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Lattice scattering mobility of a two-dimensional electron gas in GaAs

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Theory of electron mobility is presented for a two-dimensional electron gas in a polar material. Calculated values of mobility are given for gallium arsenide for temperatures of 77–300 K and surface electron concentrations of (0.5–1.2) $\times 10^{16}$ m^{-2}. The mobility values are much lower than the bulk values and are also strongly dependent on electron concentration.

INTRODUCTION

The properties of a two-dimensional electron gas[1] (2D EG) formed in the inversion and accumulation layers of metal-oxide–semiconductor field-effect transistors (MOSFET's) have received a good deal of attention from many workers[2] in recent years. Most of the studies are, however, related with 2D EG in Si-SiO_2 system, because Si MOSFET's are readily available due to advances in Si technology. Some work on III-V and II-VI compounds was also undertaken[3-7] by using common insulators to isolate the gate electrode from the semiconductor. However, 2D EG in GaAs could not be produced by these methods. The first realization of 2D EG on GaAs surface is due to Störmer *et al.*[8] who employed a GaAs-GaAlAs heterostructure. The authors have observed Shubnikov–de Haas (SdH) oscillations at 4.2 K and below and reported the mobility values at He temperature.

With the successful experimental realization of the two-dimensional electron gas in GaAs, it is only natural to expect that considerable attention will now be directed to the study of different electronic properties of 2D EG in GaAs and also in other polar materials. A particularly interesting field of research will be the study of mobility of 2D EG and its nature of variation in comparison to the bulk values. In fact, Dingle *et al.*[9] have already reported the values of mobility of 2D EG formed in a GaAs-GaAlAs superlattice. In this paper we address ourselves to the study of mobility theoretically. The relevant lattice scattering mobility will be due to acoustic and polar-optic phonon scattering. Acoustic phonon scattering has been treated in connection with 2D EG in Si.[10] Polar-optic phonon scattering for 2D EG has not, however, been considered in the literature at all. The present paper is, therefore, the first report of its kind. In the following we shall first derive the collision operator for polar-mode scattering for 2D EG. An iterative method of solving the Boltzmann equation in presence of acoustic and polar-optic scattering will then be described. The calculated results for mobility of 2D EG in GaAs will then be presented and discussed.

THEORY

We shall treat the problem under triangular potential approximation[11] and assume that all the electrons occupy the lowest subband. The energy of the carriers is then expressed as

$$E(\vec{k}) = E_0 + \frac{\hbar^2 k^2}{2m^*}, \tag{1}$$

where $\vec{k}$ is the wave vector and m^* is the effective mass of the electron parallel to the surface. E_0 is the energy of the lowest subband given by[11]

$$E_0 = \frac{(e\hbar F_s)^{2/3}}{(2m^*)^{1/3}} S_0, \tag{2}$$

S_0 being the first root of the Airy function.

The surface electric field F_s appearing in (2) is related to the inversion-layer concentration N_{inv} by

$$F_s = \frac{eN_{\text{inv}}}{\epsilon_{\text{sc}}}, \tag{3}$$

ϵ_{sc} being the permittivity of semiconductor. If $f(\vec{k})$ is the distribution function of the electrons, then its time rate of change due to collision may be expressed as[12]

$$\left.\frac{\partial f(\vec{k})}{\partial t}\right|_{\text{coll}} = \frac{A}{(2\pi)^2}\sum_{\vec{k}'}\{P(\vec{k},\vec{k}')f(\vec{k})[1-f(\vec{k}')] - P(\vec{k}',\vec{k})f(\vec{k}')[1-f(\vec{k})]\}\,d\vec{k}', \tag{4}$$

where A is the surface area and $P(\vec{k},\vec{k}')$ is the probability of transition from $\vec{k}$ to $\vec{k}'$.

In the presence of a drift field F applied parallel to the surface, we may expand the distribution function as

$$f(\vec{k}) = f_0(E) - \frac{e\hbar F}{m^*} k\cos\theta\ \phi(E)\frac{\partial f_0(E)}{\partial E}, \tag{5}$$

where $f_0(E)$ is the equilibrium Fermi distribution, θ is the angle between $\vec{F}$ and $\vec{k}$, and $\phi(E)$ is the perturbation function to be determined from Boltzmann equation.

The principle of detailed balance gives

$$P(\vec{k},\vec{k}')f_0(E)[1-f_0(E')] = P(\vec{k}',\vec{k})f_0(E')[1-f_0(E)]\,. \tag{6}$$

Using (6) and (5) in (4) and retaining only the terms containing F, we get

$$\left.\frac{\partial f(\vec{k})}{\partial t}\right|_{\text{coll}} = \frac{Ae\hbar F}{4\pi^2 k_B T m^*}\int P(\vec{k},\vec{k}')f_0(E)[1-f_0(E')] \times [\phi(E)k\cos\theta - \phi(E')k'\cos\theta']\,d\vec{k}'\,, \tag{7}$$

where k_B is the Boltzmann constant and T is the temperature. For polar-optic scattering[13]

$$P(\vec{k},\vec{k}') = \frac{2\pi}{\hbar}\frac{e^2\hbar\omega_0}{2V\epsilon_0}\left(\frac{1}{K_\infty}-\frac{1}{K_S}\right)\frac{1}{|\vec{k}'-\vec{k}|^2} \times (N+\tfrac{1}{2}\mp\tfrac{1}{2})\delta(E-E'\mp\hbar\omega_0)\,, \tag{8}$$

where the upper and lower signs refer to the emission and absorption processes, respectively. In (8) $\hbar\omega_0$ is the energy of the phonon, V is the volume, K_∞ and K_S are, respectively, the high-frequency and static dielectric constant, and N is the phonon occupation number. Also,

$$k'\cos\theta' = k'\cos\beta\cos\theta + k'\sin\beta\sin\theta\,, \tag{9}$$

$$d\vec{k}' = k'dk'd\beta\,, \tag{10}$$

where β is the angle between $\vec{k}$ and $\vec{k}'$.

Substituting (8)–(10) in (7), carrying out the integration over β between 0 and 2π, converting k and k' to E and E' and integrating over E' by using the δ function,[14] we arrive finally at

$$\left.\frac{\partial f(\vec{k})}{\partial t}\right|_{\text{coll po}} = -\frac{e\hbar F}{m^*}\frac{\partial f_0}{\partial E}k\cos\theta\,\frac{e^2}{4\hbar d\epsilon_0}\left(\frac{1}{K_\infty}-\frac{1}{K_S}\right)\frac{1}{1-f_0(E)} \times\Big(\{(N+1)[1-f_0(E-\hbar\omega_0)]u(E-E_0-\hbar\omega_0)+N[1-f_0(E+\hbar\omega_0)]u(E-E_0+\hbar\omega_0)\}\phi(E) -\{(N+1)[1-f_0(E-\hbar\omega_0)][(E-\hbar\omega_0)/E]u(E-E_0-\hbar\omega_0)\}\phi(E-\hbar\omega_0) -\{N[1-f_0(E+\hbar\omega_0)]u(E-E_0+\hbar\omega_0)\}\phi(E+\hbar\omega_0)\Big)\,, \tag{11}$$

u being a step function [$u(x)=0$ (1) for $x\leq 1$ (>1)]. For acoustic phonon scattering

$$\left.\frac{\partial f(\vec{k})}{\partial t}\right|_{\text{coll ac}} = -\frac{e\hbar F}{m^*}\frac{\partial f_0}{\partial E}k\cos\theta\,\frac{\phi(E)}{\tau_{\text{ac}}}\,, \tag{12}$$

where the relaxation time τ_{ac} for acoustic-phonon scattering is given as[10]

$$\frac{1}{\tau_{\text{ac}}} = \frac{m^*E_1^2k_BT}{\hbar^3\rho c_l^2 d}\,, \tag{13}$$

where E_1 is the deformation potential constant, ρ is the mass density, and C_l is the longitudinal velocity of sound.

In (11) and (13) d is the thickness of the inversion layer and when only the lowest subband is occupied is given by

$$d = \frac{E_0}{eF_s}\,. \tag{14}$$

Using (11) and (12) in the Boltzmann equation, we finally get

$$1 = S_0(E)\phi(E) - S_a(E)\phi(E+\hbar\omega_0) - S_e(E)\phi(E-\hbar\omega_0)\,. \tag{15}$$

The solution of the above equation by an iterative procedure is quite well-known.[15] Our present method is essentially the one described by Nag.[16]

The starting values $\phi^0(E)$ are obtained by putting S_a and S_e terms equal to zero. Thus

$$\phi^0(E+l\hbar\omega_0) = S_0^{-1}(E+l\hbar\omega_0)\,,\quad l=0,1,2,\ldots\,. \tag{16}$$

In the $(m+1)$th step of iteration, ϕ is obtained from

$$\phi^{m+1}(E+l\hbar\omega_0) = S_0^{-1}(E+l\hbar\omega_0) \times\{1+S_a(E+l\hbar\omega_0)\phi^m[E+(l+1)\hbar\omega_0] +S_e(E+l\hbar\omega_0)\phi^m[E+(l-1)\hbar\omega_0]\}\,. \tag{17}$$

The mobility is calculated from

$$\mu = -\frac{e}{m^*}\int_{E_0}^{\infty}\phi(E)(E-E_0)\frac{\partial f_0}{\partial E}dE\left(\int_{E_0}^{\infty}f_0(E)dE\right)^{-1}\,. \tag{18}$$

RESULTS

We have calculated the mobility of electrons in GaAs by using the following values of parameters: $m^*=0.072m_0$, $K_S=13.13$, $K=11.1$, $E_1=12$ eV[14], $\rho=5.36\times10^3$ kg m^{-3}, $C_l=5.24\times10^3$ m sec^{-1}, $\hbar\omega_0/k_B=416$ K.

The calculated values of mobility for a surface carrier concentration of 10^{16} m^{-2} are given in Fig. 1 for the range of temperature 77–300 K. It has

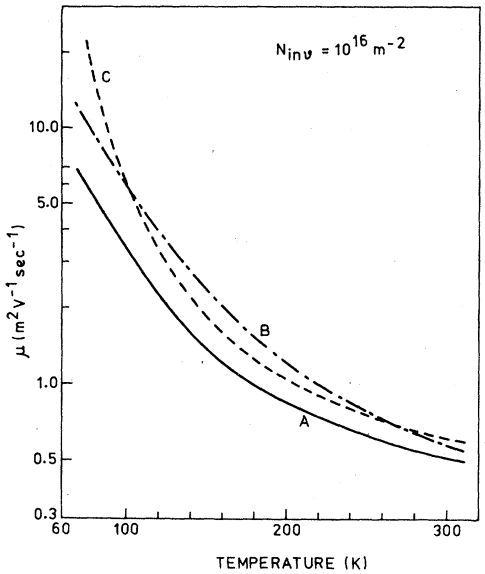

FIG. 1. Variation of mobility in GaAs with temperature. Curve A: mobility limited by acoustic and polar-optic-phonon scattering, degenerate statistics. Curve B: acoustic and polar, nondegenerate statistics. Curve C: only polar-optic, degenerate statistics.

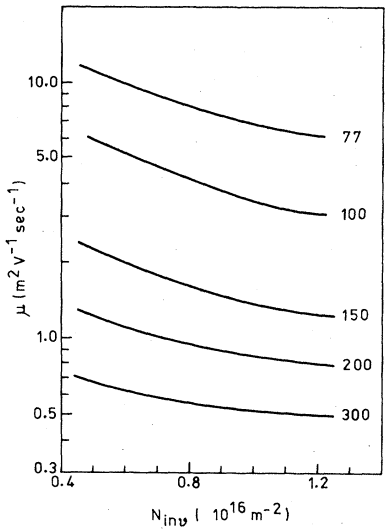

FIG. 2. Variation of mobility with inversion-layer concentration for different values of temperature.

been found that for lower temperatures (15) converges after five or six steps of iteration, while nearly twenty steps are needed for room temperature. By comparing the present values with the values of lattice mobility for bulk GaAs,[15] we notice that the values for 2D EG are reduced by a factor of about two. We have also calculated the results by assuming nondegenerate statistics. The curve given in Fig. 1 suggests that the calculated values will be in error if degenerate statistics are not applied. We have also given the value of the polar-mode-scattering-limited mobility in Fig. 1 which is found to contribute about 80% of the lattice mobility. The effect of changing the concentration on values of mobility is shown in Fig. 2 and the mobility is seen to decrease with increasing surface-carrier concentration.

CONCLUSION

An expression for the collision operator for polar-optic phonon scattering in 2D EG has been derived and the values of surface-carrier mobility in GaAs due to lattice scattering has been presented for the temperatures of 77–300 K. The results could not be compared with experiments as experimental values of mobility are not yet reported in the literature. We, however, note two significant characteristics of the mobility. The mobility is lower than in the bulk and is strongly concentration dependent.

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