closure relation for the spherical harmonics and hence the expression given on the right-hand side of Eq. (19) is obtained. The remaining contributions can be written most economically in terms of two sets of operators

$$\vec{P}(l,\hat{\xi}) = \frac{l\hat{\xi}+\vec{T}(\hat{\xi})}{(2l+1)^{1/2}} \, ,$$
$$\vec{Q}(l,\hat{\xi}) = \frac{-(l+1)\hat{\xi}+\vec{T}(\hat{\xi})}{(2l+1)^{1/2}} \, . \tag{A7}$$

With these notations we obtain

$$J_{ij} = \hat{\xi}_i\hat{\xi}_j\delta(\vec{\xi}-\vec{\xi}') + \sum_{l=1}^{\infty}\frac{\xi^{l-1}}{\xi'^{l+2}}\Theta(\xi'-\xi)\sum_{m=-l}^{+l}[Q_i(l,\hat{\xi}')Y_{lm}(\hat{\xi}')]^*P_j(l,\hat{\xi})Y_{lm}(\hat{\xi})$$
$$+\sum_{l=1}^{\infty}\frac{\xi'^{l-1}}{\xi^{l+2}}\Theta(\xi-\xi')\sum_{m=-l}^{+l}[P_i(l,\hat{\xi}')Y_{lm}(\hat{\xi}')]^*Q_j(l,\hat{\xi})Y_{lm}(\hat{\xi}) \, . \tag{A8}$$

Using formulas of the kind

$$(2l+1)^{1/2}\sin\theta e^{i\phi}Y_{lm} = c_{l,-m}Y_{l-1,m+1} - c_{l+2,m}Y_{l+1,m+1} \, ,$$

where

$$c_{l,m} = \left[\frac{(l+m)(l+m-1)}{2l-1}\right]^{1/2} \, ,$$

which can be obtained from the recurrence relations of the associated Legendre functions, we find the following results:

$$P_3Y_{lm} = \left[\frac{(l+m)(l-m)}{2l-1}\right]^{1/2} Y_{l-1,m} \, , \tag{A9}$$

$$Q_3Y_{lm} = -\left[\frac{(l+1+m)(l+1-m)}{2l+3}\right]^{1/2} Y_{l+1,m} \, , \tag{A10}$$

$$P_1Y_{lm} = \tfrac{1}{2}(c_{l,-m}Y_{l-1,m+1} - c_{l,m}Y_{l-1,m-1}) \, , \tag{A11}$$

$$P_2Y_{lm} = (1/2i)(c_{l,-m}Y_{l-1,m+1} + c_{l,m}Y_{l-1,m-1}) \, , \tag{A12}$$

$$Q_1Y_{lm} = \tfrac{1}{2}(c_{l+2,m}Y_{l+1,m+1} - c_{l+2,-m}Y_{l+1,m-1}) \, , \tag{A13}$$

$$Q_2Y_{lm} = (1/2i)(c_{l+2,m}Y_{l+1,m+1} + c_{l+2,-m}Y_{l+1,m-1}) \, . \tag{A14}$$

Because of $P_iY_{00}=0$, the summation in Eq. (A8) begins with $l=1$.

In order to justify Eq. (19) we need to show that the terms just derived give only negligible contributions to Eq. (18).

For simplicity we consider only the linearized version of Eq. (18). When linearized Eq. (18) depends on the direction of $\hat{\xi}'$ only through J_{ij}. Therefore, terms in Eq. (A8) containing $Q_i(l,\hat{\xi}')\,Y_{lm}(\vec{\xi}')$ vanish upon integrating with respect to angles and terms containing $P_i(l,\hat{\xi}')Y_{lm}(\hat{\xi}')$ contribute only if $l=1$. Hence, J_{ij} depends only on $Y_{2m}(\xi)$. Expanding $\exp[i\vec{\xi}\cdot\vec{\Pi}(\vec{R})]$ in terms of spherical Bessel functions and integrating with respect to the direction of $\hat{\xi}$ the only nonvanishing term is proportional to j_2. To prove our point we, therefore, need to consider integrals of the type

$$S_{2n+2} = \int_0^\infty d\xi'\,\xi'^2\overline{G}^0_{\sigma\sigma}(\xi',\omega_n) \times \int_0^\infty d\xi\,\xi^2\frac{1}{\xi^3}\Theta(\xi-\xi') \times \overline{G}^0_{\sigma'\sigma'}(\xi,-\omega_n)\xi^{2n+2} \tag{A15}$$

multiplying the operator $(\vec{\Pi}^2)^{n+1}$. We discuss the case $n=0$ only, because higher powers of ξ in the integrand do not change our argument.

Inserting the normal-state Green's function Eq. (9) we obtain after neglecting the Pauli term and expanding the square root:

$$S_2 = \left[\frac{m}{2\pi}\right]^2\left[\frac{v_F}{2|\omega_n|}\right]^3\frac{\omega_n}{iE_F} \, . \tag{A16}$$

For comparison, the first term on the right-hand side of (A8) yields

$$S_2' = \int_0^\infty d\xi\,\xi^2\overline{G}^0_{\sigma\sigma}(\xi,\omega_n)\overline{G}^0_{\xi'\xi'}(\xi,-\omega_n)\xi^2 = \left[\frac{m}{2\pi}\right]^2 2\left[\frac{v_F}{2|\omega_n|}\right]^3 \, . \tag{A17}$$

This is larger than S_2 by a factor of the order of E_F/T, which shows that Eq. (19) is an excellent approximation, at least when applied to Eq. (18).

APPENDIX B

In this appendix we shall derive Eq. (21) from Eq. (15) without explicit use of the approximation Eq. (19). Instead, we shall employ approximations based on the properties of the normal-state Green's function in Fourier representation Eq. (10). The treatment presented here can be applied to pairing in any l state.

We linearize Eq. (15), neglect the Pauli term to save writing, insert Eq. (10) and expand $\exp(i\vec{\xi}\cdot\vec{\Pi})$:

$$\Delta_{\sigma\sigma'}(\vec{R},\vec{k})=\int\frac{d^3k'}{(2\pi)^3}V(\vec{k}-\vec{k}')T\sum_{\omega_n}\bar{G}^0_{\sigma\sigma}(\vec{k}',\omega_n)\int d^3\xi e^{-i\vec{k}'\cdot\vec{\xi}}I(\xi) \quad , \tag{B1}$$

where

$$\begin{aligned}I(\xi)&=\int\frac{d^3p}{(2\pi)^3}\bar{G}^0_{\sigma'\sigma'}(\vec{p},-\omega_n)\sum_{N=0}^{\infty}\frac{1}{N!}(i\vec{\xi}\cdot\vec{\Pi})^N e^{i\vec{p}\cdot\vec{\xi}}\Delta_{\sigma\sigma'}(\vec{R},\vec{k}')\\&=\int\frac{d^3p}{(2\pi)^3}\bar{G}^0_{\sigma'\sigma'}(\vec{p},-\omega_n)\sum_{N=0}^{\infty}\frac{1}{N!}(\vec{\nabla}_p\cdot\vec{\Pi})^N e^{i\vec{p}\cdot\vec{\xi}}\Delta_{\sigma\sigma'}(\vec{R},\vec{k}')\\&=\sum_{N=0}^{\infty}\frac{1}{N!}\int\frac{d^3p}{(2\pi)^3}e^{i\vec{p}\cdot\vec{\xi}}(-\vec{\nabla}_p\cdot\vec{\Pi})^N\bar{G}^0_{\sigma'\sigma'}(\vec{p},-\omega_n)\Delta_{\sigma\sigma'}(\vec{R},\vec{k}') \quad .\end{aligned} \tag{B2}$$

We now use the approximation

$$\vec{\nabla}_p\bar{G}^0(\vec{p},\omega_n)\simeq\frac{\vec{v}_F(\hat{p})}{(i\omega_n-p^2/2m+E_F)^2} \quad . \tag{B3}$$

This gives

$$\begin{aligned}\Delta_{\sigma\sigma'}(\vec{R},\vec{k})&=T\sum_{\omega_n}\int\frac{d^3k'}{(2\pi)^3}V(\vec{k}-\vec{k}')\sum_{N=0}^{\infty}\frac{1}{k'^2/2m-E_F-i\omega_n}\frac{[\vec{v}_F(\hat{k}')\cdot\vec{\Pi}(\vec{R})]^N}{(k'^2/2m-E_F+i\omega_n)^{N+1}}\Delta_{\sigma\sigma'}(\vec{R},\vec{k}')\\&\approx\pi T\sum_{\omega_n}\frac{mk_F}{2\pi^2}\int\frac{d\Omega_{k'}}{4\pi}V(\vec{k}_F-\vec{k}_F')\sum_{N=0}^{\infty}\left(\frac{1}{2i}\right)^N\frac{(\mathrm{sgn}\omega_n)^N}{|\omega_n|^{N+1}}[\vec{v}_F(\hat{k}')\cdot\vec{\Pi}(\vec{R})]^N\Delta_{\sigma\sigma'}(\vec{R},\vec{k}') \quad .\end{aligned} \tag{B4}$$

Rewriting this as

$$\Delta_{\sigma\sigma'}(\vec{R},\vec{k})=\pi T\sum_{\omega_n}\frac{mk_F}{2\pi^2}\int\frac{d\Omega_{k'}}{4\pi}V(\vec{k}_F-\vec{k}_F')\int_0^{\infty}dt\,e^{-|\omega_n|t}\sum_{N=0}^{\infty}\frac{1}{N!}\left(\frac{t}{2i}\mathrm{sgn}\omega_n\vec{v}_F(\hat{k}')\cdot\vec{\Pi}(\vec{R})\right)^N\Delta_{\sigma\sigma'}(\vec{R},\vec{k}') \quad ,$$

we can reintroduce the vector $\vec{k}'$ through

$$\vec{k}'=\tfrac{1}{2}t\vec{v}_F(\hat{k}') \quad ,$$

and thus obtain

$$\Delta_{\sigma\sigma'}(\vec{R},\vec{k})=\frac{m^2}{4\pi^2}T\sum_{\omega_n}\int d^3k'\,k'^{-2}V(\vec{k}_F-\vec{k}_F')\exp(-2|\omega_n|k'/v_F)\exp[-i\,\mathrm{sgn}\omega_n\,\vec{k}'\cdot\vec{\Pi}(\vec{R})]\Delta_{\sigma\sigma'}(\vec{R},\vec{k}') \quad .$$

Assuming p-wave pairing [Eqs. (16) and (17)] this reduces to Eq. (21).

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