PHYSICAL REVIEW B VOLUME 22, NUMBER 6 15 SEPTEMBER 1980

Study of electron-phonon interaction and magneto-optical anomalies in two-dimensionally confined systems

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(Received 21 January 1980)

This paper presents a study of the nature of electron-optical phonon coupling in two-dimensionally confined systems (e.g., inversion or accumulation layers in metal-oxide-semiconductors, heterojunctions, and superlattices of polar semiconductors) and shows that magneto-optical anomalies in two dimensions provide a powerful tool for their investigation. Several models of electron-optical phonon coupling are studied. These include coupling with (i) three-dimensional optical phonon with fixed k_z, the wave vector normal to the two-dimensional plane of confinement, (ii) three-dimensional optical phonons with arbitrary k_z due to the loss of wave-vector selection rules in finite structures, (iii) interface optical-phonon modes, and (iv) purely two-dimensional optical-phonon modes. Where appropriate, zone-folding effects are explicitly taken into account. Resonant magneto-optical absorption studies are shown to be a means of unambiguously probing the electron-phonon coupling in these novel materials. Within the framework of the Fröhlich model and including only the resonant self-energy diagram in the theory, it is shown that at resonance the relevant cyclotron peak splits into a doublet. The resonance effect is found to be much sharper in two dimensions compared to the bulk case, a consequence of the lack of k_z continuum of electronic energy. For a given magnetic field of strength B, the resonant splitting is proportional to $B^{3/4}$ for two-dimensionally confined carriers, in contrast to the $B^{1/2}$ behavior in the bulk. The proportionality factor depends on the specific phonon mechanism considered. The spectral weights of the split peaks are evaluated. Allowing, phenomenologically, finite width of the resonant Landau levels, as well as for the Landau level from which optical transition originates, the expressions for optical absorption and Raman scattering are obtained. The significance of electron-light vertex correction is investigated and shown to be important under certain conditions. The possibility of a resonant splitting in two dimensions in direct intersubband transition without any magnetic field is also investigated.

I. INTRODUCTION

In the last ten years there has been an increasing interest[1] in the study of two-dimensionally confined electronic systems occurring near various semiconductor interfaces and on the surface of liquid helium. This interest is partly motivated by the technological significance of some of these two-dimensional systems, in particular the metal-oxide–semiconductor (MOS) structure, but is also due to the possibility of applying physical laws to real systems with dimensionality less than three. Many of these systems provide us with an essentially two-dimensional electron or hole gas (2DEG) in which the effective-charge carrier density can be experimentally varied over a wide range. This in itself is of considerable fundamental interest since carrier density is the crucial parameter in studying many-body interaction effects.[2] Furthermore, by decreasing the electron density sufficiently, one expects to effect Wigner crystallization[3] in such systems.

Silicon MOS inversion layers and the electrons trapped on liquid-helium surface by the image potential have so far been the most studied examples of two-dimensional electronic systems. The former system provides us with a quantum electron gas whereas the latter is a classical electron gas. The electron-electron interaction effects have been studied as a function of electron density and temperature, and the exchange and correlation effects have been found to be important in determining the electronic structure of inversion and accumulation layers in silicon MOS systems, both at zero[4] and at finite[5,6] temperatures. Evidence for Mott-Anderson localization has also been seen in the same system at low densities and temperatures.[7] Recently, Wigner crystallization in the classical regime has been observed in electrons on the surface of liquid helium.[8]

The scope of studying two-dimensionally confined charge carriers has recently been enhanced by the advances made in the experimental realization of such systems in superlattices, heterojunctions, and quantum wells made of lattice-matched semiconductors.[9,10] These new systems offer not simply another means of achieving 2DEG, but of creating situations with potentially new physics. Of the two superlattice systems on which studies have been reported so far, namely the GaAs-$Ga_xAl_{1-x}As$ system[11–13] and the InAs-GaSb system,[14] the former has been combined with the notion of modulated doping to create a 2DEG confined in the GaAs layers by the barriers provided by the adjacent $Ga_xAl_{1-x}As$ layers. This spatial separation of charge from the ionized donors (in $Ga_xAl_{1-x}As$)

results in electron mobilities higher than the Brooks-Herring limit for bulk semiconductors.[13] One of the important features of these new systems is that they are made of III-V compound semiconducting materials which are weakly polar in nature. Thus the electron-optical phonon interaction is expected to play some role in determining the properties of these systems. However, unlike the silicon inversion layers, the small effective mass and reasonably large static lattice dielectric constants of these materials make it unlikely that electron-electron interaction effects would be an important factor in deciding their properties. The electron-acoustic phonon (bulk and interface) interaction has been shown[15,16] to be of significance in understanding the dc as well as finite frequency transport and cyclotron resonance properties of the silicon inversion layers. It may also be of relevance to the newer class of systems of interest in this paper. At low temperatures, however, the electron-optical phonon interaction is not expected to play an important role in silicon in view of the small coupling strength and large optical-phonon frequencies involved. Thus this new class of lattice-matched III-V compound semiconductor heterojunctions, superlattices, and quantum wells along with the inversion layer formed on the InSb surface (and the anticipated GaAs MOS structure) provide us with systems in which the electron-optical phonon interaction and its influence can be studied in reduced dimensionality.

There is already some evidence[17-19] pointing to the possibility that the electron-optical phonon interaction does indeed play a significant role in the properties of these systems. It has also been conjectured that the nature of electron-optical phonon coupling in these systems is quite different from the bulk situation. There is strong evidence[19] toward the involvement of an LO phonon in GaAs quantum-well laser operation. Laser operation can occur below a confined-particle transition with phonon participation. In constrast, it should be noted that the LO-phonon sideband plays no significant role in the laser operation of bulk GaAs, which is expected because of the small electron-phonon coupling of this weakly polar material. The prominence of phonon-assisted recombination in quantum-well heterostructure has been ascribed to the breakdown of translational invariance and momentum-conserving selection rules. Recently, resonance-enhanced normal[17] and umklapp[18] Raman processes have been observed in GaAs-$Ga_xAl_{1-x}As$ superlattices resulting from the wave-vector-dependent Fröhlich-type electron–LO-phonon interaction.[20] Zone-folding effects in a superlattice and the loss of translational invariance in z direction in a quantum well[19] have been speculated to lead to an enhancement in electron-optical phonon coupling by opening up several phonon channels which would otherwise have been absent or inactive due to selection rules.

In this paper we study electron–LO-phonon interaction in these novel two-dimensional systems, emphasizing in partiuclar the features pertaining to two-dimensional confinement. Coupling of the confined charge carriers to ordinary bulk phonons, as well as to interface or other types of two-dimensionally confined phonons, is considered in Sec. II. Effects of zone folding, specific to superlattice structures, are explicitly taken into account. We find that under certain circumstances the coupling strength may indeed be enhanced due to the novel structure of these systems. We shall use Fröhlich theory[21] in describing the electron-LO phonon coupling in these systems. It should, however, be remarked that the continuum approximation implicit in the Fröhlich theory may not be adequate to describe electron-LO phonon coupling in superlattice structure due to the rather large size of the unit cell in the z direction (taken to be the direction in which the superlattice is grown).

After developing various models for electron–LO-phonon interaction, in Sec. III we shall consider the problem of resonant coupling of electronic Landau levels with the LO phonons in the presence of a static magnetic field normal to the two-dimensional plane of confinement. The study of the phenomenon of resonant coupling of Landau level with the longitudinal-optical phonons in a bulk semiconductor received considerable impetus with the observation[22] of anomalous magneto-optical interband absorption behavior in InSb, reported a little over a decade ago. Subsequent studies clarified various aspects of this phenomenon as it affects impurity absorption,[23] cyclotron resonance,[24] and combined resonance.[25] A systematic theoretical study[26-28] of the phenomenon in free-carrier systems has led to a classification of the relative importance of self-energy and electron-light vertex corrections in the various phenomena noted above. It should, however, be noted that the resonant coupling of Landau levels with LO phonons was first discussed[29] in the context of the dc magnetoresistance, the phenomenon in that context being called the resonant magnetophonon effect. Both, the magneto-optical anomalies and the resonant magnetophonon effects being manifestations of the same basic physics, are undoubtedly present in the two-dimensionally confined systems of interest in this paper. We will, however, focus our attention on the finite-frequency manifestation of this resonant phenomenon since the magneto-optical anomaly, we show, offers a unique means of extracting information regarding electron–LO-

phonon coupling in these new and novel two-dimensionally confined systems. We show that the situation in the two-dimensionally confined systems is different from the bulk and the results presented in this paper[30] reveal several points of departure from those for the bulk materials.[31] In particular, the resonance splitting of a Landau level is shown to be much sharper in two dimensions compared to the bulk case.

In Sec. IV, we calculate the magneto-optical absorption in the presence of the resonant coupling between LO phonons and the Landau levels. We allow for collisional damping in our theory by introducing a finite width to the Landau levels in a phenomenological fashion. We show that magneto-absorption should exhibit anomalies in the form of split peaks near the resonance and that the splitting is proportional to the effective electron-phonon coupling strength. We conclude this section with a consideration of the effect of including phonon-induced electron-light vertex correction in the theory.

We conclude in Sec. V by emphasizing how an experimental investigation of anomalous magneto-absorption (cyclotron resonance) in intersubband electronic transitions under resonance condition can be successfully used to extract information about electron-phonon coupling strengths in these two-dimensionally confined polar systems.

II. ELECTRON-LO-PHONON INTERACTION IN 2DEG

The carriers in these confined systems occupy two-dimensional subbands where their motion parallel to the interface is "free," whereas their motion in z direction (normal to the interface) is confined. It is important, however, to clearly recognize certain differences between a single two-dimensionally confined layer (such as inversion layers, accumulation layers in heterostructures, or a few quantum wells) and a repeated structure of these layers such as the superlattice (see Fig. 1). In the former case the notion of a wave vector $\vec{k}_z$ in the direction normal to the two-dimensional layer does not exist, whereas in the latter case it does. Another difference of practical significance is the possible importance of the full Bloch waveform of the electron wave function in superlattices. As we have noted before, this may make the usual continuum approximation implicit in the Fröhlich model inadequate for describing the electron-phonon coupling in superlattices. For the inversion layers, the description of the z dependence of the electron charge distribution has been sought in terms of an envelope function characteristic only of a particular subband not independent of $\vec{k}_{\parallel}$, the wave vector in the two-dimensional plane. The inversion or accumulation layer problem is usually treated within effective-mass approximation[32] and a continuum description of electron-phonon interaction remains valid. However, for superlattices the charge distribution within the unit cell may depend strongly upon the particular $\vec{k}_{\parallel}$ value via the Bloch form of the wave function.[33] The electron-optical phonon coupling in such a situation needs to be carefully investigated.

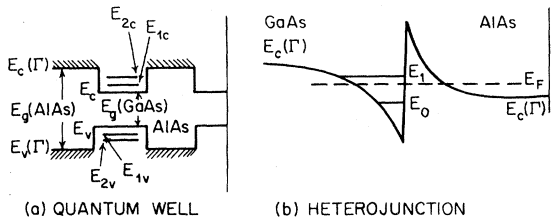

FIG. 1. Schematic representation of two-dimensionally confined electronic states in (a) GaAs|$Ga_xAl_{1-x}As$ quantum well and (b) GaAs|$Ga_xAl_{1-x}As$ heterojunction (this is similar to the inversion or accumulation layers in MOS systems). Each two-dimensional subband (E_1, E_2, etc.) shown splits into Landau levels under a normal magnetic field.

The electronic charge density at any point $\vec{R} \equiv (\vec{r}, z)$ is given by

$$\rho_{el}(\vec{r}, z) = -\frac{1}{A}\sum_{i,j}\sum_{\vec{q},\vec{q}'} \xi^*_{i\vec{q}}(z)\xi_{j\vec{q}'}(z') \times e^{-i(\vec{q}-\vec{q}')\cdot\vec{r}} C^\dagger_{qi} C_{q'j}, \qquad (1)$$

where $\vec{r}$ and $\vec{q}, \vec{q}'$ are the two-dimensional position and wave vectors, respectively. A is the total area and $C^\dagger_{qi}(C_{qi})$ is the creation (annihilation) operator for an electron (or a hole) with two-dimensional momentum $\vec{q}$ in the ith subband. We take $e=1$ throughout this section. For a finite structure (in the z direction) such as inversion layers, i stands only for the quantized subband (denoting quantization of z motion in the potential well), whereas for a repeated structure like the superlattice it may be considered to incorporate the wave vector $\vec{k}_z$. $\xi_{i\vec{q}}(z)$ in Eq. (1) is the envelope wave function describing the z motion of the quantized subbands (the motion in the two-dimensional x-y plane being assumed to be described by the plane wave $e^{i\vec{q}\cdot\vec{r}}$). For the sake of simplicity we assume $\xi_{i\vec{q}}(z)$ to be independent of $\vec{q}$. This is strictly valid for the inversion layer situation due to the vanishing of wave functions at the $z=0$ plane. However, no generality is lost by setting $\xi_{iq}(z) \equiv \xi_i(z)$. Inclusion of $\vec{q}$ dependence in ξ within the formalism developed here can be achieved without much difficulty.

Equation (1) for the electronic charge distribu-

tion suggests that its coupling with the ionic charge may be dependent upon the particular distribution that the latter may take. Consequently in the following, we consider the form of electron-phonon coupling arising from phonon modes in two broad categories as follows: (i) coupling of the confined electrons with bulk LO-phonon modes and (ii) coupling with quasi-two-dimensional phonon modes, i.e., phonon modes which reflect peaking of the ionic charge displacement within the confined region in which the electrons reside.

(i) *Coupling with bulk LO phonons.* The simplest case to consider is the coupling of those confined electrons to bulk LO phonons. We assume that the bulk phonons are unaffected by the existing one-dimensional potential in these systems. Then the electron-phonon coupling with a two-dimensional momentum-exchange $\vec{k}$ can be easily obtained by using the Fröhlich model.[21] We get

$$M_{ij}(\vec{k}, k_z) = [(2\pi/V)\omega_{LO}(\epsilon_\infty^{-1} - \epsilon_0^{-1})]^{1/2} \times f_{ij}(k_z)(k^2 + k_z^2)^{-1}, \quad (2)$$

where

$$f_{ij}(k_z) = \int dz\, \xi_i^*(z) e^{ik_z z} \xi_j(z). \quad (3)$$

Equation (2) gives the matrix element for coupling between a phonon of momentum $(\vec{k}, k_z)$, with k_z as the z component of phonon momentum, and a two-dimensionally confined electron which scatters from subband i to subband j with emission (or absorption) of a three-dimensional phonon. V is the total volume and ω_{LO}, ϵ_∞, and ϵ_0 are the LO-phonon frequency and the high- and low-frequency dielectric constants, respectively. The leading order-coupled electron-phonon processes involve the square of the matrix element in Eq. (2). Such an effective coupling strength, for example $V_{ijlm}(\vec{k},k_z)$, is thus given by

$$V_{ijlm}(\vec{k}, k_z) = M_{ij}(\vec{k}, k_z) M_{lm}^*(\vec{k}, k_z). \quad (4)$$

It signifies scattering of an electron from i to j with emission of an LO phonon of momentum $(\vec{k}, k_z)$ and the subsequent absorption of the phonon by another electron which gets scattered from l to m.

For systems without translational invariance in the z direction, k_z in Eqs. (2)–(4) becomes completely arbitrary. In that case, one sums over k_z in Eq. (4) to obtain the effective electron-phonon coupling with the exchange of two-dimension momentum $\vec{k}$. Neglecting LO-phonon dispersion and summing over all k_z, we find,

$$V_{ijlm}(\vec{k}) = \left(\frac{2\pi}{A}\omega_{LO}(\epsilon_\infty^{-1} - \epsilon_0^{-1})\right)\frac{f_{ijlm}(\vec{k})}{2k}, \quad (5)$$

where

$$f_{ijlm}(\vec{k}) = \int dz \int dz'\, \xi_i^*(z)\xi_j(z) e^{-k|z-z'|} \xi_l^*(z')\xi_m(z'). \quad (6)$$

The integral over z, z' extends over a small region of space due to the two-dimensional confinement in these systems. For comparison, we recall that the Fröhlich interaction between electrons and LO phonons in three dimensions, is given by

$$V(\vec{q}) = (2\pi/V)\omega_{LO}(\epsilon_\infty^{-1} - \epsilon_0^{-1})(1/q^2), \quad (7)$$

where $\vec{q}$ in Eq. (7) is the three-dimensional momentum exchange.

For the case of the superlattice, in addition to the modifications in the electron-phonon coupling strength arising from charge confinement effects, we also have the formation of mini-Brillouin zones in the k_z direction due to the introduction of the new superlattice period. However, noting the expected[17,18] insignificant influence of this minizone formation on the bulk optical-phonon branch, we may, in considering virtual processes requiring summation over all k_z, choose to consider it to lie within the original Brillouin zone, thus allowing for umklapp processes for electron-phonon scattering. Alternatively, one could incorporate the entire bulk optical-phonon branch via a summation over the appropriate number of phonon modes within the minizone. This is referred to as the zone-folding effect. In general, frequencies of these modes will be different due to the dispersion of the LO-phonon mode which we assume to be small. It is important to note that since the effective coupling strength given by Eq. (5) includes a sum over all k_z (and not just the unit minizone), it contains both the normal and the umklapp processes involving phonons with arbitrary k_z, with zone-folding effects already built into it. It is only for the processes involving fixed momenta (e.g., $k_z = 0$ for processes involving zone-center phonons) that the zone-folding effects will give rise to an explicit enhancement of the electron-phonon coupling strength via an increase in the number of phonon branches alone.

(ii) *Coupling with quasi-two-dimensional photons.* We assume that the LO phonons are quasi-two-dimensional in nature like the quantized carriers themselves. In order to describe the quasi-two-dimensional phonons, we introduce a phonon envelope function $\eta_\alpha(z)$ giving the decay in z direction of polarization charge density of the αth quasi-two-dimensional LO-phonon branch. Employing the Fröhlich scheme[21] of expressing the polariza-

tion in terms of charge density, and introducing standard phonon field operators (in two dimensions), we can express the effective coupling strength between the confined electrons and the quasi-two-dimensional phonons as

$$V_{ijlm}(\vec{k}) = (2\pi/A)\omega_{LO}(\epsilon_\infty^{-1} - \epsilon_0^{-1})g_{ijlm}(\vec{k})/2k\,, \tag{8}$$

where

$$g_{ijlm}(\vec{k}) = \sum_\alpha \int dz \int dz' \xi_i^*(z)\xi_j(z)\xi_l^*(z')\xi_m(z')\left(\int dz_1 \int dz_2 \eta_\alpha(z_1)\eta_\alpha(z_2) e^{-k(|z-z_1|+|z'-z_2|)}\right). \tag{9}$$

The position of the peak in $\eta_\alpha(z)$ will naturally be governed by the specific nature of the quasi-two-dimensional vibrational modes that may exist in these systems. For example, an interface phonon is characterized by an $\eta_\alpha(z)$ which peaks at the interface. From Eq. (9) it is clearly seen that maximum electron-phonon coupling is obtained when the quasi-two-dimensional phonons are peaked at the same place where the electronic charge density [given by $|\xi(z)|^2$] reaches a maximum. The influence of the electron-phonon interaction manifested in the measurements on these systems is then via an effective coupling strength which is an average over various phonon modes and their coupling with spatial distributions of electrons.

The above considerations based on the Fröhlich model clearly bring out certain special features of the electron–LO-phonon interaction in these quasi-two-dimensionally confined systems. The form factor arises only because the carriers in these systems are quantized into two-dimensional subbands. Zone-folding effects may give rise to a strong enhancement in the coupling strength in superlattices via the formation of new phonon branches. Lack of a wave-vector-conserving selection rule in the z direction makes it possible for phonons with all k_z to take part in the interaction process. This enhances the coupling strength even further. Significant effects may arise from the coupling of the confined electrons to quasi-two-dimensional phonons (possible interface modes) which may exist in these systems.

III. LANDAU-LEVEL–LO-PHONON COUPLING IN 2DEG

In the presence of a static magnetic field B in normal direction (taken to be the z axis), each two-dimensional subband i splits into a series of Landau levels described by the Landau index n. Assuming parabolic bands, the one-electron wave function describing the carrier motion in the presence of the magnetic field is given by

$$\psi_{ink}(\vec{r}, z) = e^{iky} u_n(x + kl^2)\xi_i(z)\,, \tag{10}$$

where $\vec{r} \equiv (x, y)$ is the two-dimensional position vector and $u_n(x + kl^2)$ is the harmonic-oscillator wave function with a displaced center, $l = (c\hbar/eB)^{1/2}$ being the radius of the classical cyclotron orbit. The corresponding one-particle energies are given by

$$E_{ink} = (n + \tfrac{1}{2})\hbar\omega_{ci} + E_i\,, \tag{11}$$

where $\omega_{ci} = (eB/m_i c)$ and E_i are, respectively, the cyclotron frequency and the subband bottom energy of the ith subband; m_i being the effective mass for parallel motion. Equation (11) shows the well-known degeneracy of the Landau levels with respect to k which has lost the significance of a free momentum and now gives the position of the center of the cyclotron orbit in presence of the quantizing magnetic field.

Neglecting electron-electron interaction effects, the Hamiltonian for the interacting electron-phonon system in the presence of the quantizing magnetic field can be written as

$$H = \sum_{ink} [E_i + \hbar\omega_{ci}(n + \tfrac{1}{2})] C_{ink}^\dagger C_{ink} + \sum_{\vec{q}} \hbar\omega_{LO} b_{\vec{q}}^\dagger b_{\vec{q}} + \sum_{ii'nn'k_y\vec{q}} g_{(ii')(nn')}(\vec{q}) C_{ink}^\dagger C_{i'n'k-q_y}(b_{\vec{q}} + b_{-\vec{q}})\,. \tag{12}$$

The first term in Eq. (12) is the Hamiltonian of a free 2DEG in a magnetic field, $C_{ink}^\dagger$ (C_{ink}) being the creation (annihilation) operator for the quantum state (ink). The second term is the energy of the free LO phonons. For simplicity, we consider only one kind of LO phonons with dispersionless frequency ω_{LO}. The phonon wave vector $\vec{q}$ is a two- or three-dimensional wave vector, depending on the nature of the phonon involved.

The last term in Eq. (12) describes the interation of the Landau-level electrons with the LO phonons. In general the interaction vertex $g_{(ii')(nn')}(\vec{q})$ carries four subscripts indicating that an electron could be scattered from the Landau level n in subband i to Landau level n' in subband i' due to interaction with an LO phonon. The ef-

fective coupling in Landau representation is given by

$$g_{(ii')(nn')}(\vec{q}) = M_{ii'}(\vec{q}) \int dx\, u_n(x) e^{iq_x x} u_{n'}(x - q_y l^2), \tag{13}$$

where $M_{ii'}(\vec{q})$ is the basic electron–LO-phonon coupling strength in these two-dimensionally confined systems, as discussed in Sec. II. Putting the forms for three-dimensional bulk LO phonons and quasi-two-dimensional phonons in Eq. (13) we can evaluate the respective coupling strengths in a Landau-quantized situation.

Electron–LO-phonon interaction will change the Landau-level energies from that given by Eq. (11) by dressing the electronic Green's function of the system. Without this interaction, an electron in the state (i,n) is degenerate in energy with the composite state of an electron in $(i, n-l)$ level plus one LO phonon of energy $\omega_{LO} = \omega_{ci}$. The resonance condition $\omega_{LO} = \omega_{ci} = eB/m_i c$ which gives rise to the energy degeneracy, can be rather easily achieved in the III-V semiconductors by changing the magnetic field in the (1–10)-tesla region. Obviously this degeneracy will be lifted by the electron–LO-phonon interaction. In the resonance situation this lifting of degeneracy takes place even if the electron–LO-phonon interaction is weak. It is referred to as "resonance splitting" since the degenerate level splits into two branches. Such a resonant coupling can manifest itself in an experiment involving cyclotron transitions to the nth Landau level. Such transitions may involve initial and final Landau levels belonging to different subbands (combined intersubband cyclotron resonance[34]), or it may originate from the $n-1$ level in the same subband (ordinary cyclotron resonance). The relevant magnetoabsorption peak should split into two peaks around $\omega_{ci} = \omega_{LO}$ due to the resonant coupling, provided the broadening of the peaks is less than the energy separation of the split levels (Fig. 2). From the experimental point of view, the complications of observing cyclotron resonance in the reststrahlen region suggests the use of the combined intersubband cyclotron resonance technique. In some of these two-dimensional systems intersubband combined cyclotron resonance has already been observed.[34] Similar resonant effects have been observed in the bulk.[22,31] However, the two-dimensional confinement we are considering here gives rise to features quite different from the corresponding bulk problem.

To investigate the resonant effect quantitatively, we consider the self-energy (Fig. 3) contribution to an electron in the state (i, n) from virtual states

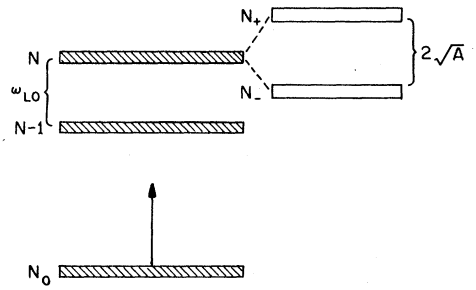

FIG. 2. Schematic representation of the resonant magnetophonon splitting of the Nth Landau level when it is degenerate in energy with $(N-1)$ level plus one LO phonon. N_0 is the level from which an optical transition may originate.

of the composite system consisting of an electron in $(i, n-1)$ and an LO phonon. In doing so we are already anticipating the resonance-splitting phenomenon since we neglect contributions from all other self-energy processes. Our rationale for doing that is the basic assumption of weak electron-phonon coupling in the systems we are considering. We assume that nonresonant self-energy corrections to electron–LO-phonon coupling in these systems are rather small (supported by the very small differences between the polaronic mass and the band mass in III-V semiconductors such as InSb, GaAs, etc.) and interesting qualitative effects arise only at the resonance which couples two adjacent Landau levels only. Of course, in principle one could consider multiphonon resonances[35] occuring at multiples of cyclotron frequency, but we neglect them as higher-order processes. The weak nature of electron–LO-phonon coupling provides the justification for neglecting vertex corrections which are essentially higher order in coupling constant.[36] The leading-order self-energy correction is then defined by the simple diagram shown in Fig. 3(a), and is given by[37]

$$\Sigma_{i,n}(i\omega_l) = -\frac{1}{\beta\hbar} \sum_{i\nu_m} \sum_{\vec{q}} G^0_{i,n-1}(i\omega_l - i\nu_m) \times |g_{(ii)(n,n-1)}(\vec{q})|^2 D_{LO}(i\nu_m, \vec{q}), \tag{14}$$

where G^0 and D_{LO} are, respectively, the noninteracting-electron Green's function and the LO-phonon propagator. $\beta = (k_B T)^{-1}$ is the inverse temperature and $i\omega_l = (2l+1)\pi i/\beta\hbar$, $i\nu_m = 2m\pi i/\beta\hbar$, with $l, m = 0, \pm 1, \pm 2, \ldots$ are, respectively, the odd and even imaginary frequencies at which fermion and boson functions are defined.[37]

We can perform the discrete frequency sum over $i\nu_m$ in Eq. (14) using the standard Poisson's sum formula.[36,37] We get

$$\Sigma_{i,n}(i\omega_l)=\sum_{\vec{q}}|g_{(i\,i)(n,n-1)}(\vec{q})|^2\left(\frac{f(\omega^0_{i,n-1})}{i\omega_l-\omega^0_{i,n-1}-\omega_{LO}}+\frac{f(-\omega^0_{i,n-1})}{i\omega_l-\omega^0_{i,n-1}+\omega_{LO}}\right), \tag{15}$$

where

$$[f(x)=e^{\beta x}+1]^{-1} \tag{16}$$

is the Fermi distribution function. Also, $\hbar\omega^0_{i,n}=E_{ink}-\mu$, where μ is the chemical potential. Replacing the internal Green's-function Landau index from $n-1$ to n' and summing over all n''s we get the result for the total self-energy contribution from all the Landau levels.

We have already assumed that only the resonant contribution to $\Sigma_{n'}$ from $n'=n-1$ is the dominant one. One might expect the diagonal contribution to the self-energy from the term $n'=n$ will be quantitatively the most dominant one. This is indeed the case away from the resonance. But the assumption of weakly polar nature of the system makes such corrections insignificant compared to the resonant contribution coming from the $n'=n-1$ term shown in Eq. (15). In Appendix A, we show explicitly that the diagonal contribution to the self-energy does not give rise to the resonance-splitting phenomenon.

The sum over $\vec{q}$ in Eq. (15) is over all three-dimensional wave vectors for coupling with bulk-like LO phonons, whereas, it is over all two-dimentional $\vec{q}$ if coupling with quasi-two-dimensional phonons is being considered. To be specific we first consider the coupling with the bulk LO phonons, in which case using Eqs. (2) and (13) we get

$$\sum_{\vec{q}}|g_{(i\,i)(n,n-1)}(\vec{q})|^2=\left(\frac{\alpha\omega^{3/2}_{LO}}{m_i^{1/2}}\right)\times\left(\sum_{\vec{Q}}f_{i,n,n-1}(\vec{Q})\right), \tag{17}$$

where

$$\alpha=\frac{e^2}{\hbar}\left(\frac{m_i}{2\hbar\omega_{LO}}\right)^{1/2}\left(\frac{1}{\epsilon_\infty}-\frac{1}{\epsilon_0}\right) \tag{18}$$

is the dimensionless Fröhlich coupling constant. The form factor $f_{i,n,n-1}(\vec{Q})$ in Eq. (17) is a function of the conserved two-dimensional wave vector $\vec{Q}$ and depends on the electronic subband quantization and the external magnetic field through the Landau indices. It is given by[38]

$$f_{i,n,n-1}(\vec{Q})=\frac{\pi}{Q}\hbar^{5/2}\left[\int dz\int dz'\,|\xi_i(z)|^2e^{-Q|z-z'|}|\xi_i(z')|^2\right]\times\frac{(n-1)!}{n!}(\tfrac{1}{2}Q^2l^2)[\mathfrak{L}^1_{n-1}(\tfrac{1}{2}Q^2l^2)]^2e^{-1/2Q^2l^2}. \tag{19}$$

For an explicit evaluation of the wave-vector sum in Eq. (17) one must know the subband structure explicitly. In the purely two-dimensional limit, the form factor within the large square brackets in Eq. (19) is unity whereas in the realistic situation of a quasi-2DEG it is less than one. $\mathfrak{L}$ in Eq. (19) is the associated Laguerre polynomial.

For our purpose we write Eq. (17) as

$$\sum_{\vec{q}}|g_{(i\,i)(n,n-1)}(\vec{q})|^2=A_{i,n}, \tag{20}$$

where

$$A_{i,n}=\left(\frac{\alpha\omega^{3/2}_{LO}}{m_i^{1/2}}\right)\sum_{\vec{Q}}\left(\frac{\pi\hbar^{5/2}}{Q}\right)\times\left(\int dz\int dz'\,|\xi_i(z)|^2e^{-Q|z-z'|}|\xi_i(z')|^2\right)\times\frac{(n-1)!}{n!}(\tfrac{1}{2}Q^2l^2)[\mathfrak{L}^1_{n-1}(\tfrac{1}{2}Q^2l^2)]^2e^{-1/2Q^2l^2}. \tag{21}$$

The important point to note is that the quantity $A_{i,n}$ is proportional to the electron-phonon coupling strength in the system. Using Eq. (21) in Eq. (15), we find that the resonant self-energy correction itself is proportional to the relevant elec-

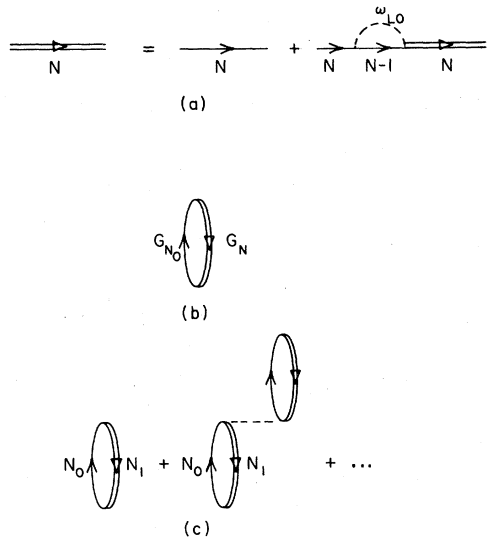

FIG. 3. (a) Dyson's equation for the resonant self-energy between the $(N-1)$ Landau level +1 LO phonon with the Nth Landau level (double and single lines denote dressed and free-electron Green's functions, respectively, whereas the dashed line is the LO-phonon propagator). (b) Lowest-order diagram for absorption in the long-wavelength limit. (c) Phonon-induced electron-light vertex correction.

tron-phonon coupling strength.

The resonant splitting phenomenon and the associated magneto-optical anomalies show up in processes involving transitions to the level n when it is resonantly coupled with the composite state of an LO phonon and the level $n-1$. Thus the physically interesting situation for the resonance process occurs when the levels $n, n-1$ are unoccupied, i.e., when they lie above the chemical potential. Using Eq. (20) in Eq. (15), and assuming $\omega^0_{i,n-1} > 0$, we get (in the $T \to 0$ limit)

$$\Sigma_{i,n}(i\omega_l) = \frac{A_{i,n}}{i\omega_l - \omega^0_{i,n-1} - \omega_{LO}} . \tag{22}$$

The singularity at $i\omega_l = \omega^0_{i,n-1} + \omega_{LO}$ in Eq. (22) is a direct manifestation of the resonant Landau-level–optical-phonon coupling in the system.

The interacting Green's function including the resonant coupling is given by a solution of Dyson's equation.[37] In our case, we include only the resonant self-energy term and Dyson's equation can be solved for the (i, n) level to give

$$G_{i,n}(i\omega_l) = [i\omega_l - \omega^0_{i,n} - \Sigma_{i,n}(i\omega_l)]^{-1} . \tag{23}$$

Use of Eq. (22) and some rearrangement of terms in Eq. (23) yields

$$G_{i,n}(i\omega_l) = (i\omega_l - \omega^0_{i,n-1} - \omega_{LO})(i\omega_l - \omega_+)^{-1}(i\omega_l - \omega_-)^{-1} , \tag{24}$$

where

$$\omega_\pm = \tfrac{1}{2}(\omega^0_{i,n} + \omega^0_{i,n-1} + \omega_{LO}) \pm \tfrac{1}{2}[(\omega^0_{i,n} - \omega^0_{i,n-1} - \omega_{LO})^2 + 4A_{i,n}]^{1/2} . \tag{25}$$

Equation (24) clearly demonstrates that the resonant coupling has given rise to two peaks in the interacting Green's function.

The strength of the quasiparticle peak is given by the spectral function[37] which is defined to be

$$\rho_{i,n}(\omega) = -2\,\mathrm{Im} G_{i,n}(i\omega_l \to \omega + i\delta)\big|_{\delta \to 0+} . \tag{26}$$

Introducing $\omega_{ci} = (\omega^0_{i,n} - \omega^0_{i,n-1})$ and $\lambda = (\omega_{ci} - \omega_{LO})$ and using Eq. (24), in Eq. (26), we get

$$\rho_{i,n}(\omega) = \pi(\omega - \omega^0_{i,n} + \lambda)(\lambda^2 + 4A_{i,n})^{-1/2} \times [\delta(\omega - \omega_+) - \delta(\omega - \omega_-)] . \tag{27}$$

From Eq. (27), the spectral weights of the peaks at ω_+ and ω_- can be readily obtained. Note that $\lambda = 0$ signifies the exact resonances ($\omega_{ci} = \omega_{LO}$) and thus we can have the following two situations:

(i) Resonant limit ($\lambda^2/A_{i,n} \ll 1$). From Eq. (27) we get

$$\rho_{i,n}(\omega) \simeq \tfrac{1}{2}\pi\left(1 + \frac{\lambda^2}{8A_{i,n}}\right)^{-1} \times \left[\left(1 + \frac{\lambda}{2\sqrt{A_{i,n}}} + \frac{\lambda^2}{8A_{i,n}}\right)\delta(\omega - \omega_+) + \left(1 - \frac{\lambda}{2\sqrt{A_{i,n}}} + \frac{\lambda^2}{8A_{i,n}}\right)\delta(\omega - \omega_-)\right] , \tag{28}$$

with

$$\omega_\pm \simeq \left(\omega^0_{i,n} - \tfrac{1}{2}\lambda \pm \sqrt{A_{i,n}} \pm \frac{\lambda^2}{8\sqrt{A_{i,n}}}\right) . \tag{29}$$

At exact resonance, $\lambda = 0$ and we get

$$\rho_{i,n}(\omega) = \tfrac{1}{2}\pi[\delta(\omega - \omega_+) + \delta(\omega - \omega_-)] \tag{30}$$

and,

$$|\omega_+ - \omega_-| = 2\sqrt{A_{i,n}} . \tag{31}$$

Thus at resonance the Landau level (i, n) splits into two symmetric peaks with equal spectral weight—the energy splitting being proportional to $A^{1/2}_{i,n}$ and hence to $\sqrt{\alpha}$. Near $\lambda \sim 0$,

$$|\omega_+ - \omega_-| \simeq 2\sqrt{A_{i,n}} + \lambda^2/4\sqrt{A_{i,n}} . \tag{32}$$

Spectral weights of the two peaks, slightly off resonance, differ from each other by a factor of $\lambda/\sqrt{A_{i,n}}$.

(ii) Nonresonant limit ($\lambda^2/A_{i,n} \gg 1$). Formally, the interacting Green's function (Eq. 24) still has two poles. But we show in the following that only one of the poles carries any significant weight and thus there is no resonance splitting phenomenon. Doing the appropriate expansion ($\lambda^2/A_{i,n} \gg 1$) in Eq. (27), we get

$$\rho_{i,n}(\omega) \simeq \pi(1 + 2A_{i,n}/\lambda^2)^{-1}[(1 + A_{i,n}/\lambda^2)\delta(\omega - \omega^0_{i,n} - A_{i,n}/\lambda) + (A_{i,n}/\lambda^2)\delta(\omega - \omega^0_{i,n} + \lambda + A_{i,n}/\lambda)] . \tag{33}$$

We immediately note that the second peak at $\omega = \omega^0_{i,n} - \lambda - A_{i,n}/\lambda$ arries an insignificantly small spectral weight $O(A_{i,n}/\lambda^2)$ compared to the other peak at $\omega = (\omega^0_{i,n} + A_{i,n}/\lambda)$ which has a strength close to unity. Thus, away from resonance, there is really one quasiparticle peak shifted from the noninteracting value $\omega^0_{i,n}$ by an amount $A^0_{i,n}/\lambda$ due to electron-phonon renormalization effect.

In the above analysis we have clearly shown that the degeneracy of the two noninteracting states—an electron in the Landau level (i, n) and the composite state of an electron in Landau level $(i, n-1)$ and an LO phonon of energy ω_{LO} at $\omega_{LO} = \omega_{ci}$—is lifted by the Landau-level–optical-phonon interac-

tion, and the simple pole of the noninteracting Green's function at $\omega^0_{i,n}$ splits into a doublet given by $(\omega^0_{i,n} \pm \sqrt{A_{i,n}})$ with equal spectral weight as a direct result of the phonon-induced resonant coupling. Optical transitions to the level (i, n) will now show two peaks corresponding to the splitting of the (i, n) level at the resonance point.

IV. RESONANT MAGNETO-OPTICAL ABSORPTION

The resonant splitting discussed in the last section can be seen in optical absorption provided certain conditions relating widths of the Landau levels and the splitting itself are satisfied. The relevant cross section for magnetoabsorption experiment (or Raman scattering) is, in the long-wavelength limit, provided by the imaginary part of the polarizability function $\Pi(\omega)$ of the coupled electron-phonon system. In the lowest order, it is given by the simple bubble shown in Fig. 3(b). As noted before, since the observation of resonant splitting is intimately connected to its magnitude being larger than the widths of the absorption peaks, we must take account of the Landau-level widths in calculating magnetoabsorption.

Note that the LO phonon themselves, even though they are responsible for the splitting phenomenon through the resonant Landau-level coupling, do not give rise to any finite imaginary part of the electronic self-energy. This is in sharp contrast to the corresponding three-dimensional bulk situation[27] and is a reflection of the quasi-zero-dimensional nature of the dynamics involved. Consequently, the resonance is much sharper in these two-dimensional systems giving rise to delta-function spectral peaks (in the absence of impurity damping). Thus the resonant-splitting phenomenon is inherently sharper in two dimensions compared to the three-dimensional situation.

We introduce damping in the theory phenomenologically by giving a width $\Gamma_{i,n}$ to the Landau-level energy $E_{i,n}$ as $(E_{i,n} + i\Gamma_{i,n})$. This phenomenological damping can be considered to be due to the background impurity and acoustic-phonon scattering. The polarizability function, $\Pi_{N_0N}(\omega)$, involving transitions from a lower Landau level $N_0 \equiv (j, n_0)$ to the upper level $N \equiv (i, n)$ at resonance $(\omega_{ci} = \omega_{LO})$, is then given by

$$\Pi_{N_0N}(\omega) \propto \int \frac{dE}{2\pi} G^0_{N_0}(E) G_N(E+\omega) = (\omega - \omega_0)(\omega - \omega_1 - i\gamma)^{-1} \times (\omega - \omega_2 - i\gamma)^{-1}, \quad (34)$$

where $\omega_0 = (\omega^0_{i,n} - \omega^0_{j,n_0})$, $\omega_{1,2} = \omega_0 \pm \sqrt{A_{i,n}}$, and γ is determined by the widths Γ_{j,n_0}, $\Gamma_{i,n}$, and $\Gamma_{i,n-1}$ of the three Landau levels involved. It is approximately given by

$$\gamma = \Gamma_{j,n_0} + \tfrac{1}{2}(\Gamma_{i,n} + \Gamma_{i,n-1}). \quad (35)$$

We point out that the Green's function of the lower level $N_0 \equiv (j, n_0)$ is taken to be noninteracting since we are interested in the resonant-coupling phenomenon only and assume the nonresonant electron-phonon renormalization effect to be small in most of these weakly polar quasi-two-dimensional systems under consideration.

Thus the behavior of the absorption spectrum, which is proportional to the imaginary part of the polarizability function, is given by

$$\mathrm{Im}\Pi_{N_0N}(\omega) \propto (\omega - \omega_0)(\omega_1 - \omega_2)^{-1} \times \left(\frac{\gamma}{(\omega - \omega_1)^2 + \gamma^2} - \frac{\gamma}{(\omega - \omega_2)^2 + \gamma^2} \right). \quad (36)$$

Expression (36) shows that for the two resonant absorption peaks to be well resolved, the resonant splitting $\delta = 2\sqrt{A_{i,n}}$ must satisfy the inequality $\delta > \gamma$. For sufficiently pure systems at low temperatures, it should be possible to satisfy this condition. Thus absorption spectra under the resonance condition would show two Lorentzian peaks [given by Eq. (36)] at $\omega_{1,2}$ indicating a splitting of the original cyclotron transition at the single frequency ω_0. This will show up in an experiment as the magnetic field is swept through the resonance value $B_{res} = m_i c\omega_{LO}/e$ to achieve the condition $\omega_{LO} = \omega_{ci} = eB/m_i c$.

In the general situation away from resonance $(\lambda = \omega_{ci} - \omega_{LO} \neq 0)$, one can evaluate the polarizability function given by Eq. (34) by using the noninteracting and the interacting Green's functions, respectively. In the absence of any damping, we get,

$$\mathrm{Im}\Pi_{N_0N}(\omega) \propto \tfrac{1}{2}(\lambda^2 + 4A_{i,n})^{-1/2} \times \{ [(\lambda^2 + 4A_{i,n})^{1/2} + \lambda]\delta(\omega - \tilde{\omega}_1) + [(\lambda^2 + 4A_{i,n})^{1/2} - \lambda]\delta(\omega - \tilde{\omega}_2) \}, \quad (37)$$

where

$$\tilde{\omega}_{1,2} = \omega_0 - \tfrac{1}{2}\lambda \pm \tfrac{1}{2}(\lambda^2 + 4A_{i,n})^{1/2}. \quad (38)$$

In writing Eqs. (37) and (38) we make use of the definitions of $\omega_\pm$ given by Eq. (25). Notice that for $\lambda = 0$ (i.e., at resonance), $\tilde{\omega}_{1,2}$ reduces to $\omega_{1,2}$ and Eq. (37) reduces to

$$\mathrm{Im}\Pi_{N_0N}(\omega) \propto \tfrac{1}{2}[\delta(\omega - \omega_1) + \delta(\omega - \omega_2)], \quad (39)$$

which is the expected $\gamma \to 0$ limit of Eq. (36). From Eq. (37) we find that, off resonance $(\lambda^2 \gg A_{i,n})$, only one of the absorption peaks (at $\tilde{\omega}_1$) is predominant, whereas near resonance $(\lambda^2 \ll A_{i,n})$, both the absorption peaks have almost equal weight.

In the above analysis the absorption peaks are assumed to be given by the poles of the conductivity function (equivalent to the polarizability function in the long-wavelength limit) described by the leading-order simple bubble diagram shown in Fig. 3(b). This is the leading-order irreducible polarization diagram.[37] The actual response function is however the corresponding reducible function, which consists of the series of bubble diagrams connected by LO-phonon lines as shown in Fig. 3(c). In the literature,[28] this has been referred to as the phonon-induced electron-light vertex correction since the electron- (external) photon vertex is being corrected by virtual phonon processes. Ordinary electron-phonon vertex correction is still being neglected as in Sec. III. The process described by Fig. 3(c) has a very simple physical meaning. The external photon induces an electron-hole bubble [Fig. 3(b)], but a chain of such bubbles may be formed by virtual phonon processes and all of them have to be taken into account in considering the system response.

Summing the geometric series for all the bubbles, we get the reducible polarizability, $\tilde{\Pi}_{N_0N}(\omega)$, of the resonance process, including the phonon-induced electron-light vertex correction. The absorption peaks are now given by the poles of $\tilde{\Pi}_{N_0N}(\omega)$ or equivalently by the equation

$$1 - g^2 D_{\mathrm{LO}}(\omega)\Pi_{N_0N}(\omega) = 0 , \tag{40}$$

where

$$g^2 = \sum_{\vec{q}} |g_{(ij)(nn_0)}(\vec{q})|^2 . \tag{41}$$

For g^2 very small, Eq. (40) gives the same absorption peaks as given by the poles of $\Pi_{N_0N}(\omega)$. In general, however, solutions of Eq. (40) will be shifted from the poles $\omega = \omega_{1,2}$ of $\Pi_{N_0N}(\omega)$ (in the absence of damping). In the general case, Eq. (40) has to be solved numerically, and for large g^2, may give results very different from those given by the poles of $\Pi_{N_0N}(\omega)$. In Appendix B, we consider a simple solution of Eq. (40) assuming the shifts produced by the phonon-induced electron-light vertex correction to be small.

V. DISCUSSIONS

In the previous sections we have shown that electron–LO-phonon interaction in quasi-two-dimensional electronic systems in the presence of a magnetic field may give rise to qualitatively new features in magnetoabsorption. Under suitable conditions, a single absorption peak will split into a doublet as the magnetic field is swept through the resonance value $B_{\mathrm{res}} = m_i \omega_{\mathrm{LO}} c/e$. The peak-to-peak splitting at resonance is given by $2\sqrt{A_{i,n}}$ and is thus proportional to the square root of electron-LO phonon coupling strength in the system. Thus the phenomenon of the resonant magnetoabsorption in two-dimensionally confined charge carrier systems offers a unique way of seeking information regarding electron–optical-phonon coupling, including features special to particular systems. This is particularly significant in view of the rather unexpected experimental indication towards an enhanced electron–LO-phonon coupling in these quasi-two-dimensional polar systems, as reflected in luminescence experiments on GaAs-$Ga_xAl_{1-x}As$ quantum wells.[19] Resonance splitting phenomenon as discussed in Sec. III and the associated magneto-optical anomalies in absorption (or Raman scattering) experiments as discussed in Sec. IV provide a means of experimentally estimating the electron–LO-phonon coupling strength. We point out that the resonance splitting phenomenon is independent of the nature of electron–LO-phonon coupling in the system and takes place even when the coupling is weak. No less significant is the fact that in these novel quasi-two-dimensional systems, there may be coupling between electrons and more than one kind of phonons (interface phonons, surface and possible others). These different LO phonons will in general have different frequencies and hence the resonance condition $\omega_{\mathrm{LO}} = \omega_{ci}$ will be achieved at different magnetic fields. Thus a careful investigation of the magneto-optical anomalies should indicate the importance of the different electron–LO-phonon coupling.

We believe that the experiment most suitable to observe the splitting phenomenon in two dimensions and to use it as a systematic tool for investigating the electron–LO-phonon coupling is the valence-to-conduction-band cyclotron resonance. This will avoid the complications arising from reststrahlen absorption in direct cyclotron resonance (from $n-1$ to n within the same subband) when $\omega_{ci} = \omega_{\mathrm{LO}}$. Furthermore, these investigations do not require free carriers in the conduction-band Landau states. Consequently linewidths arising from impurity scattering (even compared to modulation-doped situation) is minimized, along with thermal broadening by restricting measurements at low temperatures. This is in contrast to the dc manifestation of the phenomena whose existence depends upon the presence of free carriers and relatively high temperatures for a significant thermal population of relevant phonons. We therefore suggest such studies to be carried out on superlattices and heterojunctions such as the GaAs-$Al_{1-x}Ga_xAs$ system.

We have emphasized in Secs. III and IV that the resonance splitting phenomenon is much sharper

in two dimensions compared to the bulk situation due to the absence of the continuum associated with the z component of electron momentum in this case. This enables one to use the splitting in magnetoabsorption as an effective tool to estimate the electron–LO-phonon coupling in two dimensions. Another important difference is the fact that the splitting at resonance is enhanced in these two-dimensional systems compared to the three-dimensional case. From Eq. (26) and (37) it follows that the leading-order magnetic-field dependence of the resonance splitting is $B_{\rm res}^{3/4}$ whereas the dependence in the bulk case[26] is $B_{\rm res}^{1/2}$. Thus other factors being identical, the magnetic-field dependence of the resonance splitting is enhanced in two dimensions. This combined with the sharpness of the splitting phenomenon (due to lack of any z motion) argues strongly in favor of a study of magneto-optical anomalies in these systems.

There are several directions in which the results of Secs. III and IV can be refined, some of which have already been noted. For a quantitative comparison with experimental results, when they are available, one must evaluate the explicit matrix elements for the coupling strength. This involves a knowledge of the subband envelope functions which unfortunately are unavailable for the most important III-V quasi-two-dimensional systems. One might have noted that we treat the problem as a one-electron problem treating interaction with LO phonons only and neglecting all electron-electron interaction and screening effects. This is justifiable because the resonance splitting phenomenon is independent of these effects. The coupling strength will depend on screening (would in general be reduced by it). Since in our theory we do not pretend to calculate the coupling strength quantitatively (and, as we have noted earlier, this may indeed be a difficult task because Fröhlich theory itself may not be valid), we have neglected screening from our considerations. Formal inclusion of screening in the theory is straightforward[39] and we argue that the experimental investigation of the magneto-optical anomalies would yield the effective coupling strength including screening and other effects. Exchange-correlation effects from electron-electron interaction are not expected to be important in these systems due to the small effective mass and large dielectric functions and consequently the small r_s values involved. In the actual experimental situation, neither of these two effects would probably play any important role since the valence-to-conduction-intersubband magnetoabsorption experiment in GaAs-$Ga_xAl_{1-x}As$ or InAs-GaSb structures are performed in the absence of any mobile charge carriers. We have also neglected electron-phonon vertex correction in our work. This is less justifiable from a strictly theoretical ground since the usual arguments of Migdal's theorem[40] for the smallness of such terms are invalid[41] for the high-frequency optical phonons. In addition, the validity of Migdal's theorem in these novel two-dimensional systems has been questioned[42] on the basis of the fact that the relevant electron and phonon energies in these systems may be comparable. Consistency thus demands that the theory includes all ladder-bubble vertex corrections to the simple bubble of Fig. 3(b). This is an extremely difficult task and we argue that as long as the electron-phonon coupling constant is small, these higher-order diagrams are negligible and we can work within Migdal's theorem. If an experimental analysis along the line suggested here indicates the coupling strength to be appreciable, one must try to include the electron-phonon vertex correction in the theory. Before concluding we remark that the level-splitting phenomenon due to the lifting of degeneracy by the electron–LO-phonon interaction should in principle be present in these quasi-two-dimensional systems even in the absence of a magnetic field. The degeneracy is then between an electron in the ith subband and the composite state of an electron in $(i-1)$ subband plus an LO phonon of energy $\omega_{\rm LO}$ under the condition $\omega_{\rm LO}=E_i-E_{i-1}$. But the continuum associated with the free two-dimensional motion within each subband makes the level-crossing phenomenon in the absence of any magnetic field inherently much less sharp than the corresponding situation in presence of the magnetic field discussed in this paper. In Appendix C we give a brief description of the situation in the absence of any magnetic field.

Finally, it should be remarked that a resonant Landau level-optical phonon coupling and the consequent level-splitting phenomenon is, in principle, possible even when $\omega_{\rm LO}=n\omega_{ci}$, where n is any integer (not necessarily unity) whence the Nth and the $(N+n)$th Landau levels (in the ith two-dimensional subband) are coupled by an optical phonon. In the explicit analysis given in this paper, we consider the most usual case of $n=1$. However, the same analysis applies equally well when n is some other integer. The spectral weights of the corresponding splitting for $n>1$ is expected to be less than that for $n=1$ since it is energetically much more favorable to couple two adjacent landau levels. Actual observability in magneto-optical absorption of such "higher" $(n>1)$ resonances depends crucially on the spectral weights and on whether transitions to such coupled levels for $n>1$ are possible under any particular experimental situation. Recently static magneto-phonon resonances have been observed[43] in two-dimensional systems.

Since the effect described in the paper is a high-frequency generalization of the static resonances, it can be hoped that the resonant splitting in magnetoabsorption would soon be seen in these systems and would lead to a precise estimate of the electron–optical-phonon coupling strength in these confined systems.

ACKNOWLEDGMENTS

This work was supported by AFOSR (Contract No. 78-3530) and by ONR (Contract No. N00014-77-C-0397). One of us (A.M.) acknowledges the support of the Alfred P. Sloan Foundation.

APPENDIX A

Here we consider the diagonal contribution to the Landau-level self-energy $\Sigma_{i,nn'}(i\omega_l)$ and show that it does not lead to any resonance splitting phenomenon. Away from resonance however, this gives the leading-order correction to the Landau-level energy $E_{i,n}=E_i+(n+\frac{1}{2})\hbar\omega_{c_i}$. In general, a leading-order self-energy contribution $\Sigma_{ij,nn'}(i\omega_l)$ is given by

$$\Sigma_{ij,nn'}(i\omega_l)=-\frac{1}{\beta\hbar}\sum_{i\nu_m}\sum_{\vec{q}} G^0_{j,n'}(i\omega_l-i\nu_m) \times |g_{(ij)(nn')}(\vec{q})|^2 D_{LO}(i\nu_m,\vec{q}). \tag{A1}$$

This most general self-energy element can thus be off diagonal in both subband and Landau-level indices. We can neglect the off-diagonal elements in subband indices as being insignificant compared to the intrasubband diagonal contribution[5] and write $\Sigma_{ij,nn'}\equiv\Sigma_{i,nn'}\delta_{ij}$. In Secs. III and IV of the main text of the paper we are concerned with the resonant contribution to the self-energy given by the $n'=n-1$ Landau level under the condition $\omega_{ci}=\omega_{LO}$.

The diagonal contribution to the self-energy is obtained by putting $i=j$ and $n'=n$ in (A1) and after doing the frequency sum we get

$$\Sigma_{i,nn}(i\omega_l)=A_{inn}(i\omega_l-\omega^0_{i,n}-\omega_{LO})^{-1}, \tag{A2}$$

where A_{inn} is obtained by substituting n for $(n-1)$ in Eqs. (20) and (21) of the main text. Quasiparticle peaks are of course given by the solutions of Dyson's equation. Putting Eq. (A2) in the Dyson equation for the Green's function of the (i,n) level and taking the imaginary part, we get the following spectral function:

$$\rho_{i,nn}(\omega)=\pi(\omega^2_{LO}-4A_{inn})^{-1/2}(\omega-\omega^0_{i,n}-\omega_{LO}) \times[\delta(\omega-\omega^0_{i,n}-x_1)-\delta(\omega-\omega^0_{i,n}-x_2)], \tag{A3}$$

where

$$x_{1,2}=\tfrac{1}{2}[\omega_{LO}\pm(\omega^2_{LO}-4A_{inn})^{1/2}]. \tag{A4}$$

Other quantities in Eq. (A3) are the same as those defined in the main text. Clearly in the weak-coupling limit ($\omega^2_{LO}\gg 4A_{inn}$), Eq. (A3) reduces to

$$\rho_{i,nn}(\omega)\simeq\frac{2\pi}{(1-2A_{inn}/\omega^2_{LO})} \times\left[\left(1-\frac{A_{inn}}{\omega^2_{LO}}\right)\delta(\omega-\omega^0_{i,n}-x_2) -\frac{A_{inn}}{\omega^2_{LO}}\delta(\omega-\omega^0_{i,n}-x_1)\right], \tag{A5}$$

with

$$x_{1,2}\simeq\omega_{LO}\left(1-\frac{A_{inn}}{\omega^2_{LO}}\right),\ \frac{A_{inn}}{\omega_{LO}}. \tag{A6}$$

Thus, provided $A_{inn}\ll\omega^2_{LO}$, the spectral function (as renormalized by the diagonal part of self-energy) has just one peak at $\omega=\omega^0_{i,n}+x_2\simeq\omega^0_{i,n}+O(A_{inn}/\omega_{LO})$. Thus the diagonal part of self-energy, as expected, takes no part in the resonance splitting phenomenon. Its only role is to renormalize the Landau-level energies by small corrections in the weak-coupling limit.

APPENDIX B

The absorption peaks, including the phonon-induced electron-light vertex correction, are given by Eq. (40) of the main text. Neglecting the phenomenological damping, we can write Eq. (40) as

$$1-g^2\frac{2\omega_{LO}}{(\omega^2-\omega^2_{LO})}C\left(\frac{1}{\omega-\omega_1}+\frac{1}{\omega-\omega_2}\right)=0, \tag{B1}$$

where C is the constant of proportionality in Eq. (34) which in general depends on Landau indices. Equation (B1) is a general quartic equation in ω and may have more than two solutions. Also, absorption (given by $\mathrm{Im}\tilde{\Pi}$ where $\tilde{\Pi}$ is defined in the text) may show interesting structure as a function of ω. In this appendix we assume g^2 to be very small (a reasonable assumption as argued in the text) and shall calculate leading-order correction to the absorption peaks $\omega=\omega_{1,2}$ in presence of the phonon-induced electron-light vertex correction. We assume

$$\omega=z+\eta, \tag{B2}$$

where $z=\omega_{1,2}$ are the solutions to Eq. (B1) when $g^2\equiv 0$. The basic assumption is $|\eta|\ll|z|$. Putting Eq. (B2) into Eq. (B1) and neglecting terms of $O(\eta^2)$ and higher, we get

$$\eta=\frac{g^2C_1[2z-(\omega_1+\omega_2)]}{4z^3-3z^2(\omega_1+\omega_2)+2z(\omega_1\omega_2-\omega_{LO}^2)}, \quad \text{(B3)}$$

where

$$C_1=2\omega_{LO}C. \quad \text{(B4)}$$

Thus the absorption peaks in the presence of phonon-induced electron-light vertex correction are at frequencies

$$\omega=z+\eta=\omega_{1,2}+\eta_{1,2}, \quad \text{(B5)}$$

where

$$\eta_1=\frac{g^2C_1(\omega_1-\omega_2)}{\omega_1(\omega_1^2-\omega_1\omega_2-2\omega_{LO}^2)} \quad \text{(B6)}$$

and η_2 is obtained by interchanging $\omega_1 \leftrightarrow \omega_2$ in Eq. (B6). Clearly the assumption $|\eta| \ll |z|$ is valid if g^2 is very small.

APPENDIX C

In this appendix we consider the self-energy corrections to carriers confined in two-dimensional subbands due to electron–LO-phonon interaction in the absence of a magnetic field and investigate whether a resonance situation is possible when two subbands i and j are coupled by LO-phonon-mediated intersubband interaction with $\hbar\omega_{LO}=(E_i-E_j)$.

The leading-order self-energy correction to the ith subband due to electron-phonon interaction in the absence of a magnetic field is given by

$$\Sigma_{ij}(\vec{k},i\omega_l)=-\frac{1}{\beta\hbar}\sum_{i\nu_m}\int\frac{d^2q}{(2\pi)^2}G_j^0(\vec{k}-\vec{q},i\omega_n-i\nu_m)\times V_{ijji}(\vec{q})D_{LO}(i\nu_m), \quad \text{(C1)}$$

where $V_{ijlm}(\vec{q})$ is the effective electron-electron interaction in two dimensions mediated by LO-phonon coupling and is given by Eq. (5) or Eq. (8) of the main text depending on the kind of LO phonon involved. G_j^0 in Eq. (C1) is the noninteracting electron Green's function for the jth subband and is given by

$$G_j^0(\vec{k},i\omega_l)=[i\omega_l-\hbar^{-1}(E_j+\hbar^2k^2/2m_j-\mu)]^{-1}. \quad \text{(C2)}$$

$\vec{k}$ is a two-dimensional wave vector and μ is the chemical potential.

We point out that it is the off-diagonal component (V_{ijji} with $i\neq j$) of the interaction that may give rise to the resonant coupling in this situation as well. Again, away from resonance ($\hbar\omega_{LO}\neq E_i-E_j$) such self-energy corrections will be negligible compared to the diagonal term.

To evaluate the self-energy given by Eq. (C1) we express $V_{ijji}(\vec{q})$ as

$$V_{ijji}(\vec{q})=F_{ijji}(q)/q, \quad \text{(C3)}$$

where $F_{ijji}(q)$ contains matrix elements taken with subband wave functions along with a multiplicative polar coupling. We assume $F_{ijji}(q)$ to be a slowly varying function of the wave number so that it can be taken out of $\vec{q}$ integration in an averaged sense. With these assumptions we can calculate $\Sigma_{ij}(\vec{k},i\omega_l)$ given by Eq. (C1). Actually, electron-phonon self-energies are not very $\vec{k}$ dependent and hence it suffices to calculate $\Sigma_{ij}(\vec{k},i\omega_l)$ for $\vec{k}=0$. We get, in the $T\to 0$ limit,

$$\Sigma_{ij}(\vec{k}=0,i\omega_l=\omega+i\delta)=\Sigma_1+i\Sigma_2, \quad \text{(C4)}$$

where

$$\Sigma_1=F\left(\frac{1}{q_1}\ln\left|\frac{q_1-k_F}{q_1+k_F}\right|+\frac{1}{q_2}\ln\left|\frac{q_2+k_F}{q_2-k_F}\right|\right), \quad \text{(C5)}$$

$$\Sigma_2=-\left(\frac{\pi\hbar F}{m_j}\right)\left\{H(\omega-\omega_{LO})\Big/\left[\frac{2\hbar}{m_j}\left(\omega+\frac{\hbar k_F^2}{2m_j}-\omega_{LO}\right)\right]^{1/2}+H(-\omega-\omega_{LO})\Big/\left[\frac{2\hbar}{m_j}\left(\omega+\frac{\hbar k_F^2}{2m_j}+\omega_{LO}\right)\right]^{1/2}\right\}, \quad \text{(C6)}$$

where F is some averaged value of F_{ijji} and $q_{1,2}$ are given by the solutions of the equation (H is the Heaviside unit step function)

$$q_{1,2}^2=k_F^2+(2m_j/\hbar)(\omega\mp\omega_{LO}), \quad \text{(C7)}$$

with

$$\hbar^2k_F^2=2m_j(\mu-E_j). \quad \text{(C8)}$$

We are clearly concerned with the case where $E_j<\mu<E_i$.

In the resonant situation $\hbar\omega_{LO}=E_i-E_j$ and using this in Eqs. (C7) and (C8) we get

$$\hbar^2q_{1,2}^2=2m_j(\hbar\omega-E_i+\mu),\quad 2m_j(\hbar\omega-E_i+\mu-2E_j). \quad \text{(C9)}$$

Use of Eq. (C9) in the expression for the self-energy shows that no singular behavior of the resonance self-energy results as a function of frequency. This should be contrasted with the case in the presence of a magnetic field [cf. Eq. (22)] which is very singular at $i\omega_n=\omega_{i,n}^0$ if $\omega_{LO}=\omega_{ci}$. The quasiparticle spectral function in this situation is clearly not a delta function (due to the

actual damping caused by the LO phonons) and even though the Dyson equation for the quasiparticle cannot be explicitly solved in this case [due to the logarithmic functions in Eq. (C5)], it is clear that the sharp resonance splitting phenomenon of the situation under an external magnetic field is just not present in this case.

It is obvious that without the magnetic field, the two-dimensional continuum associated with the wave vector $\vec{q}$ of the electrons makes it impossible to have a sharp resonance splitting phenomenon. In presence of the magnetic field the problem is a quasi-zero-dimensional problem and the resonance structure is sharp.

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