Two-dimensional electron gas in a magnetic field: Polarizability

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Due to the complete quantization of the energy spectrum of the two-dimensional electron gas, even in weak (normal) magnetic fields, its properties are not separable into monotonic and oscillatory components as in three dimensions. This is exhibited explicitly for the longitudinal polarizability.

Stemming from experimental studies of inversion layers and electron surface layers on liquid helium in the middle 1960's, interest in the two-dimensional (2D) electron gas in a magnetic field has increased, since many of these experiments require the presence of moderate or strong magnetic fields perpendicular to the electron plane. The 2D electron gas is also an attractive model many-body system: the calculation of ground-state properties is, in a sense, simpler and freer of awkward divergences than for the corresponding three-dimensional (3D) model, and these properties are substantially different in 2D and 3D in a way we shall try to make clear.

In three dimensions the electron gas in a uniform magnetic field has a combined continuous and discrete energy-level spectrum. As a consequence all the properties of the system can be separated into parts which are analytic (monotonic) or oscillatory with respect to the magnetic field; this has been understood mathematically for nearly 50 years. In particular the monotonic field dependence arises from the continuum portion of the energy spectrum and is obtained by making a power-series expansion (in terms of the magnetic field strength) at some point in the calculation. However, the spectrum of the 2D electron gas is completely quantized even in the presence of an infinitesimal magnetic field. This can be shown to result in nonanalytic behavior in terms of the field strength and, if the ground-state properties are analyzed by the same procedure as in three dimensions, the introduction of the corresponding power series must lead to spurious divergences. (Such a procedure may be justifiable in a finite-temperature formalism,[1] where the divergences are handled by introducing a temperature-dependent cutoff, but then the zero-temperature limit cannot be taken.) We shall substantiate these remarks by presenting an exact expression for the real part of the longitudinal ground-state polarizability $\chi_1(\vec{q},\omega)$ in the random-phase approximation (RPA) which extends Stern's expression[2] to an arbitrary magnetic field. For simplicity free-electron parameters are used, $\hbar=1$ and the magnetic field strength is characterized by the cyclotron frequency ω_c. In addition $\zeta=k_F^2/2m$ denotes the chemical potential and $G=2me^2N/k_Fq^3$, where N is the electron surface density, is Stern's parameter.

The longitudinal dielectric response of the 2D electron gas has been studied in the random-phase approximation by Horing and Yildiz[3] who express the polarizability as a complicated multiple integral. By a procedure which differs somewhat from theirs we have in essence performed all but one of the integrations and obtained:

$$\chi_1(\vec{q},\omega)=-\frac{G\omega_c}{Nk_F^2}\int_0^\infty \cos(\omega t)\exp[a(\cos\omega_c t-1)] \times\sin(a\sin\omega_c t)g_n(\omega_c t)\,dt \quad , (1)$$

where $a=q^2/2m\omega_c$, n is the integer part of ζ/ω_c,

$$g_n(x)=L_n[2a(1-\cos x)]+2L_{n-1}^1[2a(1-\cos x)] \quad , \tag{2}$$

and L denotes a standard Laguerre polynomial. Our main point is that χ_1 is manifestly nonanalytic at $\omega_c=0$ and cannot be expanded in a power series about this point.[4] However, χ_1 is continuous at $\omega_c=0$ as may be seen by recovering Stern's expression by means of the classical asymptotic estimate

$$n^{-\alpha}L_n^\alpha(x/n)\sim x^{-\alpha/2}J_\alpha(2x^{1/2}), \tag{3}$$
$$n\to\infty,\quad |x| \text{ bounded} \quad .$$

This shows that

$$\lim_{\omega_c\to 0}\chi_1(\vec{q},\omega)=-2G\int_0^\infty \frac{dt}{t}\cos(ut)\sin(zt)J_1(t) \tag{4}$$

in terms of the standard variables $u=m\omega/qk_F$, $z=q/2k_F$. The integral in Eq. (4) is

$\frac{1}{2}[h(z+u)+h(z-u)]$, where

$$h(x)=\mathrm{sgn}x\int_0^\infty \frac{dt}{t}\sin|x|tJ_1(t)$$
$$=-\mathrm{sgn}x[|x|-(x^2-1)^{1/2}\Theta(|x|-1)] \quad (5)$$

is a tabulated Fourier transform and Θ denotes the unit step function. Therefore

$$\lim_{\omega_c\to 0}\chi_1(\vec{q},\omega)=G\{2z-\mathrm{sgn}(z+u)\Theta(|z+u|-1)\times[(z+u)^2-1]^{1/2}-\mathrm{sgn}(z-u)\Theta\times(|z-u|-1)[(z-u)^2-1]^{1/2}\}\ , \quad (6)$$

which is precisely Stern's result.

The expression (1) clearly displays features characterizing a system with discrete energy levels. Thus, if ω is a multiple of ω_c, by decomposing the range of integration into segments of length $2\pi/\omega_c$ and invoking periodicity we find that χ_1 has a δ function singularity. When ω is not an integer multiple of ω_c Eq. (1) can be reduced to

$$\chi_1(\vec{q},\omega)=\frac{G}{2Nk_F^2}e^{-a}\mathrm{cosec}(\omega\pi/\omega_c)\times\int_0^{2\pi}\sin\left[\frac{\omega}{\omega_c}(t-\pi)\right]e^{a\cos t}\times\sin(a\sin t)g_n(t)\,dt\ . \quad (7)$$

In particular, in the high-field limit, $g_n(t)=1$. In this case Eq. (7) can be expressed as an incomplete γ function and reduces to an expression obtained independently by Horing.[5]

The same analytic structure has been explicitly worked out for each term of the RPA ground state energy of the 2D electron gas and will be discussed fully in a forthcoming report.

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[1]A. Isihara and T. Toyoda, Phys. Rev. B 19, 831 (1979).
[2]F. Stern, Phys. Rev. Lett. 18, 546 (1967).
[3]N. J. M. Horing and M. M. Yildiz, Ann. Phys. (N.Y.) 97, 216 (1976).
[4]Such an expansion is in essence made in Ref. 1 to obtain the low-field behavior of the correlation energy.
[5]N. J. Horing (private communication).