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Theory of optical-environment-dependent spontaneous-emission rates for emitters in thin layers

W. Lukosz
Swiss Federal Institute of Technology, Professur für Optik, ETH, 8093 Zürich, Switzerland
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The spontaneous-emission rates of emitters embedded in a thin layer 0 of a loss-free dielectric depend on their optical environment, i.e., on the optical properties of the two media 1 and 2 adjoining layer 0. For electric- and magnetic-dipole transitions the spontaneous-emission rates, normalized with respect to those in an infinite medium 0, are expressed in terms of the Fresnel reflection coefficients for plane and evanescent waves incident from medium 0 on the interfaces to the planar-stratified or homogeneous media 1 and 2, respectively. From this result which is valid for arbitrary layer thicknesses d_0 we derive an approximation for extremely thin layers of optical thickness $n_0 d_0 \ll \lambda/8$ (where λ is the emission wavelength) between two homogeneous loss-free dielectric media 1 and 2. For the normalized spontaneous-emission rates as functions of the refractive indices n_j of media $j = 0$, 1, and 2 analytical expressions are obtained. We have used these expressions previously without proof [Phys. Rev. B 21, 4814 (1980)] to analyze experimentally observed changes in the fluorescence lifetime of Eu^{3+} ions in varying optical environments, which yielded the quantum efficiency of the emitting state.

I. INTRODUCTION

Experiments have shown that the fluorescence lifetime of emitters can be changed by varying their optical environment.[1-4] In our experiments[1] the emitters (Eu^{3+} ions) were embedded in a very thin layer 0 of a loss-free dielectric; that part of their optical environment which we varied were the two homogeneous loss-free dielectric media 1 and 2 adjoining to layer 0. For the analysis of our experiments we used analytical expressions for the spontaneous-emission rates for electric- (and magnetic) dipole transitions in emitting systems embedded in a layer of refractive index n_0 and of optical thickness $n_0 d_0 \ll \lambda/8$ (where λ is the emission wavelength), between two half-spaces 1 and 2 with refractive indices n_1 and n_2, respectively. We stated these theoretical results in Ref. 1 without any proof.

The object of this paper is to derive the analytical expressions for the normalized spontaneous-emission rates for emitting systems in extremely thin layers as approximations from the theory for arbitrary film thicknesses d_0. In Sec. II we explain with an argument based on the correspondence principle why the optical-environment-dependent normalized spontaneous-emission rates can be calculated by classical electrodynamics, viz., by calculation of the total power radiated by a classical dipole in the given optical environment. In Sec. III we solve the em boundary-value problem for a radiating dipole in layer 0 of arbitrary thickness d_0, by extension of a method[5] we had previously used for a dipole in front of a single interface. We use a special combination of a one-component electric Hertz vector and a one-component magnetic Hertz vector to represent the field of an electric or a magnetic dipole. Physically, this representation corresponds to a decomposition of the near and far field of the dipole into s- and p-polarized plane and evanescent waves. The final results for the normalized spontaneous-emission rates are given in Sec. IV; they are expressed in terms of the reflection coefficients for plane and evanescent waves incident from medium 0 on the interface to the adjoining media 1 and 2, respectively. The physical effects responsible for the dependence of the spontaneous-emission rates on the optical environment are briefly discussed. In Sec. V we consider dipoles located at the interface between two media. We derive "boundary conditions" for the power radiated by the dipole moved across the interface in a thought experiment. In Sec. VI we derive the approximation for the spontaneous-emission rates of emitters in extremely thin layers 0, i.e., for $n_0 d_0 \ll \lambda/8$.

II. CLASSICAL CALCULATION OF THE RATIO OF SPONTANEOUS-EMISSION RATES IN DIFFERENT OPTICAL ENVIRONMENTS

We consider an atomic system (emitter) D at position $\vec{x}_0(x_0 = 0, y_0 = 0, z_0)$ in a loss-free dielectric thin layer 0 of refractive index n_0 and thickness d_0 between two adjoining media 1 and 2 (cf. Fig. 1). All three media 0, 1, and 2 are assumed to be linear, isotropic, and nonmagnetic ($\mu = 1$). We further assume that media 1 and 2 are planar stratified in the z direction, which includes as a special case that they are homogeneous (absorbing) half-spaces with (complex) refractive index n_j at the emission wavelength λ.

We investigate how the spontaneous-emission rate A for an electric- or magnetic-dipole transition in the atomic system D depends on its optical environment, i. e., on the optical properties of media 1 and 2. The transition dipole moment is assumed to lie in the x-z plane, θ is the angle between the dipole moment and the z axis. The spontaneous-emission rate $A(z_0)$ times the photon energy $E=\hbar\omega$ is equal to the expectation value $L(z_0)$ of the energy emitted in unit time per excited system:

$$\hbar\omega A(z_0)=L(z_0) \tag{2.1}$$

In other words, $L(z_0)$ is the total power radiated by a transition dipole. According to the correspondence principle L will be given by the expression for the total power radiated by a classical dipole in which the classical dipole moment has to be replaced by the transition dipole moment. The classical dipole is located at the position $\vec{x}_0(0,0,z_0)$ of the emitter D, it oscillates with fixed frequency equal to the transition frequency ω, and with fixed dipole moment and orientation θ.

We are interested only in the normalized spontaneous-emission rate $A(z_0)/A_\infty(n_0)$, defined as the ratio of the spontaneous-emission rates of the same emitter D in the optical environment shown in Fig. 1 and in an infinite homogeneous medium of refractive index n_0, respectively. From Eq. (2.1) we derive

$$A(z_0)/A_\infty(n_0)=L(z_0)/L_\infty(n_0)\,, \tag{2.2}$$

where $L_\infty(n_0)$ is the total power radiated by the dipole in the infinite medium 0. Since we assumed linear constitutive relations for media 0, 1, and 2, the dipole field is linear in the dipole moment. Consequently, the radiated power is proportional to the square of the dipole moment in any optical environment. Therefore, the normalized radiated power $L(z_0)/L_\infty(n_0)$ and, because of Eq. (2.2), the normalized spontaneous-emission rate $A(z_0)/A_\infty(n_0)$ do not depend on the (transition) dipole moment. We implicitly assumed further that the transition dipole moment and the transition frequency ω are not changed by a variation of the optical environment of the system. Therefore, the normalized spontaneous-emission rates can be calculated by classical electrodynamics, i. e., by solving the boundary value problem for the dipole in the layer 0.

In our experiments[1] the optical environment of one and the same ensemble of emitters was varied. The ratio of observed fluorescence lifetimes in two different optical environments (denoted by a prime and no prime, respectively) were compared with the theory. The ratio of the radiative lifetimes in the primed and unprimed

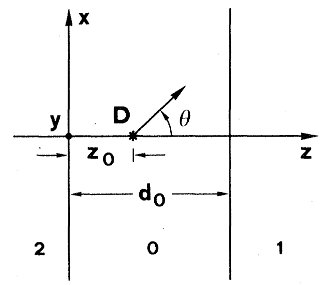

FIG. 1. Emitter (electric or magnetic dipole) D in loss-free dielectric thin layer 0 of thickness d_0 between two adjoining media 1 and 2. θ, angle between transition dipole moment and z axis.

environment is given by the inverse ratio of the (normalized) spontaneous-emission rates

$$\tau_r'(z_0)/\tau_r(z_0)=A(z_0)/A'(z_0)=L(z_0)/L'(z_0)\,. \tag{2.3}$$

If nonradiative transitions compete with the radiative transitions the quantum efficiency of the emitting state can be obtained from a comparison of experiment and theory.

III. RADIATING ELECTRIC AND MAGNETIC DIPOLES IN A THIN LAYER

We solve the boundary-value problem for a radiating dipole in layer 0. We employ a method we used previously for a dipole in front of a single interface.[5]

A. Description of em fields by two scalars

Any monochromatic em field can be represented by two scalar functions $\phi^{(E)}(\vec{x})$ and $\phi^{(H)}(\vec{x})$ which are the z components of an electric and of a magnetic Hertz vector, respectively,

$$\vec{\pi}^{(E)}(\vec{x})=(0,0,\phi^{(E)}(\vec{x}))\,, \tag{3.1}$$

$$\vec{\pi}^{(H)}(\vec{x})=(0,0,\phi^{(H)}(\vec{x}))\,. \tag{3.2}$$

We omit the time dependence $\exp(-i\omega t)$ of all field quantities. We use SI units. Both scalars satisfy the homogeneous Helmholtz equation

$$(\Delta+\epsilon k^2)\phi^{(E,H)}(\vec{x})=0,\quad k=\omega/c \tag{3.3}$$

in source-free regions of a homogeneous medium with dielectric permittivity ϵ. The field strengths are

$$\vec{E}(\vec{x})=i\omega\vec{\nabla}\times\vec{\pi}^{(H)}(\vec{x})+(\epsilon_0\epsilon)^{-1}\vec{\nabla}\times\vec{\nabla}\times\vec{\pi}^{(E)}(\vec{x})\,, \tag{3.4}$$

$$\vec{H}(\vec{x})=(\mu_0)^{-1}\vec{\nabla}\times\vec{\nabla}\times\vec{\pi}^{(H)}(\vec{x})-i\omega\vec{\nabla}\times\vec{\pi}^{(E)}(\vec{x})\,. \tag{3.5}$$

This representation is adapted to the geometry of our problem, viz., to the planar stratification of the media in the z direction. We will use this representation for the fields in media 0, 1, and 2. As shown in Ref. 5, the scalar $\phi^{(H)}(\vec{x}) = \phi^{(H)} \exp(i\vec{k}\cdot\vec{x})$ represents a plane or evanescent s-polarized wave. Its electric field $\vec{E}(\vec{x})$ is perpendicular to the plane of incidence, which is defined by the wave vector $\vec{k}$ and the unit vector $\hat{z}$ in the z direction. The scalar $\phi^{(E)}(\vec{x}) = \phi^{(E)}\exp(i\vec{k}\cdot\vec{x})$ represents a p-polarized wave; its magnetic field $\vec{H}(\vec{x})$ is perpendicular to the plane of incidence, while $\vec{E}(\vec{x})$ lies in this plane.

B. Decomposition of the dipole field into s- and p-polarized plane and evanescent waves

We consider a dipole in an infinite medium 0. We present the dipole field as a superposition of s- and p-polarized plane and evanescent waves. This representation is valid both in the far zone and in the near zone of the dipole.

The electric (point) dipole oscillates with the electric-dipole moment $\vec{p}(t) = \vec{p}_0 \cos\omega t$, where $\vec{p}_0(p_0 \sin\theta,\ 0,\ p_0\cos\theta)$. The magnetic dipole oscillates with the magnetic-dipole moment $\vec{m}(t) = \vec{m}_0 \cos\omega t$, where $\vec{m}_0(m_0\sin\theta,\ 0,\ m_0\cos\theta)$. The field of this electric or magnetic dipole of orientation θ located at $\vec{x}_0(0,0,z_0)$ in an infinite medium 0 is represented by the scalars $\phi_\infty^{(E)}(\vec{x})$ and $\phi_\infty^{(H)}(\vec{x})$, where

$$\phi_\infty^{(H,E)}(\vec{x}) = \iint_{-\infty}^{+\infty} \tilde{\phi}_{\infty,\pm}^{(H,E)}(k_x,k_y) \times \exp\{i[k_x x + k_y y \pm k_{z,0}(z - z_0)]\}\,dk_x dk_y \tag{3.6}$$

with

$$k_{z,0} = \begin{cases} +(k_0^2 - k_x^2 - k_y^2)^{1/2} & \text{if } (k_x^2+k_y^2)^{1/2} \le k_0 \\ +i(k_x^2+k_y^2-k_0^2)^{1/2} & \text{if } (k_x^2+k_y^2)^{1/2} > k_0\,, \end{cases} \tag{3.7}$$

$k_0 = n_0 k$, and $k = \omega/c = 2\pi/\lambda$. For the electric dipole

$$\tilde{\phi}_{\infty,\pm}^{(H)}(k_x,k_y) = -i(8\pi^2)^{-1}\mu_0 p_0 \omega k_y [k_{z,0}(k_x^2+k_y^2)]^{-1}\sin\theta\,, \tag{3.8}$$

and

$$\tilde{\phi}_{\infty,\pm}^{(E)}(k_x,k_y) = i(8\pi^2)^{-1}p_0[(k_{z,0})^{-1}\cos\theta \mp k_x(k_x^2+k_y^2)^{-1}\sin\theta]\,, \tag{3.9}$$

while for the magnetic dipole

$$\tilde{\phi}_{\infty,\pm}^{(H)}(k_x,k_y) = i(8\pi^2)^{-1}m_0 \times[(k_{z,0})^{-1}\cos\theta \mp k_x(k_x^2+k_y^2)^{-1}\sin\theta]\,, \tag{3.10}$$

and

$$\tilde{\phi}_{\infty,\pm}^{(E)}(k_x,k_y) = i(8\pi^2)^{-1}\epsilon_0 n_0^2 m_0 \omega k_y[k_{z,0}(k_x^2+k_y^2)]^{-1}\sin\theta\,. \tag{3.11}$$

Details of the calculation leading to Eqs. (3.8)–(3.11) are given in Ref. 5 (for the magnetic dipole only).[6]

Physically, the Fourier integral (3.6) represents the dipole field as a superposition of s- and p-polarized plane waves ($k_x^2+k_y^2 \le k_0^2$) and of evanescent waves ($k_x^2+k_y^2 > k_0^2$). In Eq. (3.6)—and Eqs. (3.8)–(3.11)—the upper and the lower sign, respectively, has to be chosen in the half-spaces $z \ge z_0$ and $z < z_0$.[7] A plane wave emitted into the half-space $z \ge z_0$ has the wave vector $\vec{k}(k_x, k_y, +k_{z,0})$ and the amplitude $\phi_{\infty,+}^{(H,E)}(k_x,k_y)$, one emitted into the half-space $z < z_0$ has the wave vector $\vec{k}(k_x, k_y, -k_{z,0})$ and the amplitude $\phi_{\infty,-}^{(H,E)}(k_x,k_y)$. The evanescent waves decay with distance from the plane $z = z_0$ in both half-spaces.

C. Solution of the boundary value problem

We use the following ansatz. The field in the layer 0 ($0 \le z \le d_0$) is

$$\phi_0^{(H,E)}(\vec{x}) = \phi_\infty^{(H,E)}(\vec{x}) + \iint_{-\infty}^{+\infty} \tilde{\phi}_{0,+}^{(H,E)}(k_x,k_y)\exp[i(k_x x + k_y y + k_{z,0}z)]\,dk_x dk_y + \iint_{-\infty}^{+\infty} \tilde{\phi}_{0,-}^{(H,E)}(k_x,k_y)\exp[i(k_x x + k_y y - k_{z,0}z)]\,dk_x dk_y\,, \tag{3.12}$$

where $k_{z,0}$ is given by Eq. (3.7) and $\phi_\infty^{(H,E)}(\vec{x})$ by Eqs. (3.6)–(3.11). The waves $\tilde{\phi}_{\infty,\pm}(k_x,k_y)$ emitted by the dipole are reflected at the interfaces 0/1 and 0/2, and are then reflected back and forth between the two interfaces. The second and third term in Eq. (3.12) represent the fields resulting from those multiple reflections. The Fourier component $\tilde{\phi}_{0,\pm}(k_x,k_y)\exp(i\vec{k}\cdot\vec{x})$ represents for $k_x^2+k_y^2 \le k_0^2$ plane waves with wave vectors $\vec{k}(k_x, k_y, \pm k_{z,0})$ which propagate to the right and the left, respectively, in the coordinate system shown in Fig. 1. For $k_x^2+k_y^2 > k_0^2$, it represents evanscent waves whose amplitudes decay in the $+z$ and the $-z$ direction, respectively. The fields in the homogeneous media 1 ($z \geq d_0$) and 2 ($z \leq 0$) are

$$\phi_1^{(H,E)}(\vec{x}) = \iint_{-\infty}^{+\infty} \tilde{\phi}_1^{(H,E)}(k_x, k_y) \times \exp\{i[k_x x + k_y y + k_{z,1}(z-d_0)]\} dk_x dk_y \quad (3.13)$$

and

$$\phi_2^{(H,E)}(\vec{x}) = \iint_{-\infty}^{+\infty} \tilde{\phi}_2^{(H,E)}(k_x, k_y) \times \exp[i(k_x x + k_y y - k_{z,2} z)] dk_x dk_y, \quad (3.14)$$

respectively, where

$$k_{z,j} = (k_j^2 - k_x^2 - k_y^2)^{1/2} \quad (3.15)$$

with $k_j = n_j k$ and $j = 1, 2$. The square root with nonnegative real and imaginary part has to be chosen. If medium j is loss-free, $k_{z,j}$ is given by Eq. (3.7), in which the subscript 0 has to be replaced by j. With this choice of the sign of $k_{z,j}$ Eqs. (3.13) and (3.14) represent plane waves which propagate into medium j, and evanescent or inhomogeneous waves which decay in medium j with increasing distance from the interface to layer 0.

From the electromagnetic boundary conditions and Eqs. (3.4) and (3.5) it follows that

$$\phi^{(E)}(\vec{x}),\ \frac{1}{\epsilon}\frac{\partial\phi^{(E)}}{\partial z}(\vec{x}),\quad \phi^{(H)}(\vec{x}),\ \frac{\partial\phi^{(H)}}{\partial z}(\vec{x}) \quad (3.16)$$

are continuous across a plane interface $z = \text{const}$ between two different media. The boundary conditions (3.16) at the interface $z=0$ and $z=d_0$ expressed with Eqs. (3.6), and (3.12)–(3.14) yield four equations for the unknown amplitudes $\tilde{\phi}_{0,+}(k_x,k_y)$, $\tilde{\phi}_{0,-}(k_x,k_y)$, $\tilde{\phi}_1(k_x,k_y)$, and $\tilde{\phi}_2(k_x,k_y)$ of the s- and p-polarized waves. By solving these sets of equations which we do not write down explicitly, we obtain

$$\tilde{\phi}_{0,+}^{(H,E)}(k_x,k_y) = m^{(s,p)} r_{0,2}^{(s,p)} \times \{\tilde{\phi}_{\infty,-}^{(H,E)}(k_x,k_y)\exp(ik_{z,0}z_0) + r_{0,1}^{(s,p)}\tilde{\phi}_{\infty,+}^{(H,E)}(k_x,k_y)\exp[ik_{z,0}(2d_0 - z_0)]\}, \quad (3.17)$$

$$\tilde{\phi}_{0,-}^{(H,E)}(k_x,k_y) = m^{(s,p)} r_{0,1}^{(s,p)}\exp(2ik_{z,0}d_0) \times [\tilde{\phi}_{\infty,+}^{(H,E)}(k_x,k_y)\exp(-ik_{z,0}z_0) + r_{0,2}^{(s,p)}\tilde{\phi}_{\infty,-}^{(H,E)}(k_x,k_y)\exp(ik_{z,0}z_0)], \quad (3.18)$$

where

$$m^{(s,p)} = [1 - r_{0,1}^{(s,p)} r_{0,2}^{(s,p)}\exp(2ik_{z,0}d_0)]^{-1}, \quad (3.19)$$

and

$$r_{0,j}^{(s,p)} = \frac{(\epsilon_j/\epsilon_0)^\rho k_{z,0} - k_{z,j}}{(\epsilon_j/\epsilon_0)^\rho k_{z,0} + k_{z,j}} \quad (3.20)$$

with $\epsilon_j = (n_j)^2$, $j = 1, 2$, and $\rho = 0$ for s- and $\rho = 1$ for p-polarized waves. Equations (3.17) and (3.18) hold true with the superscripts (H) and (s) for s-polarized waves, and with superscripts (E) and (p) for p-polarized waves. We also obtained the amplitudes $\tilde{\phi}_1^{(H,E)}(k_x,k_y)$ and $\tilde{\phi}_2^{(H,E)}(k_x,k_y)$ which we do not write down explicitly, since the fields in media 1 and 2 are not required for the calculation of the total power radiated by the dipole in Sec. IV.

We calculate the electric field at the position of the electric dipole,

$$\vec{E}(\vec{x}_0) = \vec{E}_\infty(\vec{x}_0) + \vec{E}_r(\vec{x}_0), \quad (3.21)$$

where $\vec{E}_\infty(\vec{x}_0)$ is the field of the dipole in the infinite medium 0 and $\vec{E}_r(\vec{x}_0)$ is the contribution of the waves reflected at the interfaces. The real part of $\vec{E}_\infty(\vec{x})$ becomes singular at $\vec{x} = \vec{x}_0$, but the imaginary part is finite:

$$\text{Im}[\vec{E}_\infty(\vec{x}_0)] = (6\pi\epsilon_0 n_0^2)^{-1}\vec{p}_0 k_0^3 \quad (3.22)$$

(cf. Ref. 5). With Eqs. (3.8), (3.9), (3.12), (3.17), and (3.18) we calculated the Cartesian components of $\vec{E}_r(\vec{x}_0)$ from Eq. (3.4); the final results expressed in normalized form are

$$E_{r,x}(\vec{x}_0)/\text{Im}[E_\infty(\vec{x}_0)] = \tfrac{3}{8}i(k_0)^{-3}\sin\theta \times \int_0^\infty [(k_{z,0})^2 r_-^{(p)} + (k_0)^2 r_+^{(s)}] \times (k_{z,0})^{-1} d(\kappa^2), \quad (3.23)$$

$$E_{r,y}(\vec{x}_0) = 0, \quad (3.24)$$

$$E_{r,z}(\vec{x}_0)/\text{Im}[E_\infty(\vec{x}_0)] = \tfrac{3}{4}i(k_0)^{-3}\cos\theta\int_0^\infty r_+^{(p)}(k_{z,0})^{-1}\kappa^2 d(\kappa^2), \quad (3.25)$$

where $\kappa^2 \equiv k_x^2 + k_y^2$, and

$$r_\pm^{(s,p)} = m^{(s,p)}\{2r_{0,1}^{(s,p)} r_{0,2}^{(s,p)}\exp(2ik_{z,0}d_0) \pm r_{0,1}^{(s,p)}\exp[2ik_{z,0}(d_0 - z_0)] \pm r_{0,2}^{(s,p)}\exp(2ik_{z,0}z_0)\}. \quad (3.26)$$

The magnetic field at the position of the magnetic dipole is

$$\vec{H}(\vec{x}_0)=\vec{H}_\infty(\vec{x}_0)+\vec{H}_r(\vec{x}_0)\,. \tag{3.27}$$

The real part of $\vec{H}_\infty(x)$ becomes singular at $\vec{x}=\vec{x}_0$, but its imaginary part is finite:

$$\mathrm{Im}\,[\vec{H}_\infty(\vec{x}_0)]=(6\pi\mu_0)^{-1}\vec{m}_0 k_0^3 \tag{3.28}$$

(cf. Ref. 5). With Eqs. (3.10), (3.11), (3.12), (3.17), and (3.18) we calculated $\vec{H}_r(\vec{x}_0)$ from Eq. (3.5); the Cartesian components of $\vec{H}_r(\vec{x}_0)/\,\mathrm{Im}\,[H_\infty(\vec{x}_0)]$ are given by Eqs. (3.23)–(3.25) with interchanged polarization indices $(s)\longleftrightarrow(p)$.

In the above rigorous solution of the em boundary value problem for a dipole in layer 0, the media 1 and 2 were assumed to be homogeneous. This assumption leads to the Fresnel formula (3.20) for the reflection coefficients $r_{0,j}^{(s,p)}$. The field in layer 0 is determined by the amplitudes of the s- and p-polarized plane and evanescent waves emitted by the dipole, and the reflections of these waves between the interfaces $z=0$ and $z=d_0$ to media 1 and 2, respectively. From this physical interpretation of the theory it is evident that the results for the field in layer 0—in particular Eqs. (3.23)–(3.26)—remain valid if medium j ($j=1,2$) is not homogeneous but planar stratified. In this case we have to insert for $r_{0,j}^{(s,p)}$ the reflection coefficients describing the reflection of an s- or p-polarized plane or evanescent wave incident from medium 0 on to the stratified medium j. The calculation of the resulting reflection coefficient for a layer system is a standard problem in thin-film physics.

IV. SPONTANEOUS-EMISSION RATES OF EMITTERS IN LAYERS OF ARBITRARY THICKNESS

With the results of Sec. III we calculate the normalized total power $L(z_0)/L_\infty(n_0)$ radiated by an electric or a magnetic dipole in layer 0. According to Eq. (2.2) we thus obtain the normalized spontaneous-emission rate $A(z_0)/A_\infty(n_0)$ for electric and magnetic dipole transitions in the emitting system at $\vec{x}_0(0,0,z_0)$ in layer 0.

We use the same method to calculate L as in the case of a dipole in front of a single interface.[5] The total power L radiated by the dipole is defined as the integral over the normal component of the time-averaged energy flow density $\vec{S}=\vec{E}\times\vec{H}$ through a surface enclosing the source

$$L=\oint\!\!\!\oint(\langle\vec{S}\rangle\cdot\hat{n})d\sigma$$

$$=-\tfrac{1}{2}\,\mathrm{Re}\left(\iiint\vec{E}(\vec{x})\cdot\vec{I}^*(\vec{x})d^3x\right). \tag{4.1}$$

The second line in Eq. (4.1) follows from Poynting's theorem; it leads to a simple way to calculate L. The radiated power L is equal to the negative of the work done per unit time by the field on the source; $I(\vec{x})$ is the current density in the source. With Eq. (4.1) we find for the electric and magnetic dipoles

$$L_e=\tfrac{1}{2}\omega\vec{p}_0\cdot\mathrm{Im}[\vec{E}(\vec{x}_0)]\,, \tag{4.2}$$

$$L_m=\tfrac{1}{2}\omega\vec{m}_0\cdot\mathrm{Im}[\vec{H}(\vec{x}_0)]\,, \tag{4.3}$$

respectively, where $\mathrm{Im}[\cdots]$ denotes the imaginary part of $[\cdots]$. The power radiated by electric (magnetic) dipole is proportional to that component of the electric (magnetic) field in direction of the dipole axis at the position $\vec{x}_0$ of the dipole which is 90° out of phase with the oscillating dipole moment. Inserting Eqs. (3.22) and (3.28) into Eqs. (4.2) and (4.3), respectively, we obtain for the powers radiated by a dipole in an infinite medium of refractive index n_0

$$L_\infty(n_0)=(n_0)^q L_{\mathrm{vac}}\,, \tag{4.4}$$

where $q=1$ for electric and $q=3$ for magnetic dipoles, and L_{vac} is the power radiated by the same dipole in vacuum

$$[L_{\mathrm{vac}}]_e=p_0^2\omega^4/12\pi\epsilon_0 c^3\,, \tag{4.5}$$

$$[L_{\mathrm{vac}}]_m=m_0^2\omega^4/12\pi\mu_0 c^3\,. \tag{4.6}$$

For a dipole in layer 0 we find from Eqs. (4.2) and (4.3) with Eqs. (3.21) and (3.27)

$$\left(\frac{L(z_0)}{L_\infty(n_0)}\right)_e=1+\frac{\vec{p}_0\cdot\mathrm{Im}[\vec{E}_r(\vec{x}_0)]}{\vec{p}_0\cdot\mathrm{Im}[\vec{E}_\infty(\vec{x}_0)]}\,, \tag{4.7}$$

$$\left(\frac{L(z_0)}{L_\infty(n_0)}\right)_m=1+\frac{\vec{m}_0\cdot\mathrm{Im}[\vec{H}_r(\vec{x}_0)]}{\vec{m}_0\cdot\mathrm{Im}[\vec{H}_\infty(\vec{x}_0)]}\,. \tag{4.8}$$

Inserting Eqs. (3.23)–(3.25) into (4.7) we obtain for an electric dipole of orientation θ

$$[L(z_0)]_{e,\theta}=\cos^2\theta[L(z_0)]_{e,\perp}+\sin^2\theta[L(z_0)]_{e,\parallel} \tag{4.9}$$

where $\perp$ and $\parallel$, respectively, denote a dipole oriented perpendicular ($\theta=0$) and parallel ($\theta=90°$) to the layer. We further obtain

$$[L(z_0)/L_\infty(n_0)]_{e,\perp}=1+\tfrac{3}{4}(k_0)^{-3}\,\mathrm{Re}\left(\int_0^\infty r_+^{(p)}(k_{z,0})^{-1}\kappa^2 d(\kappa^2)\right), \tag{4.10}$$

and

$$[L(z_0)/L_\infty(n_0)]_{e,\parallel}=1+\tfrac{3}{8}(k_0)^{-3}\,\mathrm{Re}\left(\int^\infty[(k_{z,0})^2 r_-^{(p)}+(k_0)^2 r_+^{(s)}](k_{z,0})^{-1}d(\kappa^2)\right), \tag{4.11}$$

where $r_{\pm}^{(s,p)}(\kappa)$ is given by Eq. (3.26). For the total power radiated by magnetic dipoles Eqs. (4.9)–(4.11) hold true with interchanged polarization indices:

$$e \to m\,,\qquad (s)\to(p),\quad (p)\to(s)\,. \tag{4.12}$$

From Eqs. (4.9)–(4.12) follows that the normalized total radiated powers $L(z_0)/L_\infty(n_0)$ for an electric or a magnetic dipole of any orientation θ depend only on the reduced distance $n_0 z_0/\lambda$, on the reduced optical film thickness $n_0 d_0/\lambda$, and on the reflection coefficients $r_{0,j}^{(s,p)}$ $(j=1,2)$ which in turn depend only on the (complex) relative refractive indices n_1/n_0 and n_2/n_0 in the case where media 1 and 2 are homogeneous.

In the special case where $n_0=n_1$, the dipole is located in medium 1 at distance z_0 from the interface to medium 2 and Eqs. (4.10) and (4.11) reduce to

$$[L(z_0)/L_\infty(n_1)]_{e,\perp} = 1+\tfrac{3}{4}(k_1)^{-3}\,\mathrm{Re}\left(\int_0^\infty r_{1,2}^{(p)}(k_{z,1})^{-1}\exp(2ik_{z,1}z_0)\kappa^2 d(\kappa^2)\right), \tag{4.13}$$

$$[L(z_0)/L_\infty(n_1)]_{e,\parallel} = 1+\tfrac{3}{8}(k_1)^{-3}\,\mathrm{Re}\left(\int_0^\infty [(k_1)^2 r_{1,2}^{(s)} - (k_{z,1})^2 r_{1,2}^{(p)}](k_{z,1})^{-1}\exp(2ik_{z,1}z_0) d(\kappa^2)\right), \tag{4.14}$$

while for magnetic dipoles analogous expressions hold with interchanged polarization indices $(s)\to(p)$. Equations (4.13) and (4.14) agree with Eqs. (3.15)–(3.17) of Ref. 5.

Our results (4.10) and (4.11) for the dipole in layer 0—expressed with Eq. (2.3) in terms of radiative lifetimes—agree with those of Chance *et al.*[4] [their equations (2.47) and (2.48)] who gave a derivation for electric dipoles only. The approach used by these authors differs from ours in the following respects: They use the model of a harmonically bound charge for the electric dipole. The oscillation of the charge under the reaction of the reflected field yields the radiation damping of the dipole. The classical model is applied without further theoretical justification. They solved the em boundary value problem by a different technique.

We give a physical interpretation of Eqs. (4.10)–(4.12). The power emitted by the dipole in the form of plane waves in layer 0 [integration interval $0\le\kappa=(k_x^2+k_y^2)^{1/2}\le k_0$] is determined by the following effects:

(i) The wide-angle interferences of the emitted plane waves $\tilde{\phi}_{\infty,+}(k_x,k_y)$ and $\tilde{\phi}_{\infty,-}(k_x,k_y)$ with wave vectors $\vec{k}(k_x,k_y,k_z=+k_{z,0})$ and $\vec{k}(k_x,k_y,k_z=-k_{z,0})$, respectively. The wide-angle interferences arise only if the dipole is near to a single partially reflecting plane interface or between two such interfaces. The directly emitted wave $\tilde{\phi}_{\infty,+}(k_x,k_y)$ interferes with that part of the wave $\tilde{\phi}_{\infty,-}(k_x,k_y)$ which is reflected from the interface 0/2 (cf. Fig. 2 of Ref. 5); also the directly emitted wave $\tilde{\phi}_{\infty,-}(k_x,k_y)$ interferes with that part of the wave $\tilde{\phi}_{\infty,+}(k_x,k_y)$ reflected from the interface 0/1.

(ii) The waves resulting from these wide-angle interferences are then reflected back and forth between the interfaces 0/1 and 0/2. This leads to the multiple-beam interference factor m given by Eq. (3.19) which appears in Eq. (3.26). The energy emitted by the dipole is radiated into media 1 and 2 in the form of plane or inhomogeneous waves, respectively, depending on whether the medium is loss-free or absorbing. If $n_0>n_1,n_2$, and if d_0 exceeds the cutoff thickness, layer 0 is a planar dielectric waveguide whose guided modes are excited by the dipole. Then, energy is also carried away in the waveguide.

The evanescent waves in the dipole's near field play no role for its emission in an infinite medium. But for a dipole in a thin layer they can contribute to the radiated power (integration interval $k_0<\kappa<\infty$). Assume that one of the adjoining media, say, medium 2, is a loss-free dielectric, optically denser than medium 0, i.e., $n_2>n_0$. Then, evanescent waves with $k_0<\kappa\le k_2$ which reach the interface 0/2 are refracted; the transmitted waves in medium 2 are plane waves with angles of refraction $\alpha_2=\arcsin(\kappa/k_2)$ exceeding the critical angle $\alpha_{2,c}=\arcsin(n_0/n_2)$ [cf. Fig. 3 of Ref. 5]. We have shown that this emission process can contribute very effectively to the total power radiated by a dipole in front of an interface to a denser medium.[5] For dipoles in a thin layer 0 this emission process is influenced by the following two effects which are analogous to the wide-angle interferences and the multiple-beam interferences of the emitted plane waves.

(a) Interferences involving the two evanescent waves $\tilde{\phi}_{\infty,+}(k_x,k_y)$ and $\tilde{\phi}_{\infty,-}(k_x,k_y)$ which for a dipole in an infinite medium 0 exist separated in the half-spaces $z \geq z_0$ and $z < z_0$, respectively. The interference effect results from the presence of the interfaces 0/1 and 0/2. Interfering waves are the evanescent wave $\tilde{\phi}_{\infty,-}(k_x, k_y)$ and that part of the wave $\tilde{\phi}_{\infty,+}(k_x, k_y)$ reflected from the interface 0/1, which after this reflection also decays in the $-z$ direction. The two superposed waves impinge on to the interface to the (denser) medium 2.

(b) The resulting wave is then reflected back and forth between the interfaces 0/1 and 0/2.

Different processes in which the evanescent waves contribute to the radiated power $L(z_0)$ are treated by Chance *et al.*[4]: If medium j ($j = 1, 2$) is absorbing, the evanescent waves transport energy from the dipole to the absorber, in particular, if the dielectric permittivity of medium j is negative $[\epsilon_j < -(n_0)^2]$, energy is absorbed by surface (plasmon) polaritons.

From our derivation of Eqs. (4.10)–(4.12) it follows (cf. end of Sec. III) that they are valid not only for homogeneous media 1 and 2, but also when one of the two media, or both, are planar stratified in the z direction. For $r_{0,j}^{(s,p)}$ the resulting reflection coefficients of the layer system have to be inserted. The properties of media 1 and 2 influence the radiated power only through the reflection coefficients.

V. DIPOLES AT AN INTERFACE: BOUNDARY CONDITIONS FOR THE RADIATED POWER

We will compare the total power radiated by the same dipole located on different sides of one and the same interface. We assume the dipole to be located first in layer 0 at the interface to the loss-free homogeneous medium 1 ($z_0 = d_0 - 0$), then in medium 1 at the interface to layer 0 ($z_0 = d_0 + 0$).

For electric dipoles in layer 0, i.e., for $0 \leq z_0 \leq d_0$, Eqs. (4.10) and (4.11) can be written in the following form:

$$\left(\frac{L(z_0)}{L_\infty(n_0)}\right)_{e,\perp} = \tfrac{3}{4}(k_0)^{-3}\,\mathrm{Re}\left(\int_0^\infty (1 + r_+^{(p)})(k_{z,0})^{-1}\kappa^2 d(\kappa^2)\right), \tag{5.1}$$

$$\left(\frac{L(z_0)}{L_\infty(n_0)}\right)_{e,\parallel} = \tfrac{3}{8}(k_0)^{-3}\,\mathrm{Re}\left(\int_0^\infty [(k_{z,0})^2(1 + r_-^{(p)}) + (k_0)^2(1 + r_+^{(s)})](k_{z,0})^{-1} d(\kappa^2)\right). \tag{5.2}$$

For a dipole located in medium 1, i.e., for $z_0 \geq d_0$, we find with Eqs. (4.13) and (4.14)

$$\left(\frac{L(z_0)}{L_\infty(n_1)}\right)_{e,\perp} = \tfrac{3}{4}(k_1)^{-3}\,\mathrm{Re}\left(\int_0^\infty \{1 + r_{1,0,2}^{(p)} \exp[2ik_{z,1}(z_0 - d_0)]\}(k_{z,1})^{-1}\kappa^2 d(\kappa^2)\right), \tag{5.3}$$

and

$$\left(\frac{L(z_0)}{L_\infty(n_1)}\right)_{e,\parallel} = \tfrac{3}{8}(k_1)^{-3}\,\mathrm{Re}\left(\int_0^\infty ((k_{z,1})^2\{1 - r_{1,0,2}^{(p)} \exp[2ik_{z,1}(z_0 - d_0)]\} + (k_1)^2\{1 + r_{1,0,2}^{(s)} \exp[2ik_{z,1}(z_0 - d_0)]\})(k_{z,1})^{-1} d(\kappa^2)\right), \tag{5.4}$$

where

$$r_{1,0,2}^{(s,p)} = \frac{r_{1,0}^{(s,p)} + r_{0,2}^{(s,p)} \exp(2ik_{z,0}d_0)}{1 + r_{1,0}^{(s,p)} r_{0,2}^{(s,p)} \exp(2ik_{z,0}d_0)} \tag{5.5}$$

are the reflection coefficients for a plane or evanescent wave incident from medium 1 on to layer 0. For magnetic dipoles Eqs. (5.1)–(5.4) hold with interchanged polarization indices $(s) \leftrightarrow (p)$.

We now consider dipoles on the interface $z = d_0$. For $z_0 = d_0$ Eq. (3.26) yields

$$1 + r_\pm^{(s,p)} = (1 \pm r_{0,1}^{(s,p)}) \frac{1 \pm r_{0,2}^{(s,p)} \exp(2ik_{z,0}d_0)}{1 - r_{0,1}^{(s,p)} r_{0,2}^{(s,p)} \exp(2ik_{z,0}d_0)}. \tag{5.6}$$

With the expression (3.20) for the reflection coefficients $r_{0,j}^{(s,p)}$, Eq. (5.6) becomes

$$1 + r_\pm^{(s,p)} = (\epsilon_1/\epsilon_0)^{\pm p}(k_{z,0}/k_{z,1})^{\pm 1}(1 \pm r_{1,0,2}^{(s,p)}). \tag{5.7}$$

With the result (5.7) we obtain by comparing Eqs. (5.1) and (5.2) with Eqs. (5.3) and (5.4) our final results:

$$[L(z_0 = d_0 - 0)]_{e,\perp} = (n_1/n_0)^4 [L(z_0 = d_0 + 0)]_{e,\perp}, \tag{5.8}$$

$$[L(z_0 = d_0 - 0)]_{e,\parallel} = [L(z_0 = d_0 + 0)]_{e,\parallel}, \tag{5.9}$$

and

$$[L(z_0 = d_0 - 0)]_m = [L(z_0 = d_0 + 0)]_m. \tag{5.10}$$

Equation (5.10) holds for magnetic dipoles of arbitrary orientation θ.

Let us consider a thought experiment in which a dipole is moved across the interface between media 0 and 1, the dipole moment and orientation θ of the dipole axis being kept constant. For a magnetic dipole with arbitrary orientation ($0 \leq \theta \leq 180°$), and the parallel electric $(e, \|)$ dipole the radiated power does not change, i.e., L is continuous across the interface. For the perpendicular electric $(e, \perp)$ dipole $\epsilon^2 L$ is continuous; for the $(e, \perp)$ dipole the radiated power is lower in the optically denser medium and higher in the rarer medium. The boundary conditions (5.8)–(5.10) for L are not only valid for dipoles on an interface between the planar stratified media 1/0/2 considered in this section; they are valid for dipoles on the interface between any two linear, isotropic, loss-free, and nonmagnetic media, since they can be shown to follow from the em boundary conditions, viz. the continuity of $E_\|$, $H_\|$, $B_\perp$, and $D_\perp$ across the interface.[8]

VI. SPONTANEOUS-EMISSION RATES FOR EMITTERS IN EXTREMELY THIN LAYERS

We derive an approximation for the normalized spontaneous-emission rate for emitters embedded in extremely thin layers. With $z_0 \to 0$ and $d_0 \to 0$ we obtain from Eq. (3.26) the approximation

$$r_\pm^{(s,p)} = \frac{2r_{0,1}^{(s,p)} r_{0,2}^{(s,p)} \pm r_{0,1}^{(s,p)} \pm r_{0,2}^{(s,p)}}{1 - r_{0,1}^{(s,p)} r_{0,2}^{(s,p)}}. \tag{6.1}$$

Inserting the expressions (3.20) for the reflection coefficients $r_{0,j}^{(s,p)}$ into Eq. (6.1) we find

$$1 + r_\pm^{(s,p)} = (\epsilon_1/\epsilon_0)^{\pm\rho}(k_{z,0}/k_{z,1})^{\pm 1}(1 \pm r_{1,2}^{(s,p)}), \tag{6.2}$$

where $\rho = 0$ and $\rho = 1$, respectively, for s- and p-polarized waves. Note that $r_{1,2}^{(s,p)}$ are the reflection coefficients for the interface between medium 1 and the directly adjoining medium 2, no intermediate layer 0 being present. Inserting Eq. (6.2) into Eqs. (4.10) and (4.11) we find by comparison with Eqs. (4.12)–(4.14)

$$[L_{1,0,2}/L_\infty(n_0)] = (n_1/n_0)^\gamma [L_{1,2}(z_0 = 0)/L_\infty(n_1)], \tag{6.3}$$

where $\gamma = 5$ for $(e, \perp)$, $\gamma = 1$ for $(e, \|)$, and $\gamma = 3$ for $(m, \perp)$ and $(m, \|)$ dipoles.[9] Equation (6.3) expresses the normalized power radiated by a dipole in an extremely thin layer 0 between the homogeneous loss-free dielectric media 1 and 2 in terms of the normalized power radiated by the same dipole located in medium 1 at the interface ($z_0 = 0$) to the directly adjoining medium 2. With Eq. (4.4) for $L_\infty(n)$, Eq. (6.3) yields

$$[L_{1,0,2}]_{e,\perp} = (n_1/n_0)^4 [L_{1,2}(z_0 = 0)]_{e,\perp}, \tag{6.4}$$

$$[L_{1,0,2}]_{e,\|} = [L_{1,2}(z_0 = 0)]_{e,\|}, \tag{6.5}$$

and

$$[L_{1,0,2}]_m = [L_{1,2}(z_0 = 0)]_m. \tag{6.6}$$

Equation (6.6) is valid for both $(m, \perp)$ and $(m, \|)$ dipoles, and, because of Eqs. (4.9) and (4.12) also for magnetic dipoles of arbitrary orientation θ.

Equations (6.4)–(6.6) can also be derived from the following consideration: According to Sec. V, $\epsilon^2 L(z_0)$ for an $(e, \perp)$ dipole and $L(z_0)$ for $(e, \|)$ and m dipoles are continuous across an interface between two media. We assume that for a dipole in an extremely thin layer the radiated power $L_{1,0,2}$ is independent of z_0. Therefore, $L_{1,0,2}$ can be obtained from the value of $L(z_0 = d_0)$ for a dipole in medium 1 at the interface to layer 0. For extremely small d_0 the existence of layer 0 between media 1 and 2 can be neglected [i.e., the reflection coefficients $r_{1,0,2}^{(s,p)}$ given by Eq. (5.5) are $r_{1,0,2}^{(s,p)} \simeq r_{1,2}^{(s,p)}$], and we obtain Eqs. (6.4)–(6.6).

According to Eqs. (6.4)–(6.6) the power $L_{1,0,2}$ radiated by a dipole in an extremely thin layer is independent of its position z_0. For $(e, \perp)$ dipoles only, $L_{1,0,2}$ depends on the refractive index n_0 of layer 0. For $(e, \|)$ dipoles and magnetic dipoles of any orientation θ, $L_{1,0,2}$ is independent of the properties of layer 0. We obtain analytical expressions for $L_{1,0,2}/L_\infty(n_0)$ when we insert into Eq. (6.3) the expressions for $L_{1,2}(z_0 = 0)/L_\infty(n_1)$ which we have derived in Ref. 10. The functions $L_{1,2}(z_0 = 0)/L_\infty(n_1) \equiv l^{(0)}(n)$ for $(e, \perp)$, $(e, \|)$, $(m, \|)$ dipoles (the latter notation was used in Ref. 10) depend only on the relative refractive index $n \equiv n_2/n_1$. Therefore, the normalized radiated powers $L_{1,0,2}/L_\infty(n_0)$ are functions of the relative refractive indices n_1/n_0 and n_2/n_0 only; they are symmetric in n_1 and n_2. From the symmetry of the physical situation it is obvious that an interchange of media 1 and 2 can not affect the radiated power $L_{1,0,2}$. In Eqs. (6.3)–(6.6) the indices 1 and 2 can be interchanged. Then, $L_{1,0,2}$ is expressed in terms of the power $L_{2,1}(z_0 = 0)$ radiated by the dipole located in medium 2 at the interface to the directly adjoining medium 1. [The resulting expressions are consistent with Eqs. (6.3)–(6.6) because $\epsilon^2 L$ for $(e, \perp)$ and L for $(e, \|)$ and m dipoles are continuous across the interface 1/2; this leads to $l^{(0)}(1/n) = n^{-\gamma} l^{(0)}(n)$ with values of γ given immediately below Eq. (6.3).]

In the special case where $n_1 = n_2$ we have $L_{1,2}(z_0) = L_\infty(n_1)$, and, consequently,

$$[L_{1,0,2}]_{e,\perp} = (n_1/n_0)^4 [L_\infty(n_1)]_{e,\perp}, \tag{6.7}$$

and

$$[L_{1,0,2}] = [L_\infty(n_1)] \tag{6.8}$$

for $(e, \|)$ and m dipoles. Parallel electric dipoles and magnetic dipoles of any orientation θ in layer 0 radiate as if they were located in an infinite medium of index n_1. But for the perpendicular electric dipole in layer 0 the radiation rate is higher if $n_0 < n_1$ and lower, if $n_0 > n_1$ than that in the infinite medium of index n_1.

We estimate the layer thicknesses d_0 for which the approximations (6.3)–(6.6) are valid. We obtained Eq. (6.1) from Eq. (3.26) with the approximations $\exp(2ik_{z,0}d_0) \simeq 1$, $\exp(2ik_{z,0}z_0) \simeq 1$, and $\exp[2ik_{z,0}(d_0 - z_0)] \simeq 1$, where $z_0 \leq d_0$. For plane waves emitted by the dipole we have $2k_{z,0}d_0 = 4\pi (n_0 d_0/\lambda)\cos\alpha_0$, where α_0 is the angle between the wave vector and the z axis; the approximations are justified if $n_0 d_0 \ll \lambda/8$. For the evanescent waves in the dipole field in layer 0 the exponential functions become negative exponentials as, for example, $\exp(-2|k_{z,0}|d_0)$ with $2|k_{z,0}|d_0 = 4\pi (n_0 d_0/\lambda)[(k_x^2 + k_y^2)/k_0^2 - 1]^{1/2}$. If one of the adjoining media (say medium 2) is optically denser than layer 0 (i.e., $n_2 > n_0$) the evanescent waves in the dipole field with $k_0 < (k_x^2 + k_y^2)^{1/2} \leq k_2$ appear as plane waves in medium 2 with angles of refraction α_2 exceeding the critical angle $\alpha_{2,c} = \arcsin(n_0/n_2)$. This process can contribute very efficiently to the total power radiated by the dipole (cf. Refs. 5 and 10). The approximations are justified if $d_0(n_2^2 - n_0^2)^{1/2} \ll \lambda/8$. If $n_0 > n_1, n_2$ the evanescent waves do not contribute to the power radiated by the dipole. In a subsequent paper we will investigate the range of applicability of the approximations (6.4)–(6.6) by comparing values $L_{1,0,2}$ calculated from them with values $L(z_0)$ calculated from Eqs. (4.10)–(4.12) by computer.

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[2] A list of references to related experimental and theoretical work is given in Ref. 1, and in the review articles Refs. 3 and 4.

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[4] R. R. Chance, A. Prock, and R. Silbey, in *Advances in Chemical Physics*, edited by I. Prigogine and S. A. Rice (Wiley, New York, 1978), Vol. XXXVII, p. 1.

[5] W. Lukosz and R. E. Kunz, J. Opt. Soc. Am. 67, 1607 (1977).

[6] Note that in Ref. 5 the dipole is located in an infinite medium designated as medium 1. In the present paper we use the subscripts + and − at $\tilde{\phi}_{\infty,\pm}^{(H,E)}(k_x, k_y)$ to distinguish the amplitudes in the half-spaces $z \geq z_0$ and $z \leq z_0$, respectively. These subscripts ± were omitted in Ref. 5; there the distinction between the amplitudes in the two half-spaces is expressed by Eq. (2.19).

[7] Although the scalars $\phi^{(E)}(\vec{x})$ for an electric dipole and $\phi^{(H)}(\vec{x})$ for a magnetic dipole are discontinuous across the plane $z = z_0$ if $\Theta \neq 0$, the em fields $\vec{E}(\vec{x})$ and $\vec{H}(\vec{x})$ are continuous everywhere.

[8] W. Lukosz (unpublished).

[9] We use the subscripts 1, 0, 2 only in this Sec. VI to designate a dipole in the (extremely thin) layer 0 between media 1 and 2.

[10] W. Lukosz and R. E. Kunz, Opt. Commun. 20, 195 (1977).