Coexistence of p-state superconductivity and itinerant ferromagnetism

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The possibility of a p-wave "equal-spin-pairing" superconducting state is predicted in itinerant ferromagnets. The pairing interaction is mediated by the exchange of longitudinal spin fluctuations, and the resulting state is analogous to the $A1$ phase of superfluid ^{3}He. We describe the system in terms of a Hubbard-type exchange interaction constant $\bar{I}$ and a Stoner enhancement factor $S=(1-\bar{I})^{-1}$. As $\bar{I}$ is varied and the ferromagnetic transition is approached from either the ferromagnetic or paramagnetic side, the p-state transition temperature goes through a maximum and then falls to zero. Rough calculations of the transition temperature indicate that this state should be observable in very clean samples of weak itinerant ferromagnets at currently attainable temperatures. Applications to $ZrZn_2$ and Ni are discussed.

I. INTRODUCTION

The p-wave superconducting transition temperature for paramagnon-induced pairing in nearly ferromagnetic systems has been shown to go through a maximum and then approach zero as the system becomes ferromagnetic.[1–3] Here one is considering a system at very low temperature in which the ferromagnetic transition is brought about by increasing the strength of the exchange interaction. The coexistence of p-wave pairing (or triplet pairing in general) is, however, not excluded by this result. We show that the exchange of longitudinal "paramagons" can lead to an "equal-spin-pairing" (ESP) state in itinerant ferromagnets which is similar to the $A1$ phase of ^{3}He in a magnetic field. As in the paramagnetic case, the p state T_c of this phase exhibits a maximum and falls to zero as the magnetic transition is approached from the ferromagnetic side. We should emphasize that we are considering coexistence in an *itinerant* system in which the *same* electrons (usually d electrons in a transition metal) are responsible for *both* the superconductivity and the ferromagnetism.

We assume that the relevant magnetic behavior of the system is adequately described by a RPA-(random-phase approximation)–mean-field theory in terms of a Hubbard-type exchange interaction parameter $\bar{I}$ and the Stoner enhancement factor $S=(1-\bar{I})^{-1}$, the magnetic transition occurring at $\bar{I}=1$. At the very low temperatures appropriate for p-wave pairing this approach receives support from the work of Hertz[4] who showed that, for a zero temperature nearly ferromagnetic system, the RPA is qualitatively correct in that the mean-field critical exponents are obtained. The RPA is presumably also a reasonable approximation for a weak ferromagnet at temperatures much less than the Curie temperature. In the limit of very weak ferromagnetism where $\bar{I}\to 1$ from above and T_C is thus very small, we should really go beyond the RPA and at least employ the corrections due to the coupling among the spin fluctuations introduced by Moriya.[5] However, since we are primarily interested in obtaining a qualitative picture of the behavior of the system and an order of magnitude estimate of T_c, the RPA should be sufficient. We have also assumed a spherical Fermi surface.

We consider in this paper only the ESP state where the pairing is between parallel-spin electrons. In the paramagnetic phase at $T=T_c$, the three spin components of the triplet are equivalent in the absence of a magnetic field in the sense that all components yield the same T_c. This will be evident from the RPA calculation. In the ferromagnetic region, on the other hand, one must consider parallel- and antiparallel-spin pairing separately. The theory of the parallel-spin pairing developed here is analogous to the theory of superfluid ^{3}He in a magnetic field ($A1$ phase) as discussed by Levin.[6] Triplet antiparallel-spin pairing cannot however by strictly ruled out. In this case exchange of spin waves can lead to attraction.[7] However, this pairing and that due to phonons is reduced by the exchange splitting of the Fermi surface which cuts out a part of the attractive low-frequency spin-wave and phonon spectra. To minimize this reduction, the electrons must be paired with a finite total momentum $\vec{Q}$.[7] As a consequence the phase-space volume available for the scattering processes leading to the Cooper pairing is reduced. The optimum value of $\vec{Q}$ minimizes both of these reductions simultaneously. Furthermore, the nonzero value of the pairing momentum $\vec{Q}$ generates

a supercurrent that must be compensated for by the total current of the unpaired electrons so that Bloch's theorem is fulfilled. Fulde and Ferrell[8] succeeded in constructing a depaired superconducting ground state that exhibits zero current assuming a constant BCS pairing interaction. For the time being we assume that the transition temperature of a Fulde-Ferrell state is lower than that of the ESP state we investigate. This assumption probably fails *very* close to $\bar{I}=1$ where the p wave T_c approaches zero. This is because the spin-wave energy (and hence T_c) approaches zero somewhat more slowly than the effective longitudinal spin-fluctuation energy. This question is rather academic since p-state pairing would probably only be observable (if at all) near the maximum of the T_c vs $\bar{I}$ curve where our assumption should be valid.

This work was undertaken, in part, due to the current interest in $ZrZn_2$. Several groups are apparently planning experimental investigations of the low-temperature properties of this material.[9] $ZrZn_2$ is one of the few known weak itinerant ferromagnets and has the interesting property of becoming paramagnetic at a critical pressure P_c of about 8 kbar.[10] Varying the external pressure would thus in principle allow one to effectively vary $\bar{I}$ through the point $\bar{I}=1 (P=P_c)$. That p-wave pairing due to the spin-fluctuation (SF) exchange mechanism[1-3] might be observed in the nearly ferromagnetic $P \gtrsim P_c$ region of $ZrZn_2$ was first pointed out by Leggett[11] and later discussed by several authors.[12,13] Our present work suggests that p-state pairing could also occur in the weakly ferromagnetic region $P \lesssim P_c$. Indeed, the maximum of T_c^1 for $ZrZn_2$ seems to lie near zero external pressure. Since the elements Zr and Zn are superconductors, the s-wave phonon pairing interaction in $ZrZn_2$ is expected to also be large and attractive. Thus one might expect s-wave superconductivity to appear in $ZrZn_2$ at pressures well above P_c where $\bar{I} \ll 1$ and therefore the spin fluctuations which presumably suppress singlet pairing[14] are no longer dominant. Singlet pairing is also suppressed in the ferromagnetic region by both spin-wave exchange[7] and by the longitudinal SF's considered here indicating that if superconductivy can coexist at all with itinerant ferromagnetism, it it would most likely be of the triplet type. Thus we can speculate on the progression of states that might be observed in clean $ZrZn_2$ at very low (probably <0.5 K) temperature as the pressure is increased from zero to $P \gg P_c$: itinerant ferromagnetism, itinerant ferromagnetism plus p-state pairing, paramagnetic p-wave pairing, and paramagnetic s-wave pairing.

Since p-state pairing is strongly suppressed by impurity scattering (and almost anything else one can think of), it is not clear that $ZrZn_2$ can be made clean enough to actually observe the above effects. Thus we have also investigated the possibility of an ESP state in Ni which can be made quite clean: In measurements of the de Haas–van Alphen effect,[15] single crystalline samples are used in which the residual resistance ratio is about 10^4 with a Dingle scattering temperature $T_D \lesssim 0.5$ K. Unfortunately, the majority spin band in Ni is probably full which turns out to be quite detrimental to SF pairing in either band. Not being deterred by experimental facts, however, we have used our model to simulate a hypothetical Ni in which the majority band is not completely filled. This yields a T_c^1 on the order of 20 mK. While not directly applicable to Ni, this calculation may still find application in other intermediate strength itinerant ferromagnetics or possibly in Ni under pressure.

In Sec. II we discuss the paramagnon-mediated pair interaction in the paramagnetic and ferromagnetic phases. In Sec. III, T_c is calculated in a weak coupling approximation and the results for a model ferromagnet are discussed. In Sec. IV application is made to $ZrZn_2$ and Ni and in Sec. V our results are briefly summarized.

II. SPIN-FLUCTUATION PAIRING

We assume a one-band, spherical Fermi-surface model and, since we are primarily interested in determining whether a p-wave superconducting state can exist and, if so, at approximately what temperature, we consider the exact finite temperature gap equation linearized at T_c,[16]

$$\Delta^{\sigma\sigma'}(\vec{p},\omega_\nu) = T_c \sum_{\nu'} \sum_{\vec{k}} V^{\sigma\sigma'}(\vec{p},\omega_\nu;\vec{k},\omega_{\nu'}) G_\sigma(k,\omega_{\nu'}) G_{\sigma'}(-\vec{k},-\omega_{\nu'}) \Delta^{\sigma\sigma'}(\vec{k},\omega_{\nu'}) \quad . \tag{2.1}$$

Here G_σ is the exact single-particle propagator and $V^{\sigma\sigma'}(p,k)$ is the irreducible interaction for a Cooper pair with spin (σ,σ') scattering from $(\vec{p},-\vec{p})$ to $(\vec{k},-\vec{k})$ and $\sigma \equiv (\pm 1) \equiv (\uparrow,\downarrow)$. Both G and V are to be evaluated in the nonsuperconducting state at $T=T_c$ and, since T_c is expected to be very small, we calculate V at zero temperature as BCS do. The interaction V contains a phonon contribution. In general, that is, when V_{phonon} is not strongly dependent on $\vec{p}$ and $\vec{k}$, the p-wave component should be much smaller than the s-wave component[13]; therefore we neglect it and consider V to be due entirely to spin fluctuations.

In the next section we will argue that a weak coupling solution of Eq. (2.1) is sufficient for our present purposes. In this case the quantity of primary importance for general l-state pairing in the paramagnetic phase is the lth angular momentum

component with respect to the angle between $\vec{p}$ and $\vec{k}$ of the static limit of V with $|\vec{p}|=|\vec{k}|=k_F$. Generalizing to include both parallel-spin pairing in the ferromagnetic region and general triplet pairing in the paramagnetic region we define a BCS-type pairing parameter $\lambda_l^{\sigma,L}$ by

$$\lambda_l^{\sigma,L} \equiv N_\sigma(0) V_l^{\sigma\sigma} \ , \tag{2.2}$$

with

$$V_l^{\sigma\sigma} = \int_0^{2k_{F\sigma}} \frac{q dq}{2k_{F\sigma}^2} P_l\left(1-\frac{q^2}{2k_{F\sigma}^2}\right) \times V^{\sigma\sigma}(|\vec{q}|, q_0=0)\Big|_{|\vec{p}|=|\vec{k}|=k_{F\sigma}} \ . \tag{2.3}$$

Here $N_\sigma(0)$ is the density of states at the spin-σ Fermi surface, $q=|\vec{q}|=|\vec{p}-\vec{k}|$, and the superscript L reminds us that this contribution is due to longitudinal spin fluctuations. For triplet pairing in the paramagnetic phase the spin indices can be dropped [see Eq. (2.11)].

A. Paramagnetic region

We briefly review the spin-fluctuation theory of the pairing interaction in a paramagnetic system to provide some background and to illustrate some of the new features that appear in itinerant ferromagnets. We assume that a repulsive Hubbard-type contact interaction I acts between particles of opposite spin. This model is often employed to simulate the intra-atomic Coulomb interaction in narrow-band transition metals. More generally, I should be considered as an approximation to the exact irreducible particle-hole 4-point interaction vertex.

The RPA susceptibility is obtained by summing the particle-hole ladder diagrams which yields the well-known result

$$\chi(\vec{q},q_0) = \frac{\chi_0(\vec{q},q_0)}{1-I\chi_0(\vec{q},q_0)} \ , \tag{2.4}$$

where

$$\chi_0(\vec{q},q_0) = i\int \frac{d^4k}{(2\pi)^4} G_0(k+q) G_0(k) \tag{2.5}$$

is the susceptibility of the noninteracting system. Using the result

$$\lim_{q\to 0}\lim_{q_0\to 0} \chi_0(\vec{q},q_0) = N(0) = mk_F/2\pi^2 \tag{2.6}$$

along with Eq. (2.4) we define the Stoner enhancement factor S:

$$S \equiv \lim_{q\to 0}\lim_{q_0\to 0} [\chi(\vec{q},q_0)/\chi_0(\vec{q},q_0)] = (1-\bar{I})^{-1} \ , \tag{2.7}$$

where

$$\bar{I} \equiv N(0) I \ . \tag{2.8}$$

The Stoner (Hartree-Fock) criterion for the ferromagnetic instability is $\bar{I}\to 1$. For $\bar{I}\lesssim 1$, the susceptibility is strongly enhanced for *small* $\vec{q}$ and q_0 indicating the presence of large spin fluctuations or "paramagnons." When calculating properties such as the effective mass or pairing interaction for such a nearly ferromagnetic system the contributions involving iterated particle-hole diagrams must be included.

We consider first the SF-induced pair interaction V which is shown for an antiparallel-spin pair in Fig. 1. The bubble and ladder diagrams belong to the two general classes shown in Fig. 1 of Ref. 2. It is often useful to classify these diagrams according to their properties in the particle-hole channel: The ladder diagrams have momentum transfer $p+k$ in the particle-hole channel and represent transverse spin fluctuations, i.e., particle-hole triplets ($S_z=\pm 1$). The bubble diagrams, however, have $S_z=0$ and momentum transfer $p-k$ in the particle-hole channel and contain both singlet (density fluctuation) and triplet (longitudinal spin-fluctuation) contributions. The diagrams of Fig. 1 are easily summed with the result

$$V^{\uparrow\downarrow}(p,k) = -I - \frac{1}{2}\left[\frac{I^2\chi_0}{1-I\chi_0} - \frac{I^2\chi_0}{1+I\chi_0}\right]_{p-k} - \left[\frac{I^2\chi_0}{1-I\chi_0}\right]_{p+k} \ , \tag{2.9}$$

where we have written the contribution of the bubble diagrams so that the separation into particle-hole singlet and triplet is evident.

The parallel-spin interaction $V^{\uparrow\uparrow}$ is obtained in a similar manner by summing bubble diagrams with *odd* numbers of bubbles. The ladder diagrams are

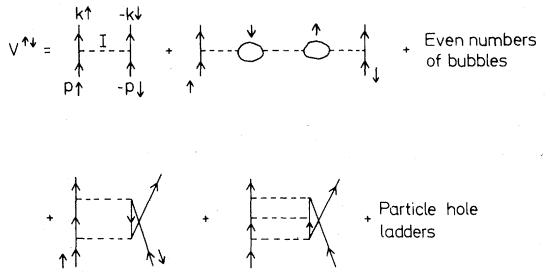

FIG. 1. The spin-fluctuation contribution to the irreducible, antiparallel-spin, pair interaction $V^{\uparrow\downarrow}(p,k)$ for the contact model.

absent due to the Pauli principle

$$V^{\uparrow\uparrow}(p-k)=\frac{1}{2}\left[\frac{I^2\chi_0}{1+I\chi_0}+\frac{I^2\chi_0}{1-I\chi_0}\right]_{p-k} . \quad (2.10)$$

Again the longitudinal spin fluctuations and the density fluctuations separate. $V^{\uparrow\uparrow}$ is obviously a particle-particle triplet interaction and thus contains only odd-l partial-wave components. $V^{\uparrow\downarrow}$ on the other hand is a combination of particle-particle singlet and triplet and all l values can contribute. In order to calculate V_l from Eq. (2.3) we write $V^{\uparrow\downarrow}$ as a function only of $p-k$: Consideration of Eq. (2.3) shows that we may replace $\vec{p}+\vec{k}\rightarrow\vec{p}-\vec{k}$ in the last term if we multiply the corresponding contribution to $V_l^{\uparrow\downarrow}$ by $(-1)^l$. It is thus easily verified from Eqs. (2.9) and (2.10) that for triplet pairing

$$V_l^{\uparrow\downarrow}=V_l^{\uparrow\uparrow}(l \text{ odd}) . \quad (2.11)$$

Thus, in the absence of any long-range magnetic order, $T_c^{\uparrow}$ is the same for parallel- and antiparallel-spin pairs as could have been expected from symmetry grounds. One can also easily show that $V_l^{\uparrow\downarrow}$ is large and negative (repulsive) for singlet (even-l) pairing when S is large.

B. Ferromagnetic region

Now we must attach a spin index to the G_0 function in Eq. (2.5) and replace $\chi_0\rightarrow\chi_0^{\sigma\sigma'}$ where $\chi_0^{\uparrow\downarrow}\neq\chi_0^{\uparrow}\neq\chi_0^{\downarrow}$. The interaction functions corresponding to Eqs. (2.9) and (2.10) are

$$V^{\uparrow\downarrow}(p,k)=-I-\left[\frac{I^3\chi_0^{\downarrow}\chi_0^{\uparrow}}{1-I^2\chi_0^{\downarrow}\chi_0^{\uparrow}}\right]_{p-k}-\left[\frac{I^2\chi_0^{\uparrow\downarrow}}{1-\chi I_0^{\uparrow\downarrow}}\right]_{p+k} , \quad (2.12)$$

$$V^{\sigma\sigma}(p-k)=\left[\frac{I^2\chi_0^{-\sigma}}{1-I^2\chi_0^{\sigma}\chi_0^{-\sigma}}\right]_{p-k} \quad (\sigma=\uparrow \text{ or } \downarrow) , \quad (2.13)$$

where we have defined $\chi_0^{\sigma}\equiv\chi_0^{\sigma\sigma}$. The longitudinal spin fluctuations and the density fluctuations can no longer be separated and Eq. (2.11) does not hold. As we shall see shortly, for $\bar{I}\gtrsim 1$ (weak ferromagnetism), $V^{\sigma\sigma}$ is strongly peaked for $|\vec{p}-\vec{k}|\rightarrow 0$ which leads to an enhanced p-state pairing interaction much as in the paramagnetic phase. We point out also that, due to the requirement of spin conservation, exchange of spin waves or transverse spin fluctuations does not contribute to the ESP interaction $V^{\sigma\sigma}$. We also note for future reference that the RPA diagrams can also be summed if I is allowed to be a function of the total momentum in the particle-hole channel.

Brinkman and Engelsberg[17] (hereafter BE) have considered the RPA equations for a more general momentum-dependent irreducible particle-hole interaction. In fact, our $V^{\uparrow\uparrow}$ is just their $t_{\uparrow\uparrow}^0$ [BE Eq. (2.8)] specialized to a contact interaction. In the general case ladder diagrams as well as bubbles can contribute to $V^{\uparrow\uparrow}$. We will follow closely the notation of BE and take over directly a number of their results.

We restrict our attention now to the calculation of $\lambda_l^{\sigma,L}$ from Eqs. (2.2) and (2.13). For this we require $\chi_0^{\sigma}(\vec{q},q_0)$ for $q_0=0$. The k_0 integration of Eq. (2.5) yields

$$\chi_0^{\sigma}(\vec{q},q_0)=\int\frac{d^3k}{(2\pi)^3}\frac{\Theta(-\epsilon_{k+q,\sigma})-\Theta(\epsilon_{k,\sigma})}{q_0+\epsilon_{k,\sigma}-\epsilon_{k+q,\sigma}} , \quad (2.14)$$

where Θ is the unit step function and

$$\epsilon_{k,\sigma}=k^2/2m-\tfrac{1}{2}I(N_{\sigma}+N_{-\sigma})+\tfrac{1}{2}I(N_{-\sigma}-N_{\sigma})-\mu \quad (2.15)$$

is the Hartree-Fock single-particle energy measured relative to the chemical potential μ and N_{σ} is the number of particles with spin σ [not to be confused with the density of states $N_{\sigma}(0)$]. Evaluation of Eq. (2.14) for $q_0=0$ leads to

$$\chi_0^{\sigma}(q)\equiv\chi_0^{\sigma}(\vec{q},0)=N_{\sigma}(0)u_{\sigma}(q) , \quad (2.16)$$

where

$$u_{\sigma}(q)=\frac{1}{2}\left[1+\frac{k_{F\sigma}}{q}\left(1-\frac{q^2}{4k_{F\sigma}^2}\right)\ln\left|\frac{1+q/2k_{F\sigma}}{1-q/2k_{F\sigma}}\right|\right] \quad (2.17)$$

is the Lindhard function for the σ spin band, and

$$N_{\sigma}(0)=mk_{F\sigma}/2\pi^2 . \quad (2.18)$$

Given the spin-σ Fermi momentum $k_{F\sigma}$, $\lambda_l^{\sigma,L}$ can be found from Eqs. (2.2), (2.3), (2.13), and (2.16)–(2.18).

For our application to a model system in which $\bar{I}$ can be varied through the magnetic transition, it is convenient to express $k_{F\sigma}$ in terms of $\bar{I}$ and the Fermi momentum k_F of the (perhaps hypothetical) paramagnetic phase. Demanding that the particle densities be equal in both phases yields the relation

$$k_F^3=\tfrac{1}{2}(k_{F\uparrow}^3+k_{F\downarrow}^3) . \quad (2.19)$$

A second condition is that the energy of a spin-up electron at the up Fermi surface should equal the energy of a down-spin electron at the down Fermi surface: $\epsilon_{k_{F\uparrow}}=\epsilon_{k_{F\downarrow}}$. Using Eq. (2.15) with $N_{\sigma}/V=\frac{1}{2}(k_{F\sigma}^3/3\pi^2)$ and Eq. (2.8) this condition yields

$$k_{F\downarrow}^2-k_{F\uparrow}^2=2\bar{I}(k_{F\downarrow}^3-k_{F\uparrow}^3)/3k_F . \quad (2.20)$$

Although we could solve Eqs. (2.19) and (2.20) directly for $k_{F\uparrow}$ and $k_{F\downarrow}$, it is useful to introduce the auxiliary quantities k_F' and η employed by BE:

$$k_{F\sigma}=k_F'(1-\sigma\eta) \ . \tag{2.21}$$

The parameter η is a measure of the "strength" of the ferromagnetism and has the range $0\leq\eta\leq 1$, with $\eta=1$ corresponding to $k_{F\uparrow}=0$ and all spins down. In a nearly free-electron gas (or in a nondegenerate band), weak and strong ferromagnetism can be roughly defined as $\eta\gtrsim 0$ and $\eta\lesssim 1$, respectively. With the help of Eq. (2.21), the equilibrium conditions [Eqs. (2.19) and (2.20)] can be written as

$$k_F'=k_F(1+3\eta^2)^{-1/3} \ , \tag{2.22}$$

$$(1+3\eta^2)=\bar{I}^3(1+\eta^2/3)^3 \ . \tag{2.23}$$

From Eq. (2.23) one finds that the region $0\leq\eta\leq 1$ corresponds to $1\leq\bar{I}\leq\frac{3}{4}4^{1/3}\cong 1.19$. For $\bar{I}\geq\frac{3}{4}4^{1/3}$ the magnetization is saturated with all electrons in the majority ($\downarrow$) band. Using Eqs. (2.21) and (2.22) we can write

$$k_{F\sigma}=k_F(1-\sigma\eta)/(1+3\eta^2)^{1/3} \tag{2.24}$$

and η can be expressed in terms of $\bar{I}$ through Eq. (2.23). These equations were employed in our numerical computations

C. $\lambda_1^{\sigma,L}$ in the weak ferromagnetic limit

We can calculate $\lambda_1^{\sigma,L}$ exactly in the limit of very weak ferromagnetism, $\eta\to 0$, if we employ the small-q approximation for $\chi_0^\sigma(q)$:

$$\chi_0^\sigma(q)=N_\sigma(0)[1-\tfrac{1}{12}(q/k_{F\sigma})^2+O(q^4)] \ . \tag{2.25}$$

Expanding Eq. (2.24) for small η and inserting the result in Eqs. (2.18) and (2.25) we have

$$N_\sigma(0)=N(0)(1-\sigma\eta-\eta^2+\sigma\eta^3+2\eta^4\cdots) \tag{2.26}$$

and

$$\chi_0^\sigma(q)\cong N(0)[1-\sigma\eta-\eta^2+\sigma\eta^3+2\eta^4-\tfrac{1}{12}\bar{q}^2(1+\sigma\eta)] \ , \tag{2.27}$$

where $\bar{q}=q/k_F$. From Eqs. (2.8), (2.13), (2.26), and (2.27) we find

$$N_\sigma(0)V^{\sigma\sigma}(q)\cong \frac{\bar{I}^2[1-3\eta^2-\frac{1}{12}\bar{q}^2(1-2\sigma\eta)]}{1-\bar{I}^2[1-3\eta^2+7\eta^4-\frac{1}{12}\bar{q}^2[2+O(\eta^2)]]} \ . \tag{2.28}$$

We neglect the σ-dependent term in the numerator of Eq. (2.28). Numerical computation has justified this approximation. This has the result that the leading contribution to $\lambda_1^{\sigma,L}$ does not depend on σ, a situation analogous to that in the $A1$ phase of ^{3}He where the "strong coupling" contributions to λ are independent of the magnetic field to leading order in the field.[6]

Expanding Eq. (2.23) we have

$$\bar{I}-1=\tfrac{2}{3}\eta^2-\tfrac{11}{9}\eta^4+O(\eta^6) \ . \tag{2.29}$$

Employing this expansion in Eq. (2.28) yields

$$N_\sigma(0)V^{\sigma\sigma}(q)\cong\frac{\frac{1}{2}\bar{I}^2(1-3\eta^2-\frac{1}{12}\bar{q}^2)}{\frac{5}{4}(\bar{I}-1)(1+\frac{37}{30}\eta^2)+\frac{1}{12}\bar{I}\bar{q}^2} \ , \tag{2.30}$$

which is valid for $\bar{I}\to 1$, $\eta\to 0$, and $\bar{q}\to 0$. We now insert $N_\sigma(0)V^{\sigma\sigma}(q)$ into Eq. (2.3) and observe that, at the level of approximation in Eq. (2.30), we may replace $\bar{q}\to q/k_{F\sigma}$. With the substitution $x=q/2k_{F\sigma}$, the integral does not depend on σ and, neglecting the η^2 term in Eq. (2.30), we find, for the singular terms as $\eta\to 0$

$$\lambda_1^{\sigma,L}=-\tfrac{3}{2}\bar{I}^2\ln(\bar{I}-1) \quad (\bar{I}\to 1) \tag{2.31}$$

and thus the p-wave pairing interaction diverges logarithmically as the magnetic transition is approached from the magnetic side.

We note in passing that for *strong* ferromagnetism, $\eta\to 1$, $\lambda_1^{\sigma,L}$ vanishes due to the factor $N_\sigma(0)N_{-\sigma}(0)\propto(1-\eta^2)$.

D. Effective-mass and renormalization effects

In a nearly ferromagnetic system the value of T_c is strongly influenced by single-particle renormalization effects.[18,19] This is also true for weak itinerant ferromagnets. Indeed, the reason that the divergence of λ_1 as $\bar{I}\to 1$ does not lead to a divergence of T_c^1 is due, roughly speaking, to cancellation with the effective mass which also diverges. The relevant quantity for T_c is actually the renormalization constant at the Fermi surface $z_{k_F}=1/Z(0)$ defined by

$$Z(0)=\left[1-\frac{\partial\Sigma(k_F,k_0)}{\partial k_0}\right]_{k_0=0} \ , \tag{2.32}$$

where Σ is the single-particle self-energy. Making the usual assumption that Σ is a very slowly varying function of momentum, one obtains the important SF relation for a nearly ferromagnetic system

$$m^*/m\cong Z(0)=1+\lambda_0 \ , \tag{2.33}$$

where we have indicated the connection with the s-wave pairing interaction parameter.[2] $Z(0)$ was calculated in the paramagnetic region in a one-paramagnon exchange approximation to Σ first by Doniach and

Engelsberg[20] and subsequently by a number of authors (see Ref. 2 and the references therein) including BE who also considered the ferromagnetic phase. These calculations showed that the one-paramagnon exchange approximation with a contact interaction overestimates m^*/m and thus presumably also λ_1. The simplest extension of this model is to allow the irreducible particle-hole interaction to have a small but finite range in position space. The effect of $\bar{I}$ becoming now momentum dependent is to weaken the spin fluctuations thus reducing m^*/m and, unfortunately, λ_1 also. This procedure appears to work fairly well in Pd where a range of about 1.6 Å allows a good fit to several measured quantities.[2]

We therefore generalize our previous equations for $V^{\sigma\sigma'}(q)$ by making the replacement $I \to I(q)$ with

$$I(q) = I(1+b^2q^2)^{-1} \ . \tag{2.34}$$

The Fourier transform of $I(q)$ has Yukawa form with a range given by b. In the paramagnetic region the calculation of m^*/m proceeds as in Ref. 2 and the parameters $\bar{I}$ and b can be determined from the measured Stoner factor and effective mass. In the ferromagnetic region we generalize the result of BE to the case of a momentum dependent $\bar{I}$. The effective mass is now spin dependent and the contributions from longitudinal and transverse spin fluctuations must be treated spearately. Equation (2.33) becomes

$$(m^*/m)_\sigma \cong Z(0)_\sigma = 1 + \lambda_0^{\sigma,L} + \lambda_0^{\sigma,T} \equiv 1 + \lambda_0^\sigma \ . \tag{2.35}$$

The longitudinal part is simply the s-wave component of Eq. (2.13) calculated from Eqs. (2.2) and (2.3). In the limit $\bar{I} \to 1$, the Legendre polynomial in Eq. (2.3) can be replaced by $(-1)^l$ due to the peaking of V for small q and we find

$$\lambda_0^{\sigma,L} = -\lambda_1^{\sigma,L} \quad (\bar{I} \to 1) \ . \tag{2.36}$$

We calculate the transverse contribution as in BE with the exception that we employ Eq. (2.34) expanded to lowest order in q^2. Equation (4.13) of BE becomes[21]

$$\lambda_0^{\sigma,T} = \frac{-6\bar{I}' \ln\eta}{\bar{I}'(1-\frac{1}{5}\eta^2) + 12b^2k_F'^2(1-\sigma\eta)} \ , \tag{2.37}$$

where we have made use of Eqs. (2.22) and (2.23) in simplifying BE Eq. (4.13). Here $\bar{I}' = (k_F'/k_F)\bar{I}$. For $\bar{I} \to 1$, we have

$$\lambda_0^{\sigma,T} = \frac{-3\ln(\bar{I}-1)}{1+12b^2k_F^2} \quad (\bar{I} \to 1) \ . \tag{2.38}$$

For $\bar{I} \to 1$ and $b = 0$, the total effective mass is

$$(m^*/m)_\sigma = -\tfrac{9}{2}\ln(\bar{I}-1) \quad (\bar{I} \to 1), \quad (b=0) \tag{2.39}$$

in agreement with BE.

In our numerical calculation of $Z(0)$ for use in the T_c equation, we have employed Eq. (2.37) for $\lambda_0^{\sigma,T}$ and solved Eqs. (2.2), (2.3), and (2.13) exactly for $\lambda_0^{\sigma,L}$. This is not really consistent since Eq. (2.37) is based on a small-q, small-η approximation. Due to the phenomenological nature of the theory and the fact that experimental effective masses are not yet available for the systems of interest, a more accurate evaluation of $\lambda_0^{\sigma,T}$ does not seem worthwhile at present.

III. CALCULATION OF T_c^l

We solve Eq. (2.1) with $\sigma = \sigma'$ in a "renormalized weak coupling" approximation in which the frequency dependence of Δ and $V^{\sigma\sigma}$ is neglected and the single-particle propagator has the form

$$G_\sigma(\vec{k}, \omega_\nu) = Z(0)_\sigma^{-1}(i\omega_\nu - \epsilon_{k\sigma}^*)^{-1} \ ,$$

where $\epsilon_{k\sigma}^*$ is given by Eq. (2.15) with $m \to m^*$. This approximation retains the most important renormalization and effective-mass effects. The frequency sum in Eq. (2.1) yields

$$\Delta^\sigma(\vec{p}) = \int \frac{d^3k}{(2\pi)^3} \frac{V^{\sigma\sigma}(\vec{p},\vec{k}) \tanh(\epsilon_{k\sigma}^*/2k_BT_C)}{2\epsilon_{k\sigma}^* Z(0)^2} \Delta^\sigma(\vec{k}) \ , \tag{3.1}$$

where we have defined $\Delta^\sigma(\vec{p}) = \Delta^{\sigma\sigma}(\vec{p}, 0)$. Including the 2nd term on the right-hand side of Eq. (2.15) in the chemical potential we can write

$$\epsilon_{k\sigma}^* = \epsilon_k^* + \sigma\delta \ , \tag{3.2}$$

where $\epsilon_k^* = k^2/2m^* - \mu$ and $\delta = (I/2)(N_\downarrow - N_\uparrow)$. Assuming that $V^{\sigma\sigma}(\vec{p},\vec{k})$ is a function only of the magnitudes of $\vec{p}$ and $\vec{k}$ and the angle between them (true for the SF interaction), we set $\Delta^\sigma(\vec{p}) = \Delta_l^\sigma(p) Y_l^m(\hat{p})$ and expand Eq. (3.1) in spherical harmonics. Converting to an integral over $\epsilon_{k\sigma}^*$ and using Eq. (3.2) we find

$$\Delta_l^\sigma(p) = \frac{1}{2}\int_{-\mu+\sigma\delta}^{\infty} N(\epsilon_{k\sigma}^* - \sigma\delta) d\epsilon_k^* \frac{\tanh(\epsilon_{k\sigma}^*/2k_BT_c)}{Z(0)^2\epsilon_{k\sigma}^*} V_l^{\sigma\sigma}(p,k)\Delta_l(k) \ . \tag{3.3}$$

We now parametrize $V_l^{\sigma\sigma}$ in the manner of BCS:

$$V_l^{\sigma\sigma}(p,k) = V_l^{\sigma\sigma}\Theta(\omega_c - |\epsilon_{k\sigma}^*|)\Theta(\omega_c - |\epsilon_{p\sigma}^*|) \tag{3.4}$$

with $V_l^{\sigma\sigma}$ defined by Eq. (2.3). The cutoff ω_c simulates in a crude way the fact that $V^{\sigma\sigma}$ is in reality a strong function of frequency, being sharply peaked at small energy transfer for $\bar{I} \approx 1$. For $k_B T_c \ll \omega_c \ll \mu$, Eqs. (3.3) and (3.4) lead to

$$k_B T_c^{l,\sigma} = 1.14\omega_c \exp[-Z(0)^2/N(-\sigma\delta)V_l^{\sigma\sigma}] \quad . \tag{3.5}$$

Observing that $N(-\sigma\delta) = N_\sigma^*(0) \equiv (m^*/m)_\sigma N_\sigma(0)$, and using Eq. (2.35) we have

$$k_B T_c^{l,\sigma} = 1.14\omega_c \exp[-(1+\lambda_0^\sigma)/\lambda_l^{\sigma,L}] \quad . \tag{3.6}$$

Our "derivation" of Eq. (3.6) is admittedly not very satisfying for a nearly (or weakly) ferromagnetic system where the effective interaction and self-energy are strongly energy dependent. We should really employ the methods of conventional strong coupling superconductivity[22] which were first applied to the paramagnon model by Berk and Schrieffer[14] for s-state pairing and later generalized to arbitrary l pairing by Fay and Layzer.[18,19] Extensive numerical calculations have recently been done for p-wave pairing in the nearly ferromagnetic region by Levin and Valls.[3] The strong coupling calculations show that, to a good approximation for small T_c, the general form of Eq. (3.6) is retained but the prefactor $1.14\omega_c$ is replaced by a factor $T_0^l(\bar{I})$ which must be computed numerically. The approach we follow however is simply to use Eq. (3.6) as it stands. In the paramagnetic region we take $\omega_c = (1-\bar{I})E_F$ which is a rough measure of the maximum paramagnon energy and in the ferromagnetic region we use the related quantity κ_0^2 given by BE below their Eq. (4.16):

$$\omega_c^{\text{ferro}} = \kappa_0^2 E_F = [(1-\bar{I}') + \tfrac{1}{2}\eta^2 \bar{I}'^2]E_F \quad . \tag{3.7}$$

For $\bar{I} \to 1$, $\kappa_0^2 \to \frac{5}{4}(\bar{I}-1)$.

With these choices for ω_c one sees clearly from Eq. (3.6) together with Eqs. (2.31), (2.35), and (2.39) that, as $\bar{I} \to 1$ (in either phase), $T_c^{l,\sigma}$ goes through a maximum and then approaches zero due to the vanishing of the ω_c factor. Comparison of our results with those of Ref. 3 shows that while our choice of ω_c is not a very good quantitative approximation to $T_0^l(\bar{I})$ in the paramagnetic region, the qualitative behavior is correct. Since our numerical values of T_c^l in the paramagnetic phase are almost an order of magnitude smaller than those of Ref. 3, the $T_c^{l,\sigma}$ values we report for the ferromagnetic region most likely *underestimate* the actual transition temperatures also.

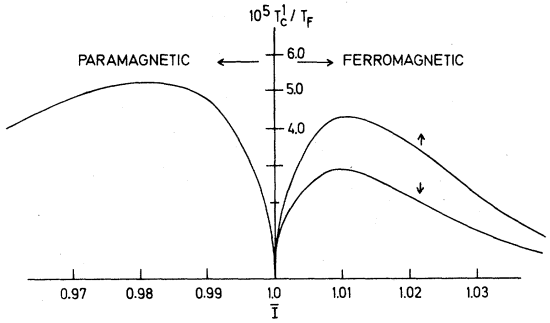

FIG. 2. The p-state superconducting transition temperature as a function of the exchange interaction parameter $\bar{I}$ with range $b = 0.5k_F^{-1}$.

We would like to point out that none of the T_c calculations mentioned (strong or weak coupling) have consistently included vertex corrections[19,23] which are probably important for $\bar{I} \approx 1$. In view of this and the lack of experimental information about the parameters $\bar{I}$ and m^*/m in real itinerant ferromagnetics, we believe our weak coupling solution is adequate for our present purposes.

In our numerical T_c calculations we have also included a phonon contribution to the effective-mass enhancement $\lambda_0^{\text{ph}} = 0.35$. Not having detailed experimental information, we have used the value appropri-

TABLE I. Properties of the maximum of the T_c^l vs $\bar{I}$ curve in the paramagnetic region for various values of the range b of the exchange interaction $\bar{I}$.

bk_F	$\bar{I}$	m^*/m	λ_1	$T_c^l/T_F(10^5)$
0.00	0.989	14.40	2.54	3.78
0.25	0.986	9.20	1.62	4.93
0.50	0.983	5.08	0.87	5.21
0.75	0.986	3.53	0.57	3.01
1.00	0.990	2.80	0.42	1.15
1.25	0.994	2.46	0.33	0.35
1.50	0.997	2.25	0.28	0.09
1.75	0.998	2.07	0.23	0.02
2.00	0.999	1.99	0.20	0.005

TABLE II. Properties of the maximum of $T_c^{l,\sigma}$ in the ferromagnetic region.

	Minority spin ($\sigma=\uparrow$)				Majority spin ($\sigma=\downarrow$)			
bk_F	$\bar{I}$	m^*/m	$\lambda_1^{\uparrow,L}$	$T_c^{1,\uparrow}/T_F(10^5)$	$\bar{I}$	m^*/m	$\lambda_1^{\downarrow,L}$	$T_c^{1,\downarrow}/T_F(10^5)$
0.00	1.002	26.5	4.57	0.90	1.0042	20.1	3.16	0.92
0.25	1.004	15.1	2.66	1.87	1.0065	11.2	1.83	1.82
0.50	1.011	6.7	1.15	4.30	1.0079	5.6	0.95	2.85
0.75	1.018	4.1	0.67	5.19	1.0079	3.6	0.57	1.92
1.00	1.025	3.0	0.44	3.81	1.0042	3.0	0.46	0.81
1.25	1.030	2.4	0.32	1.95	1.0024	2.6	0.37	0.28
1.50	1.040	2.1	0.24	0.75	1.0015	2.3	0.29	0.08
1.75	1.051	1.9	0.19	0.23	1.0011	2.1	0.24	0.02
2.00	1.063	1.8	0.15	0.06	1.0003	2.1	0.24	0.01

ate for Pd.[2] This has the effect of further reducing our calculated T_c.

Typical numerical results for $T_c^{l,\sigma}/T_F$ vs $\bar{I}$ are shown in Fig. 2 for a range parameter $b=0.5k_F^{-1}$ which is on the order of 0.5–1.0 Å for the systems of interest. The value of b for a particular physical system could be fixed to fit the measured effective mass. In Tables I and II we give, for various values of b, some of the properties associated with the maxima of the T_c/T_F vs $\bar{I}$ curves. As b increases from zero, the maxima in the paramagnetic region and for majority-spin pairing in the ferromagnetic region generally move toward $\bar{I}=1$, while the minority spin maximum moves to larger $\bar{I}$. In the paramagnetic region the largest $T_c^l \cong 5.2\times10^{-5}T_F$ occurs for $b\cong0.5k_F^{-1}$ and $\bar{I}\cong0.983(S=60)$. The largest $T_c^{l,\sigma}$ in the ferromagnetic region is $5.5\times10^{-5}T_F$ at $b\cong0.75k_F^{-1}$ and $\bar{I}\cong1.018$ for pairing in the minority band ($\sigma=\uparrow$) and $2.8\times10^{-5}T_F$ at $b=0.5k_F^{-1}$ and $\bar{I}\cong1.008$ in the majority band ($\sigma=\downarrow$). Very close to $\bar{I}=1$, curves for both spin bands fall together in agreement with the analytic results.

IV. APPLICATIONS

A. $ZrZn_2$

As mentioned in the Introduction, increasing the external pressure on $ZrZn_2$ should effectively decrease the parameter $\bar{I}$. In order to determine whether p-state pairing might be observed in the pure ferromagnetic phase we need an estimate of $\bar{I}$ at zero pressure. This can be obtained from the Curie temperature T_M and the degeneracy temperature T_F. Stoner theory provides the relation $\bar{I}=1+(\pi^2/12)\times(T_M/T_F)^2$.[24] Corrections to this result are important and have been studied for weak ferromagnetics by Moriya.[5] An approximate expression containing Moriya's corrections is[25]

$$\bar{I}\cong1+10.6(T_M/T_F)^{4/3}\ . \tag{4.1}$$

There is unfortunately some uncertainty about the parameter T_F for $ZrZn_2$. For $T_M=35$ K and $T_F=1$ eV,[26] Eq. (4.1) yields $\bar{I}=1.0046$ which is very close to the value given by Wohlfarth.[27] Enz and Matthias[12] estimate T_F at 0.2 eV from which Eq. (4.1) gives $\bar{I}=1.04$. From Table II we see that, for reasonable values of b, these numbers correspond to T_c values in the vicinity of the maximum. The actual value of T_c depends on T_F for which estimates range from 260 K (Ref. 27) to 1.2×10^4 K.[26] Thus, considering that our weak coupling approximation may somewhat underestimate $T_c^{l,\sigma}$, values as high as 1 K might be possible if, and this seems to be a big if, $ZrZn_2$ can be made extremely pure.

B. Nickel

From an experimental point of view Ni is attractive because it appears that real single crystals can be grown which are ultrapure, corresponding to a residual resistance ratio $r\approx10^4$–10^5.[15] Furthermore, a great deal of theoretical and experimental effort is being made to obtain a consistent picture of the one-electron band structure for the ferromagnetic ground state. There is now a consensus that the d electrons in Ni, being responsible for the ferromagnetism, can be regarded as *itinerant* electrons.[28,29] Taking into account both d and s electrons, Wang and Callaway[30] have recently calculated self-consistently the energy bands of ferromagnetic Ni. Their results for the cross sections of the Fermi surface in the (100) and (110) planes are in good agreement with de Haas–van Alphen measurements for both the d and the sp electrons. It is found that the majority spin

(here $\downarrow$) d band is full and that the exchange splitting of the d band is not strongly dependent on the momentum $\vec{k}$ and has the magnitude $\Delta E_{ex} \cong 0.7$ eV. The calculated magneton number is found to be close to the experimental value of 0.56. Experimentally, ΔE_{ex} and the Stoner gap δ (the energy separation between the highest occupied majority spin d band and the Fermi energy) have recently been obtained from angle-resolved polarization-dependent photoelectron spectroscopy.[31,32] The experimental values depend somewhat on the single-crystal surface. For the (111) surface, $\Delta E_{ex} = 0.31$ eV and $\delta \approx 100-250$ meV, as compared to the theoretical values of 0.7 and 0.4 eV, respectively. There is a considerable discrepancy between the theoretical and experimental values of the two salient parameters of the spin-polarized band structure, ΔE_{ex} and δ, and it is not clear at this time to what extent the discrepancy can be accounted for by many-body effects.[33]

Despite this difficulty, certain qualitative features are clear: The majority spin d band is full and the majority spin sp-d band has a density of states at the Fermi energy that is $\sim \frac{1}{8}$ of the total minority spin density of states at E_F.[34] Because of the full majority spin d band, these d electrons cannot be paired and, since $\chi^{\downarrow}_{0d}$ is zero in Eq. (2.13), they also cannot participate in the pairing between minority spin d electrons. The minority spin d electrons, however, can still pair due to the spin-fluctuation contribution to $V^{\downarrow\downarrow}$ that involves the excitation of $\downarrow$ electrons in the sp-d band. At E_F the density of states of the $\uparrow$ electrons in the sp-d band is almost entirely of "d character," as is seen from the orbital decomposition of $N_{\uparrow}(E_F)$.[34] Hence it is reasonable for our purposes to employ a Ni band-structure model consisting of a $3d$ band only (that is, ignoring the sp-d band) which has a minority and majority spin Fermi surface with the density-of-states values $N_{\uparrow}(E_F)$ and $N_{\downarrow}(E_F)$ given by $N_{\uparrow,\downarrow;\mathrm{tot}}(E_F)$ of Ref. 34, for example. The parabolic $3d$ band is threefold degenerate; we count in terms of holes in this band. We assume that in the paramagnetic phase the number of $3d$ holes per Ni atom is $N = N_{\downarrow} + N_{\uparrow} = 0.6$. In the ferromagnetic phase we take $N_{\downarrow} - N_{\uparrow} = 0.5$, corresponding approximately to the experimental magneton number. The exchange splitting is given by $\Delta E_{ex} = (N_{\downarrow} - N_{\uparrow}) I_{\mathrm{eff}}$, where $I_{\mathrm{eff}} = \frac{2}{3}$ eV yields the experimental value $\Delta E_{ex} = 0.31$ eV. Now the spin-polarization parameter η_{Ni} is given by Eqs. (2.20) and (2.23). We find $\eta_{\mathrm{Ni}} = 0.38$, corresponding to $(N_{\downarrow} - N_{\uparrow})/N = 0.833$ and $\bar{I} = I_{\mathrm{eff}} N(E_F) = 1.076$. Using these parameter values and taking $T_F = 2994$ K (corresponding to $m^* = 9.8 m_0$ and a degeneracy of 3), we get for $T_c^{\downarrow}$ the values given in Table III. As a function of the range b of the interaction $\bar{I}$, $T_c^{\downarrow}$ exhibits a maximum which for the minority spin electrons occurs at $b \approx k_F^{-1}$. The value $T_c^{\downarrow} \approx 0.023$ K can be easily attained with present cryogenics but it depends of course rather sensitively on our model assumptions for the electronic structure of ferromagnetic nickel. Neverthe-

TABLE III. Numerical results for the effective mass, p-wave pairing interaction, and the p-wave transition temperature for various values of the range b for the value $\bar{I} = 1.076$ corresponding to our nickel model.

bk_F	σ	m^*/m	λ_1^{σ}	$T_c^{\downarrow,\sigma}$ (K)
0.00	$\uparrow$	13.05	0.367	3.0×10^{-12}
0.00	$\downarrow$	6.15	0.311	9.0×10^{-7}
0.25	$\uparrow$	9.03	0.384	2.1×10^{-8}
0.25	$\downarrow$	4.39	0.278	4.8×10^{-4}
0.50	$\uparrow$	5.37	0.385	2.8×10^{-4}
0.50	$\downarrow$	2.84	0.210	4.5×10^{-4}
0.75	$\uparrow$	3.69	0.348	8.4×10^{-3}
0.75	$\downarrow$	2.18	0.151	1.8×10^{-4}
1.00	$\uparrow$	2.86	0.298	0.023
1.00	$\downarrow$	1.86	0.110	1.4×10^{-5}
1.25	$\uparrow$	2.39	0.250	0.024
1.25	$\downarrow$	1.70	0.082	3×10^{-7}
1.50	$\uparrow$	2.11	0.209	0.014
1.50	$\downarrow$	1.60	0.063	3×10^{-9}
1.75	$\uparrow$	1.92	0.176	0.006
1.75	$\downarrow$	1.54	0.049	1×10^{-11}
2.00	$\uparrow$	1.80	0.149	0.002
2.00	$\downarrow$	1.50	0.040	$\sim 10^{-14}$

less, the results may find application in Ni under ambient conditions and under pressure where $N_\downarrow(E_F)/N_\uparrow(E_F)$ becomes larger,[34] in certain intermetallic compounds, or perhaps, with some modifications, in Co or Fe. These metals do have more complicated Fermi surfaces than Ni but, in contrast to Ni, the majority spin d bands are only partially filled so that $\chi^{\downarrow}_{0d}$ is not necessarily small and the majority spin d electrons can make a pertinent SF contribution to the pairing of minority spin electrons.

V. CONCLUSIONS

We have shown that exchange of longitudinal spin fluctuations can lead to an ESP p-wave superconducting state in an itinerant ferromagnet. As possible experimental candidates we have suggested $ZrZn_2$ and Ni. Since we have limited our calculation to $T = T_c$, we cannot make any definite predictions concerning the detailed nature of the proposed condensed state. We should mention, however, that even in the absence of an external magnetic field, a Meissner-Ochsenfeld effect due to the influence of the spontaneous magnetization on the orbital motion of the triplet pair is expected to occur. Ginsburg[35] has given a theory of the Meissner effect for a singlet ferromagnetic superconductor which includes the combined effect of the internal field due to the magnetic ordering and an externally applied magnetic field. His theory also applies to the triplet case. In the limit of a weak itinerant ferromagnetic the internal field is small and a bulk condensed state should be possible. For a stronger itinerant ferromagnetic, such as our Ni model, the Meissner effect could be rendered harmless by using a thin film whose critical field is much larger than the spontaneous magnetization. For Ni, this is found to be the case when the film thickness is ~ 50 Å.[35]

Finally, it does not seem relevant to speak of a Clogston effect[36] that could provide an upper limit for the critical field as occurs in a BCS-singlet superconductor.

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