Micromagnetics of twisted amorphous ribbons

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Solutions of the micromagnetic equations for a twisted ribbon are given on the assumption of isotropic magnetostriction and lack of crystalline anisotropy. This is the case of amorphous magnetic materials. Both exchange forces and tension-induced anisotropy are considered together with the torsion effects, showing the small influence of the exchange term in most practical cases. Results agree with remanence and Matteucci-effect measurements in annealed samples of Metglas 2826.

I. INTRODUCTION

Torsion in a thin ribbon produces an inhomogeneous distribution of shearing stresses which varies linearly along the thickness (x axis in Fig. 1); see Sec. II A). In a twisted magnetostrictive material, this stress distribution will give rise to an inhomogeneous magnetoelastic anisotropy. If the magnetostriction is not isotropic and crystalline anisotropy is present, the equilibrium position of the atomic moments is difficult to determine. Amorphous materials exhibit no crystalline anisotropy, and their magnetostriction is fully isotropic, so that the easy directions will lie along the tension or compression lines into which the shear stress can be decomposed, depending upon the sign of the magnetostriction constant λ_s. However, at $x=0$, exchange forces will tend to smooth the transition between the two easy directions, which make an angle of $\frac{1}{2}\pi$ rad at this point (see Sec. II).

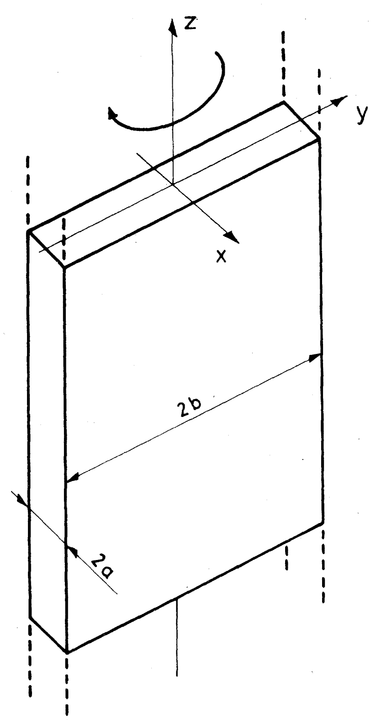

FIG. 1. Orientation of a ribbon of thickness $2a$ and width $2b$ with respect to the xyz axis. Torsion is applied about the z axis.

For this reason, determination of the arrangement of the atomic moments in a twisted amorphous ribbon requires a micromagnetic calculation, similar to that of a wall between ferromagnetic domains, in which both exchange and anisotropy are involved. This distribution being known, it is easy to calculate the longitudinal remanence as well as the transverse or "circular" one.[1] These quantities can be experimentally checked by measuring the usual magnetization curves and the Matteucci effect, respectively.[2–4]

Because experimental arrangements usually involve some tension stress in order to keep the sample in a fixed position during the application of torsion, such a situation has been also considered in the present work.

II. THEORETICAL MODEL

A. Exchange-force contribution

In a first step of calculation, only exchange forces together with torsion-induced anisotropy are taken into account. If the ribbon is assumed to be infinitely wide, i.e., $b/a >> 1$, the complicated stress distribution caused by torsion[5] is reduced to a single component

$$\sigma_{yz}=2\mu\xi ax=\tau x \quad , \tag{1}$$

where μ is the shear modulus of the material, ξ is the torsion angle per unit length, and x is measured in units of the half thickness a, so that τ is the maximum value of the stress occurring at the ribbon surface.

The magnetoelastic energy density can now be found by introducing the value of σ_{yz} in the general expression

$$W_{\mathrm{me}} = -\tfrac{3}{2}\lambda_{100}(\sigma_x\alpha_1^2+\sigma_y\alpha_2^2+\sigma_z\alpha_3^2) - 3\lambda_{111}(\sigma_{xy}\alpha_1\alpha_2+\sigma_{yz}\alpha_2\alpha_3+\sigma_{zx}\alpha_3\alpha_1) \quad (2)$$

valid for a cubic crystal[6] and setting $\lambda_{100}=\lambda_{111}=\lambda_s$ for isotropic magnetostriction in our case. This magnetoelastic term is favorable for the magnetization to lie in the yz plane, as is the demagnetizing factor along the x direction. So if the magnetization vector M_s makes an angle θ with the z axis at a point x (see Fig. 2), we can write

$$W_{\mathrm{me}} = -(\tfrac{3}{2})\lambda_s\tau x \sin 2\theta \ , \quad (3)$$

which implies that the easy directions are at $\pm 45°$ to the z axis. The exchange term is now

$$W_{\mathrm{ex}} = \left[\frac{C}{2a^2}\right]\left[\frac{d\theta}{dx}\right]^2 \ , \quad (4)$$

C being the exchange constant. The energy per unit area due to the magnetization distribution can be found by integrating the total energy density along the thickness

$$E = 2a\int_0^1 (W_{\mathrm{me}}+W_{\mathrm{ex}})\,dx \ . \quad (5)$$

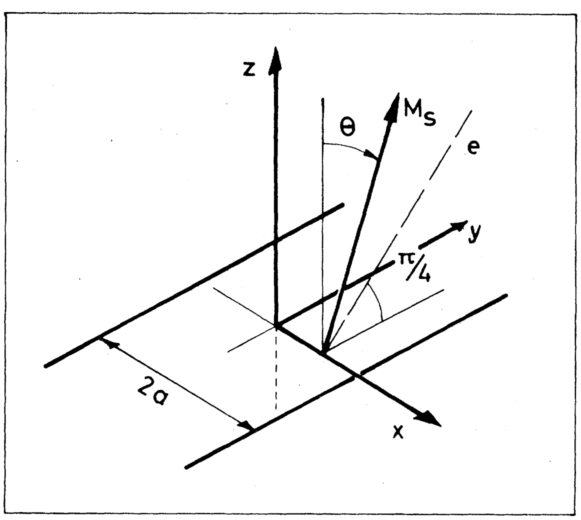

FIG. 2. Equilibrium position of the magnetization vector M_s. $2a$ is the ribbon thickness and e the easy direction arising from torsion.

Minimization of this energy leads to the following Euler equation:

$$\left[\frac{C}{a^2}\right]\left[\frac{d^2\theta}{dx^2}\right] + 3\lambda_s\tau x\cos 2\theta = 0 \ . \quad (6)$$

Adequate boundary conditions are[7]

$$\theta(0)=0,\quad \theta'(\pm 1)=0 \quad (7a)$$

or

$$\theta(0)=\pi,\quad \theta'(\pm 1)=0 \quad (7b)$$

opposite directions being equivalent under the influence of stress. Equations (7a) and (7b) give the same results with a shift of π rad. The possibility that $\theta(0)=\frac{1}{2}\pi$ or $\frac{3}{2}\pi$ will not be taken into account, owing to the demagnetizing factor along the y axis which is much greater than in the z direction.

Equation (6) has been solved by the finite difference method[8] using a constant grid interval of 0.02 for x. A limiting value of 10^{-3} for $\theta'(1)$ was imposed. No significant difference was found by increasing the number of points in the calculation nor by lowering the final value of $\theta'(1)$.

Results for different values of the parameter

$$T = 3\lambda_s\tau a^2/C \quad (8)$$

are shown in Fig. 3 for $0<x<1$. As a consequence of the symmetry of the problem, the magnetization distribution on the $-1<x<0$ part of the ribbon will be the same but inverted, so that a kind of 90° wall centered at the plane xy is formed by the effect of torsion.

When a magnetic field sufficiently high to saturate the sample is applied along the z axis and then removed, the remanence can be calculated as

$$M_r = M_s\int_0^1 \cos\theta\,dx \quad (9)$$

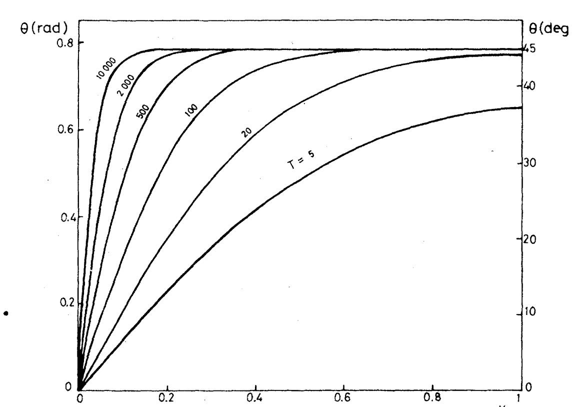

FIG 3. Variation of the angle between the magnetization vector and the ribbon axis for different values of the torsion-to-exchange ratio T.

giving a value of $0.77 M_s$ for $T=100$. Considering the experimentally obtained value of the shear modulus of about 4×10^{10} Nm^{-2}, a saturation magnetostriction constant of about 10^{-5} (Ref. 9) and taking the exchange constant of the iron $C=2\times10^{-11}$ Jm^{-1}, the mentioned value of $T=100$ corresponds to a twist of about 1° in a sample 10 cm long and 50 μm thick such as those used in the measurements (see Sec. III). It is experimentally found that such a small twist does not produce appreciable change in the magnetization, so that another mechanism must be responsible for the observed behavior.

B. Influence of the tension-induced anisotropy

As mentioned above, the usual arrangements used in torsion experiments include also a tension stress. On the other hand, in amorphous materials any kind of anisotropy must be stress-induced, so that we shall always write an energy term of the form:

$$W_{\mathrm{me}} = -\tfrac{3}{2}\lambda_s \sigma \cos^2\theta \quad . \tag{10}$$

As deduced from Eq. (2) when θ is the angle between the magnetization vector and the direction of the applied stress σ.

For a material of positive magnetostriction and with an applied tension along its axis, the angle θ coincides with that appearing in Fig. 2. The equilibrium position of the atomic moments in the ribbon can be found by the same procedure used in Sec. II A, and leads to the following equation:

$$\left(\frac{d^2\theta}{dx^2}\right) = S\sin 2\theta - Tx\cos 2\theta \quad . \tag{11}$$

Here S is the ratio between the tension-induced anisotropy and the exchange term

$$S = \tfrac{3}{2}\lambda_s \sigma a^2/C \tag{12}$$

and T has the same value as in the preceding section. The boundary conditions are those given in expression (7).

Numerical solutions of Eq. (11) can be introduced in expression (9) to get the longitudinal remanence, which is found to increase quickly with S, for a given value of T. Nevertheless, the relative influence of

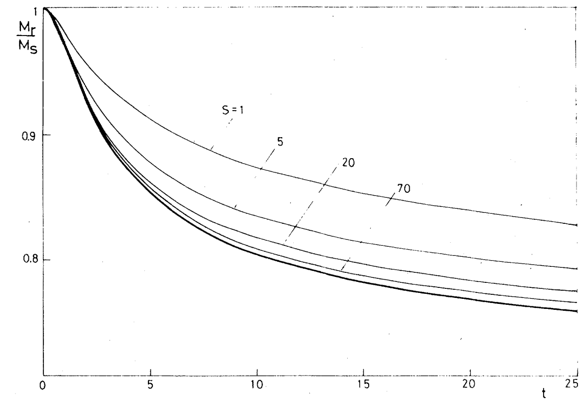

FIG. 4. Remanence of the longitudinal magnetization vs reduced torsion t (see the text), for different values of the tension to exchange ratio S. The lowest curve, represented with a thick line, corresponds to a zero-exchange term and is given by expression (15).

the exchange and tension terms can be emphasized by taking as a constant the ratio between the tension and torsion stresses. This is done in Fig. 4, where curves for different values of S are plotted as a function of parameter t which expresses the torsion stress in units of the applied tension:

$$t = T/S = 2\tau/\sigma \quad . \tag{13}$$

For increasing values of S, the remanence curves tend to a situation at which exchange contribution can be disregarded. Although this will be the case only for $S=\infty$, in practice, this situation is reached for a value as low as 100 for S, corresponding to a weight of 1 g attached at the end of a sample 1.2 mm wide and 50 μm thick, with the values of λ_s and C previously used.

If, as seen, exchange forces are not important to explain the magnetization distribution in a twisted ribbon, they can be dropped out from Eq. (11), and then the equilibrium angle θ for the magnetization vector is given by

$$\theta = \tfrac{1}{2}\tan^{-1}(tx) \quad . \tag{14}$$

Now an analytical expression for the remanence in the z direction can be found by performing the integration in Eq. (9). This gives

$$M_r = \left(\frac{M_s}{\sqrt{2}t}\right)\left([1+t^2-(1+t^2)^{1/2}]^{1/2} - \tfrac{1}{2}\ln\{2(1+t^2)^{1/2} - 2[1+t^2-(1+t^2)^{1/2}]^{1/2} - 1\}\right) \tag{15}$$

represented as a thick curve in Fig. 4. In turn the expression

$$M_s\int_0^1 \sin\theta\, dx = \left(\frac{M_s}{\sqrt{2}t}\right)\left([1+t^2+(1+t^2)^{1/2}]^{1/2} - \ln\{[1+(1+t^2)^{1/2}]^{1/2} + (1+t^2)^{1/4}\} - 0.533\right) \tag{16}$$

will account for an averaged transverse magnetization at the remanence. This corresponds to the circular magnetization in ferromagnetic wires and its variation produces an emf appearing at the ends of the sample, known as the Matteucci effect, which allows its measurement.

III. EXPERIMENTAL RESULTS

Measurements of both longitudinal and circular remanence have been performed in annealed samples of Metglas 2826. They were ribbons 10 cm long, 1.2 mm wide, and 0.05 mm thick. As received, some anisotropy is present in the samples as a result of the fabrication procedure.[10–13] After annealing 30 min at 320 °C in a high-purity argon atmosphere, most of this anisotropy disappeared, and saturation occurs at fields lower than 250 Am^{-1}, in contrast with the 5×10^3 Am^{-1} fields which were needed without the annealing. Samples were shown still to be amorphous after heating, by x-ray diffraction.

During measurement the samples were kept at a constant tension and the earth's field was accurately compensated. The torsion mechanism was able to detect 1° of torsion. The applied field was supplied by a Helmholtz system operating at 50 Hz, and the longitudinal magnetization measured by means of a secondary coil of 500 turns connected in opposition with a compensating coil of the same characteristics. For measuring the Matteucci effect, the emf appearing at the ends of the ribbon was used. In both cases the signal was integrated by means of an electronic fluxmeter and displayed on an oscilloscope. For high torsion, hysteresis loops are fully squared both in longitudinal magnetization and Matteucci effect; however, for low torsion the shape effect is important, owing to the high permeability of the samples, and remanence was taken as the extrapolation to zero field of the apparent saturation zone in the observed loops.[14,15]

Results for three values of the applied tension are plotted in Fig. 5. They show good agreement with the predictions of Eqs. (15) and (16). Fitting of the experimental data, however, needs minor changes in the strength of the nominally applied tension when a shear modulus of 3.7×10^{10} Nm^{-2} was used in the formula. This value was experimentally obtained, by means of a torsion pendulum,[16] so that discrepancies must be due to some remaining anisotropy in the annealed samples.

Agreement between the theoretical and experimental Matteucci-effect remanence implies that the ribbon behaves as a coil of area $0.64al$, a being the half thickness and l the length of the sample. On the other hand, this proportionality factor between the emf induced in the ribbon and the time derivative of the average transverse or "circular" magnetization, can be obtained by solving Maxwell's equations in some

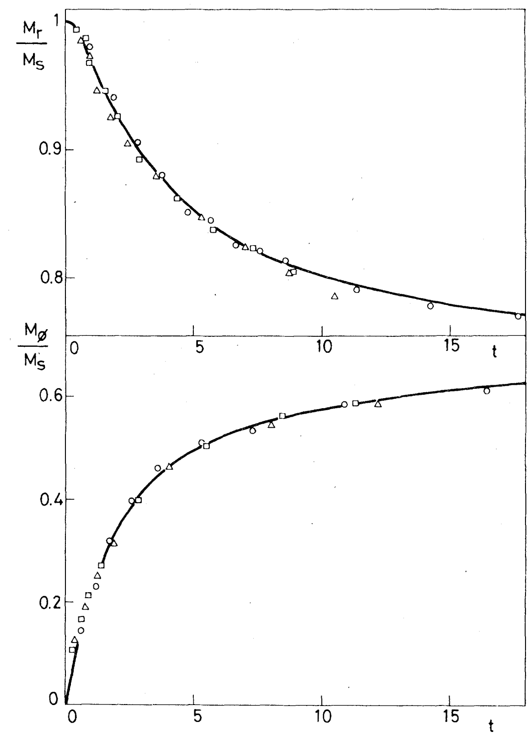

FIG. 5. Remanence of the longitudinal magnetization (upper), and Matteucci effect (lower) as a function of the applied torsion, for different values of the tension stress. (○) 8, (△) 16, (□) 24 $\mathrm{N\,mm}^{-2}$. Full lines are given by expressions (15) and (16).

simple cases. The fact that M_y has opposite signs for $x>0$ and $x<0$ can be taken into account by averaging only in the half thickness of the ribbon. Therefore, supposing $A_z(x)$ to be the only nonvanishing component of the potential vector, the procedure outlined in Ref. 1 gives for this factor $\frac{1}{2}al$ when M_y is constant and $\frac{2}{3}al$ for M_y increasing linearly along the thickness. The experimental value is close to the latter, as expected.

IV. CONCLUSIONS

It has been found that torsion in a ribbon induces an inhomogeneous distribution of the anisotropy, so that the magnetization varies continuously along the thickness of the sample. In such a situation, the magnetization process cannot be described in terms of domain theory, solutions of micromagnetic equations being needed.

The macroscopic 90° wall formed in the sample is determined by the stress-induced anisotropy, exchange forces being negligible for the usual dimensions. For annealed samples, such as those studied, only applied stresses must be taken into account in order to explain the observed behavior. As prepared, however, amorphous ferromagnets exhibit a complicated distribution of stresses, and a deeper treatment must be done in order to theorize their response to torsion.

On the other hand, the influence of the exchange term depends upon the square of the thickness, so that the thinner the sample, the stronger the exchange contribution. In this way, a study performed on samples of decreasing thickness down to thin films should be of interest for determining the influence of the exchange interactions.

Although a simplified model has been recently proposed[17] by the authors, further work on the dynamic magnetization process is now being carried out on the basis of the static magnetization distribution developed in this paper.

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