New theory of flicker noise

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It is shown that the flicker noise arises in islands, of any size (from a few nm to several μm), which are enclosed by a potential-energy barrier or which, especially for the smallest volumes, present an energy well for the carriers. The essential feature of such islands, which may be originated by many types of structural defects of the conducting medium, is that the current entering them, due to the thermionic and tunnel emissions in the first case and to generation-recombination processes in the second one, is an exponential function of a random variable energy ϕ. In fact, from that and from the Maxwell and Poisson equations, it follows that the fluctuations of the island charge obey a stochastic relaxation equation whose relaxation time τ, according to the value range of ϕ, has a huge dispersion, even from 10^{-12} to 10^8 sec. Moreover, it is shown that the random driving source of the equation is the shot noise across the island surface and that its spectrum, according the Schottky and Nyquist theorems, is inversely proportional to τ itself. The charge fluctuation in its turn induces a fluctuating current dipole vector whose effects on the voltage noise at the device terminals are computed by means of the impedance field method. Finally, by summing the independent contributions of the islands, a voltage noise spectrum of the type Γ/f^{γ} is achieved, down to however low a frequency f, and its parameters Γ and γ, with $0.7<\gamma<1.3$, are computed as functions of the ϕ distribution and of other system quantities, such as volume, carrier number, temperature, current, and frequency itself.

I. INTRODUCTION

In a previous work[1] a cause of flicker noise in the unipolar electron devices has been identified in the dielectric relaxation phenomena arising in islands which have an ion density about twice that of the surrounding medium and an extension, along the current direction, comparable with the Debye length. The model is based on the assumption, justified by means of considerations of irreversible thermodynamics, that the fluctuations of the electric field parallel to the average current are negligible with respect to the perpendicular ones.

In the present paper it is shown that the flicker noise arises also in islands of any shape and size, from a few nm to several μm, which are enclosed by a potential-energy barrier or which, especially in the case of the smallest volumes, present an energy well for the carriers. The essential feature of such islands, which may be originated by any type of structural defects of the medium, is that the current entering them is an exponential function of a random variable energy ϕ. In reality, it is such a dependence that creates the wide dispersion of correlation times necessary to account for the flicker noise.

In fact, from the continuity and Poisson equations it is deduced that the fluctuations of the island charge obey a stochastic relaxation equation whose relaxation time τ, according to the ϕ dispersion, may assume values from 10^{-12} to 10^8 sec. It is also shown that the random driving source of the equation is constituted by the shot current across the island surface and that, according to the Nyquist and Schottky theorems, it has a spectrum inversely proportional to τ itself.

The fluctuations of the island charge induces a fluctuating current dipole whose effects on the voltage noise at the device terminals are computed by means of the impedance field method.[1]

Finally, by summing the independent contributions of the islands, a voltage noise spectrum of the type Γ/f^{γ}, down to however low a frequency f, is obtained, and the dependence of its parameters Γ and γ, with $0.7<\gamma<1.3$, on the ϕ distribution and on the system quantities, such as volume, carrier number, temperature, current, and frequency itself, is deduced. The theoretical results so obtained agree well with the very numerous experimental measures of the flicker noise.

II. PHYSICAL MODEL

A. Islands

Let a zone or island of a conducting medium be enclosed by a potential-energy barrier or let it present an energy well for the charge carrier as shown in Fig. 1(a). Such a situation may occur in many physical systems. For instance, metal thin-film resistors present islands of various shape and size, which are separated from other islands and from continuous parts of the film by a semi-insulating zone of any width.[2]

The same situation occurs in a sharper way in the thick-film resistors prepared with the proper commercial pastes or inks, which are constituted of conducting grains, ranging in size from 10 to 100 nm, separated among themselves by barriers

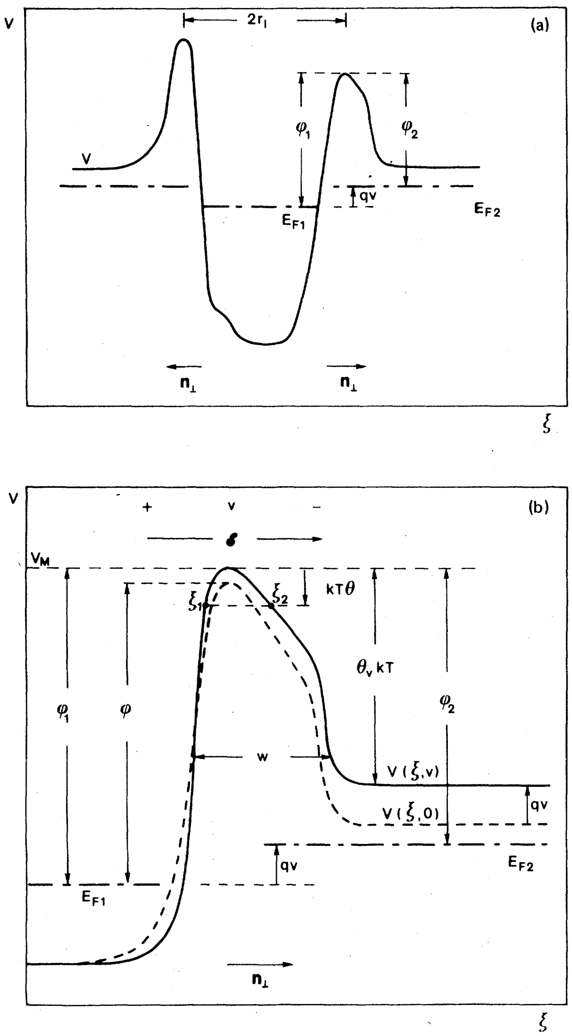

FIG. 1. (a) Potential-energy diagram of an island and (b) its right barrier in a different scale.

provided by dielectric layers.[3]

Zones able to release and to receive carriers across a closed energy barrier or into an energy well may be constituted of agglomerates of localized states, due to any cause, allocated both in the bulk and on the surface of the device or in the oxides coating its parts.

An important case of islands enclosed by an energy barrier or able to store carriers in an energy well (Fig. 1) is given by the microdefects which are formed in the semiconductor devices during both the crystal growth and the device fabrication.[4] In fact, the investigations by means of the transmission electron microscope show that the swirl defects are composed of perfect dislocation loops of any size, from a few nm to several μm. Such dislocation loops, arising from the condensation of the thermal point defects (vacancies and self-interstitials), as well as other defect clusters produced by impurity segregations and by precipitates or induced by misfits and deformations during post-growth processing, create for the charge carriers local potential energies which, among others, may have the shape of Fig. 1(a) characterized by a well or by a closed barrier.

B. Thermionic and tunnel emissions

When the characteristic size $2r_I$ of the island [Fig. 1(a)] is much greater than the wavelength of the states localized inside it, the energy-band theory and the effective-mass approach may be used in order to compute the density $\vec{J}$ of the current which crosses the energy barrier (Fig. 1).

The transport mechanisms of the current are the thermionic and tunnel emissions and $\vec{J}$ becomes[5]

$$\vec{J} = A^*T^2\alpha[\exp(-\varphi_2/kT) - \exp(-\varphi_1/kT)]\vec{n}_\perp + \vec{\xi}\,, \tag{2.1}$$

where $\vec{\xi}$ is the stochastic component of the current due to the independent random passages of the carriers across the energy barrier and

$$\alpha = \int_{-\infty}^{\theta_v} \frac{\exp(\theta)}{1+\exp[F(|\theta|)\,|\theta|/\theta]}\,d\theta\,. \tag{2.2}$$

In (2.1) and (2.2), $A^* = 4\pi q m^* k^2/h^3$ is the Richardson constant, q and m^* are the charge and the effective mass of the electron, respectively, k and h are the Boltzmann and Planck constants, respectively, T is the absolute temperature, $\varphi_i(v)$ is the energy barrier height evaluated from the Fermi level E_{Fi} of the side i $(i = 1, 2)$ [Fig. 1(b)], and φ is its value when the voltage v applied to the barrier of width $w(v)$ is null. The function $F(|\theta|)$ is defined by

$$F(|\theta|) = \frac{4\pi(2m^*)^{1/2}}{h}\int_{\xi_1(|\theta|)}^{\xi_2(|\theta|)} [kT|\theta| - V_M + V(\xi)]^{1/2}\,d\xi\,, \tag{2.3}$$

in which V_M is the maximum of the potential energy $V(\xi)$, $\xi_i(|\theta|)$ is the abscissa of the classical turning point [Fig. 1(b)], and $kT\theta$ is the electron energy evaluated downward from the maximum V_M [Fig. 1(b)] in the direction, represented by the unit vector $\vec{n}_\perp$, perpendicular to the energy barrier surfaces in the considered point.

The coefficient α, which takes into account both the quantum-mechanical reflection and tunneling through the barrier,[5,6] according to (2.2) and (2.3), becomes $\alpha = \int_{-\infty}^{0}\exp(\theta)d\theta = 1$ when, for every energy $kT\theta$, the distance $(\xi_2 - \xi_1)$ between the classical turning points is much greater than the "wavelength" inside the barrier, that is, when the thermionic emission prevails on the quantum effects.

The fluctuation $\Delta\vec{J}$ of $\vec{J}$ due to $\vec{\xi}$ and to fluctuation Δv of v around its time average $\bar{v}$, according to (2.1)–(2.3), becomes

$$\Delta\vec{J} = g\,\Delta v \vec{n}_\perp + \vec{\xi}\,, \qquad (2.4)$$

where, for

$$\bar{v} \ll kT/q\,, \qquad (2.5)$$

the conductance for unit surface g, since $\varphi_1 - \varphi_2 = qv$ [see Fig. 1(b)], is given by

$$g = \frac{\partial \vec{J}}{\partial v}\bigg|_{v=\bar{v}} = qA^* k^{-1} T\alpha(\bar{v}) \exp\left(\frac{-\varphi}{kT}\right)\,. \qquad (2.6)$$

On the other hand, in the barrier depletion layer the charge density is practically independent of v, and the fluctuation $\Delta\vec{\mathcal{E}}$, independent of ξ, of the electric field $\vec{\mathcal{E}}$ becomes

$$\Delta v = w(\bar{v})\vec{n}_\perp \cdot \Delta\vec{\mathcal{E}}\,, \qquad (2.7)$$

so that (2.4) may be written also in the form

$$\Delta\vec{J} = gw\Delta\vec{\mathcal{E}}_\perp + \vec{\xi}\,, \qquad (2.8)$$

where $\Delta\vec{\mathcal{E}}_\perp = (\vec{n}_\perp \cdot \Delta\vec{\mathcal{E}})\vec{n}_\perp$ is the $\Delta\vec{\mathcal{E}}$ component along the $\vec{n}_\perp$ direction.

The condition (2.5) for w of the order of few nm is verified for values up to 10^4 V/cm of the electric field $\bar{\mathcal{E}}_v$, externally applied across the depletion layer, because, at least when w is independent of v, it is $\bar{v} = w\bar{\mathcal{E}}_v$.

The fluctuation Δi of the current

$$i = -\int_{A_I} d\vec{A} \cdot \vec{J}\,, \qquad (2.9)$$

entering the island across any closed surface A_I which belongs to the depletion layer and is perpendicular to $\vec{n}_\perp$ in each point, according to (2.8), becomes

$$\Delta i = -\int_{A_I} d\vec{A} \cdot (gw\Delta\vec{\mathcal{E}}) + \eta\,, \qquad (2.10)$$

where $d\vec{A} = \vec{n}_\perp dA$ is the vector element of area and

$$\eta = -\int_{A_I} d\vec{A} \cdot \vec{\xi} \qquad (2.11)$$

is the stochastic current crossing A_I due to the shot noise.

When Δv is constant on A_I from (2.7) and (2.10), it also follows that

$$\Delta i = -G\Delta v + \eta\,, \qquad (2.12)$$

where

$$G = \int_{A_I} g\, dA \qquad (2.13)$$

is the island conductance.

Other conduction processes in a closed insulating shell have conductivity (gw) and conductance G which, according to (2.6), (2.8), and (2.13), depend exponentially on a quantity φ. This happens for the Frenkel-Poole emission[7] for which φ represents the trapped electron energy with respect to the conduction-band edge for the thermally stimulated current,[7] and for the ionic current[7] for which φ is the activation energy. Also the hopping conduction obeys the exponential law (2.6) in which the variable exponent is the impurity distance.[7]

Also, in the metal-semiconductor contacts, in the p-n and n-n^+ junctions, the current is given by (2.1). However, in these cases the depletion layer w is comparable with the device dimensions and, hence, only a very few islands completely surrounded by a high-energy barrier may eventually exist in the device itself.

C. Generation and recombination processes

When the characteristic dimension $2r_I$ of the island is comparable with the wavelength of the states which are localized in it and which are characterized by discrete energy levels E_j, the current i entering the island may be computed by means of the capture cross section σ_j of each jth level.

In fact, at the thermodynamic equilibrium, the mean currents i^+ and i^- entering and leaving the island, respectively, are both equal to

$$i_0 = \sum_j q\bar{n} v_m \sigma_j N_j \exp\left(\frac{E_j - E_F}{kT}\right)\,, \qquad (2.14)$$

where $\bar{n}$ is the time average of density n of the conduction-band electrons, v_m is their mean velocity, and $N_j \exp[(E_j - E_F)/kT]$ is the number of the unoccupied states in the jth level. When the mean potential energy inside the island varies with $-qv$ because it is storing a positive excess charge, i^- remains unchanged, whereas i^+ becomes $i^+ = i_0\exp(-qv/kT)$ as a consequence of the energy-level shift of $-qv$.

Therefore the net current $i = i^+ - i^- + \eta$ entering the island becomes

$$i = i_0[\exp(-qv/kT) - 1] + \eta\,, \qquad (2.15)$$

while its fluctuation is given again by (2.12) where the conductance G, when $\bar{v}$ satisfies (2.5), now has the form

$$G = qi_0/kT\,. \qquad (2.16)$$

For nondegenerate material, since the Fermi level E_F is below the conduction-band edge E_c, one also has $v_m = (4kT/m\pi)^{1/2}$, $n = N_c \exp[(E_F - E_c)/kT]$ with $N_c = 2(2\pi m^* kT/h^2)^{3/2}$, so that i_0, according to (2.14), becomes

$$i_0 = A^*T^2 \sum_j 2^{3/2} \sigma_j N_j \exp\left(\frac{-\varphi_j}{kT}\right) , \qquad (2.17)$$

where $\varphi_j = E_c - E_j$ is the activation energy of the level E_j, and N_j is its state number.

The island surface A_I which intervenes in (2.9) and in the following relationships is the smallest closed surface enclosing all the electrons belonging to the levels E_j. In general, A_I is different from σ_j.

III. STOCHASTIC RELAXATION EQUATION

A. Thermionic and tunnel emissions

Let Ω_I and $Q = \int_{\Omega_I} \rho d\Omega$ be the volume and the total electric charge, respectively, of the island enclosed by surface A_I. From Gauss's theorem and the charge conservation principle, or, less directly from the integration on Ω_I of the Poisson and Maxwell equations,

$$\vec{\nabla} \cdot (\epsilon \vec{\mathcal{E}}) = \rho , \qquad (3.1)$$

$$\vec{\nabla} \times \vec{H} = \frac{\partial(\epsilon \vec{\mathcal{E}})}{\partial t} + \vec{J} , \qquad (3.2)$$

where ϵ is the dielectric constant, ρ is the charge density, and $\vec{H}$ is the magnetic field, one obtains that

$$Q = \int_{A_I} d\vec{A} \cdot (\epsilon \vec{\mathcal{E}}) , \qquad (3.3)$$

$$\frac{\partial Q}{\partial t} = -\int_{A_I} d\vec{A} \cdot \vec{J} , \qquad (3.4)$$

which, for the fluctuations ΔQ, give

$$\Delta Q = \int_{A_I} d\vec{A} \cdot \Delta(\epsilon \vec{\mathcal{E}}) , \qquad (3.5)$$

$$\frac{\partial \Delta Q}{\partial t} = -\int_{A_I} d\vec{A} \cdot \Delta \vec{J} = \Delta i . \qquad (3.6)$$

When ϵ and (gw) are constant on A_I, Eqs. (3.5), (3.6), and (2.10) yield for ΔQ the stochastic relaxation equation

$$\frac{\partial \Delta Q}{\partial t} = -\frac{\Delta Q}{\tau} + \eta , \qquad (3.7)$$

where the relaxation time τ is given by

$$\tau = \epsilon / wg . \qquad (3.8)$$

In general, Eq. (3.7) holds true more especially when Δv has the same value in all the points of A_I. In fact, in this case, which occurs when the carrier densities inside and outside the island are relatively high and the distances among the island themselves are much greater than the screening length λ (see the following section), from (3.5), (3.6), (2.7), and (2.12), one again obtains (3.7) where τ is now given by the more general relationship

$$\tau = C/G , \qquad (3.9)$$

in which the conductance G is given by (2.13) and the capacitance C becomes

$$C = \int_{A_I} \frac{\epsilon}{w} dA = \frac{\epsilon A_I}{w_I} . \qquad (3.10)$$

The third term of (3.10), that holds true when ϵ is constant on A_I, the "mean" value w_I of w is given by

$$\frac{1}{w_I} = \frac{1}{A_I} \int_{A_I} \frac{1}{w} dA . \qquad (3.11)$$

B. Generation-recombination processes

In each point $\vec{r}$ external to the island let the variation $\Delta\rho(\vec{r})$ of the electric charge density be proportional to the variation $\Delta v_i(\vec{r})$ of the electric potential, and let the island and the symmetry of $\Delta\rho(\vec{r})$ around it be spherical. Under these conditions the integration of Poisson's equation (3.1) gives

$$\Delta v_i = \Delta v (r_I / r) \exp[(r_I - r)/\lambda] , \qquad (3.12)$$

where λ is the screening length, r_I is the radius of the island, and $\vec{r}$ is computed from its center.

Since $\Delta \vec{\mathcal{E}}(\vec{r}) = -(\partial \Delta v_i / \partial r) \vec{r}/r$, from (2.9), (2.12), (2.15), (3.5), (3.6), and (3.12) one again obtains (3.7) and (3.9) where the conductance G is now given by (2.16) and the capacitance C becomes

$$C = (\epsilon A_I / \lambda)(1 + \lambda / r_I) , \qquad (3.13)$$

in which the screening length λ, for the instance of degenerate conducting materials, is given by

$$\lambda = \left(\frac{2\epsilon(E_F - E_c)}{3q^2 \bar{n}}\right)^{1/2} . \qquad (3.14)$$

For example, for $r_I = 6$ nm, $\lambda = 1.5$ nm, and a relative dielectric constant $\epsilon_r = 12$, (3.13) yields $C = 4 \times 10^{-5}$ pF. The same value is given by (3.10) for $w_I = 1.2$ nm. In this case the storage of an electron in the island produces a variation $\Delta v = -4$ mV of its electric potential.

C. Relaxation time

From (3.8) and (2.6) and, in general, from (3.9), (3.10), (2.6), and (2.13) the relaxation time becomes

$$\tau = \tau_0 \exp(\phi / kT) , \qquad (3.15)$$

where

$$\tau_0 = \epsilon k / \langle w_I \rangle q A^* T , \qquad (3.16)$$

and

$$\phi = -kT\left\{\ln\left[\frac{1}{A_I}\int_{A_I}\alpha\exp\left(\frac{-\varphi}{kT}\right)dA\right] + \ln\frac{w_I}{\langle w_I\rangle}\right\} \quad (3.17)$$

in which $\langle w_I\rangle$ is the mean value of w_I on the island ensemble.

When (2.16), (2.17), and (3.13) hold true, $\langle w_I\rangle$ in (3.16) is replaced by λ, and ϕ becomes

$$\phi = -kT\left[\ln\sum_j 2^{3/2}\frac{\sigma_j}{A_I}N_j\exp\left(\frac{-\varphi_j}{kT}\right) + \ln\frac{r_I}{r_I+\lambda}\right], \quad (3.18)$$

whereas, for degenerate materials, (2.14), (2.16), and (3.13) yield

$$\tau_0 = \epsilon kT/\lambda q^2\bar{n}v_m, \quad (3.19)$$

and

$$\phi = -kT\left[\ln\sum_j\frac{\sigma_j}{A_I}N_j\exp\left(-\frac{E_F-E_j}{kT}\right) + \ln\frac{r_I}{r_I+\lambda}\right]. \quad (3.20)$$

If A_I has a zone A_m in which the barrier height φ reaches a value φ_m a few kT lower than its values in the other points of A_I and, but not necessarily, in A_m the barrier thickness w also attains its minimum width w_m, from (3.17) one obtains, in particular,

$$\phi \simeq \varphi_m + kT\ln[\langle w_I\rangle A_I/w_I A_m\alpha(w_m)]. \quad (3.21)$$

From such a relationship and, more in general from (3.17), (3.18), and (3.20), it follows that for the same device and even for the same cause creating the closed energy barriers against the carrier motion, the energy $\phi>0$, from which τ depends exponentially, may assume at random any value in a wide interval, for instance from 0.1 to 1.30 eV.

Since, for example, for $\epsilon_r=12$, $\langle w_I\rangle=1.2$ nm, and $T=300$ K, (3.17) give $\tau_0=2.12\times10^{-14}$ sec, such a large dispersion of ϕ, according to (3.15), leads to a dispersion of τ from 10^{-12} to 1.3×10^{8} sec, that is, of 20 decades.

Such a huge dispersion of τ is the first theoretical result able to account for the existence, experimentally verified, of the $1/f^{\gamma}$ spectrum of the flicker noise even over thirteen decades of frequency and down to 6×10^{-6} Hz.[8]

IV. ISLAND NOISE SPECTRUM

The power spectral density S_Q of the fluctuations ΔQ of the charge enclosed in the island, according to (3.7) and to the Langevin approach, is given by the Lorentzian spectrum

$$S_Q = S_\eta/(\tau^{-2}+\omega^2), \quad (4.1)$$

where S_η is the spectrum of the random source η and $\omega=2\pi f$. Since the carrier passages across the island surface A_I in both directions constitute a series of independent events occurring at random, the spectrum S_η, according to Schottky's theorem and to (2.1) and (2.11), becomes

$$S_\eta = 2q\int_{A_I}A^*T^2\alpha\left[\exp\left(\frac{-\varphi_1}{kT}\right)+\exp\left(\frac{-\varphi_2}{kT}\right)\right]dA, \quad (4.2)$$

where α, φ_1, and φ_2 are computed for $v=\bar{v}$. Then from (2.5), (2.6), (2.13), and (4.2) it follows that

$$S_\eta = 4kTG, \quad (4.3)$$

that is, for the nearly thermodynamic equilibrium of the barrier expressed by (2.5), the shot noise across the island surface, according to (4.3) and to Nyquist's theorem, coincides with the thermal noise of the conductance G.

From (2.5), (2.15), and (2.16) it follows that (4.3) holds true also for the generation-recombination case. From (3.9) and (4.3) one also obtains that

$$S_\eta = 4kTC/\tau, \quad (4.4)$$

which, together with (4.1) and (3.15), leads to the spectrum

$$S_Q = \frac{4kTC\tau}{1+\tau^2\omega^2} = \frac{2kTC}{\omega\cosh[(\phi-\phi_F)/kT]}, \quad (4.5)$$

where

$$\phi_F = -kT\ln(\tau_0\omega). \quad (4.6)$$

Finally, in accordance with the thermodynamic approach of the charge fluctuations of a capacitance,[9] (4.5) leads to the variance

$$\langle\Delta Q^2\rangle_{av} = kTC \quad (4.7)$$

of the island charge.

V. CURRENT DIPOLE VECTOR

A. Current dipole vector

The fluctuations of the island charge induce variations of the charge outside the island itself and they originate a fluctuating current dipole vector. In order to show this effect, consider a shell which has a thickness of a few λ, a volume Ω_E, and an internal surface coincident with the external one A_I of the island. Since, according to (3.12), the fluctuations $\Delta\vec{\mathcal{E}}$ due to fluctuations ΔQ of the island are negligible on the external surface of the shell, from (3.1) it follows that

$$\int_{\Omega_E}\Delta\rho\, d^3x = -\int_{\Omega_I}\Delta\rho\, d^3x = -\Delta Q. \quad (5.1)$$

On the other hand the current dipole vector $\Delta\vec{P}$ originated by the current fluctuations $\Delta\vec{J}$ in $(\Omega_E+\Omega_I)$ is defined by

$$\Delta\vec{P}=\int_{\Omega_I}\Delta\vec{J}\,d^3x+\int_{\Omega_E}\Delta\vec{J}\,d^3x\,, \tag{5.2}$$

where, in the resistive unipolar devices, the current $\vec{J}$, outside the barrier depletion layer, is given by

$$\vec{J}=-qn\vec{p}+\vec{\zeta}\,, \tag{5.3}$$

in which $p=-\mu(\mathcal{E})\vec{\mathcal{E}}$ is the carrier drift velocity and μ is the static mobility.

Therefore, since $\Delta\rho=-q\Delta n$, from (5.3) it follows that

$$\Delta\vec{J}=\Delta\rho\langle\vec{p}\rangle+q\bar{n}(\bar{\mu}+\bar{\mu}'\mathcal{E})\Delta\vec{\mathcal{E}}+\vec{\zeta}\,,$$

where $\bar{\mu}=\mu(\vec{\mathcal{E}})$ and $\bar{\mu}'=(\partial\mu/\partial\mathcal{E})|_{\mathcal{E}=\bar{\mathcal{E}}}$, and then from (5.2)

$$\Delta\vec{P}=\int_{\Omega_I}\Delta\rho\langle\vec{p}\rangle d^3x+\int_{\Omega_E}\Delta\rho\langle\vec{p}\rangle d^3x\,, \tag{5.4}$$

because, owing to the symmetries of $\Delta\vec{\mathcal{E}}$ around the island and the random distribution of $\vec{\zeta}$ in the space, their contributions are negligible. $\langle\vec{p}\rangle$ is the time average of $\vec{p}$.

By replacing $\langle\vec{p}\rangle$ with its space averages $\langle\bar{\vec{p}}_E\rangle$ and $\langle\bar{\vec{p}}_I\rangle$ in Ω_E and Ω_I, respectively, from (5.1) and (5.4) one has

$$\Delta\vec{P}\simeq\Delta Q\langle\bar{\vec{p}}\rangle \tag{5.5}$$

where

$$\langle\bar{\vec{p}}\rangle=\langle\bar{\vec{p}}_I\rangle-\langle\bar{\vec{p}}_E\rangle\,. \tag{5.6}$$

Since the energy barrier or the generation-recombination process makes $\langle\vec{J}\rangle\simeq0$ inside the island, from (5.3) and (5.6) it follows also that

$$\langle\bar{\vec{p}}\rangle=-\bar{\bar{\mu}}\langle\bar{\vec{\mathcal{E}}}\rangle=-\langle\bar{\vec{J}}\rangle/q\bar{\bar{n}}\,, \tag{5.7}$$

where the time and space averages $\bar{\bar{\mu}}$, $\langle\bar{\vec{\mathcal{E}}}\rangle$, $\langle\bar{\vec{J}}\rangle$, and $\bar{\bar{n}}$ are computed in Ω_E.

B. Impedance field method

The fluctuating current dipole $\Delta\vec{P}$ produces a voltage noise at the terminals A and B of the device. In order to compute it, let $V'=ZI'$ be the phasor of the voltage induced between A and B when a current of phasor I' is injected in the point $\vec{r}$ and is extracted from B. The impedance Z is a complex function of $\vec{r}$ and ω which in the low-frequency band of the flicker noise is normally independent of ω itself.

The phasor $\Delta V'$ at frequency f induced between A and B by $\Delta\vec{P}$, according to (5.5), becomes $\Delta V'=\Delta Q'\langle\bar{\vec{p}}\rangle\cdot\vec{\nabla}Z$, where $\Delta Q'$ is the phasor of the f component of ΔQ. Therefore the power spectrum S_{VI} of the voltage noise generated at the terminals by the fluctuations ΔQ in the island is given by

$$S_{VI}=|\langle\bar{\vec{p}}\rangle\cdot\vec{\nabla}Z|^2S_Q\,, \tag{5.8}$$

whereas the total one S_V, due to the uncorrelated contributions of all the separate islands, becomes

$$S_V=\int S_{VI}\,D\,d^3x\,dC\,d\phi\,, \tag{5.9}$$

in which $D(\vec{r},C,\phi)$ is the island density in the space (x_1,x_2,x_3,C,ϕ).

VI. FLICKER NOISE

A. Dependence on the frequency

The quantities C and ϕ, which, according to (3.10), (3.13), and (3.17)–(3.21) depend on each other through A_I and w_I or r_I, may be considered as independent variables because from (3.17)–(3.21) it follows that, in reality, ϕ is a slowly varying function of A_I and w_I or r_I themselves.

Therefore, according to (4.5) and (5.7)–(5.9), the power spectral density S_V becomes

$$S_V=\frac{1}{f}\left(\frac{kT}{q}\right)^2M(\phi_F)\,, \tag{6.1}$$

where

$$M(\phi_F)=\frac{1}{\pi kT}\int\frac{P(\phi)}{\cosh[(\phi-\phi_F)/kT]}\,d\phi\,, \tag{6.2}$$

in which the "distribution" $P(\phi)$ of ϕ is given by

$$P=\int|\langle\bar{\vec{J}}\rangle\cdot\vec{\nabla}Z|^2\bar{\bar{n}}^{-2}C\,D\,dC\,d^3x\,; \tag{6.3}$$

both M and P are dimensionless functions.

The integral (6.2) gives

$$M(\phi_F)=P(\phi_F)+\sum_{\nu=1}^{\infty}\frac{F_\nu}{(2\nu)!}\left(\frac{\pi kT}{2}\right)^{2\nu}\frac{d^{2\nu}P(\phi)}{d\phi^{2\nu}}\bigg|_{\phi=\phi_F}\,, \tag{6.4}$$

where F_ν is the νth Euler number.

By making the Taylor-series expansion of $\ln M(\phi_F)$ with respect to ϕ_F about $\phi_F=\phi_F^*=-kT\ln(\tau_0^*f^*)$, where τ_0^* and f^* are given reference values of τ_0 and f, respectively, from (4.6) and (6.1) one obtains

$$S_V=\frac{1}{f^\gamma}\left(\frac{kT}{q}\right)^2\left(\frac{\tau_0^*f^*}{\tau_0}\right)^\delta M(\phi_F^*), \tag{6.5}$$

where

$$\delta=kT\frac{d\ln M(\phi_F)}{d\phi_F}\bigg|_{\phi_F=\phi_F^*}\,, \tag{6.6}$$

and

$$\gamma=1+\delta\,, \tag{6.7}$$

across ϕ_F^*, are functions of f^*.

When the "width" of $P(\phi)$ is much larger than kT, (6.4) becomes

$$M(\phi_F) \simeq P(\phi_F). \tag{6.8}$$

Moreover, if ϕ has a normal distribution $P(\phi)$ with mean ϕ_P and variance ψ^2, from (6.6) and (6.8) one also obtains

$$\delta = kT(\phi_P - \phi_F^*)/\psi^2. \tag{6.9}$$

From (6.9) it follows that for low values of ψ and high ones of $|\phi_P - 2\phi_F^*|$, δ depends appreciably on the temperature, and it may be considered as a constant only on few decades around f^*. Furthermore from (6.9), for $\psi \leq 0.26$ eV, $|\phi_P - \phi_F^*| \geq 0.26$ eV and $T = 300$ K, one has that $|\delta| \geq 0.1$.

For large values of ψ, however, δ and γ become independent of T and f^*, and $\gamma \simeq 1$. For example, for $\psi = 0.5$ eV and $\phi_P \simeq \phi_F^*$ [for $f^* = 1$ Hz, $\tau_0^* = \tau_0 = 2.12 \times 10^{-14}$ sec, and $T = 300$ K, it is $\phi_F^*(1\text{ Hz}) = 0.815$ eV], from (6.9) it follows that $\gamma = 1 \pm 0.037$ for $10^{-6} < f^* < 10^6$ Hz.

Therefore the flicker noise may have spectra of $1/f^\gamma$ type with $\gamma \simeq 1$ also on bandwidths of many decades and down to any frequency, however low, to which it may be physically measured. However, since the dispersion and the distribution of ϕ are finite, from (4.5), (4.7), and (6.1)–(6.3) it follows that the variance

$$\langle \Delta V^2 \rangle_{\text{av}} = kTq^{-2} \int P(\phi) d\phi \tag{6.10}$$

of the voltage fluctuations is always finite.

All the previous theoretical results agree well with the experimental values of γ obtained from the flicker noise measurements on very many electron devices.[10,11] In particular, as stated, the spectrum $1/f^\gamma$ with $\gamma = 1$ has been verified even over thirteen decades and down to 6×10^{-6} Hz.[8]

B. Dependence on the current

In the three-dimensional case and when $\bar{\bar{n}}$ depends on $\langle \vec{J} \rangle$, from (6.1)–(6.4) it is not easy to obtain the dependence of S_V on the average direct current $\bar{I}$ which crosses the device. However, it is possible for the one-dimensional devices for which one has $\vec{\nabla} Z \simeq \vec{i}_1(\sigma_d S)^{-1}$, $\langle \vec{J} \rangle = \vec{i}_1 \bar{I}/S$, and $d^3x = S dx_1$, where σ_d is the differential conductance and $S(x_1)$ is the device section. In this case, from (6.3) one has

$$P = \bar{I}^2 \int_0^L \int \left(\frac{CD}{\bar{\bar{n}} \sigma_d^2 S^3} dC \right) dx_1, \tag{6.11}$$

where L is the device length.

When $\bar{\bar{n}}$ and σ_d are independent of $\bar{I}$, (6.1)–(6.3) and (6.11) give a spectrum S_V proportional to $\bar{I}^2$. Also this result agrees with the experiments relative to many unipolar electron devices.[10-12]

C. Dependence on the volume and the carrier number

When the device is macroscopically homogeneous and it has a constant section S one can put $\langle \vec{\mathcal{E}} \rangle \simeq \bar{V}/L$, $\sigma_d = q\mu_m n_m$, and $n_m = N/\Omega$, where $\bar{V}$ is the average voltage across the device, Ω is the sample volume, N is the total number of the free carriers, and μ_m is their mean mobility that, in general, is different from the local one $\bar{\bar{\mu}}$ around the islands. For such resistive systems, (5.7) and (6.3) yield

$$P = \frac{\bar{V}^2}{\Omega} \left(\frac{\mu}{\mu_m n_m} \right)^2 \int CD\, dC, \tag{6.12}$$

which, together with (6.5) and (6.8), leads to

$$S_V = \frac{\beta \bar{V}^2}{n_m \Omega f^\gamma} = \frac{\beta \bar{V}^2}{N f^\gamma}, \tag{6.13}$$

where

$$\beta = \left(\frac{kT\bar{\bar{\mu}}}{q\mu_m} \right)^2 \left(\frac{\tau_0^* f^*}{\tau_0} \right)^\delta \frac{1}{n_m} \int CD(\phi_F^*, C) dC. \tag{6.14}$$

For the (degenerate) conductors $n_m \propto \bar{n} \simeq \bar{\bar{n}} \propto (E_F - E_c)^{3/2}$, $v_m \propto (E_F - E_c)^2$ and, moreover, $\lambda \simeq 1$ Å $\ll r_I$.

Therefore from (3.13), (3.14), (3.19), and (6.14) one has $\beta \propto n_m^b$ with $b = (13\delta - 5)/6$, so that for $|\delta| \leq 0.1$, $0.5 \leq |b| \leq 1$, whereas for $\delta \geq 0.2$, $b \leq 0.4$. Moreover, n_m varies little from one metal to another; consequently for the conductors the coefficient β is nearly independent of N. This result and (6.13) agree with the experimental measurements and with Hooge's empirical formula.[12] More generally, the dependence on $\bar{V}^2$, Ω, and f given by (6.13) has been universally found in the resistive devices.[10,12]

The integral in the expression (6.14) of β is difficult to be computed in a general form because its integrand strongly depends on the device technology.

D. Dependence on the temperature

The spectrum S_V of the flicker noise depends, in general, in a very complex way on the temperature. In fact, according to (6.1)–(6.4), it depends on T both directly and indirectly through various quantities such as Z, $\bar{\bar{n}}$, D, and so on. When such an indirect dependence across P can be neglected and (6.8) holds true, according to (6.1), the dependence on T^2 and the one through ϕ_F remain; ϕ_F, by virtue of (4.6), is a function of T also across $\tau_0(T) \propto T^c$ [see (3.16) and (3.19)].

In this case, for a given frequency f, S_V has a maximum at the temperature T_M given by

$$T_M = -[\phi_P + (\phi_P^2 + 8\psi^2)^{1/2}]/2k \ln[\tau_0(T_M)\omega], \tag{6.15}$$

obtained from (4.6), (6.1), and (6.8) with the assumption of a normal distribution $P(\phi)$ and of $\phi_F \gg ckT$. Also the complex dependence of S_V on T and the existence for a given frequency of a maximum in the plot of S_V vs T, as well as the dependence of γ on T [see (6.7) and (6.9)], have been experimentally observed.[10,11]

VII. CONCLUSIONS

A new and simple model of the flicker noise has been deduced in a straightforward way without any particular assumption or approximation from general laws, theorems, and methods. The physical basis of the theory and the origin of the flicker noise are put in the existence, in the conducting medium, of islands of any shape, dimension, and size, enclosed by an energy barrier or containing an energy well.

The transport mechanisms of the carriers between the island and the surrounding medium are the tunnel and thermionic emissions (TTE) in the first case and generation-recombination processes (GRP) in the second one. In both cases they lead to a conductance between island and its surroundings which depends exponentially on a random variable energy. Moreover, the island ability of storing and releasing an electric charge with a proportional variation of its potential determines a capacitance which, together with the conductance, creates island relaxation times that may have the wide dispersion necessary to explain the flicker noise.

The bases and the tools used to carry out the analysis, other than the TTE and GRP theories, are the charge conservation principle, the Gauss, Schottky, and Nyquist theorems, the Langevin approach, the current dipole model, the impedance field method, and the thermodynamic theory of the fluctuations.

The model yields the spectrum of the flicker noise and its dependence, other than on the frequency, on the current, voltage, temperature, shape, volume, and carrier number of the device. The theoretical results so obtained agree well with experimental measurements relative to the unipolar electron devices.

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[1]B. Pellegrini (unpublished).

[2]L. L. Karmerski and D. M. Racine, J. Appl. Phys. 40, 791 (1975).

[3]G. E. Pike and C. H. Seager, J. Appl. Phys. 48, 5152 (1977).

[4]A. J. R. De Kock, *Proceedings of the Fourteenth International Conference on Physics of Semiconductors*, edited by B. L. H. Wilson (The Institute of Physics, Bristol, 1978), pp. 103–111.

[5]B. Pellegrini, Solid State Electron. 18, 417 (1975).

[6]A more general expression of the current, which takes into account the effective mass anisotropy and Fermi-Dirac statistics, is reported in B. Pellegrini, Solid State Electron. 18, 791 (1975).

[7]A. G. Milnes, *Deep impurity in Semiconductors* (Wiley, New York, 1973).

[8]T. E. Firle and H. Winston, J. Appl. Phys. 26, 716 (1955).

[9]K. M. Van Vliet and J. R. Fasset, in *Fluctuation Phenomena in Solids*, edited by R. E. Burgess (Academic, New York, 1965), p. 290.

[10]Many references for the experimental results on the flicker noise are reported in Ref. 1.

[11]P. Dutta, P. Dimon, and P. M. Horn, Phys. Rev. Lett. 43, 646 (1979).

[12]F. N. Hooge, Physica 60, 130 (1972).