Theoretical study of the linear dispersion relation and the stationary energy-transport velocity of a coupled polar LO phonon-plasmon system

O. Keller
Group of Physics, Institute 7, Aalborg University Center, DK 9100 Aalborg, Denmark
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A semiclassical investigation of the linear dispersion relation and the stationary energy-transport velocity associated with coupled LO phonon-plasmon waves in polar semiconductors is made. The influence of a free-carrier drift velocity on the frequency dispersion of the modes is considered. The complex wave vector of the coupled mode is obtained from the roots of an effective dielectric function which has been calculated on the basis of the appropriate linearized Boltzmann transport equation, the Maxwell equations, the long-wavelength equation of motion of the polar-optical lattice vibrations, and the Born and Huang expression for the lattice polarization. The minor contribution arising from the collision-drag effect associated with impurity scattering is included in the general framework of the theory. With main emphasis on small or even vanishing electronic drift velocities, inverse dispersion relations valid for low-density Maxwellian plasmas are obtained and discussed. The limiting cases, outside the quantum region, where local or extremely nonlocal approaches can be adopted, are examined. By means of a simple nonlinear Boltzmann equation the energy-balance equation associated with the LO phonon-plasmon mode is derived. A general expression for the stationary energy-transport velocity associated with damped (or amplified) coupled modes is established. Finally, the basic concepts of velocity of energy propagation are applied to classical plasmas.

I. INTRODUCTION

In polar semiconductors the free charge-carrier system is coupled to the longitudinal-optical lattice vibrations via the quasistatic, nonradiative longitudinal electric field associated with the LO modes.[1-3] The coupling modifies the dispersion relation for both the plasma oscillations and the optic modes of vibration, especially if the free-carrier plasma frequency is comparable to the LO phonon frequency, the collective modes cease to by *phononlike* or *plasmonlike* in a broad wave-vector range. From a theoretical point of view the investigation of the dynamical properties of the coupled LO phonon-plasmon system conveniently is based on three different approaches. Thus, if the mean free path of the conduction electrons is small compared to the wavelength of the coupled mode a macroscopic classical analysis can be used. When nonlocal electronic transport effects are of importance, the LO phonon-plasmon system can be investigated by means of the Boltzmann transport equation, assuming the characteristic electron de Broglie wavelength to be small compared to the wavelength of the coupled mode. Finally, for coupled-mode wave vectors which are not small compared to the Fermi wave vector of the electrons, a quantum-mechanical description must be applied.

The purpose of the present paper is, on the basis of the semiclassical approximation, to study the dispersion relation of the coupled LO phonon-plasmon mode and the stationary energy-transport velocity associated with the propagation of the wave.

Quantum-mechanical studies of the dispersion relation have been given by Varga[4] and Kim *et al.*[5] in the case where no dc current is flowing through the crystal. In the quantum theories of Spector[6] and Gunn[7] the dispersion relation is used to calculate the real and imaginary parts of the wave vector of the coupled modes in the presence of a dc electric field.

Spector[8,9] also has considered the interaction of conduction electrons with acoustic waves in the presence of an external field on the basis of the Boltzmann equation. Although the basic concepts in his semiclassical investigations display points of resemblance to our theory, the qualitative differences between optical and acoustical lattice vibrations are so pronounced that only formally can one be guided by the work of Spector. For instance, if one considers acoustic waves in the long-wavelength limit it is, for not too low temperatures, permissible to assume that the free-carrier plasma is collision dominated, whereas with respect to optical lattice waves, often the plasma can be considered as collisionless at long wavelengths. Furthermore, Spector's method is less general than the present theory in the sense that it neglects the reaction of the conduction-electron system on the acoustic wave. Expressed in a different way, Spector uses an impressed-wave method; that is, he regards the wave number as a constant and the attenuation of the acoustic wave is taken into account by calculating the power transferred to the electrons. In the present paper the attenuation (or gain), and the wave numbers of the coupled LO phonon-plasmon mode are calculated with the aid of the dispersion relation, relating the real and imaginary parts of the wave vector of the coupled mode to the (real) frequency of the wave. This kind of approach is of course necessary when the LO phonon and the plasmon are strongly coupled.

A phenomenological approach to the investigation of optical lattice-wave amplification in semiconductors which neglect nonlocal transport effects has been undertaken by Woodruff.[10]

The most significant new material presented in this paper is (i) the introduction of collision drag for the optical-phonon case, (ii) the exact solution of the dc part of the Boltzmann equation in the relaxation-time approximation, and (iii) the analysis of the stationary energy transport.

In Sec. II the framework of the theory is given. Combining the two inhomogeneous field equations from the theory of classical electrodynamics, the equation of motion for the optical lattice vibrations, the well-known constitutive equation of Born and Huang[11] describing the lattice polarization, and the constitutive equation for the longitudinal free-carrier current density, the dispersion relation can be derived from the usual condition that the effective complex dielectric function vanishes. The strict additivity of the ionic and free-carrier contributions to the dielectric function noted by Varga[4] is destroyed in the present analysis. This is due to the fact that the collision-drag effect[12] has been incorporated in the equation of motion of the ionic lattice. In the calculations it has been assumed that point impurities are the dominating scattering sources for the conduction electrons. In the treatment of Woodruff the collision drag was neglected.

In Sec. III the response functions relating the self-consistent ac electric field, the free-carrier density fluctuations, and the mean displacement amplitude of the ions in the unit cell to the ac part of the free-carrier current density are calculated by means of a linearized Boltzmann transport equation. Although we treat the conduction electrons as obeying Maxwell-Boltzmann statistics, and thus restrict the application of our theory to low-density plasmas in semiconducting crystals, the collision-drag effect, which roughly speaking is supposed to be of importance in high-density plasmas only, is retained for completeness. In a subsequent paper we shall study the dynamical properties of the LO phonon-plasmon system at degenerate free-carrier concentrations. The Boltzmann-equation calculation presented in this paper is more general than that of Spector[8] because we solve the dc part of the transport equation exactly and next use this dc distribtuion function to evaluate the explicit form of the ac distribution function. In the limit where the free-carrier drift velocity is very small, the dc part of the free-carrier distribution function approaches that obtained by Spector,[8] i.e., a displaced Maxwell-Boltzmann distribution. Also, the ac part of the free-carrier distribution function takes the appropriate form at low drift velocities.

In Sec. IV the linear dispersion relation is studied in the important cases where (i) the free-carrier drift velocity vanishes and (ii) the drift velocity is so small that the dc distribution function is just a displaced Maxwell-Boltzmann distribution. Explicit expressions are derived for the real and imaginary parts of the LO phonon-plasmon wave number as functions of the mode frequency in the regions of long and short wavelengths. In the appropriate limits our results for the effective conductivity are reduced to those obtained by Lindhard[13] and Spector.[8] As required, the equations for the frequency dispersion of the complex wave number also contain the familiar results of Spector[8] and Woodruff[10] as limiting cases.

In Sec. V the stationary energy-transport velocity associated with the LO phonon-plasmon wave is examined. For an absorbing (or amplifying) medium, the group velocity can no longer be identified with the velocity of energy propagation.[14-16] Under steady-state conditions, given by the requirement that the cycle-averaged energy density in the mode is time independent, one can define the stationary energy-transport velocity as the cycle-averaged Poynting vector of the coupled LO phonon-plasmon mode divided by the cycle-averaged stored-energy density of the wave.[17,18] Considered somewhat unambiguously, the Poynting vector is composed of contributions from the radiative part of the electromagnetic field, from the ionic system, and from the energy flux carried by the conduction electrons. In the present case the contributions from the electromagnetic field and the ionic vibrations are absent since purely longitudinal plane waves do not radiate electromagnetic energy, and since the LO phonon frequency is assumed to be independent of the wave vector of the mode. The calculation of the induced Poynting vector and the induced-energy density of the free-carrier system is obtained by means of an approximate solution to the nonlinear Boltzmann equation. Next, on the basis of the Boltzmann transport equation the energy-balance equation associated with the LO phonon-plasmon mode is derived. To the stored-energy density of the mode, both the material oscillators, bound as well as free, and the electromagnetic field contribute. Finally, an explicit expression for the stationary energy-transport velocity is given in the case where the collision-drag effect can be neglected and the external dc electric field is absent. In a subsequent paper numerical results for low-density plasmas in semiconductors will be presented and compared to experimental results.

II. FRAMEWORK OF THE THEORY

The present description of the dynamical properties of a coupled LO phonon-plasmon system is based on a dispersion relation which is obtained by a linearization of the corresponding set of coupled (differential) equations. The linearization leads to the existence of elementary solutions in the form

$$A_1(\vec{r}, t) = A_1(\vec{Q}, \Omega)\exp[i(\vec{Q}\cdot\vec{r} - \Omega t)] \tag{1}$$

with the understanding that the real part of Eq. (1) should be taken to obtain the appropriate field components. Solutions more general than those of Eq. (1) are obtained by superposition to form Fourier integrals to fit the boundary or initial conditions with use of the Fourier inversion theorem. Since the equations controlling the dynamics of the system are linear, the inverse dispersion relation

$$\vec{Q} = \vec{Q}(\Omega) \tag{2}$$

is reduced to a relation between the complex wave vector ($\vec{Q}$) and the complex angular frequency (Ω) of the mode. In a more general nonlinear dispersion theory the amplitude appears in the dispersion relation which now is associated with periodic, nonsinusoidal wave trains.[19] As limiting cases of these wave trains, solitary waves are found.[19] Because we are interested in steady-state mode propagation, Ω is taken as a real quantity. To account for amplitude attenuation (or amplification) in space, a nonzero imaginary part of $\vec{Q}$ must be retained.

Restricting the analysis to pure longitudinal modes in nonmagnetic materials, one obtains from the two inhomogeneous macroscopic Maxwell equations

$$\vec{\nabla}\times\vec{B} - \frac{1}{c_0^2}\frac{\partial\vec{E}}{\partial t} = \mu_0\left(\vec{J} + \frac{\partial\vec{P}}{\partial t}\right) \tag{3}$$

and

$$\vec{\nabla}\cdot\vec{E} = (1/\epsilon_0)(\rho - \vec{\nabla}\cdot\vec{P}) \tag{4}$$

the following relations between the Fourier amplitudes of the electric field ($\vec{E}_1$), the polarization ($\vec{P}_1$), the free-carrier density (N_1), and the free-carrier current density ($\vec{J}_1$):

$$i\Omega[\epsilon_0\vec{E}_1(\vec{Q}, \Omega) + \vec{P}_1(\vec{Q}, \Omega)] = \vec{J}_1(\vec{Q}, \Omega) \tag{5}$$

and

$$iQ[\epsilon_0 E_1(\vec{Q}, \Omega) + P_1(\vec{Q}, \Omega)] = -eN_1(\vec{Q}, \Omega), \tag{6}$$

where e denotes the numerical magnitude of the free-electron charge. The macroscopic polarization of the medium is throughout the work assumed to be adequately determined in the dipole approximation.

To proceed further, an equation of motion for the ionic lattice vibrations must be established. For simplicity, the discussion is limited to cubic crystals containing two nonequivalent ions in the primitive unit cell, and it is assumed that the wavelength of the optical mode is sufficiently long that the purely elastic part of the restoring force can be considered as independent of the wave vector of the mode. With these restrictions the equation of motion for the relative longitudinal displacement ($\vec{W}_1$) of the two sublattices takes the form

$$\frac{\partial^2\vec{W}_1}{\partial t^2} + \Gamma\frac{\partial\vec{W}_1}{\partial t} + \omega_{TO}^2\vec{W}_1 = \frac{1}{M}(e^*\vec{E}_1 + \vec{F}_1^{cd}), \tag{7}$$

where M is the reduced mass of the two ions in the unit cell, Γ is a phenomenological wave-vector-independent damping constant accounting for loss of energy from the ionic motion by coupling to excitations in the solid other than those associated with the free-carrier system, ω_{TO} is the long-wavelength transverse-optical phonon frequency, and e^* is the effective charge[20] of the mode. The force term $\vec{F}_1^{cd}$ arises from the fact that the velocity distribution towards which the conduction electrons relax is a Fermi-Dirac (or eventually a Maxwell-Boltzmann) distribution centered, not at the origin of the velocity space, but at a point equal to some characteristic velocity related to the instantaneous local lattice-displacement velocities of the two ionic sublattices. The interaction mechanism described by the term $\vec{F}_1^{cd}$ is analogous to the collision-drag effect[9,12] well known from the theory of ultrasonic absorption in metals and semiconductors. To the author's knowledge the collision-drag effect has been neglected in previous studies of coupled LO phonon-plasmon systems. In Sec. IV it will be discussed under which conditions the contributions arising from the collision-drag effect can be neglected in the LO phonon-plasmon dispersion relation. The Fourier amplitude of the relative ion displacement obeys the equation

$$M(-\Omega^2 - i\Gamma\Omega + \omega_{TO}^2)\vec{W}_1(\vec{Q}, \Omega) = e^*\vec{E}_1(\vec{Q}, \Omega) + \vec{F}_1^{cd}(\vec{Q}, \Omega). \tag{8}$$

By assuming that (i) point impurities are the dominating scattering source for the conduction electrons and (ii) these impurities are uniformly distributed throughout the crystal with equal densities on the two sublattices, a phenomenological expression for $\vec{F}_1^{cd}(\vec{Q}, \Omega)$ can be obtained as follows. The conduction electrons will relax toward a distribution centered at the local average velocity of the two ions in the unit cell. This velocity is denoted by $\partial\vec{\xi}_1/\partial t$. Since it is required that the center of mass in the cell remains fixed, i.e.,

$$M^+\vec{U}_1^+ + M^-\vec{U}_1^- = 0\,, \tag{9}$$

where M^+, M^- and $\vec{U}_1^+$, $\vec{U}_1^-$ denote the masses and displacements of the positive and negative ions, the mean displacement

$$\vec{\xi}_1 = \tfrac{1}{2}(\vec{U}_1^+ + \vec{U}_1^-) \tag{10}$$

and the relative displacement

$$\vec{W}_1 = \vec{U}_1^+ - \vec{U}_1^- \tag{11}$$

are related by the equation

$$\vec{\xi}_1 = \frac{M}{2}\left(\frac{1}{M^+} - \frac{1}{M^-}\right)\vec{W}_1\,. \tag{12}$$

Because the local conduction-electron velocity averaged over the velocity distribution of the electrons before a collision $(\langle\vec{v}\rangle)$ differs from that after the collision, which is just $\partial\vec{\xi}_1/\partial t$, the mean velocity of the lattice, the average loss in momentum per electron per unit time is $m^*(\langle\vec{v}\rangle - \partial\vec{\xi}_1/\partial t)/\tau$, where m^* and τ denote the effective mass and the energy independent relaxation time of the free carriers, respectively. Since the rate of loss in the local momentum density of the conduction-electron system equals the local force density exerted on the lattice, one obtains the following expression for the local collision-drag force:

$$\vec{F}^{cd}(\vec{r}, t) = \frac{N_0 + N_1(\vec{r}, t)}{N_i}\,\frac{m^*}{\tau}\left(\langle\vec{v}(\vec{r}, t)\rangle - \frac{\partial\vec{\xi}_1(\vec{r}, t)}{\partial t}\right), \tag{13}$$

where N_0 is the free-carrier density in thermal equilibrium and N_i is the number of unit cells per unit volume. Equation (13) shows that the Fourier amplitude of the collision-drag force is given by

$$\vec{F}_1^{cd}(\vec{Q}, \Omega) = (m^*/eN_i\tau) \times[ieN_0\Omega\vec{\xi}_1(\vec{Q}, \Omega) - \vec{J}_1(\vec{Q}, \Omega)]\,. \tag{14}$$

Combining Eqs. (8), (12), and (14), the equation of motion of the relative ion displacement is equivalent to

$$M\left[-\Omega^2 - i\Omega\left(\Gamma + \frac{1}{\tau}\frac{N_0}{N_i}\,\frac{m^*(M^- - M^+)}{2M^+M^-}\right) + \omega_{TO}^2\right]\vec{W}_1(\vec{Q}, \Omega) = e^*\vec{E}_1(\vec{Q}, \Omega) - (m^*/eN_i\tau)\vec{J}_1(\vec{Q}, \Omega)\,. \tag{15}$$

To complete the framework of the theory one needs constitutive equations for the solid involving the polarization and the current density. In the present approach we shall describe the lattice polarization by the familiar equation of Born and Huang,[9]

$$\vec{P}_1(\vec{Q}, \Omega) = N_i e^*\vec{W}_1(\vec{Q}, \Omega) + \epsilon_0\chi^\infty\vec{E}_1(\vec{Q}, \Omega)\,, \tag{16}$$

where χ^∞ is the high-frequency linear dielectric susceptibility of the crystal. For our purpose, the attractiveness of the formulation given in Eq. (16) lies in the fact that although it incorporates the deformability and polarizability of the ions, it involves the macroscopic electric field only.

On the basis of a Boltzmann-equation calculation the constitutive relation for the Fourier amplitude of the longitudinal free-carrier current density takes the form

$$\vec{J}_1(\vec{Q}, \Omega) = \Sigma(\vec{Q}, \Omega)\vec{E}_1(\vec{Q}, \Omega) - e(\Omega/Q)N_1(\vec{Q}, \Omega)\vec{R}(\vec{Q}, \Omega) - \frac{i}{2}eN_0\Omega\frac{M^+ - M^-}{M^+ + M^-}\,T(\vec{Q}, \Omega)\vec{W}_1(\vec{Q}, \Omega)\,. \tag{17}$$

The explicit expressions for the frequency and wave-vector-dependent quantities Σ, $\vec{R}$, and T will be derived in the next section.

The two inhomogeneous field equations from the theory of classical electrodynamics, the equation of motion for the relative displacement of the two ions in the unit cell, plus the constitutive equations of the material [i.e., Eqs. (5), (6), (15)–(17)] involve the five unknown Fourier amplitudes E_1, P_1, N_1, J_1, and W_1. By using the condition that the determinant of these algebraic equations must equal zero in order to obtain a nontrivial solution, one can derive the LO phonon-plasmon dispersion relation $\Omega = \Omega(\vec{Q})$ in implicit form. After some straightforward but tedious manipulations one gets

$$\epsilon_{eff}(Q, \Omega) + i[\sigma_{eff}(Q, \Omega)/\epsilon_0\Omega] = 0\,, \tag{18}$$

where the superfluous vectorial specification of the direction of phase propagation has been suppressed. Above, we have introduced an effective long-wavelength frequency- and wave-vector-dependent dielectric function of the lattice via

$$\epsilon_{eff}(Q, \Omega) = \epsilon_r^\infty\,\frac{\Omega^2 + i\Gamma_{eff}^{(1)}\Omega - \omega_{LO}^2}{\Omega^2 + i\Gamma_{eff}^{(2)}(Q, \Omega)\Omega - \omega_{TO}^2}\,, \tag{19}$$

and an effective frequency- and wave-vector-dependent longitudinal conductivity by

$$\sigma_{\text{eff}}(Q,\Omega)=\Sigma(Q,\Omega)\frac{1-\epsilon_0\epsilon_r^\infty\Omega/S_{0i}}{1-R(Q,\Omega)-\Sigma(Q,\Omega)/S_{0i}}, \quad (20)$$

where $\epsilon_r^\infty=1+\chi^\infty$ is the relative high-frequency linear dielectric constant, and the quantity S_{0i} is given by

$$S_{0i}=\frac{N_i}{N_0}\frac{e^*}{e}\sigma_0\left(=\frac{N_i e e^* \tau}{m^*}\right), \quad (21)$$

denoting the dc conductivity of the conduction electrons by σ_0. In usual notation

$$\omega_{\text{LO}}=[\omega_{\text{TO}}^2+N_i(e^*)^2/\epsilon_0\epsilon_r^\infty M]^{1/2} \quad (22)$$

denotes the longitudinal-optical phonon frequency. It should be stressed that although *the form* of the dielectric function ϵ_{eff} is identical almost to that of the conventional long-wavelength dielectric function of the lattice, it only is *phononlike* because it contains the coupling to the free-carrier distribution in the effective damping coefficients

$$\Gamma_{\text{eff}}^{(1)}=\Gamma-\frac{1}{\tau}\frac{N_0}{N_i}\frac{m^*(M^+-M^-)}{2M^+M^-} \quad (23)$$

and

$$\Gamma_{\text{eff}}^{(2)}(Q,\Omega)=\Gamma-\frac{1}{\tau}\frac{N_0}{N_i}\frac{m^*(M^+-M^-)}{2M^+M^-}\times\left(1-S_{0i}\frac{T(Q,\Omega)}{\Sigma(Q,\Omega)}\right). \quad (24)$$

Thus the collision-drag effect destroys the strict additivity of the ionic and free-carrier contributions to the dielectric functions noted by Varga.[4] Furthermore, spatial dispersion effects are introduced in the phonon-like part of the long-wavelength dielectric constant due to the collision-drag effect. Note that $\Gamma_{\text{eff}}^{(2)}\to\Gamma_{\text{eff}}^{(1)}$ in the limit $T/\Sigma\to 0$.

Facets of the dispersion relation of the LO phonon-plasmon system, given implicitly by the usual condition [Eq. (18)] that the total dielectric function vanishes, will be studied in Sec. IV.

III. DETERMINATION OF THE MATERIAL DESCRIPTORS Σ, R, AND T

To include nonlocal free-carrier transport effects in our analysis of the LO phonon-plasmon system we solve the linearized Boltzmann equation for the conduction electrons in the presence of an optical lattice mode. Since we exclude a quantum-mechanical description of the electronic motion, our treatment will be valid only as long as the characteristic electron de Broglie wavelength is short compared to the wavelength of the LO phonon-plasmon wave. It is assumed that the free carriers are subjected to an external homogeneous dc electric field $\vec{E}_0$.

Inserting a phenomenological collision term to simulate the effects of electron-impurity scattering, the dynamic changes in the free-carrier distribution function $f(\vec{r},\vec{v},t)$ are determined by the Boltzmann equation

$$\frac{\partial f}{\partial t}+\vec{v}\cdot\frac{\partial f}{\partial\vec{r}}-\frac{e}{m^*}(\vec{E}_0+\vec{E}_1)\cdot\frac{\partial f}{\partial\vec{v}}=-\frac{f-f_s}{\tau}, \quad (25)$$

where f_s is the distribution function to which the electrons relax in the presence of the longitudinal-optical lattice wave. Since (i) the impurities are uniformly distributed with equal densities on the two sublattices and (ii) the scattering is local and therefore does not change the local electron density, the distribution function f_s is given by

$$f_s(\vec{r},\vec{v},t)=\tfrac{1}{2}\left[f_0\left(\vec{v}-\frac{\partial\vec{U}_1^+}{\partial t},N_0+N_1\right)+f_0\left(\vec{v}-\frac{\partial\vec{U}_1^-}{\partial t},N_0+N_1\right)\right]\cong\left(1+\frac{N_1}{N_0}\right)f_0(\vec{v})-\frac{\partial\vec{\xi}_1}{\partial t}\cdot\frac{\partial f_0(\vec{v})}{\partial\vec{v}}, \quad (26)$$

where $f_0(\vec{v})$ is the thermal-equilibrium distribution function of the electrons. In deriving the second step in Eq. (26) it has been assumed that $\partial\vec{U}_1^+/\partial t$, $\partial\vec{U}_1^-/\partial t\ll\langle\vec{v}\rangle$ and $N_1\ll N_0$. The last term on the right-hand side of Eq. (26) arises from the collision-drag effect.

Of interest in the present work is classical, low-density plasma. Thus, treating the electrons as obeying Maxwell-Boltzmann statistics, one has

$$f_0(\vec{v})=N_0\left(\frac{m^*}{2\pi k_B T}\right)^{3/2}\exp\left(-\frac{m^*v^2}{2k_B T}\right), \quad (27)$$

where k_B is Boltzmann's constant and T is the absolute temperature.

To solve the Boltzmann equation we make the ansatz

$$f(\vec{r},\vec{v},t)=f_{\text{dc}}(\vec{v})+g(\vec{v})\exp[i(\vec{Q}\cdot\vec{r}-\Omega t)], \quad (28)$$

where the first term represents the free-carrier distribution in the presence of the dc electric field, but in the absence of the longitudinal-optical lattice wave, and the second term represents the part of the distribution function which is induced by the wave. For simplicity, the free-carrier drift velocity given by

$$\vec{v}_d=-\frac{e\tau}{m^*}\vec{E}_0 \quad (29)$$

is assumed to be parallel to the wave vector $\vec{Q}$ of the coupled LO phonon-plasmon mode.

A. Solution of the linearized dc part of the Boltzmann equation

By neglecting the nonlinear contribution to the dc part of the Boltzmann equation present in Eq. (25) and by choosing the drift velocity of the free electrons in the direction of the positive z axis of our Cartesian coordinate system, it is seen that $f_{dc}(\vec{v})$ obeys the inhomogeneous first-order differential equation

$$v_d \frac{\partial f_{dc}(\vec{v})}{\partial v_z} + f_{dc}(\vec{v}) = f_0(\vec{v}) . \tag{30}$$

The solution of Eq. (30) satisfying the boundary condition $f_{dc}(v_z \to -\infty) = 0$ is conveniently written

$$f_{dc}(v_z) = v_d^{-1} \int_{-\infty}^{v_z} f_0(v_z') \exp\left(-\int_{v_z'}^{v_z} \frac{dv_z''}{v_d}\right) dv_z' , \tag{31}$$

where the dependence of the distribution function on the x and y components of the electronic velocity has been suppressed because this dependence is unaffected by the impressed external dc electric field. Inserting the Maxwellian velocity distribution in Eq. (31) one obtains the exact solution

$$f_{dc}(\vec{v}) = \frac{(2\pi)^{-1} N_0}{v_d v_{th}^2} \exp\left[-\left(\frac{v_\perp}{v_{th}}\right)^2 - \frac{v_z}{v_d} + \left(\frac{v_{th}}{2v_d}\right)^2\right] \operatorname{erfc}(u) , \tag{32}$$

where we have introduced the component of the electron velocity perpendicular to the dc field, $v_\perp = (v_x^2 + v_y^2)^{1/2}$, the thermal electron velocity

$$v_{th} = (2k_B T/m^*)^{1/2} , \tag{33}$$

and the complementary error function[19]

$$\operatorname{erfc}(u) = \frac{2}{\sqrt{\pi}} \int_u^\infty e^{-t^2}\, dt \tag{34}$$

of the (real) argument

$$u = v_{th}/2v_d - v_z/v_{th} . \tag{35}$$

Alternatively, the dc part of the free-carrier distribution function can be written on the illustrating form

$$f_{dc}(\vec{v}) = f_0(\vec{v} - \vec{v}_d) \Delta(v_z, v_d) , \tag{36}$$

where $f_0(\vec{v} - \vec{v}_d)$ is just a Maxwell-Boltzmann distribution displaced an amount $\vec{v}_d$ in velocity space. In order to get the correct dc distribution function this drifted Maxwell-Boltzmann distribution must be corrected by the factor

$$\Delta(v_z, v_d) = \sqrt{\pi}\, \frac{v_{th}}{2v_d} F(u) \exp\left[\left(\frac{v_d}{v_{th}}\right)^2 \left(1 - \frac{2v_z}{v_d}\right)\right] , \tag{37}$$

where

$$F(u) = \exp(u^2) \operatorname{erfc}(u) . \tag{38}$$

Excluding from our considerations the small number of conduction electrons in the spectrum having very high velocities, i.e., $|v_z| \gg v_{th}$, one obtains[21] for $v_d \ll v_{th}$, $F(u) \approx (2/\sqrt{\pi})(v_d/v_{th})$. Inserting this value of $F(u)$ in Eq. (37), it appears, to lowest order in v_d/v_{th}, that $\Delta = 1$. This implies that the dc part of the distribution function for small drift velocities equals a drifted Maxwell-Boltzmann distribution, a result obtained previously by Spector.[8]

B. Solution of the linearized ac part of the Boltzmann equation

Linearizing the ac part of the transport equation one obtains the following inhomogeneous first-order differential equation for the Fourier amplitude of the distribution function:

$$\left(1 + i(\vec{Q}\cdot\vec{v} - \Omega)\tau + \vec{v}_d \cdot \frac{\partial}{\partial \vec{v}}\right) g(\vec{v}) = \frac{e\tau}{m^*} \vec{E}_1(\vec{Q}, \Omega) \cdot \frac{\partial f_{dc}(\vec{v})}{\partial \vec{v}} + \frac{N_1(\vec{Q}, \Omega)}{N_0} f_0(\vec{v}) + i\Omega \vec{\xi}_1(\vec{Q}, \Omega) \cdot \frac{\partial f_0(\vec{v})}{\partial \vec{v}} . \tag{39}$$

The formal solution of Eq. (39) is given by

$$g(\vec{v}) = \frac{\tau}{v_d} \int_{-\infty}^{v_z} \left[\left(\frac{e}{m^*} E_1 \frac{\partial f_{dc}(\vec{v}')}{\partial v_z'} + \frac{N_1}{N_0} \frac{f_0(\vec{v}')}{\tau} + i\frac{\Omega}{\tau} \xi_1 \frac{\partial f_0(\vec{v}')}{\partial v_z'}\right) \exp\left(-\int_{v_z'}^{v_z} [1 + i(Qv_z'' - \Omega)\tau] \frac{dv_z''}{v_d}\right)\right] dv_z' . \tag{40}$$

To derive an explicit expression for $g(\vec{v})$ one utilizes

$$\frac{\partial f_{dc}(\vec{v})}{\partial v_z} = \frac{f_0(\vec{v})}{v_d} \left(1 - \frac{\sqrt{\pi}}{2} \frac{v_{th}}{v_d} F(u)\right) . \tag{41}$$

Then, by combining Eqs. (27), (35), (38), (40), and (41) one obtains the result

$$g(\vec{v}) = g_E(\vec{v}) + g_N(\vec{v}) + g_\xi(\vec{v}) , \tag{42}$$

where

$$g_E(\vec{v}) = \frac{N_0 e\tau}{2\pi m^* (v_d v_{th})^2} \left(1 - iQl\frac{v_{th}}{2v_d}\right)^{-1/2} \exp\left(-\frac{v_\perp^2 + v_z^2}{v_{th}^2}\right) \left(F(\alpha) - \frac{v_{th}}{v_d} G(\alpha)\right) E_1(Q, \Omega) , \tag{43}$$

$$g_N(\vec{v})=\frac{1}{2\pi v_d v_{th}^2}\left(1-iQl\frac{v_{th}}{2v_d}\right)^{-1/2}\exp\left(-\frac{v_\perp^2+v_z^2}{v_{th}^2}\right)F(\alpha)N_1(Q,\Omega)\,, \tag{44}$$

and

$$g_\xi(\vec{v})=\frac{iN_0\Omega}{\pi^{3/2}v_d v_{th}^3}\left(1-iQl\frac{v_{th}}{2v_d}\right)^{-1}\exp\left(-\frac{v_\perp^2+v_z^2}{v_{th}^2}\right)\left[1-(1-i\Omega\tau)\frac{v_{th}}{v_d}\left(1-iQl\frac{v_{th}}{2v_d}\right)^{-1/2}\frac{\sqrt{\pi}}{2}F(\alpha)\right]\xi_1(Q,\Omega)\,. \tag{45}$$

In the above equation for the amplitude of the free-carrier ac distribution we have introduced the function $G(\alpha)$ by the integral representation

$$G(\alpha)=\exp(\alpha^2)\int_\alpha^\infty F(u')\exp[-(\alpha')^2]d\alpha'\,, \tag{46}$$

where

$$\alpha=(1-i\Omega\tau)\frac{v_{th}}{2v_d}\left(1-iQl\frac{v_{th}}{2v_d}\right)^{-1/2}-\left(1-iQl\frac{v_{th}}{2v_d}\right)^{1/2}\frac{v_z}{v_{th}}\,. \tag{47}$$

The transformations $u\to u'$ and $\alpha\to\alpha'$ are obtained by the replacement $v_z\to v_z'$. The electron mean free path (l) has been introduced using $l=v_{th}\tau$.

It should be noticed that the expression derived for the free-carrier distribution function in this section is also valid for acoustic lattice waves which generate longitudinal electric fields. For long-wavelength acoustic phonons the deformation of the energy bands can be incorporated in the expression for $g(\vec{v})$ by making the following replacement for the self-consistent electric-field amplitude[9]:

$$\vec{E}_1(\vec{Q},\Omega)\to\vec{E}_1(\vec{Q},\Omega)-(1/e)\vec{Q}\vec{Q}\cdot\overleftrightarrow{\Xi}\cdot\vec{\xi}_1(\vec{Q},\Omega)\,, \tag{48}$$

where $\overleftrightarrow{\Xi}$ is the deformation-potential tensor and $\vec{\xi}_1$ now denotes the displacement amplitude of the acoustic wave.

In the limit of small drift velocities, where the dc part of the distribution function is just a drifted Maxwell-Boltzmann distribution, the result for $g(\vec{v})$ is reduced to that obtained by Spector previously.[8]

C. Linear constitutive equation for the free-carrier current density

The amplitude of the longitudinal ac current density induced by the optical lattice wave is given by

$$J_1(Q,\Omega)=-e\int v_z g(\vec{v})d^3v\,. \tag{49}$$

By making use of the formulas[8]

$$\int_{-\infty}^{\infty}\exp(-ax^2)F(b-cx)dx=\left(\frac{\pi}{a-c^2}\right)^{1/2}\times F\left(\frac{b}{(1-c^2/a)^{1/2}}\right) \tag{50}$$

and

$$\int_{-\infty}^{\infty}x\exp(-ax^2)F(b-cx)dx=\frac{c}{a^{1/2}(a-c^2)}\times\left[1-\frac{\pi^{1/2}b}{(1-c^2/a)^{1/2}}F\left(\frac{b}{(1-c^2/a)^{1/2}}\right)\right]\,, \tag{51}$$

straightforward calculations show that the linear constitutive equation for the current density takes the form

$$J_1=\Sigma E_1-e(\Omega/Q)RN_1+ie\Omega N_0T\xi_1\,, \tag{52}$$

where the material descriptors of conduction (Σ), diffusion (R), and collision drag (T) are given by

$$\Sigma(Q,\Omega)=\sigma_0\left[\left(iQl\frac{v_{th}}{2v_d}\right)^{1/2}\frac{yI}{1-i\Omega\tau}+\frac{i}{Ql}\frac{v_{th}}{v_d}[1-\pi^{1/2}yF(y)]\right]\,, \tag{53}$$

$$R(Q,\Omega)=(i\Omega\tau)^{-1}[1-\pi^{1/2}yF(y)]\,, \tag{54}$$

and

$$T(Q,\Omega)=2(1-i\Omega\tau)^{-1}y^2[1-\pi^{1/2}yF(y)]\,. \tag{55}$$

For brevity, the quantities

$$y=\frac{1-i\Omega\tau}{Ql}\left(1+\frac{i}{Ql}\frac{2v_d}{v_{th}}\right)^{-1/2} \tag{56}$$

and

$$I=v_d^{-2}\int_{-\infty}^{\infty}v_z\exp\left[-\left(\frac{v_z}{v_{th}}\right)^2\right]G(\alpha)dv_z \tag{57}$$

have been introduced in Eqs. (53)–(55). Inserting Eq. (12) into Eq. (52) one obtains the constitutive relation for the free-carrier current density postulated in Eq. (17).

IV. LINEAR DISPERSION RELATION

By means of the explicit expressions for the material descriptors $\Sigma(Q,\Omega)$, $R(Q,\Omega)$, and $T(Q,\Omega)$, which were obtained in Sec. III, the frequency- and wave-vector-dependent response functions $\epsilon_{eff}(Q,\Omega)$ and $\sigma_{eff}(Q,\Omega)$ can be obtained. In turn, the real and imaginary parts of the LO phonon-plasmon wave number can be determined implicitly as functions of the coupled-mode frequency using the condition $\sigma_{eff} = i\epsilon_0 \epsilon_{eff} \Omega$. To obtain, in the most general case, the inverse dispersion relation one must take recourse to numerical methods. In this section we shall undertake a qualitative study of the linear dispersion relation in some important special cases. Numerical results will be presented in a forthcoming paper.

A. No free-carrier drift velocity

1. Arbitrary wavelengths

In the limit where the external dc electric field vanishes, i.e., $v_d \to 0$, the material descriptors now denoted by subscripts zero are given by

$$\Sigma_0(Q,\Omega) = \frac{2\sigma_0 z^2}{1 - i\Omega\tau}\left(\frac{\sqrt{\pi}}{i} z w(z) - 1\right), \tag{58}$$

$$R_0(Q,\Omega) = -(i\Omega\tau)^{-1}[(\sqrt{\pi}/i) z w(z) - 1], \tag{59}$$

and

$$T_0(Q,\Omega) = \frac{2z^2}{1 - i\Omega\tau}\left(\frac{\sqrt{\pi}}{i} z w(z) - 1\right), \tag{60}$$

where

$$z = (i + \Omega\tau)/Ql \tag{61}$$

and

$$w(z) = \frac{i}{\pi}\int_{-\infty}^{\infty} \frac{e^{-t^2} dt}{z - t}. \tag{62}$$

For $\mathrm{Im}(z) > 0$, $w(z)$ is related to the complementary error function of complex argument via[21]

$$w(z) = \exp(-z^2)\,\mathrm{erfc}(-iz) = F(z/i). \tag{63}$$

Without the external dc electric field one obtains

$$\Gamma_{eff}^{(2)} \to \Gamma_{eff}^{(0)} = \Gamma - \frac{1}{\tau}\frac{N_0}{N_i}\left(1 - \frac{N_i}{N_0}\frac{e^*}{e}\right)\frac{m^*(M^+ - M^-)}{2M^+M^-}, \tag{64}$$

which shows that the effective dielectric function ϵ_{eff} is independent of the wave vector of the mode. Furthermore, the contribution from the collision-drag effect to ϵ_{eff} can be neglected if

$$\Gamma\tau \gg \frac{m^* |M^+ - M^-|}{2M^+M^-}\frac{N_0}{N_i}\left|1 - \frac{N_i}{N_0}\frac{e^*}{e}\right|, \tag{65}$$

assuming $(N_i/N_0)(e^*/e) \geq 2$ or ≤ 0. If these last inequalities do not hold, one should, according to Eq. (23), replace the factor $|1 - (N_i/N_0)(e^*/e)|$ by unity to obtain the appropriate condition for the negelct of the collision drag. In lightly doped (i.e., $N_0 \cong 10^{20} - 10^{22}\ \mathrm{m}^{-3}$) polar III-V semiconducting compounds such as InSb and GaAs, for which numerical results will be presented in a forthcoming paper, the collision-drag effect can be neglected with confidence.

By inserting into Eq. (18) the wave-vector-independent effective dielectric function

$$\epsilon_{eff}^{(0)}(\Omega) = \epsilon_r^\infty \frac{\Omega^2 + i\Gamma_{eff}^{(1)}\Omega - \omega_{LO}^2}{\Omega^2 + i\Gamma_{eff}^{(0)}\Omega - \omega_{TO}^2} \tag{66}$$

and the effective conductivity

$$\sigma_{eff}^{(0)}(Q,\Omega) = \frac{2\sigma_0 z^2}{1 - i\Omega\tau}\left(1 - \frac{eN_0}{e^*N_i}\frac{\Omega}{\omega_p^\infty}(\omega_p^\infty\tau)^{-1}\right)\left[(i\Omega\tau)^{-1} + \left(\frac{\pi^{1/2}}{i} z w(z) - 1\right)^{-1} - \frac{2z^2}{1 - i\Omega\tau}\frac{eN_0}{e^*N_i}\right]^{-1}, \tag{67}$$

the implicit form of the dispersion relation is obtained. In the above equation the high-frequency screened free-carrier plasma frequency

$$\omega_p^\infty = (N_0 e^2/\epsilon_0 \epsilon_r^\infty m^*)^{1/2} \tag{68}$$

has been introduced.

2. Long wavelengths

In the limit of long wavelengths, i.e., in the local regime $Ql \ll 1$, where consequently $|z| \gg 1$, one gets by means of the asymptotic expansion[21]

$$w(z) \sim \frac{i}{z\sqrt{\pi}}\left(1 + \sum_{m=1}^{\infty} \frac{(1)(3)\cdots(2m-1)}{(2z^2)^m}\right), \quad |\arg z| < \frac{3\pi}{4} \tag{69}$$

the following expression for the effective conductivity

$$\sigma_{eff}^{(0)}(Q,\Omega) = \frac{\sigma_0 d}{1 - i\Omega\tau}\left(1 - \frac{eN_0}{e^*N_i}\frac{\Omega}{\omega_p^\infty}(\omega_p^\infty\tau)^{-1}\right)\left[1 - \frac{d}{2}\left(3 - \frac{1}{i\Omega\tau}\right)\left(\frac{Ql}{1 - i\Omega\tau}\right)^2 + \frac{3d^2}{2}\left(\frac{3}{2} + d^{-1} - \frac{1}{i\Omega\tau} - \frac{1}{6(\Omega\tau)^2}\right)\left(\frac{Ql}{1 - i\Omega\tau}\right)^4 + O((Ql)^6)\right] \tag{70}$$

with the abbreviation

$$d = \left(1 - \frac{1}{1 - i\Omega\tau}\frac{eN_0}{e^*N_i}\right)^{-1}. \tag{71}$$

If the collision-drag effect can be neglected Eq. (70) is simplified to the form

$$\sigma_{\rm eff}^{(0)}(Q,\Omega)=\frac{\sigma_0}{1-i\Omega\tau}\left[1-\tfrac{1}{2}\left(3-\frac{1}{i\Omega\tau}\right)\left(\frac{Ql}{1-i\Omega\tau}\right)^2+\tfrac{1}{4}\left(15-\frac{6}{i\Omega\tau}-\frac{1}{(\Omega\tau)^2}\right)\left(\frac{Ql}{1-i\Omega\tau}\right)^4+O((Ql)^6)\right]. \tag{72}$$

The result in Eq. (72) can be compared with that obtained by Spector[8] in his study of the interaction of acoustic waves and conduction electrons. For acoustic waves one has $\Omega\tau=Ql(V_p/V_{\rm th})\ll 1$ for $Ql\ll 1$, V_p being the phase velocity of the soundlike mode. Neglecting terms of order $(Ql)^4$ and higher in $\sigma_{\rm eff}^{(0)}$, one obtains precisely the results of Spector.[8]

Combining Eqs. (18), (66), (70), and (71), one obtains to second order in Ql the inverse dispersion relation

$$Q=(v_{\rm th}\tau)^{-1}\left(2\Omega\tau\frac{1-i\Omega\tau}{1-3i\Omega\tau}\right)^{1/2}\left[\Omega\tau+i\left(1-\frac{eN_0}{e^*N_i}\right)\right]^{1/2}$$
$$\times\left(\frac{\Omega\tau+i(1-eN_0/e^*N_i)}{(\omega_p^\infty)^2\tau/\Omega-eN_0/e^*N_i}\,\frac{\Omega^2+i\Gamma_{\rm eff}^{(1)}\Omega-\omega_{\rm LO}^2}{\Omega^2+i\Gamma_{\rm eff}^{(0)}\Omega-\omega_{\rm TO}^2}-1\right)^{1/2},\quad Ql\ll 1. \tag{73}$$

3. Short wavelengths

In the limit of short wavelengths, i.e., in the extremely nonlocal regime $Ql\gg 1$, but still outside the quantum regime, one has $|z|\ll 1$ so that it is appropriate to use the series expansion[21]

$$w(z)=\sum_{n=0}^{\infty}\frac{(iz)^n}{\Gamma(\frac{1}{2}n+1)}, \tag{74}$$

where Γ denotes the Γ function. On the basis of Eq. (74) the effective conductivity takes the form

$$\sigma_{\rm eff}^{(0)}(Q,\Omega)=\frac{-2i\Omega\tau\sigma_0}{(Ql)^2}\left(1-\frac{eN_0}{e^*N_i}\frac{\Omega}{\omega_p^\infty}(\omega_p^\infty\tau)^{-1}\right)\left(1+i\sqrt{\pi}\,\frac{\Omega\tau}{Ql}+i[\pi-2(2-d^{-1})(1-i\Omega\tau)]\frac{\Omega\tau}{(Ql)^2}+O((Ql)^{-3})\right). \tag{75}$$

By combining Eqs. (18), (66), and (75) one obtains to lowest order in $(Ql)^{-1}$ an inverse dispersion relation

$$Q=i\sqrt{2}\,\frac{\omega_p^\infty}{v_{\rm th}}\left[\frac{\Omega^2+i\Gamma_{\rm eff}^{(0)}\Omega-\omega_{\rm TO}^2}{\Omega^2+i\Gamma_{\rm eff}^{(1)}\Omega-\omega_{\rm LO}^2}\left(1-\frac{eN_0}{e^*N_i}\frac{\Omega}{(\omega_p^\infty)^2\tau}\right)\right]^{1/2}. \tag{76}$$

It should be remembered that particle-hole excitations giving rise to Landau damping[22] are neglected in the present semiclassical treatment.

B. Small free-carrier drift velocity

1. Arbitrary wavelengths

Let us consider free-carrier drift velocities so small that the conduction-electron distribution function is accurately described by a drifted Maxwell-Boltzmann distribution. Under these conditions the material descriptors $R(Q,\Omega)$ and $T(Q,\Omega)$ are still given by Eqs. (54) and (55). However, the descriptor $\Sigma(Q,\Omega)$ given by Eq. (53) will take the reduced form

$$\Sigma(Q,\Omega)=[2\sigma_0/(1-i\Omega\tau)]y^2[1-\pi^{1/2}xF(x)]. \tag{77}$$

where

$$x=\frac{1-i(\Omega-Qv_d)\tau}{Ql[1+i(2v_d/v_{\rm th})(Ql)^{-1}]^{1/2}}, \tag{78}$$

in accordance with the analysis of Spector.[8] Inserting the effective damping coefficient

$$\Gamma_{\rm eff}^{(2)}(Q,\Omega)=\Gamma-\frac{1}{\tau}\frac{N_0}{N_i}\frac{m^*(M^+-M^-)}{2M^+M^-}\times\left(1-\frac{N_ie^*}{N_0e}\,\frac{1-\pi^{1/2}yF(y)}{1-\pi^{1/2}xF(x)}\right) \tag{79}$$

in Eq. (19), and the above simplified expression for Σ into Eq. (20), the appropriate form of the dispersion relation for small free-carrier drift velocities can be obtained numerically via Eq. (18).

2. Long wavelengths

In the local regime, i.e., for $Ql\ll 1$ or equivalently $|z|\gg 1$, one has

$$\frac{x}{y}=1-z^{-1}\frac{v_d}{v_{\rm th}}\approx 1, \tag{80}$$

so that $\Gamma_{\rm eff}^{(2)}\approx\Gamma_{\rm eff}^{(0)}$ and consequently $\epsilon_{\rm eff}(Q,\Omega)\approx\epsilon_{\rm eff}^{(0)}(\Omega)$. The long-wavelength approximation to the effective conductivity can be obtained by making the replacement $z\to iy$ in Eq. (67). Doing this one has

$$\sigma_{eff}(Q,\Omega)=\frac{2\sigma_0 y^2}{1-i\Omega\tau}\left(1-\frac{eN_0}{e^*N_i}\frac{\Omega}{(\omega_p^\infty)^2\tau}\right)\left([1-\pi^{1/2}yF(y)]^{-1}-(i\Omega\tau)^{-1}-\frac{2y^2}{1-i\Omega\tau}\frac{eN_0}{e^*N_i}\right)^{-1}. \tag{81}$$

A further simplification of the expression for σ_{eff} can be achieved if $|y|\gg 1$. Thus, by using the asymptotic expansion in Eq. (69) for $F(y)=w(iy)$ one finds

$$\sigma_{eff}(Q,\Omega)=\frac{\sigma_0 d}{1-i\Omega\tau}\left(1-\frac{eN_0}{e^*N_i}\frac{\Omega}{(\omega_p^\infty)^2\tau}\right)\left[1-\frac{d}{2}\left(3-\frac{1}{i\Omega\tau}\right)\left(\frac{Ql}{1-i\Omega\tau}\right)^2\left(1+\frac{2iv_d}{v_{th}}(Ql)^{-1}\right)\right.$$
$$\left.+\frac{3d^2}{2}\left(\frac{3}{2}+d^{-1}-\frac{1}{i\Omega\tau}-\frac{1}{6(\Omega\tau)^2}\right)\left(\frac{Ql}{1-i\Omega\tau}\right)^4\left(1+\frac{2iv_d}{v_{th}}(Ql)^{-1}\right)^2+O(y^{-6})\right]. \tag{82}$$

In the limit $v_d\to 0$ Eq. (82) reproduces the result in Eq. (70).

By combining Eqs. (18), (66), and (82) to second order in y^{-1}, one obtains the following important dispersion relation at long wavelengths:

$$Ql=\frac{v_d}{iv_{th}}\pm\left\{\frac{2i\Omega\tau(1-i\Omega\tau)^2}{d(1-3i\Omega\tau)}\left[\frac{\Omega(\Omega+i/\tau)}{d(\omega_p^\infty)^2}\left(1-\frac{eN_0}{e^*N_i}\frac{\Omega}{(\omega_p^\infty)^2\tau}\right)^{-1}\frac{\Omega^2+i\Gamma_{eff}^{(1)}\Omega-\omega_{LO}^2}{\Omega^2+i\Gamma_{eff}^{(0)}\Omega-\omega_{TO}^2}-1\right]-\left(\frac{v_d}{v_{th}}\right)^2\right\}^{1/2}. \tag{83}$$

If no electrons are trapped and one substitutes the relations $\omega_D\equiv v_d^2/D_n=2(v_d/v_{th})^2/\tau$ and $\omega_c\equiv\sigma_0/(\epsilon_0\epsilon_r^\infty)$ for the diffusion frequency (ω_D) and the dielectric relaxation frequency (ω_c) into Eq. (83), one obtains, neglecting the collision drag, essentially the results of Woodruff[10] in the limit where the plasma is collision dominated.

3. *Short wavelengths*

In the extremely nonlocal regime ($Ql\gg 1$) one has for small drift velocities ($v_d/v_{th}\ll 1$)

$$x\cong y+\frac{v_d}{v_{th}}\cong\frac{z}{i}+\frac{v_d}{v_{th}}, \tag{84}$$

which shows that $|y|\ll 1$ and $|x|\ll 1$. Thus, by using the series expansion in Eq. (74), it follows that the effective damping coefficient $\Gamma_{eff}^{(2)}(Q,\Omega)$ approaches a wave-vector- and frequency-independent damping constant $\Gamma_{eff}^{(d)}$ given by

$$\Gamma_{eff}^{(d)}(v_d)=\Gamma-\frac{1}{\tau}\frac{N_0}{N_i}\frac{m^*(M^+-M^-)}{2M^+M^-}\times\left[1-\frac{N_ie^*}{N_0e}\left(1+\pi^{1/2}\frac{v_d}{v_{th}}\right)\right] \tag{85}$$

in the approximation $w(z)=1$. Notice that $\Gamma_{eff}^{(d)}$ is a linear function of the free-carrier drift velocity. Now, the phononlike wave-vector-independent dielectric fucntion becomes

$$\epsilon_{eff}(\Omega)=\epsilon_r^\infty\frac{\Omega^2+i\Gamma_{eff}^{(1)}\Omega-\omega_{LO}^2}{\Omega^2+i\Gamma_{eff}^{(d)}(v_d)\Omega-\omega_{TO}^2}. \tag{86}$$

On the basis of Eq. (84) the effective conductivity takes the form

$$\sigma_{eff}(Q,\Omega)=\frac{-2i\Omega\tau\sigma_0}{(Ql)^2}\left(1-\frac{eN_0}{e^*N_i}\frac{\Omega}{(\omega_p^\infty)^2\tau}\right)\times\left[1+i\frac{\pi^{1/2}}{v_{th}}\left(\frac{\Omega}{Q}-v_d\right)+O\left((Ql)^{-2},\left(\frac{v_d}{v_{th}}\right)^2,(Ql)^{-1}\frac{v_d}{v_{th}}\right)\right] \tag{87}$$

to third order in the quantities $(Ql)^{-1}$, v_d/v_{th}, and products of these. The above results, being valid for arbitrary magnitudes of $\Omega\tau$, agrees with that obtained by Spector[8] if one neglects the collision-drag effect. In the limit $v_d\to 0$, Eq. (87) becomes identical to Eq. (75), as required. Note that $\sigma_{eff}(Q,\Omega)$ is a linear function of v_d in the present approximation.

The inverse dispersion relation, determined by the usual condition $\sigma_{eff}=i\epsilon_0\epsilon_{eff}\Omega$, is obtained by solving the cubic equation

$$(Ql)^3+2\left((\omega_p^\infty\tau)^2-\frac{eN_0}{e^*N_i}(\Omega\tau)\right)\frac{\Omega^2+i\Gamma_{eff}^{(d)}(v_d)\Omega-\omega_{TO}^2}{\Omega^2+i\Gamma_{eff}^{(1)}\Omega-\omega_{LO}^2}\times\left[\left(1-\pi^{1/2}\frac{v_d}{v_{th}}\right)Ql+i\pi^{1/2}\Omega\tau\right]=0. \tag{88}$$

V. STATIONARY ENERGY-TRANSPORT VELOCITY

In Sec. III we solved the linearized Boltzmann transport equation in order to obtain explicit expressions for the material descriptors Σ, R, and T. However, to study the energy transport associated with the LO phonon-plasmon wave one cannot neglect the nonlinearities in the transport equation. In the present work we shall examine the energy-transport velocity by retaining in the dc part of the Boltzmann equation the contribution arising from the nonlinear term $\vec{E}_1(\vec{r},t)$

$\cdot\, \partial g(\vec{r}, \vec{v}, t)/\partial \vec{v}$ only. Thus, second and higher harmonics in the free-carrier distribution function and the electric field are neglected. Furthermore, the small contribution from the collision-drag effect will be neglected in our calculation of the energy-transport velocity.

A. Solution of the nonlinear Boltzmann equation

To solve the nonlinear Boltzmann equation we make the ansatz given in Eq. (28). Retaining the dc contribution arising from the nonlinear term $\vec{E}_1 \cdot \partial g/\partial \vec{v}$ only, the ac part of the free-carrier distribution function still is given by the Fourier amplitude in Eq. (39). The dc part of the Boltzmann equation is changed to

$$\vec{v}_d \cdot \frac{\partial f_{dc}(\vec{v})}{\partial \vec{v}} + f_{dc}(\vec{v}) = f_0(\vec{v}) + \frac{e\tau}{2m^*}\mathrm{Re}\left(\vec{E}_1^*(\vec{Q}, \Omega)\cdot \frac{\partial g(\vec{v})}{\partial \vec{v}}\right) \times \exp(-2z\,\mathrm{Im}Q)\,. \qquad (89)$$

To determine approximately $f_{dc}(\vec{v})$ and $g(\vec{v})$ from the coupled differential equations (39) and (89) the dc part of the distribution function is split into a sum of a large real linear term (f_{dc}^{L}) and a small complex nonlinear term (f_{dc}^{NL}). Thus,

$$f_{dc}(\vec{v}) = f_{dc}^{L}(\vec{v}) + \tfrac{1}{2}\{f_{dc}^{NL}(\vec{v}) + [f_{dc}^{NL}(\vec{v})]^*\}\,. \qquad (90)$$

Inserting Eq. (90) into Eq. (89) one gets

$$\vec{v}_d \cdot \frac{\partial f_{dc}^{L}(\vec{v})}{\partial \vec{v}} + f_{dc}^{L}(\vec{v}) = f_0(\vec{v}) \qquad (91)$$

and

$$\vec{v}_d \cdot \frac{\partial f_{dc}^{NL}(\vec{v})}{\partial \vec{v}} + f_{dc}^{NL}(\vec{v}) = \frac{e\tau}{2m^*}\vec{E}_1^*(\vec{Q}, \Omega)\cdot \frac{\partial g(\vec{v})}{\partial \vec{v}} \times \exp(-2z\,\mathrm{Im}Q)\,. \qquad (92)$$

Now, the procedure for obtaining the nonlinear part of the dc free-carrier distribution is straightforward. Since the differential equation (91) determining the linear part of the dc distribution function is identical to Eq. (30), $f_{dc}^{L}(\vec{v})$ is given by Eq. (32). Inserting $f_{dc}(\vec{v}) \cong f_{dc}^{L}(\vec{v})$ into the differential equation for the ac part of the free-carrier distribution function, i.e., Eq. (39), one obtain the solution given in Eq. (42) for $g(\vec{v})$. By using this approximative solution for $g(\vec{v})$ on the right-hand side of Eq. (92), the formal solution of this differential equation takes the form

$$f_{dc}^{NL}(\vec{v}) = \frac{e\tau}{2m^* v_d}E_1^*(Q, \Omega) \times \int_{-\infty}^{v_z} \frac{\partial g(\vec{v}')}{\partial v_z'} \exp\left(-\int_{v_z'}^{v_z}\frac{dv_z''}{v_d}\right) dv_z' \times \exp(-2z\,\mathrm{Im}Q)\,. \qquad (93)$$

By integration by parts the LO phonon-plasmon induced part of the dc distribution equivalently can be written

$$f_{dc}^{NL}(\vec{v}) = \frac{1}{2}\frac{E_1^*(Q, \Omega)}{E_0} \times \left[g(\vec{v}) - \frac{1}{v_d}\int_{-\infty}^{v_z}\exp\left(\frac{v_z' - v_z}{v_d}\right) g(\vec{v}')\,dv_z'\right] \times \exp(-2z\,\mathrm{Im}Q)\,. \qquad (94)$$

In the remaining part of this section the contribution from the collision-drag effect to $g(\vec{v})$ will be neglected.

B. Energy balance

To study the energy balance in the coupled LO phonon-plasmon system the Boltzmann equation (25) is multiplied by $\frac{1}{2}m^* v^2$ and then integrated over the velocity space. Hence, one obtains

$$\frac{\partial w_f}{\partial t} + \vec{\nabla}\cdot\vec{S}_f - \frac{e}{2}(\vec{E}_0 + \vec{E}_1)\cdot\left(\int v^2 \frac{\partial f}{\partial \vec{v}} d^3v\right) + \frac{m^*}{2}\int (f - f_s)\frac{v^2}{\tau} d^3v = 0\,, \qquad (95)$$

where we have introduced the energy flux density, or Poynting vector, of the free-carrier system

$$\vec{S}_f(\vec{r}, t) = \frac{m^*}{2}\int v^2 \vec{v} f(\vec{r}, \vec{v}, t)\, d^3v \qquad (96)$$

and the free-carrier energy density

$$w_f(\vec{r}, t) = \frac{m^*}{2}\int v^2 f(\vec{r}, \vec{v}, t)\, d^3v\,. \qquad (97)$$

Since integration by parts gives

$$\int v^2 \frac{\partial f}{\partial \vec{v}} d^3v = -2\int \vec{v} f\, d^3v = \frac{2}{e}(\vec{J}_0 + \vec{J}_1)\,, \qquad (98)$$

where $\vec{J}_0$ is the dc free-carrier current density, and since from the Maxwell equation (3)

$$\vec{J}_1 = -\epsilon_0\frac{\partial \vec{E}_1}{\partial t} - \frac{\partial \vec{P}_1}{\partial t}\,, \qquad (99)$$

the nonlinear term in Eq. (95) can be transformed as

$$-\frac{e}{2}(\vec{E}_0 + \vec{E}_1)\cdot\left(\int v^2 \frac{\partial f}{\partial \vec{v}} d^3v\right) = \frac{\partial}{\partial t}(\tfrac{1}{2}\epsilon_0 E_1^2) + \vec{E}_1\cdot\frac{\partial \vec{P}_1}{\partial t} - \vec{E}_0\cdot\vec{J}_0 - \vec{E}_0\cdot\vec{J}_1 - \vec{E}_1\cdot\vec{J}_0\,. \qquad (100)$$

Combining the equation of motion for the relative displacement of the two sublattices (suppressing the collision drag) [Eq. (7)], and the Born and Huang constitutive equation for the polarization [Eq. (16)], the second term on the right-hand side of Eq. (100) can be converted to the form

$$\vec{E}_1 \cdot \frac{\partial \vec{P}_1}{\partial t} = N_i M \Gamma \left(\frac{\partial W_1}{\partial t}\right)^2 + \frac{\partial w_i}{\partial t} + \frac{\partial}{\partial t}\left(\tfrac{1}{2}\epsilon_0 \chi^\infty E_1^2\right), \quad (101)$$

where

$$w_i(\vec{r}, t) = \frac{N_i M}{2}\left[\left(\frac{\partial W_1}{\partial t}\right)^2 + \omega_{TO}^2 W_1^2\right] \quad (102)$$

is the energy density of the oscillating ions. The fourth term on the right-hand side of Eq. (100) can be written

$$-\vec{E}_0 \cdot \vec{J}_1 = \frac{\partial}{\partial t}\left[\epsilon_0(1+\chi^\infty)\vec{E}_0 \cdot \vec{E}_1\right] + N_i e^* \vec{E}_0 \cdot \frac{\partial \vec{W}_1}{\partial t} \quad (103)$$

if one uses Eqs. (16) and (99). By combining Eqs. (95), (100), (101), and (103) one obtains

$$\vec{\nabla} \cdot \vec{S}_f + \frac{\partial}{\partial t}(w_f + w_i + w_{em}) + N_i e^* \vec{E}_0 \cdot \frac{\partial \vec{W}_1}{\partial t} + eN_0 \vec{E}_1 \cdot \vec{v}_d + N_i M \Gamma \left(\frac{\partial W_1}{\partial t}\right)^2 + \frac{m^*}{2}\int (f - f_s)\frac{v^2}{\tau} d^3v - \sigma_0 |\vec{E}_0|^2 = 0, \quad (104)$$

where we have introduced the energy density of the electromagnetic field, including the energy of excitations in the dielectric background as evidenced by the presence of the high-frequency susceptibility, via

$$w_{em} = \tfrac{1}{2}\epsilon_0(1+\chi^\infty)|\vec{E}_0 + \vec{E}_1|^2 . \quad (105)$$

If the external electric field is removed, one obtains after integration of Eq. (104) over a fixed volume V of the solid

$$\int_A \vec{S}_f \cdot d\vec{A} + N_i M \Gamma \int_V \left(\frac{\partial W_1}{\partial t}\right)^2 dV + \frac{m^*}{2\tau}\int_V \int (f - f_s) v^2 d^3v \, dV = -\frac{\partial}{\partial t}\int_V (w_f + w_i + w_{em}) dV . \quad (106)$$

The interpretation of Eq. (106) from the principle of energy conservation is straightforward. Thus, the time rate-of-energy loss in the volume V due to transfer through its closed surface A (first term on the left-hand side of the equation) and dissipation via frictional damping of the harmonic oscillators plus collision losses of the free-carrier system (neglect of collision drag) (second and third terms) is balanced by the total rate-of-loss of the energy stored within V (term on right-hand side). Notice that $f_{dc}^{L} = f_0$, $f_{dc}^{NL} = f_{dc}^{NL}(\vec{v}_d = 0)$, $g = g(\vec{v}_d = 0)$, and $w_{em} = w_{em}(\vec{E}_0 = 0)$ in Eq. (106). When $\vec{E}_0 \neq 0$ the contributions $N_i e^* \vec{E}_0 \cdot (\partial \vec{W}_1/\partial t)$ representing the rate of work per unit volume done by the external dc field on the ionic oscillators, and $N_0 e \vec{E}_1 \cdot \vec{v}_d$ giving the power per unit volume transferred from the drifting carriers to the ac part of the electromagnetic field, must be incorporated in the considerations on the energy balance. However, since these products of ac and dc quantities give no cycle average they are of no significance for our study of the stationary energy-transport velocity associated with the LO phonon-plasmon mode. Also the dc joule-effect density $\vec{E}_0 \cdot \vec{J}_0$ is unimportant in our context.

C. Velocity of energy transport

1. Basic concepts

In the following the energy transport associated with the coupled LO phonon-plasmon wave is studied under steady-state conditions, given by the requirement that the cycle-averaged energy density in the wave is time independent, i.e., $\partial\langle w\rangle/\partial t = 0$, where w is the total energy density of the coupled mode.

According to Eq. (104) one has

$$\langle w\rangle = \langle w_{em}\rangle + \langle w_i\rangle + \langle w_f\rangle . \quad (107)$$

The cycle-averaged energy density in the electromagnetic field is given by

$$\langle w_{em}\rangle = \tfrac{1}{2}\epsilon_0(1+\chi^\infty)E_0^2 + \langle w_{em}^{NL}\rangle , \quad (108)$$

where the first term is the contribution from the external dc field, and the second term

$$\langle w_{em}^{NL}\rangle = \tfrac{1}{4}\epsilon_0(1+\chi^\infty)|\vec{E}_1(\vec{Q}, \Omega)|^2 \exp(-2z \,\mathrm{Im} Q) \quad (109)$$

is the contribution associated with the LO phonon-plasmon mode. The cycle-averaged energy density stored in the ionic oscillators is ($\langle w_i\rangle = \langle w_i^{NL}\rangle$)

$$\langle w_i^{NL}\rangle = \tfrac{1}{4}N_i M(\Omega^2 + \omega_{TO}^2)|\vec{W}_1(\vec{Q}, \Omega)|^2 \exp(-2z \,\mathrm{Im} Q) . \quad (110)$$

Finally, the cycle-averaged energy density of the free-carrier system is given by

$$\langle w_f\rangle = \langle w_f^{B}\rangle + \langle w_f^{L}\rangle + \langle w_f^{NL}\rangle , \quad (111)$$

where

$$\langle w_f^{B}\rangle = \frac{m^*}{2}\int v^2 f_0(\vec{v}) d^3v \quad (112)$$

is the energy density bound in the thermal bath of the free carriers,

$$\langle w_f^{L}\rangle = \frac{m^*}{2}\int v^2[f_{dc}^{L}(\vec{v}) - f_0(\vec{v})] d^3v \quad (113)$$

is the energy density induced by the external dc field, and

$$\langle w_f^{\rm NL}\rangle = \frac{m^*}{2}\int v^2[f_{\rm dc}^{\rm NL}(\vec{v})d^3v + {\rm c.c.}]\exp(-2z\,{\rm Im}Q) \tag{114}$$

is the energy density associated with the bunching of the free carriers. Note that any partitioning of the energy of the system between the material excitations and the electromagnetic field is inevitably associated with an element of arbitrariness, because of the impossibility of assigning the interaction energy to either source. Only the total energy is unambiguously defined for such a system.

The cycle-averaged Poynting vector of the free-carrier system can be decomposed into

$$\langle \vec{S}_f\rangle = \langle \vec{S}_f^{\rm L}\rangle + \langle \vec{S}_f^{\rm NL}\rangle , \tag{115}$$

where

$$\langle \vec{S}_f^{\rm L}\rangle = \frac{m^*}{2}\int v^2\vec{v}[f_{\rm dc}^{\rm L}(\vec{v}) - f_0(\vec{v})]d^3v \tag{116}$$

is the energy flux density induced by the external dc electric field, and

$$\langle \vec{S}_f^{\rm NL}\rangle = \frac{m^*}{4}\int v^2\vec{v}[f_{\rm dc}^{\rm NL}(\vec{v})d^3v + {\rm c.c.}]\exp(-2z\,{\rm Im}Q) \tag{117}$$

is the part of the Poynting vector which is associated with the LO phonon-plasmon mode and which arises from the nonlinearity of the system. In general, in a coupled matter-electromagnetic wave, the total Poynting vector is composed of a contribution from the material and one from the electromagnetic field. However, since we are considering purely longitudinal modes the electromagnetic radiation field is absent. Thus, the electromagnetic Poynting vector equals zero. The energy flux associated with the ion system also vanishes, because it has been assumed that the frequency of the LO phonon is independent of its wavelength. In parallel to a previous comment on the enrgy density of the mode it should be stressed that only the total Poynting vector is unambiguously defined for a coupled matter-electromagnetic wave.

For an absorbing (or amplifying) medium, the group velocity

$$\vec{v}_g = \vec{\nabla}_{\vec{Q}}\Omega(\vec{Q}) \tag{118}$$

cannot be identified with the velocity of energy propagation.[14-18] Instead, under steady-state conditions, the energy-transport velocity ($\vec{v}_E$) of the LO phonon-plasmon mode can be defined as the time-averaged rate of energy flow per unit area associated with the wave divided by the time-averaged stored-energy density of the wave,[17,18] i.e.,

$$\vec{v}_E = \frac{\langle \vec{S}_f^{\rm NL}\rangle}{\langle w_{em}^{\rm NL}\rangle + \langle w_i^{\rm NL}\rangle + \langle w_f^{\rm NL}\rangle}\,. \tag{119}$$

2. *No external electric field*

In the absence of the external dc electric field the Fourier amplitude of the free-carrier distribution function is given by

$$g(\vec{v}) = \frac{N_1(Q,\Omega)/N_0 - (e\tau v_z/k_BT)E_1(Q,\Omega)}{1 + i(Qv_z - \Omega)\tau}f_0(\vec{v}) \tag{120}$$

if the collision-drag effect is neglected. On the basis of Eq. (120) the nonlinear part of the dc free-carrier distribution is obtained via

$$f_{\rm dc}^{\rm NL}(\vec{v}) = \frac{e\tau}{2m^*}\vec{E}_1^*(Q,\Omega)\cdot\frac{\partial g(\vec{v})}{\partial\vec{v}}\exp(-2z\,{\rm Im}Q)\,. \tag{121}$$

By inserting Eq. (121) into Eqs. (114) and (117), and then making use of the formulas

$$\int_{-\infty}^{\infty}\frac{te^{-t^2}}{z-t}dt = \frac{\pi}{i}zw(z) - \pi^{1/2}, \tag{122}$$

$$\int_{-\infty}^{\infty}\frac{t^2e^{-t^2}}{z-t}dt = \frac{\pi}{i}z^2w(z) - \pi^{1/2}z\,, \tag{123}$$

and

$$\int_{-\infty}^{\infty}\frac{t^3e^{-t^2}}{z-t}dt = \frac{\pi}{i}z^3w(z) - \pi^{1/2}z^2 - \frac{\pi^{1/2}}{2}, \tag{124}$$

one obtains for the energy density associated with the free-carrier bunching the result

$$\langle w_f^{\rm NL}\rangle = \left[\frac{eN_0}{4iQ}E_1^*(Q,\Omega)\left(\frac{\pi^{1/2}}{i}zw(z) - 1\right)\left(\frac{N_1(Q,\Omega)}{N_0} - \frac{2e\tau z}{m^*v_{\rm th}}E_1(Q,\Omega)\right) + {\rm c.c.}\right]\exp(-2z\,{\rm Im}Q)\,, \tag{125}$$

and for the Poynting vector of the free-carrier system the equation

$$\langle \vec{S}_f^{\rm NL}\rangle = \left\{\hat{z}\frac{eN_0v_{\rm th}}{4iQ}E_1^*(Q,\Omega)\left[\left(\frac{\pi^{1/2}}{i}(3z^2+1)w(z) - 3z\right)\frac{N_1(Q,\Omega)}{N_0} - \frac{2e\tau}{m^*v_{\rm th}}\left(\frac{\pi^{1/2}}{i}(3z^2+1)zw(z) - 3z^2 - \tfrac{5}{2}\right)E_1(Q,\Omega)\right] + {\rm c.c.}\right\}\exp(-2z\,{\rm Im}Q)\,, \tag{126}$$

where $\hat{z}$ is a unit vector in the direction of the positive z axis.

To obtain the stationary energy-transport velocity it is necessary to relate $W_1(Q,\Omega)$ and $N_1(Q,\Omega)$ to $E_1(Q,\Omega)$. From the equation of motion for the lattice vibrations [Eq. (8)] it follows that

$$W_1(Q,\Omega)=\frac{e^*}{M}\frac{E_1(Q,\Omega)}{\omega_{\rm TO}^2-\Omega^2-i\Gamma\Omega} \qquad (127)$$

when $F_1^{\rm cd}=0$. By using this result with the Maxwell equation (6) and the constitutive equation (16) one finds

$$N_1(Q,\Omega)=\frac{\epsilon_0\epsilon_r^\infty Q}{ie}\left(1+\frac{(\omega_i^\infty)^2}{\omega_{\rm TO}^2-\Omega^2-i\Gamma\Omega}\right)E_1(Q,\Omega), \qquad (128)$$

where

$$\omega_i^\infty=\left(\frac{N_i(e^*)^2}{M\epsilon_0\epsilon_r^\infty}\right)^{1/2} \qquad (129)$$

denotes the high-frequency screened-ion plasma frequency.

Finally, by using the implicit form of the dispersion relation

$$\epsilon_r^\infty\frac{\Omega^2+i\Gamma\Omega-\omega_{\rm LO}^2}{\Omega^2+i\Gamma\Omega-\omega_{\rm TO}^2}+\frac{2i\sigma_0 z^2}{\epsilon_0\Omega(1-i\Omega\tau)}\left[(i\Omega\tau)^{-1}+\left(\frac{\pi^{1/2}}{i}zw(z)-1\right)^{-1}\right]^{-1}=0 \qquad (130)$$

and by combining Eqs. (109), (110), (119), and (125)–(128), on normalized form one obtains the following expression for the stationary energy-transport velocity:

$$\frac{v_E(Q,\Omega)}{v_{\rm th}}=\frac{2\,{\rm Re}[H(Q,\Omega)Y(z)-5i(\omega_p^\infty\tau)^2/Ql]}{1+\dfrac{(\omega_i^\infty)^2(\Omega^2+\omega_{\rm TO}^2)}{(\omega_{\rm TO}^2-\Omega^2)^2+(\Gamma\Omega)^2}+2\,{\rm Re}[H(Q,\Omega)X(z)]}, \qquad (131)$$

where

$$X(z)=(\pi^{1/2}/i)zw(z)-1, \qquad (132)$$

$$Y(z)=(\pi^{1/2}/i)(3z^2+1)w(z)-3z, \qquad (133)$$

and

$$H(Q,\Omega)=\frac{2i(\omega_p^\infty\tau)^2}{Ql}\,\frac{z}{1+(i\Omega\tau)^{-1}X(z)}. \qquad (134)$$

In Eq. (131) the stationary energy-transport velocity is a function of Q and Ω. To eliminate Q one has to apply the dispersion relation given in Eq. (130).

In the long-wavelength regime ($Ql\ll 1$) one obtains by means of the asymptotic expansion for $w(z)$ given in Eq. (69)

$$H(Q,\Omega)Y(z)-\frac{5i(\omega_p^\infty\tau)^2}{Ql}=\frac{i(\omega_p^\infty\tau)^2}{2(i+\Omega\tau)}\{[11-5(i\Omega\tau)^{-1}]z^{-1}+O(z^{-3})\} \qquad (135)$$

and

$$H(Q,\Omega)X(z)=\frac{i(\omega_p^\infty\tau)^2}{i+\Omega\tau}\{1+\tfrac{1}{2}[3-(i\Omega\tau)^{-1}]z^{-2}+O(z^{-4})\}. \qquad (136)$$

Inserting Eqs. (135) and (136) in Eq. (131) the expression for the normalized stationary energy-transport velocity takes the explicit form

$$\frac{v_E(Q,\Omega)}{v_{\rm th}}=\frac{17v_{\rm th}\left(\dfrac{\omega_p^\infty}{\Omega}\right)^2\dfrac{Q}{\Omega}}{1+2\left(\dfrac{\omega_p^\infty}{\Omega}\right)^2+\dfrac{(\omega_i^\infty)^2(\Omega^2+\omega_{\rm TO}^2)}{(\Omega^2-\omega_{\rm TO}^2)^2}},\quad Ql\ll 1,\quad \Omega\tau\gg 1,\quad \Gamma\to 0 \qquad (137)$$

if it is assumed that the solid-state plasma is collisionless and that the intrinsic damping of the bound harmonic oscillators can be neglected. The above expression for the energy-transport velocity holds in the frequency region where mode propagation takes place. The real wave number can be eliminated from Eq. (137) by using Eq.(73), letting $\Gamma\to 0$ and $\omega\tau\to\infty$.

VI. OUTLOOK

The present theoretical investigation of the linear dispersion relation and the stationary energy-transport velocity, associated with a coupled polar LO phonon-plasmon system under the influence of an impressed free-carrier drift velocity, has been limited in several respects. Thus, it seems natural to generalize the self-consistent field approach, based here on the Boltzmann-Vlasov equation, into the quantum regime to study the LO phonon-plasmon modes at degenerate free-carrier concentrations in polar semiconductors. At high carrier densities, where the collision-drag effect plays an essential role, the treatment of electron collisions with thermal lattice vibrations from the standpoint of a basic electron-lattice interaction theory will be appropriate. A detailed analysis of the energy-transport velocity assoicated with quasilongitudinal modes which radiate electromagnetic energy and with dispersive phonon modes which themselves transport energy will be essential. Finally, it would be interesting to study the nonstationary energy transport connected with anharmonically coupled systems of polar lattice vibrations and plasma modes, especially solitary waves.

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