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Low-field magnetoacoustic dispersion in metals

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Magnetoacoustic dispersion and attenuation for a free-electron metal are shown to be proportional, respectively, to the imaginary and real parts of a complex resistivity tensor. In general, they do not have the same magnetic field dependence, contrary to previous predictions. The stronger oscillations of the dispersion in copper in the low-field region found by other workers and also in the data presented here can be explained by considering the asymptotic expressions for the resistivity tensor for $ql \gg 1$ and $ql/\omega_c\tau \gg 1$.

I. INTRODUCTION

Rodriguez[1] showed that the oscillations in the velocity of sound in pure metals at low temperatures are closely related to the geometric oscillations in attenuation usually referred to as magnetoacoustic oscillations. For $ql \gg 1$ and $\omega_c\tau \gg 1$ he found that the velocity oscillations have the same magnetic field dependence as those in the attenuation in the free-electron model. Here $q = 2\pi/\lambda$ is the sound wave vector, l is the electron mean free path, ω_c is the electron cyclotron frequency, and τ is the electron relaxation time.

Experimental results[2,3] differ from Rodriguez's calculations in two respects: (i) the experimentally determined velocity changes tend to have more and stronger oscillations at low fields, and (ii) the line shapes of the velocity peaks are generally sharper than predicted. Beattie[4] explained the first discrepancy for small values of ql. We will show the reason for the same discrepancy for $ql \gg 1$.

Starting with Rodriguez's formalism we find that magnetoacoustic dispersion and attenuation are proportional, respectively, to the imaginary and real parts of a resistivity tensor $\underline{R}$ which do not necessarily have the same magnetic field dependence. We give an asymptotic expression for the velocity which is valid for weak fields when $ql \gg 1$, and show that it explains the earlier experimental observations[3] as well as some additional data for copper reported here.

II. THEORY

A. Derivation of the dispersion equation

We use the formalism given by Rodriguez[1] and the expressions for the field-dependent conductivity given by Cohen, Harrison, and Harrison[5] (CHH). It is assumed that a metal consists of a free-electron gas of density n plus a uniform positive background of n/z ions per unit volume, where z is the number of electrons per ion. We choose the x axis parallel to $\vec{q}$ and the z axis parallel to $\vec{B}$, the applied magnetic field.

Starting from the equation of motion for ions as given by Rodriguez, and assuming perfect screening of the ion current by the electrons, we find the following dispersion relation:

$$q_i^2 = \frac{\omega^2}{v_i^2}\left[1 + \frac{zm}{M\omega\tau}\left(iS_{ii} - T_{ii} + \frac{q_i^2 l^2}{3\omega\tau}\delta_{ix}\right)\right], \qquad (1)$$

where the subscript i represents the direction of ion motion, ω is the sound frequency, v_i is the field-independent sound velocity, m is the electron mass, M is the ion mass, and S_{ii} and T_{ii} are, respectively, the real and imaginary parts of the magnetoresistivity tensor R.

Using the relations $\alpha_i = 2\,\mathrm{Im}q_i$ and $v_i = \omega/\mathrm{Re}q_i$, we obtain

$$\alpha_i = (zm/Mv_i\tau)S_{ii} \qquad (2)$$

and

$$\frac{\Delta v_i}{v_i} = \frac{zm}{2M\omega\tau}\left(T_{ii} - \frac{q_i^2 l^2}{3\omega\tau}\delta_{ix}\right), \qquad (3)$$

to first order in S_{ii} and T_{ii}.

According to CHH,

$$S_{ii} = \sigma_0\,\mathrm{Re}(R_{ii}) - 1 \qquad (4)$$

and

$$T_{ii} = \sigma_0\,\mathrm{Im}(R_{ii}), \qquad (5)$$

where

$$R_{xx} = (\sigma_{xx} + \sigma_{xy}^2/\sigma_{yy})^{-1} + \Delta,$$
$$R_{yy} = (\sigma_{yy} + \sigma_{xy}^2/\sigma_{xx})^{-1}, \qquad (6)$$
$$R_{zz} = (\sigma_{zz})^{-1}.$$

Here $\sigma_0 = ne^2\tau/m$ and $\Delta = iq^2l^2/3\sigma_0(1 - i\omega\tau)\omega\tau$.

The σ_{ii}'s are expressed as infinite series in CHH. For example,

$$\sigma_{xx} = \frac{3\sigma_0}{q^2l^2}(1 - i\omega\tau)\left(1 - (1 - i\omega\tau)\sum_{n=-\infty}^{\infty}\frac{g_n(X)}{1 + i(n\omega_c - \omega)\tau}\right), \qquad (7)$$

where

$$g_n(X)=\int_0^{\pi/2} J_n^2(X\sin\theta)\sin\theta\, d\theta \tag{8}$$

and $X=ql/\omega_c\tau$. When $|\omega_c\tau/(1-i\omega\tau)|^2 \gg 1$, only the $n=0$ term makes a significant contribution to the σ_{ij}'s and one can show that

$$\frac{\Delta v_i}{v_i}\approx \pm\frac{zm}{2m}(S_{ii}+1)\,, \tag{9}$$

where the + is for longitudinal waves ($i=x$) and the − for transverse waves ($i=y$ or z). This is the result obtained by Rodriguez. Comparison of Eqs. (2) and (9) indicates that the velocity shift has the same functional dependence on magnetic field as the attenuation when this approximation is valid.

Beattie[4] pointed out that this approximation breaks down at low fields for $ql\approx 1$. He computed α and $\Delta v_x/v_x$ for several small values of ql, keeping the first 20 terms in Eq. (7). The resulting curves are in good agreement with the experimental results for copper under these conditions. He also computed these functions for large ql, still keeping just 20 terms. He concluded that the dispersion and attenuation had the same field dependence for $ql\gg 1$.[4,6] Oscillations in the velocity extend to lower fields than do those in the attenuation. Beattie's calculations failed in this regime because he truncated the series for the σ_{ij}'s at $n=20$. It turns out that the number of terms required for good convergence is proportional to $X=ql/\omega_c\tau$, hence many more terms must be kept for high ql and low fields. In Fig. 1 we show the results of calculations for $ql=110$ where convergence has been achieved. It is clear that the velocity oscillations extend to lower fields than those of the attenuation for this case. A physical interpretation of this result is made easier if we look at the asymptotic behavior of Eqs. (2) and (3) for $ql\gg 1, X\gg 1$.

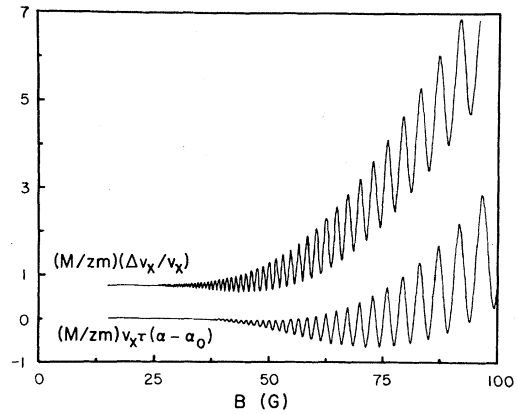

FIG. 1. Normalized velocity and attenuation of longitudinal sound as functions of applied magnetic field, calculated from Eqs. (2) and (3) for a free-electron model with electron density equal to that of copper. $ql=110$, $\omega=(2\pi)$ 52 MHz, $v_F/v_x=300$, and α_0 is the attenuation at $B=0$ $(X\to\infty)$. The velocity curve has been displaced upward for clarity.

B. Asymptotic results for longitudinal waves

Rather good asymptotic expressions for the σ_{ij}'s were obtained by Gavenda and Chang[7] (GC) for $ql\gg 1, X\gg 1$. For longitudinal waves, the normalized resistivity is given, in this limit, by

$$\sigma_0 R_{xx}\approx\frac{\sigma_0}{\sigma_{xx}}+\frac{iq^2l^2}{3(1-i\omega\tau)\omega\tau}\,. \tag{10}$$

Substitution of Eq. (10) into Eqs. (2) and (3) yields

$$\alpha_x/q_x=(zm/M)(q^2l^2/3\omega\tau)\mathrm{Re}G(X) \tag{11}$$

and

$$\Delta v_x/v_x=(zm/2M)(q^2l^2/3\omega\tau)\mathrm{Im}G(X)\,, \tag{12}$$

where

$$G(X)=\sum_{n=-\infty}^{\infty}\frac{g_n(X)}{1+i(n\omega_c-\omega)\tau}. \tag{13}$$

We have also used the fact that $|G(X)|\ll 1$.

In the limit $ql\gg 1, X\gg 1$, GC showed that

$$G(X)\approx\frac{\pi}{2ql}\left(\coth\frac{(1-i\omega\tau)\pi}{\omega_c\tau}+(\pi X)^{-1/2}\operatorname{csch}\frac{(1-i\omega\tau)\pi}{\omega_c\tau}\sin(2X-\pi/4)\right). \tag{14}$$

Substitution of Eq. (14) into Eqs. (11) and (12) yields the asymptotic expressions for attenuation and velocity:

$$\frac{\alpha_x}{q_x}=\frac{\pi zmv_F}{6Mv_x}\sinh\left(\frac{\pi}{\omega_c\tau}\right)\left[\cosh\left(\frac{\pi}{\omega_c\tau}\right)+(\pi X)^{-1/2}\cos\left(\frac{\pi\omega}{\omega_c}\right)\sin\left(2X-\frac{\pi}{4}\right)\right]\Big/\left[\sin^2\left(\frac{\pi\omega}{\omega_c}\right)+\sinh^2\left(\frac{\pi}{\omega_c\tau}\right)\right] \tag{15}$$

and

$$\frac{\Delta v_x}{v_x}=\frac{\pi zmv_F}{12Mv_x}\sin\left(\frac{\pi\omega}{\omega_o}\right)\left[\cos\left(\frac{\pi\omega}{\omega_c}\right)+(\pi X)^{-1/2}\cosh\left(\frac{\pi}{\omega_c\tau}\right)\sin\left(2X-\frac{\pi}{4}\right)\right]\Big/\left[\sin^2\left(\frac{\pi\omega}{\omega_c}\right)+\sinh^2\left(\frac{\pi}{\omega_c\tau}\right)\right] \tag{16}$$

where v_F is the electron Fermi velocity.

There are two different sets of oscillations described by these equations. Terms containing $\sin(2X-\pi/4)$ give "geometric" resonances when

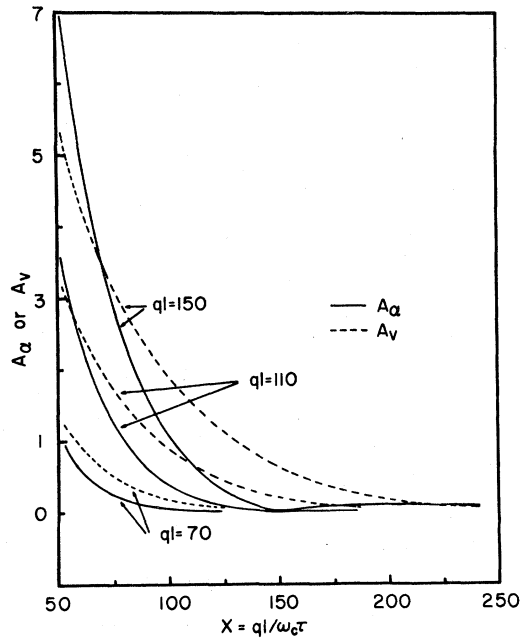

FIG. 2. Oscillation amplitudes for attenuation (A_α) and velocity (A_v) of longitudinal sound calculated from Eqs. (15) and (16) assuming $v_F/v_x = 300$.

the extremal diameter of the electron orbits is an integral multiple of the sound wavelength. Terms containing $\sin(\pi\omega/\omega_c)$ or $\cos(\pi\omega/\omega_c)$ are related to acoustic cyclotron resonance[8] (ACR). For ACR to be observable, one must have $\omega\tau > 1$, a condition which is not satisfied in most magnetoacoustic experiments. However, the ACR terms play a vital role in the rate at which geometric oscillations decay as $X \to \infty$.

For typical magnetoacoustic experiments in pure specimens, $ql \approx 100$ and $\omega\tau \approx 0.3$. Examination of Eqs. (15) and (16) reveals that the amplitude of the geometric oscillations in attenuation goes as $e^{-\pi/\omega_c\tau}\cos(\pi\omega/\omega_c)$ for $X \gtrsim ql$, while that of the velocity oscillations goes as $e^{-\pi/\omega_c\tau}\sin(\pi\omega/\omega_c)$. The former expression can also be written as $e^{-\pi/\omega_c\tau}\cos(\pi\omega\tau/\omega_c\tau)$. The exponential factor damps out the oscillations for $\pi/\omega_c\tau \gtrsim 1$. For $\omega\tau = 0.3$, the cos term will be going to zero, while the sin term in the velocity oscillations is still increasing. This explains the fact that velocity oscillations are observed to lower field values than attenuation oscillations in high ql specimens. Figure 2 shows the relative amplitudes for several values of ql. Note that the amplitude of the attenuation oscillations begins to rise again at large values of X for $ql = 150$. This agrees with observations in high-purity cadmium.[9]

III. ADDITIONAL EXPERIMENTAL EVIDENCE

In the process of studying velocity changes near

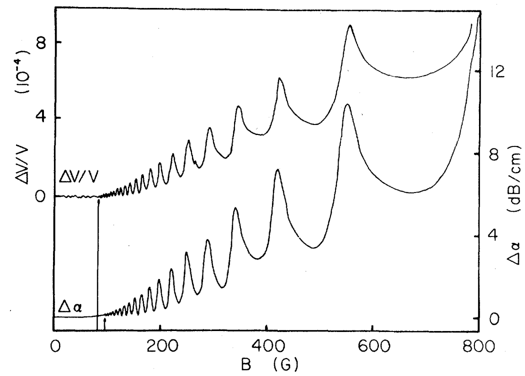

FIG. 3. Measured velocity and attenuation of longitudinal sound in copper at a temperature of 4 K and frequency of 52 MHz for $\vec{q} \parallel [\bar{1}01]$ and $\vec{B} \parallel [010]$. Arrows indicate where geometric oscillations begin.

open-orbit resonances in copper,[10] we also noted evidence for enhanced geometric oscillations in the velocity of sound at low fields. An example is shown in Fig. 3. Not only do the velocity oscillations extend to somewhat lower fields, but they decay at a slower rate than do the attenuation oscillations, in accordance with the predictions of the previous section.

The fact that the oscillations are not truly sinusoidal is evidence that the free-electron model is not adequate to describe in detail the line shapes for a real metal. However, it appears to explain adequately the field dependence of the oscillation amplitudes.

These measurements were made on a copper single crystal having a resistivity ratio of 35 000 with 52-MHz longitudinal waves propagating along $[\bar{1}01]$. The two-phase CW resonances technique employed is described in Ref. 10. The geometric oscillations come from belly orbits, since $\vec{B} \parallel [010]$.

IV. SUMMARY AND CONCLUSIONS

Discrepancies between experimental data and earlier calculations of magnetoacoustic oscillations in attenuation and dispersion have been shown to be the result of invalid approximations. Asymptotic expressions for the magnetic field dependence of attenuation and velocity are given for $ql \gg 1$, $ql/\omega_c\tau \gg 1$. From these it is evident that the amplitudes of geometric oscillations do not simply fall off exponentially at low fields in high-ql specimens; the effects of acoustic cyclotron resonance must be taken into account.

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