PHYSICAL REVIEW B VOLUME 22, NUMBER 8 15 OCTOBER 1980

Hot photoexcited electrons: Determination of the time-dependent distribution function in a quantizing magnetic field

N. Pottier and D. Calecki

Groupe de Physique des Solides de l'Ecole Normale Supérieure, Université Paris VII, 2 Place Jussieu, 75221 Paris, Cedex 05, France*

(Received 29 November 1979)

The expression of the time evolution of the distribution function of hot electrons photoexcited by a laser pulse in the conduction band of a semiconductor under extreme quantum limit conditions is calculated. The transformation, justified in previous papers, of the master equation into a Fokker-Planck equation is used. The notions of diffusion and of drift velocity in the electron energy space are rigorously introduced. Finally, the time evolution of the mean electron energy is calculated and the influence of different scattering mechanisms and of various physical parameters is discussed.

I. INTRODUCTION

The pioneer work on the determination of hot electron energy relaxation by photoluminescence is due to the work of Ulbrich.[1] Picosecond laser spectroscopy has recently been applied to the determination of electron energy relaxation in semiconductors.[2,3] The knowledge of the electron distribution function (EDF) in the conduction band and its variation with time is the first theoretical step to determine the evolution of the mean electron energy. The essential phenomena which we need to take into account for the determination of the EDF are electron scattering (including carrier-carrier interaction) and recombination. However, it is well known that the application of a quantizing magnetic field can reduce the complexity of analytical calculations[4]; this is due to the fact that high magnetic fields greatly reduce the importance of electron-electron scattering.[5] In particular, under extreme quantum limit (EQL) conditions, binary electron-electron scattering in the absence of collision broadening has no effect on the EDF. The present work is thus concerned with the time evolution of the distribution function of electrons photoexcited by a short laser pulse into the conduction band of a semiconductor under EQL conditions.

In a previous paper,[6] we carried out simple calculations in order to get qualitative results on the time evolution of the mean energy of the photoexcited electrons. We assumed that the EDF could be essentially characterized by a unique parameter, and, as usual, we used a Maxwellian EDF with a time-dependent electronic temperature $T_e(t)$ representing the mean electron energy. We wrote down the balance equations for particle number and energy and set up a differential equation for $T_e(t)$. We discussed the influence on $T_e(t)$ of the magnetic field, the photoexcitation intensity, the photoexcitation energy, the recombination time, the lattice temperature, and the role of the various types of electron-phonon scattering mechanisms.

The aim of the present paper is to give a more rigorous treatment and to eliminate the Maxwellian *a priori* assumption for the EDF. We use the method that we developed for the treatment of the stationary EDF when the photoexcitation is continuous.[7] We transform the master equation obeyed by the EDF into a solvable Fokker-Planck equation; this is possible when the electron-phonon energy transfers are small as compared to the electron energy, and is meaningful when electrons are scattered by acoustical phonons. The Fokker-Planck equation is the starting point to describe the mean electron energy evolution as a random-walk motion in energy space. We recover here the main features of an analog problem: the determination of the quantum energy distribution function of hot electrons in crossed electric and magnetic fields.[8,9]

The present paper is organized as follows: In Sec. II we describe the model and write down the master equation for the time-dependent EDF. In Sec. III we restrict ourselves to the extreme quantum limit conditions, under which the master equation can be transformed into a Fokker-Planck equation. We emphasize the analogy between the problem we consider here and a well-known Brownian motion problem, i.e., the sedimentation phenomenon of Brownian particles[10]; then we give the expression for the time-dependent EDF. Section IV contains a presentation and a discussion of the numerical results obtained for the time evolution of the mean electron energy.

II. DESCRIPTION OF THE MODEL AND MASTER EQUATION

The electron states in a magnetic field $\vec{B}$ parallel to the z axis are specified by three quantum numbers n, k_y, and k_z, summarized by the index ν. The associated energies are simply

$$\epsilon_\nu = (n+\tfrac{1}{2})\hbar\omega_c + \hbar^2 k_z^2/2m \,, \qquad (1)$$

where m is the effective mass of the assumed parabolic conduction band and $\omega_c = q_e B/m$ is the cyclotron frequency.

The time evolution of the EDF ρ_ν in the different states ν is due to the following three effects:

(i) The first one is the laser pulse, whose duration is negligible when compared to all other characteristic times of the problem such as the recombination time or the momentum and energy relaxation times of the electrons. This assumption is now realistic, since one utilizes picosecond lasers with pulses shorter than the momentum relaxation time τ_{ac} of electrons scattered by acoustical phonons [τ_{ac} is of the order of 10^{-10} s or more for InSb in EQL (see Sec. III)]; the energy relaxation time is always greater than τ_{ac} and this is also quite generally the case for the recombination time. Thus if ΔN electrons are created at time $t=0$ in the state of energy ϵ_P above the bottom of the first Landau level, the corresponding rate of variation of ρ_ν can simply be written down as

$$\left.\frac{\partial \rho_\nu}{\partial t}\right|_{\text{phot}} = \frac{\Delta N}{g(\epsilon_\nu)}\delta(\epsilon_\nu - \epsilon_P - \tfrac{1}{2}\hbar\omega_c)\,\delta(t), \tag{2}$$

where $g(\epsilon)$ is the density of states in the conduction band.

(ii) The second effect is the annihilation of electrons in the Landau states by various recombination mechanisms. We will crudely sum up the complex recombination phenomenon by a simple recombination time τ_ν.

(iii) The last effect is the electron-phonon scattering characterized by the transition probability per unit time $W_{\nu\nu'}$. In the Born approximation, for collisions with phonons,

$$W_{\nu\nu'} = \frac{2\pi}{\hbar}\sum_{\vec{q}} |\gamma_{\nu\nu'}(\vec{q})|^2 [N_q\,\delta(\epsilon_\nu - \epsilon_{\nu'} + \hbar\omega_q) + (1+N_q)\,\delta(\epsilon_\nu - \epsilon_{\nu'} - \hbar\omega_q)]\,, \tag{3}$$

where

$$|\gamma_{\nu\nu'}(\vec{q})|^2 = C^2(q)\,|\langle\nu|\exp(i\vec{q}\cdot\vec{r})|\nu'\rangle|^2 .$$

$C(q)$ is related to the particular type of electron-phonon coupling and

$$N_q = [\exp(\hbar\omega_q/kT) - 1]^{-1}$$

is the distribution function of the phonons assumed to be in thermal equilibrium at temperature T.

Thus, the total rate of change of ρ_ν is given by the master equation

$$\frac{\partial\rho_\nu}{\partial t} = \frac{\Delta N}{g(\epsilon_\nu)}\delta(\epsilon_\nu - \epsilon_P - \tfrac{1}{2}\hbar\omega_c)\,\delta(t) - \frac{\rho_\nu - \rho_\nu^0}{\tau_\nu} + \sum_{\nu'}(W_{\nu'\nu}\rho_{\nu'} - W_{\nu\nu'}\rho_\nu)\,, \tag{4}$$

where

$$\rho_\nu^0 \sim \exp(-\epsilon_\nu/kT) \tag{5}$$

is the EDF describing the thermodynamic equilibrium at temperature T, which prevails at time $t<0$ and for times $t\to\infty$ (i.e., in practice, $t \gg \tau_\nu, \tau_{ac}$).

Equation (3) can be conveniently transformed in the special case where the recombination time is the same for all the Landau states. Let us define that part $\Delta\rho_\nu(t)$ of $\rho_\nu(t)$ whose variation comes from the collision term after the laser pulse has been emitted, and such that

$$\rho_\nu(t) = \rho_\nu^0 + \Delta\rho_\nu(t)\exp(-t/\tau)\,\Theta(t)\,. \tag{6}$$

$\Theta(t)$ is the step function and τ is the ν-independent recombination time. Moreover, $\Delta\rho_\nu(t)$ satisfies the initial condition

$$\Delta\rho_\nu(0) = \frac{\Delta N}{g(\epsilon_\nu)}\delta(\epsilon_\nu - \epsilon_P - \tfrac{1}{2}\hbar\omega_c)\,. \tag{7}$$

Inserting (6) into (4), taking account of (7), and remembering that the collision term cancels when $\rho_\nu = \rho_\nu^0$ according to the detailed balance principle, we finally get

$$\frac{\partial\Delta\rho_\nu(t)}{\partial t} = \sum_{\nu'}[W_{\nu'\nu}\Delta\rho_{\nu'}(t) - W_{\nu\nu'}\Delta\rho_\nu(t)]\,. \tag{8}$$

If we had followed on with a state-dependent recombination time, we would have obtained instead of Eq. (8)

$$\frac{\partial\Delta\rho_\nu(t)}{\partial t} = \sum_{\nu'}\{W_{\nu'\nu}\Delta\rho_{\nu'}(t)\exp[-t(\tau_{\nu'}^{-1} - \tau_\nu^{-1})] - W_{\nu\nu'}\Delta\rho_\nu(t)\}\,.$$

These last three equations are entirely equivalent to the master equation (4): Equation (8) is particularly convenient for a discussion in terms of random walk and Brownian motion in the energy space. Moreover, Eq. (8) is the time-dependent analog of Eq. (13) in Ref. 9, in which a method of transformation of the master equation into a Fokker-Planck equation has been proposed. We shall briefly summarize here the steps of the transformation.

We are looking for an EDF appropriate to a situation in which the electron density is homogeneous; consequently, ρ_ν is k_y independent. Furthermore, there is no electrical current along the magnetic field, thus ρ_ν must be an even function of k_z. We may then label the electron distribution functions $\rho_\nu(t)$ and $\Delta\rho_\nu(t)$ as $\rho_n(\epsilon, t)$ and $\Delta\rho_n(\epsilon, t)$, respectively, where n is the Landau quantum number and ϵ is the electron energy. In Eq. (8) we may perform the k_y' summation. After multiplication of both terms of Eq. (8) by the density of states of the nth Landau level which is, in the absence of any collision broadening

$$g_n(\epsilon)=2\,\frac{V}{(2\pi\delta)^2}\left(\frac{2m}{\hbar^2}\right)^{1/2}[\epsilon-(n+\tfrac{1}{2})\hbar\omega_c]^{-1/2} \qquad (9)$$

[$V=L^3$ is the volume of the sample and $\delta=(\hbar/m\omega_c)^{1/2}$ the cyclotron radius], we obtain

$$\frac{\partial}{\partial t}[g_n(\epsilon)\Delta\rho_n(\epsilon,t)]=\sum_{n'}\int d\epsilon'\,[P_{n'n}(\epsilon',\epsilon)\Delta\rho_{n'}(\epsilon',t)-P_{nn'}(\epsilon,\epsilon')\Delta\rho_n(\epsilon,t)]\,, \qquad (10)$$

where

$$P_{nn'}(\epsilon,\epsilon')=g_n(\epsilon)g_{n'}(\epsilon')\,2\pi(\delta/L)^2\sum_{k_y'}W_{\nu\nu'}\,, \qquad (11)$$

the result of the k_y' summation is k_y-independent because $W_{\nu\nu'}$ depends only on the difference k_y-k_y'. The detailed form of $P_{nn'}(\epsilon,\epsilon')$ varies with the type of scattering; however, we know that $P_{nn}(\epsilon,\epsilon')$ will be appreciable only if the energy transfer $\epsilon'-\epsilon$ is of the order of the phonon energy.

Equation (10) has a very simple interpretation in terms of the current of electrons along the energy axis. Let us first consider what happens only in the nth Landau level. We define the net current of electrons belonging to the nth level that cross the energy ϵ per unit time:

$$J_n(\epsilon)=\int_{-\infty}^{+\infty}d\epsilon'\int_{-\infty}^{+\infty}d\epsilon''\,P_{nn}(\epsilon',\epsilon'')\Delta\rho_n(\epsilon')\times[\Theta(\epsilon-\epsilon')\Theta(\epsilon''-\epsilon)-\Theta(\epsilon'-\epsilon)\Theta(\epsilon-\epsilon'')] \qquad (12)$$

(Θ is the usual step function). Thus Eq. (10) is equivalent to

$$\frac{\partial}{\partial t}[g_n(\epsilon)\Delta\rho_n(\epsilon,t)]+\frac{\partial J_n(\epsilon)}{\partial\epsilon}=S_n(\epsilon,t)\,. \qquad (13)$$

We recognize here a typical conservation equation which relates the density of electrons having energy ϵ in the nth Landau level to the current of electrons in energy space belonging to this nth level. The last term of Eq. (13)

$$S_n(\epsilon,t)=\sum_{n'\neq n}\int d\epsilon'\,[P_{n'n}(\epsilon',\epsilon)\Delta\rho_{n'}(\epsilon',t)-P_{nn'}(\epsilon,\epsilon')\Delta\rho_n(\epsilon,t)] \qquad (14)$$

is the source term due to the electrons arriving in the nth level and leaving it with energy ϵ.

In general, a direct solution of Eq. (13) is impossible. A great simplification occurs when there is no source term. This can be the case in the extreme quantum limit, when the electrons stay permanently in the first Landau level.

III. EXPRESSION FOR THE TIME-DEPENDENT EDF IN THE EXTREME QUANTUM LIMIT

We shall assume from now on that the temperature is sufficiently low, the magnetic field is high enough, and the photoexcitation energy is so weak that only the first Landau level $n=0$ is occupied: $\hbar\omega_c\gg kT$, $\epsilon_P<\hbar\omega_c$.

According to a phenomenological treatment by Kurosawa and Yamada[8] and to calculations by Calecki, Lewiner, and Nozières,[9] the motion of an electron colliding with the phonons in the energy space appears as a random walk with a reflecting barrier at the bottom of the first Landau level (which from now on we take as the origin of energies). Simple results are obtained when the electron energy exchange during a collision is small compared to its average energy; this condition is fulfilled here if we neglect the effect of the optical phonons. In particular, electrons cannot occupy significantly energy states too high in the first Landau level. Before dealing more rigorously with the expression of $\Delta\rho(\epsilon,t)$ which satisfies the exact equation

$$\frac{\partial}{\partial t}[g(\epsilon)\Delta\rho(\epsilon,t)]+\frac{\partial J(\epsilon,t)}{\partial\epsilon}=0 \qquad (15)$$

(we dropped the index n which is always equal to 0 in the EQL), let us use this random-walk approximation to obtain some interesting preliminary qualitative results.

A. Expression for $J(\epsilon,t)$ in a random-walk approximation

The net effect of the electron-phonon collisions is to change the energy of the electron and to produce a particle current along the energy axis. We can evaluate this current using the same argument as Ref. 8. $J(\epsilon,t)$ comprises two parts: (i) A drift term $vg(\epsilon)\Delta\rho(\epsilon,t)$, where v is the drift velocity in the energy space resulting from the energy losses due to the collisions, and (ii) a diffusion term $D(\partial/\partial\epsilon)[g(\epsilon)\Delta\rho(\epsilon,t)]$, where D is the diffusion coefficient in energy space due to the energy fluctuations produced by the collisions. Hence

$$J(\epsilon,t)=-vg(\epsilon)\Delta\rho(\epsilon,t)-D\frac{\partial}{\partial\epsilon}[g(\epsilon)\Delta\rho(\epsilon,t)]\,. \qquad (16)$$

Moreover, we have the boundary condition

$$J(0,t)=0\,, \qquad (17)$$

since no electrons can get an energy lower than $\epsilon=0$, and the initial condition

$$g(\epsilon)\Delta\rho(\epsilon,t=0)=\Delta N\delta(\epsilon-\epsilon_P)\,. \qquad (18)$$

Now we will give more details on the two coefficients v and D. First, there is a trivial relation between v and D. There is no net current at thermal equilibrium: $J(\epsilon, t)=0$ when $\Delta\rho(\epsilon, t) \sim \exp(-\epsilon/kT)$ and we must have $D/v=kT$. This is the analog of the well-known Einstein relation for a nondegenerate electron gas between the diffusion coefficient and the mobility.

Now, we go one step further and use the classical results on random walk[10] to evaluate the drift velocity v and the diffusion coefficient D in terms of microscopic parameters. The electron energy change at each collision with a phonon is of order $\hbar s\delta^{-1}$, since phonons whose wave vector is of order δ^{-1} are the most efficient. The drift velocity v in energy space is equal to the average energy lost per unit time. Since the emission probability of a phonon is proportional to N_q, whereas the absorption probability is proportional to $1+N_q$, we have

$$v \simeq \frac{1}{\tau_{\mathrm{ac}}} \frac{\hbar s\delta^{-1}(1+N_q) - \hbar s\delta^{-1}N_q}{1+2N_q},$$

with

$$N_q \sim [\exp(\hbar s\delta^{-1}/kT) - 1]^{-1}$$

(τ_{ac} is the time between two successive collisions). In the limit $\hbar s\delta^{-1} \ll kT$, v reduces to

$$v \simeq \frac{1}{\tau_{\mathrm{ac}}} \frac{(\hbar s\delta^{-1})^2}{2kT}. \tag{19}$$

Similarly, the diffusion coefficient D is equal to $\frac{1}{2}nl^2$ where n is the number of collisions per unit time and l the change of energy per collision; with our notations, the expression of D is given by

$$D \simeq \tfrac{1}{2}(1/\tau_{\mathrm{ac}})(\hbar s\delta^{-1})^2. \tag{20}$$

We immediately recover the previous relation $D/v=kT$.

We have now to solve Eq. (15) in which the expression of $J(\epsilon, t)$ given by Eq. (16) is completely known. However we shall first justify these last results by a more rigorous approach, based on the derivation of a Fokker-Planck equation.

B. Derivation of a Fokker-Planck equation

The exact expression for $J(\epsilon, t)$ in EQL is deduced from Eq. (12):

$$J(\epsilon, t) = \int d\epsilon' \int d\epsilon'' P(\epsilon', \epsilon'')\Delta\rho(\epsilon', t) \times [\Theta(\epsilon-\epsilon')\Theta(\epsilon''-\epsilon) - \Theta(\epsilon'-\epsilon)\Theta(\epsilon-\epsilon'')], \tag{21}$$

where $P(\epsilon', \epsilon'') \equiv P_{nn'}(\epsilon', \epsilon'')$ when $n=n'=0$. In Eq. (21) we recall that $P(\epsilon', \epsilon'')$ considered as a function of ϵ' has only significant values when ϵ' is close to ϵ'' and therefore when these two energies themselves are close to ϵ. If $g(\epsilon')\Delta\rho(\epsilon', t)$ is a slowly varying function as compared to $P(\epsilon', \epsilon'')$, it can be expanded into a Taylor series around $\epsilon'=\epsilon$ which yields

$$g(\epsilon')\Delta\rho(\epsilon', t) \simeq g(\epsilon)\Delta\rho(\epsilon, t) + (\epsilon'-\epsilon)\frac{\partial}{\partial\epsilon}[g(\epsilon)\Delta\rho(\epsilon, t)] + O((\epsilon'-\epsilon)^2).$$

Moreover we can consider $P(\epsilon', \epsilon'')$ as a function of the two independent energies ϵ' and $\xi=\epsilon'-\epsilon''$, $P(\epsilon', \xi)$. Since ϵ' is very close to ϵ we can assume that $P(\epsilon', \xi) \simeq P(\epsilon, \xi)$. Owing to these two properties of $P(\epsilon', \epsilon'')$ we obtain in a straightforward manner

$$J(\epsilon, t) = -v(\epsilon)g(\epsilon)\Delta\rho(\epsilon, t) - D(\epsilon)\frac{\partial}{\partial\epsilon}[g(\epsilon)\Delta\rho(\epsilon, t)], \tag{22}$$

where

$$v(\epsilon) = \frac{1}{g(\epsilon)} \int d\xi\, \xi P(\epsilon, \xi) \tag{23a}$$

and

$$D(\epsilon) = \frac{1}{g(\epsilon)} \int d\xi\, \frac{\xi^2}{2} P(\epsilon, \xi). \tag{23b}$$

Equation (22) has exactly the same form as Eq. (16) derived by qualitative arguments based on a random-walk approach. The main difference lies in the dependence of the drift velocity and of the diffusion coefficient upon the electron energy.

In summary, as long as the electron energy transfer in a collision is small as compared to the scale of the variation of $\Delta\rho(\epsilon, t)$ with ϵ, the equation satisfied by $\Delta\rho(\epsilon, t)$ is of a Fokker-Planck type and can be written

$$\frac{\partial}{\partial t}[g(\epsilon)\Delta\rho(\epsilon, t)] = \frac{\partial}{\partial\epsilon}[v(\epsilon)g(\epsilon)\Delta\rho(\epsilon, t)] + \frac{\partial}{\partial\epsilon}\left(D(\epsilon)\frac{\partial}{\partial\epsilon}[g(\epsilon)\Delta\rho(\epsilon, t)]\right), \tag{24}$$

in which the part of the EDF $\Delta\rho(\epsilon, t)$ has to satisfy the two conditions (17) and (18).

C. Expression for the EDF

The explicit calculation of $v(\epsilon)$ and $D(\epsilon)$ has been performed in Ref. 7 in the two cases where electrons interact with acoustical phonons either through the deformation potential (ad) or the piezoelectric (ap) coupling. We give here only the final results using the same notation as in Ref. 7:

$$v(\epsilon) = g(\epsilon)M_1 \quad \text{and} \quad D(\epsilon) = g(\epsilon)M_2 \tag{25}$$

with

$$M_1 = M_0 \frac{(\hbar s \delta^{-1})^2}{kT} \quad \text{and} \quad M_2 = M_0 (\hbar s \delta^{-1})^2 , \tag{26}$$

where

$$M_0 = \frac{4\pi k T D^{(3)}}{\hbar^2 s} \quad \text{or} \quad \frac{2\pi D^{(1)} kT}{\hbar^2 \delta^{-2} s}$$

for the (ad) or (ap) cases, respectively.

Equation (24) has no direct solution when the two coefficients $v(\epsilon)$ and $D(\epsilon)$ are ϵ dependent. Only numerical calculations can be made in that case. However, if we assume that $\Delta\rho(\epsilon, t)$ varies much more rapidly than the density of states $g(\epsilon)$, we can consider this latter quantity as a quasiconstant, and a straightforward solution of (24) can be obtained. At first sight this procedure is questionable because $g(\epsilon)$ diverges as $\epsilon^{-1/2}$ when ϵ goes to zero. However, if we take into account the effect of collision broadening, the density of states becomes very smooth and this divergence entirely disappears. Anyway, no more calculations are needed to solve Eq. (24) if we replace $g(\epsilon)$ by a constant value g_0; we can take g_0 to be the density of states for an electron with the thermal energy $\frac{1}{2}kT$:

$$g_0 = \frac{2V}{(2\pi\delta)^2} \left(\frac{4m}{\hbar^2 kT} \right)^{1/2} . \tag{27}$$

The situation becomes entirely equivalent to the phenomenon of sedimentation, that is, to the Brownian motion of particles in a vessel submitted both to a diffusion process and to the effect of gravity. If the particles are initially at a height z_0 above the bottom of the vessel containing the solution and if we label $w(z, t)$ the density assumed to be uniformly distributed in the (x, y) plane, the evolution of the density is governed by the following equations[10]:

$$\frac{\partial w}{\partial t} = D \frac{\partial^2 w}{\partial z^2} + c \frac{\partial w}{\partial z} , \tag{28}$$

$$w \to \delta(z - z_0) \quad \text{as } t \to 0 , \tag{29}$$

$$D \frac{\partial w}{\partial z} + cw = 0 \quad \text{for } z = 0 \quad \text{for all } t > 0 . \tag{30}$$

D is the diffusion coefficient and c is related to the gravitational constant. The solution of Eqs. (28)–(30) can be found in Ref. 10; conveniently transposed, it gives immediately the expression for $\Delta\rho(\epsilon, t)$:

$$\Delta\rho = \frac{\Delta N}{g_0 kT} \left\{ \frac{1}{2(\pi\nu)^{1/2}} \left[\exp\left(-\frac{(x - x_0 + \nu)^2}{4\nu} \right) + \exp x_0 \exp\left(-\frac{(x + x_0 + \nu)^2}{4\nu} \right) \right] + \frac{1}{2} \exp(-x) \operatorname{erfc} \frac{x + x_0 - \nu}{(4\nu)^{1/2}} \right\} \tag{31}$$

with

$$x = \epsilon/kT, \; x_0 = \epsilon_P/kT, \quad \nu = \frac{M_1^2}{M_2}, \; g_0 t = \left(\frac{\hbar s \delta^{-1}}{kT} \right)^2 \frac{t}{\tau_{\text{ac}}} ,$$

where $\tau_{\text{ac}}^{-1} = \sum_{\nu'} W_{\nu\nu'} \simeq M_0 g_0$ is the collision frequency of an electron on acoustical phonons in EQL.

Taking account of our starting point, i.e., Eq. (6), we finally get the EDF in the EQL:

$$\rho(\epsilon, t) = \rho^0(\epsilon) + \Delta\rho(\epsilon, t) \exp(-t/\tau) \Theta(t) , \tag{32}$$

with

$$\rho^0(\epsilon) = \frac{N_0}{2V} (2\pi\delta)^2 \left(\frac{\hbar^2}{2\pi m kT} \right)^{1/2} \exp\left(-\frac{\epsilon}{kT} \right) . \tag{33}$$

D. Time evolution of the distribution function

In order to discuss the formal results (31) and (32), we consider the variations of $\rho(\epsilon, t)$ with ϵ/ϵ_P for different values of the time t in the case of InSb, a semiconductor in which EQL conditions are easily fulfilled. We forget the nonparabolic character of the conduction band of InSb and use the numerical values of the physical parameters reported in Refs. 11 and 12. We fix the temperature equal to 10 K, the magnetic field value B equal to 5 tesla, and the photoexcitation energy ϵ_P equal to $5kT$. Finally, we need a value of the collision frequency τ_{ac}^{-1} of an electron with acoustical phonons. We know that $\tau_{\text{ac}}^{-1} \simeq 2.2 \times 10^{-10}$ s for the deformation potential coupling and $\tau_{\text{ac}} \simeq 5.5 \times 10^{-10}$ s for the piezoelectric coupling.

We plotted in Fig. 1 the dimensionless quantity $kTg_0 \Delta\rho(\epsilon, t)/\Delta N$ as a function of ϵ/ϵ_P when only the ad type of scattering is taken into account and for different successive values of the time. The distribution function is initially very peaked around the energy ϵ_P (curve 1); then diffusion takes place and the peak spreads out into a wider curve. The electron energy loss by the acoustical phonons produces then a drift of the curves towards the lower energies (curves 2, 3, 4). In the meantime, the maximum flattens down and, once the probability of finding electrons near the bottom of the band becomes appreciable, the curves begin to rise upwards in this energy region because of the reflection which the electrons suffer at $\epsilon = 0$ (curve 5). Finally, we recover the equilibrium Maxwel-

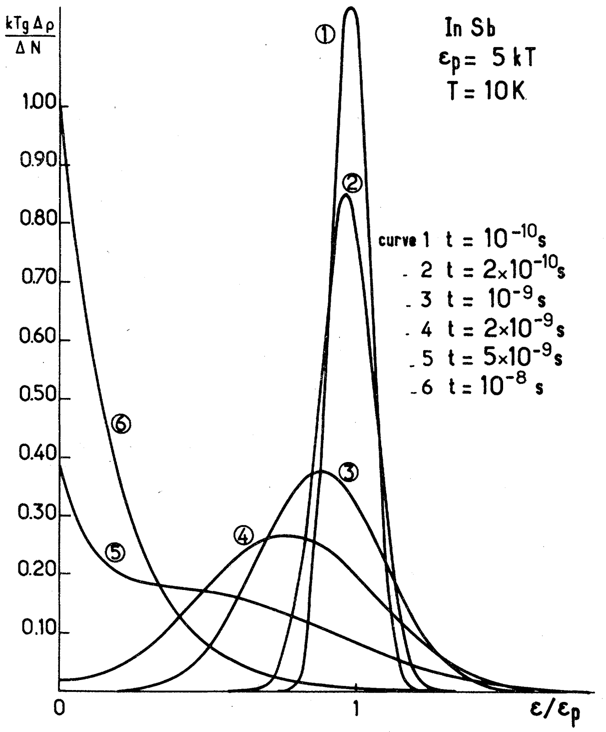

FIG. 1. Variation of the mean photoexcited number of electrons $kTg(\epsilon)\Delta\rho(\epsilon)/\Delta N$ with the reduced energy ϵ/ϵ_P at different times after the laser pulse.

lian distribution (curve 6).

We observe that the thermalization process is achieved in a mean time of order $100\tau_{ac} \simeq 10^{-8}$s in InSb. This result is roughly in agreement with the experimental results of Partl *et al.*[13] who studied the time evolution of the hot electron distribution function in EQL in the presence of crossed electric and magnetic fields. However, the photoexcited electrons can be thermalized only if the recombination time τ is larger than $100\tau_{ac}$; if, for instance, $\tau = 10\tau_{ac}$, it is obviously impossible for the photoexcited electrons to be thermalized before they recombine.

IV. TIME EVOLUTION OF THE MEAN ELECTRON ENERGY

The mean energy per electron is defined in the usual way:

$$\langle \epsilon(t) \rangle = \frac{\int_0^\infty d\epsilon\, \epsilon g(\epsilon)\rho(\epsilon, t)}{\int_0^\infty d\epsilon\, g(\epsilon)\rho(\epsilon, t)} . \qquad (34)$$

There are two distinct contributions to $\langle \epsilon(t) \rangle$: The first one comes from the thermalized electrons preexisting in the conduction band; the other one arises from the photoexcited electrons. This appears more clearly if we rewrite formula (34) as

$$\langle \epsilon(t) \rangle = \frac{\frac{1}{2} N_0 kT + \Delta N \epsilon_1 \exp(-t/\tau)}{N_0 + \Delta N \exp(-t/\tau)} \qquad (35)$$

with

$$\epsilon_1(t) = \frac{\int_0^\infty d\epsilon\, \epsilon g(\epsilon)\Delta\rho(\epsilon, t)}{\int_0^\infty d\epsilon\, g(\epsilon)\Delta\rho(\epsilon, t)} . \qquad (36)$$

$\epsilon_1(t)$ is the contribution from the photoexcited electrons to the mean energy; it must obviously satisfy the two following limit conditions:

$$\lim_{t\to 0} \epsilon_1(t) = \epsilon_P , \qquad (37a)$$

$$\lim_{t\to \infty} \epsilon_1(t) = \tfrac{1}{2} kT . \qquad (37b)$$

We must now evaluate $\epsilon_1(t)$. In the preceding section, we determined an approximate expression for $\Delta\rho(\epsilon, t)$, assuming that the electron density of states could be chosen as a constant g_0. Let us use this expression for $\Delta\rho(\epsilon, t)$ in formula (36); we easily verify that the two conditions (37a) and (37b) are satisfied. Expression (36) must give a good interpolation and we thus expect to obtain a reasonable value of $\epsilon_1(t)$ at any time.

As in Sec. III, we consider the case of InSb; we use the same value of τ_{ac} as before, but we must also know precisely the recombination time τ and the relative number of excess photoexcited electrons $A = \Delta N/N_0$. We have chosen to deal with a τ ten or a hundred times larger than τ_{ac}, that is, $\tau \simeq 10^{-9}$ s or 10^{-8} s. We have given A the two possible limit values $A = 1$ or 10^3. We plotted in Figs. 2 and 3 the variations of

$$T_e(t) - T = (2/k)\langle \epsilon(t) \rangle - T \qquad (38)$$

versus t/τ. Although the EDF is not a Maxwellian, $T_e(t)$ can be considered as a measure of the electron temperature.

We shall now discuss the characteristic features of the electronic energy decay curves of Figs. 2 and 3 and analyze the influence on these curves of the different parameters of the problem.

A. Influence of the magnetic field

We observed in Sec. III that $\Delta\rho(\epsilon, t)$ depends on the dimensionless parameter $\nu = (\hbar s \delta^{-1}/kT)^2 (t/\tau_{ac})$, which is proportional to $B^2 t$ or Bt, respectively, for the deformation potential and the piezoelectric coupling. Therefore an increase of the magnetic field has the same effect as an increase of time, i.e., produces a decrease of the electron temperature T_e. This result can be easily explained, since the mean electron energy lost per unit time due to the collisions with the phonon gas is of order $\tau_{ac}^{-1}[(\hbar s \delta^{-1})^2/2kT]$ and thus increases with B. We recover here the well-known cooling effect of the magnetic field, which is observed in Fig. 2.

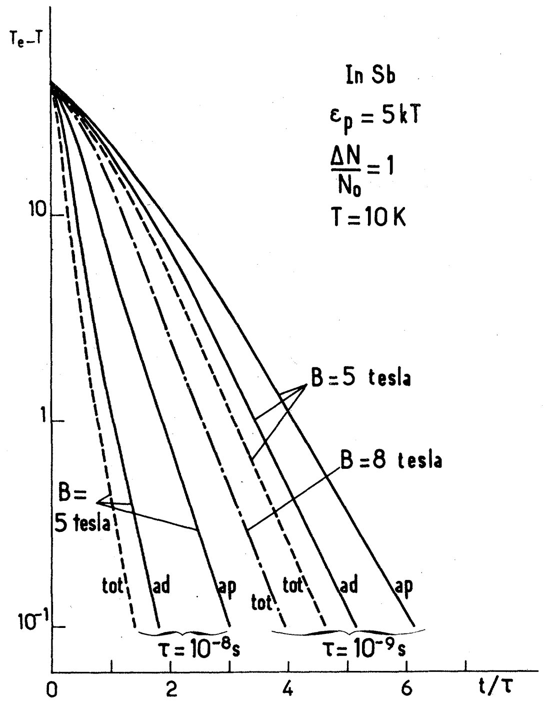

FIG. 2. Variations with time of the excess mean electron energy for different values of the recombination time: $\tau = 10^{-8}$ and 10^{-9} s. We distinguish between the contribution of two different one-phonon scattering mechanisms together with electron recombination: acoustic piezoelectric (curve ap) and acoustic deformation potential (curve ad) scattering. The dotted lines correspond to both phonons' effects and to electron recombination together at $B = 5$ tesla. The broken lines show the effect of an increased magnetic field of $B = 8$ tesla when $\tau = 10^{-9}$ s and when all phonons and recombination processes occur.

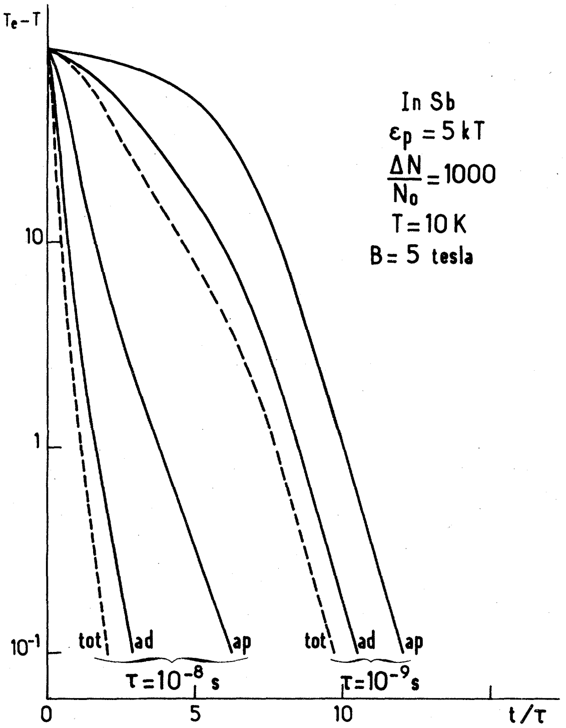

FIG. 3. Same parameters as in Fig. 2 but $A = \Delta N/N = 10^3$ and $B = 5$ tesla only.

B. Influence of the photoexcitation intensity

At first, one can recall that, in the absence of collisions, the decay of $T_e(t)$ is only due to the electron recombination and $T_e(t)$ varies with time as

$$T_e(t) - T = \left(2\frac{\epsilon_P}{k} - T\right)\frac{A\exp(-t/\tau)}{1+A\exp(-t/\tau)}\,. \qquad (39)$$

If $A\exp(-t/\tau) \ll 1$, $T_e(t) - T$ decays exponentially with a time constant equal to the recombination time τ. If, on the other hand, $A\exp(-t/\tau) \gg 1$, $T_e(t)$ varies very slowly.

These features remain when the collisions with the acoustical phonons are taken into account. Especially, when $A = 1000$ and $\tau = 10^{-9}$ s, we observe particularly well (Fig. 3) a slow variation of $T_e(t)$ for $t \lesssim \tau$ and an exponential behavior for $t \gtrsim 10\tau$, with a characteristic time which is not τ but less than τ because of the effect of the collisions. As A is decreased or τ is increased, this initial slow variation of $T_e(t)$ disappears.

C. Influence of the photoexcitation energy

The photoexcitation energy ϵ_P determines the initial value of the mean electron energy and the slope of the curves $T_e(t)$ at time $t = 0$. In particular, when ϵ_P is lower than the mean equilibrium thermal energy $\frac{1}{2}kT$, $T_e(0)$ is lower than T and $T_e(t)$ increases with time up to its limit value T (cf. Ref. 6).

D. Influence of the recombination time

Since the curves $T_e(t) - T$ of Figs. 2 and 3 are plotted versus the reduced time variable $x = t/\tau$, the recombination time τ is involved only in $\epsilon_1(t)$, defined by Eq. (36), which depends on the parameter

$$\nu = (\hbar s\delta^{-1}/kT)^2 (t/\tau_{ac}) = (\hbar s\delta^{-1}/kT)^2 (\tau/\tau_{ac})\, x\,.$$

For a given x, a larger value of τ enhances the τ/τ_{ac} value, that is the number of collisions suffered by an electron during its lifetime; this evidently leads to a more rapid decay of the electron temperature, as shown in Fig. 2.

E. Influence of the lattice temperature

The lattice temperature T is involved in the determination of the initial value of $T_e(t)$ [$T_e(0)$ evidently increases with T]. Moreover, the distribution function of the photoexcited electrons $\Delta\rho(\epsilon, t)$ depends on the dimensionless parameter $\nu = (\hbar s \delta^{-1}/kT)^2 (t/\tau_{ac})$, which is proportional to $T^{-3/2}$ for both deformation potential and piezoelectric coupling. Therefore the decay of $T_e(t)$ will be slower if the lattice temperature T increases.

F. Influence of the different types of electron-phonon scattering mechanisms

The τ_{ac} value in InSb is smaller in the case of deformation potential coupling than in the case of piezoelectric coupling. The acoustic deformation potential coupling is thus more effective than the piezoelectric coupling. This can be observed on the decay curves of $T_e(t)$ (Figs. 2 and 3). In fact, these two mechanisms occur simultaneously. More precisely, we can easily verify that it suffices to use the collision frequency $(\tau_{ac}^{-1})_{ad} + (\tau_{ac}^{-1})_{ap}$ in Eq. (31) to take into account both scattering mechanisms. So we will obtain more rapid variations of $T_e(t)$ than when they are considered separately (curves tot of Figs. 2 and 3).

V. CONCLUSION

We are now in a position to compare the results on the time evolution of the mean energy of electrons photoexcited by a laser pulse in the conduction band of a semiconductor in extreme quantum limit conditions which we obtained by two different methods:

(i) The first method, developed in Ref. 6, is a simple calculation relying on the hypothesis of a Maxwellian EDF with an electron temperature $T_e(t)$ depending on time.

(ii) The second one, proposed in the present paper, is a first-principles derivation in which the master equation for the time-dependent EDF is transformed into a solvable Fokker-Planck one.

The qualitative features of the variations of the electron temperature with time on the one hand and with various physical parameters, such as magnetic field, photoexcitation intensity, photoexcitation energy, recombination time, lattice temperature, and electron-phonon scattering mechanisms, on the other hand, are found to be the same whatever the method used. Even the orders of magnitude of the electron temperatures obtained by methods (i) and (ii) coincide.

However, if we consider the electron distribution function itself, calculation (ii) evidently shows that the Maxwellian approximation is not correct as long as $t \lesssim 100\tau_{ac}$. The agreement between the mean electron energy results obtained by methods (i) and (ii) is a consequence of the integration involved in the expression of $\langle \epsilon(t) \rangle$ which is not very sensitive to the form of the EDF.

Moreover, the first-principles derivation we propose in this paper allows us to show that the way in which the photoexcited electrons tend towards thermal equilibrium can very simply be understood in terms of a random-walk process in the energy space involving both drift and diffusion phenomena. This method thus yields a deeper insight into the microscopic phenomena involved in the energy relaxation processes, which was evidently beyond the scope of the preceding treatment.

---

*Laboratoire associé au CNRS.

[1] R. G. Ulbrich, Phys. Rev. B 8, 5719 (1973); Solid State Electron. 21, 51 (1978).

[2] C. V. Shank, D. H. Auston, E. P. Ippen, and O. Teschke, Solid State Commun. 26, 567 (1978).

[3] D. von der Linde and R. Lambrich, Phys. Rev. Lett. 42, 1090 (1979).

[4] J. R. Barker, Solid State Electron. 21, 197 (1978).

[5] A. M. Zlobin and P. S. Zyrianov, Zh. Eksp. Teor. Fiz. 58, 952 (1970) [Sov. Phys.—JETP 31, 513 (1970)]; Usp. Fiz. Nauk 104, 353 (1971) [Sov. Phys. Usp. 14, 379 (1972)].

[6] D. Calecki and N. Pottier, Solid State Electron. 22, 999 (1979).

[7] C. Lewiner, D. Calecki, and N. Pottier, Phys. Rev. B 18, 6348 (1978).

[8] T. Kurosawa, J. Phys. Soc. Jpn. 20, 937 (1965); E. Yamada and T. Kurosawa, *ibid.* 34, 603 (1973).

[9] D. Calecki, C. Lewiner, and P. Nozières, J. Phys. (Paris) 38, 169 (1977).

[10] S. Chandrasekhar, Rev. Mod. Phys. 15, 3 (1943).

[11] B. R. Nag and D. Chattopadhyay, Solid State Electron. 21, 306 (1978).

[12] P. Lavallard, R. Bichard, and C. Benoit à la Guillaume, Phys. Rev. B 16, 2804 (1973).

[13] H. Partl, W. Müller, F.Kohl, and E. Gornik, J. Phys. C 11, 1091 (1978).