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Superconducting critical currents for thick, clean superconductor–normal-metal–superconductor junctions

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Superconducting critical currents have been studied for superconductor–normal-metal–superconductor (*S-N-S*) junctions as a function of temperature, magnetic field, and thickness in order to determine the spatial variation of the superconducting pair potential in the normal-metal region. Emphasis is placed on the thick, clean limit for the normal metal. The temperature dependence of J_c obeys predictions based on the de Gennes-Werthamer theory, and the magnitude of J_c is reasonable. In a high magnetic field, J_c falls exponentially with increasing magnetic field, indicating that the inverse of the decay length of the order parameter, K_N^{-1}, is approximately linear in H.

I. INTRODUCTION

The electrical transport properties of superconductor–normal-metal–superconductor (*S-N-S*) sandwiches have proven to be very useful for studying the boundary effect of superconductor–normal-metal interfaces[1–3] both through measurements of the junction resistance R and through measurements of the superconducting critical current I_c. From the temperature dependence of the resistance, the strength of the BCS parameter $N(0)V$ in the normal metal can be estimated[2] and the electron-phonon inelastic scattering rate can be deduced.[3] From the I_c of the sandwiches, one can determine the pair potential in the middle of the junction[1] and the magnitude of the various depairing parameters such as magnetic impurities and magnetic fields.

As shown by Clarke,[1] when the normal metal is in the dirty limit, the critical current density J_c, measured near the transition temperature of the superconductor, T_{cS}, is proportional to $(1-T/T_{cS})^2$, distinctly different from that of an overdamped superconductor-insulator-superconductor (*S-I-S*) junction. At lower temperatures, $J_c \sim \exp[(-(T/T_0)^{1/2}]$ and decreases very rapidly as T is increased. His findings were analyzed with the de Gennes-Werthamer[4,5] theory (dGW) of the proximity effect, and the dependence of J_c on weak parallel magnetic field was explained by the theory of Waldram *et al.*[6] Clarke's experiment was later extended to S'-*N-S* systems by Kobayashi *et al.*[7] and to *S-M-S* systems by Paterson[8] where M is a normal metal alloyed with magnetic impurities. When the cross section of the *S-N-S* sandwich is small and N is in the dirty limit, considerable experimental work has been done,[9] and the results generally agreed well with calculations based on the Usadel equations.[10] It appears now that, in the dirty limit, there is a good understanding of the transport properties of the *S-N-S* sandwiches both experimentally and theoretically.

For the clean limit, much less is known, presumably because the thick clean barriers are more difficult to prepare and the mathematical difficulties in the theory are more serious. Experimentally, the earliest work is that of Bondarenko *et al.*[11] who measured I_c of Ta-Ag-Sn and Sn-Ag-Sn junctions and found an empirical relation:

$$I_c = 3.6\times10^{-3}(1-T/T_{cS})^2 R_N^{-1}\exp(-d_N/770) \quad ,$$

where R_N is the junction resistance (in Ω), and d_N is the barrier thickness (in Å), and I_c is in amperes. Their junctions have unknown geometries, and the measured results of I_c, limited to near T_{cS}, contradicted results of Kobayashi *et al.*[4] and did not find a satisfactory theoretical interpretation. Shepherd[12] measured Pb-Cu-Pb junctions with Cu in the clean limit at very low temperatures and found that $J_c = A\exp(-d_N T/\eta)$, where $A = 10^{10\pm1}\ \mathrm{Am}^{-2}$ and $\eta = 1.0\pm0.2\ \mu\mathrm{m\,K}$. Only a limited range of values of d_N was attempted and no comparison to theoretical calculations was made.

The purpose of this work is to study the critical currents for *S-N-S* junctions with N in the clean limit for the full range of thickness, temperature and magnetic field. Recent advances in microscopic theory by Krähenbühl and Watts-Tobin[13] permit an extension of the de Gennes-Werthamer[4,5] theory to the case of a clean normal-metal barrier so comparisons of the experimental results can be made. Heretofore, little work has been done with the magnetic field dependence of J_c for large H. Special emphasis is placed on changes in the characteristic decay length for the order parameter, $K_N^{-1}(H)$. Section II describes experimental details and Sec. III contains the experimental results. Some theoretical considerations and comparisons with experimental results are in Sec. IV and a summary is in Sec. V.

II. EXPERIMENTAL PROCEDURES

A. Sample preparation

Pb and Cd have been chosen as the two components of the *S-N-S* sandwiches because the transition temperatures T_c of 0.52 and 7.2 K are well separated and because the zero-field properties have been studied in some detail.[14–16] In addition to this, the phase diagram[17] indicates that metallic diffusion into the normal metal is not a problem. The solubility of Pb in Cd is negligible at room temperature, while Cd is lightly soluble ($\sim$1.4% at 200 °C) in Pb.[14] For this reason, 1–2% of Bi was typically added in Pb so that the entire film was in the dirty regime. The Pb-Bi alloy could be evaporated easily with uniform composition.[3] The solubility of Bi in Cd is only 0.03 at. %, and our experimental results indicated no noticeable effect due to interdiffusion.

Two different sample geometries were used. For samples requiring thin (1–5 μm) Cd layers we used successive evaporations of a Pb-Bi strip, a few hundred μm wide, a Cd disk, $\sim$2 mm in diameter, and a second Pb-Bi cross strip onto a fused quartz substrate. Contact masks were used in an evaporator with base pressure, after baking, in the low 10^{-7} Torr range. The times elapsed between evaporations of different materials were made as short as possible, typically 20 sec. The Pb-Bi alloy was directly evaporated from a Ta boat at $\sim$20 nm/sec. The evaporation of Cd was made with a Cd point source maintained at relatively low temperature and with the sublimed Cd vapor collimated by a Ta chimney. With low evaporation rate ($\sim$1 nm/sec), a smooth Cd layer was produced. Thicknesses of the films were monitored with a quartz crystal balance. For samples requiring 15- to 100-μm-thick Cd layers, the Cd was rolled to the desired thickness, cut to $\sim 1 \times 2$ cm^2 rectangular geometry, cleaned with methanol and acetone, rinsed in dilute hydrochloric acid, and further cleaned in alcohol vapor. The foil was clamped between Ta masks and mounted in an all-metal high-vacuum system with base pressure of 10^{-8} Torr and resistively heated to about 160 °C to clean the surface by evaporation. The heating period was typically 20 min. After turning off the heat supply, the chamber pressure dropped to its original value and the Pb-Bi alloy was evaporated from a diffuse source at an oblique angle along the edge of the Cd foil. Two concentric Pb-Bi disks of area 0.011 cm^2 and each with a *T*-shaped strip a few hundred μm wide, extending in opposite directions, were evaporated on the Cd. The strips were used as electrical leads. Samples produced in this fashion were usually very fragile and no more than 30% survived the subsequent handling.

Resistivities of the various materials were separately determined on the same junction or on simultaneously evaporated materials after measurements on the junction were completed. The evaporated Pb-Bi alloy had a resistivity of 2.2×10^{-8} to 4.0×10^{-8} Ω m, corresponding to mean free path of 49 to 27 nm. The evaporated Cd film had a resistivity of 3.0×10^{-9} Ω m, with variation from film to film of $\pm0.4\times10^{-9}$ Ω m. The corresponding mean free path l was thus $\simeq 600$ nm. The Cd foil had a resistivity of 1.9×10^{-9} Ω m and $l=950$ nm. Initially it was anticipated that the measured resistivity of the Cd along the film or foil might be quite different from that transverse to the film or foil, but this was not the case. Resistivities measured along the strip and perpendicular to the strip agreed within 20%.

For both types of junctions, care was taken to make the Pb films very thick (10–15 μm). It is important that the Pb film be much thicker than the coherence length even near T_{cS} so that the value of

TABLE I. Properties of materials.

	Pb	Cd (film)	Cd (foil)
ρl (Ω m^2)	1.06×10^{-15}	1.8×10^{-15}	
$\xi_0=\hbar v_F/\pi\Delta_0$ (nm)	76		
ρ (Ω m)	$(2.2\text{–}4.0)\times10^{-8\,\mathrm{a}}$	$3.0\times10^{-9}\pm0.4\times10^{-9\,\mathrm{a}}$	1.9×10^{-9}
l (nm)	49–22	600±80	950
v_F (m s^{-1})$^{\mathrm{b}}$	4.8×10^5	7.7×10^5	
D (m^2 s^{-1})$^{\mathrm{c}}$	$(7.84\text{–}3.52)\times10^{-3\,\mathrm{a}}$	$0.15\pm0.02^{\mathrm{a}}$	0.24
W_1 (μm)$^{\mathrm{d}}$		305	1200
W_2 (μm)$^{\mathrm{d}}$		220	

[a]Determined from $v_F=\pi^2k_B^2/e^2\gamma\rho l$, where γ is the coefficient of electronic specific heat, Ref. 23.
[b]Determined from $D=\pi^2k_B^2/e^2\gamma\rho$, Ref. 23.
[c]Spread of value is due to variation in evaporation condition or evaporant content from run to run.
[d]W_1, W_2 are the widths of the junction. For the foil junctions, the diameter of the junction area is given.

the energy gap deep in the Pb side is not altered from its bulk value by the proximity of the normal metal.

In the preparation of junctions for these measurements, irregularities in procedure sometimes gave junction resistivities far from the expected value of the normal metal and some criteria had to be established to sort out junctions having imperfections such as oxide barriers or shorts. In addition, junctions having presumably the same thickness of the normal metal sometimes exhibit I_c variation by 2 or 3 orders of magnitude. One generally can trace the origin of these variations of I_c to inadequate cleaning, poor vacuum, etc. One accompanying effect of small I_c was that the resistance of the junction at low temperature also became excessively large. For all junctions, the $R(T)$ curve showed a plateau between 1.5 and 4 K, where one might expect the resistance to be governed by the normal Cd layer. The criterion used here for a satisfactory junction was that the value of R between 1.5 and 4 K must agree with the calculated resistance of the normal region, R_N with 30%. Within this criterion, supercurrents of presumably identical junctions (junctions with the same R_N) still varied by a factor of 2, but the general temperature dependence of I_c remained the same. Characteristic value of the film and foil parameters are given in Table I.

B. Measurement technique

The samples were connected in series with a known resistor and the input coil of an rf superconducting quantum interference device (SQUID) in a standard feedback voltmeter configuration.[3] The voltmeter sensitivity, operating with an open loop gain of $\sim 10^5$, always was dominated by the Johnson noise of the sample and the standard resistor. Electrical leads to the film samples were Pb strips soldered onto the quartz substrate with Pb-Bi eutectic solder which adhered strongly to the substrate and had a higher T_c (~ 8.2 K) than the evaporated films. The connection to the Nb wire coil was made with a Nb-PbSn solder union spot welded to the Nb wire and soldered to the Pb strip. The foil samples were mounted on a Teflon frame and clamped so as to expose only the area for connecting leads which were Pb strips soldered with the Pb-Bi solder. With the leads connected, the Cd foil and the Teflon frame were wetted at several spots with Apiezon N-grease to improve thermal contact. Separate current and voltage leads were always used.

The cryostat was a ^{3}He refrigerator. The SQUID was placed in the ^{4}He bath maintained at 4.2 K while the sample was mounted on a copper plate connected to the ^{3}He pot. Temperature between 0.35 and 10 K can be achieved and electronically regulated to a precision $\pm 0.1\%$. A superconducting shield surrounding the sample could be used to trap a persistent magnetic field in the plane of the Pb-Cd-Pb junction. This arrangement was necessary at high magnetic fields when the SQUID became unstable in an external field. When it was necessary to change the magnetic field, the sample and the shield were heated to about 10 K, eliminating possible trapped fields. The complete insert was surrounded by a superconducting magnet and Mumetal can to reduce the ambient field.

Bias current to the sample was ramped to find the critical current (I_c) of the junction. Noise in the sample current supply did not permit measurements of I_c greater than 100 mA. The smallest detectable I_c, depending on the junction, was 20–100 μA. When measuring I_c as a function of temperature T and magnetic field H, we have chosen to fix H and vary T as a matter of convenience. At a few points T was fixed and H varied to demonstrate that the order of taking data did not matter.

III. EXPERIMENTAL RESULTS

A. Current-voltage characteristics

A typical I-V characteristic is shown in Fig. 1 by the 4.19-K data (curve a). The voltage across the junction was zero to an accuracy of 10^{-13} V up to a critical current I_c, above which the voltage rose abruptly and then tapered to a linear behavior. Detailed study of the shape of the I-V curve near I_c showed a rather poor fit to $V=(I^2-I_c^2)^{1/2}R$ for a resistively shunted junction model except near T_{cS}. The critical current I_c could be suppressed by either increasing the temperature (curve b in Fig. 1) or by the application of a magnetic field (curve c). At even

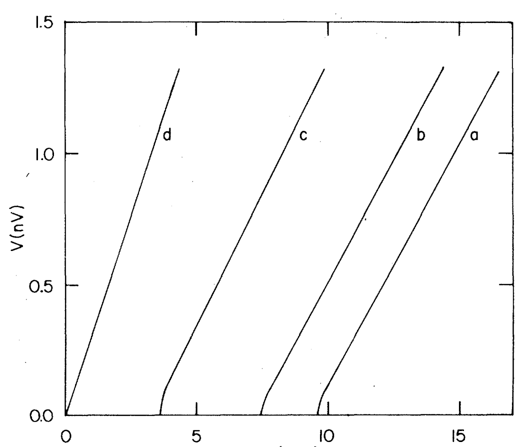

FIG. 1. Current-voltage characteristics for a junction with $d_N = 3.8$ μm. Curve a is for 4.19 K and $H=0$; b is for 4.40 K and $H=0$; c is for 4.19 K and $H=1$ Oe; d is for 6.55 K and $H=0$.

higher temperature (curve d), the I-V curve was a straight line within $\pm 1\%$ throughout the current range used.

All the I-V curves were symmetrical with respect to the origin and were nonhysteretic. An interesting feature is that the abrupt rise from $V=0$ of the I-V curve above I_c remained in the finite-magnetic-field regime and did not broaden as might be expected if inhomogeneous flux pinning were a problem.

It is important in passing to point out that the asymptotic slope (R) of the I-V curve near $T_c(\gtrsim 0.9T_c)$ and at very low temperature ($\lesssim 0.8$ K) became strongly temperature and magnetic-field dependent. Near T_{cS}, the nonequilibrium effects become important, and it has been shown by Hsiang and Clarke[3] that the passage of quasiparticle current into the superconductor results in a higher junction resistance than that of the normal region alone. Because these quasiparticles do not immediately relax to the ground state at the boundary, the effective normal-metal region includes part of the Pb. This effect is especially important for the alloyed Pb we used in our junctions. At low temperatures, the induced energy gap in Cd would provide a potential barrier for quasiparticles to undergo Andreév reflection inside the normal metal,[2] effectively reducing the differential resistance of the junction. Both of these effects depend on the magnetic field. The decay length of the nonequilibrium quasiparticle current for a type-II superconductor is reduced by the pair-breaking parameter (in this case the magnetic field) and the pair potential in the normal metal certainly is suppressed by a magnetic field. For these reasons, we choose not to use the parameter I_cR to characterize the junction as is customary in small S-N-S junction literature[9] (where the above effects, albeit small, were not taken into consideration), but rather discuss I_c and R separately. An account for the temperature and magnetic-field dependence of R will be published elsewhere.

To understand the nonlinear behavior of the I-V curve, one must consider the penetration depth[1] of a Josephson junction,

$$\lambda_J = [\hbar c^2/16\pi(\tfrac{1}{2}d_N+\lambda)eJ_c]^{1/2} \quad , \tag{3.1}$$

where d_N, λ, and J_c are the thickness of the normal barrier, the magnetic-field penetration depth, and the critical current density. This is the characteristic distance from the edge over which the supercurrent flows. In our specimens, $\frac{1}{2}d_N$ was always much larger than λ, and Eq. (3.1) can be approximated by

$$\lambda_J \simeq (\hbar c^2/8\pi d_N eJ_c)^{1/2} \quad . \tag{3.2}$$

For small critical current, when the junction width W (220 and 305 μm for the film junctions and 1200 μm for the foil junctions) is smaller than $2\lambda_J$ (corresponding to $I_c \leqslant 480$ μA for a 3-μm-thick film junction or $I_c \leqslant 46$ μA for a 18-μm-thick foil junction), the S-N-S junction essentially behaves like an overdamped Josephson junction and the "resistively shunted junction" model applies well.

In the regime where $W > 2\lambda_J$, however, one needs to take into account of the flux-flow effects. Waldram *et al.*[6] have calculated the flux-flow I-V curve for a *long* junction and found that when the length of the junction exceeded $2\lambda_J$, a persistent supercurrent will be present in parallel with the normal current even at rather high voltage. It was argued that a spatial average of the critical current density in this case would yield a supercurrent about half the maximum value at zero voltage. Our results, however, usually indicated a persistent current much higher than $\frac{1}{2}I_c$, typically 90% of I_c. Although the long-junction theory of Waldram *et al.* does not strictly apply for these nearly square junctions, the same physical effects may explain deviations from the resistively-shunted-junction model.

All of the junctions reported here are very thick compared to the order-parameter decay length. This gives a large spatial variation in the order parameter so the junctions are not ideal for observing Fraunhofer-like effects.

B. Critical currents

Critical currents for the clean thick junctions in zero applied field are very similar to the dirty-limit case and roughly follow the relation $J_c = J_0(1-T/T_{cS})^2 \exp(-K_N d_N)$. Typically J_c is on the order of 10^7 A/m^2 for a 3.4-μm-thick barrier for all temperatures less than 3 K. To analyze the temperature dependence of J_c, it is important to note that the apparatus can measure only over five orders of magnitude in J_c from 10^2 to 10^7 A/m^2 so that the entire temperature range is not accessible for any one junction. This in turn limits the temperature range for any one junction in which the measurements could be made. The functional dependence of $J_c(T)$ thus appears to have different forms because different regimes are emphasized.

At temperature near T_{cS}, Fig. 2 shows two junctions with relatively "thin" barriers. The linear behavior of $\sqrt{J_c}$ vs T indicated that J_c is proportional to $(1-T/T_{cS})^2$ in this range. The extrapolated value of T_{cS} was in good agreement with separately measured T_{cS}. At very low temperatures it is found that J_c is proportional to exp $(-T/T_0)$ as is apparent from Fig. 3 where J_c for several "thick" junctions are plotted semilogarithmically vs T. This behavior is distinctly different from that found for S-N-S junctions with N dirty[1] but it is similar to that found by Shepherd.[12] The data follow Shepherd's expression $J_c = A$ exp $(-d_N T/\eta)$ but it is found that η varies with d_N (and hence T) in a monotonic way instead of

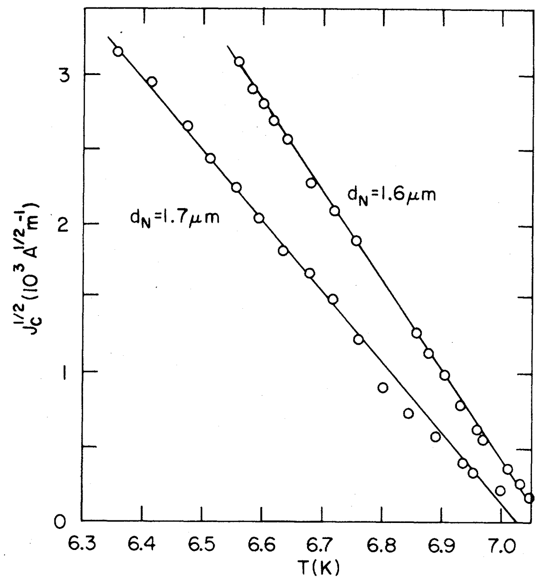

FIG. 2. $J_c^{1/2}$ for two junctions showing the temperature dependence of J_c near T_{cS} is $(1-T/T_{cS})^2$.

being constant.[12] The significance of this thickness dependence is given in Sec. IV.

For Cd barriers of intermediate thickness both the $(1-T/T_{cS})^2$ and the exp $(-K_N d_N)$ terms for J_c are important. As can be seen on Fig. 4 (open circles) $J_c(T)$ deviates from a simple exponential. If, however, one plots $\ln[J_c(1-T/T_{cS})^{-2}]$ vs T as shown by the open squares, the deviation from linearity is largely reduced. These results, which are qualitatively similar to dirty S-N-S junctions, strongly suggest that an extension of the dGW theory to the clean limit is appropriate.

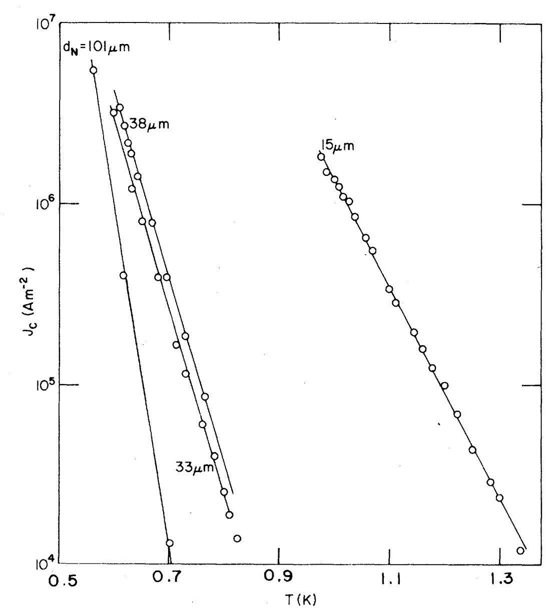

FIG. 3. J_c vs T at low temperatures for four junctions with very thick Cd layer.

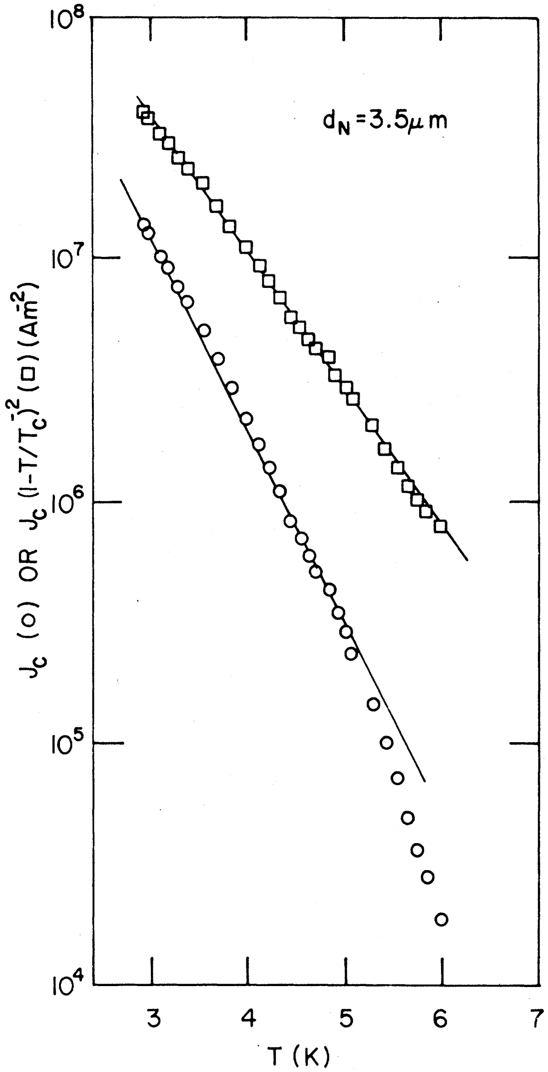

FIG. 4. J_c vs T (O) for one junction with intermediate thickness of Cd layer. Also shown is $J_c(1-T/T_{cS})^{-2}$ (□). Straight lines are drawn through the majority of the data points for purposes of comparison.

In the determination of J_c from the measured I_c, it is important to realize that the supercurrent is not uniformly distributed across the junction if the Josephson penetration depth $\lambda_J \lesssim \frac{1}{2} W$ where W is the junction width. To account for this effect, we

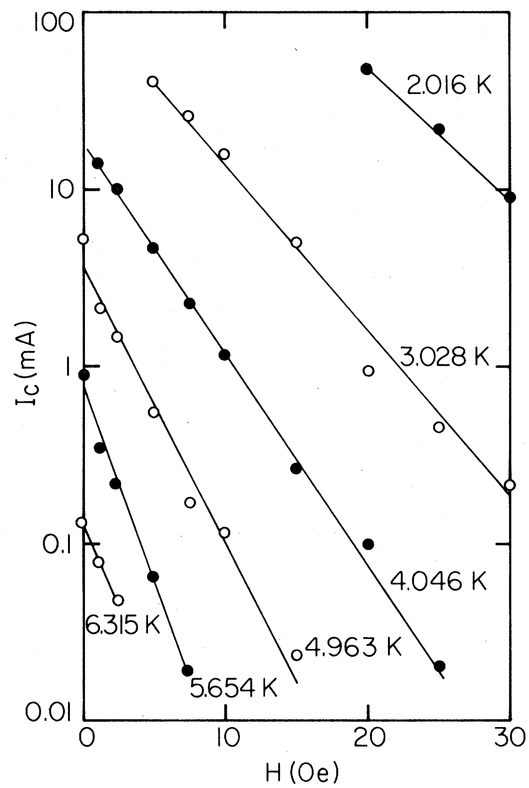

FIG. 5. Variation of I_c vs H at several temperatures for a junction of $d_N = 34$ μm.

have followed the work of Clarke[1] and assumed that the current is uniform for $\lambda_J > \frac{1}{2}W$ whereas if $\lambda_J \leq \frac{1}{2}W$, then it is assumed that the supercurrent flows uniformly along the edge within a width λ_J. Because λ_J depends on J_c, J_c was calculated self-consistently. These approximations are very good for both $\lambda_J << \frac{1}{2}W$ and $\lambda_J >> \frac{1}{2}W$. Most of the important data presented here are for the high J_c case where $\lambda_J << \frac{1}{2}W$.

When a magnetic field was applied parallel to the junction, there was a rapid decrease of J_c with increasing H. As shown in Fig. 5, J_c is approximately exponential in H for the full range of temperature studied. Plots of J_c vs H on a linear scale or on a log-log scale are clearly not straight lines. This exponential dependence of J_c on H is a dominant feature of the data not heretofore reported and it poses an interesting question about its origin. At very low fields, on the order of a few mOe, one expects a Fraunhofer diffraction pattern or a pattern similar to the predictions of Owen and Scalapino.[18] In the relatively large field range used here, from 0.1 to 30 Oe, a more plausible reason for the decay is that K_N^{-1} is modified by the magnetic field.

IV. DISCUSSION

A. Zero-magnetic-field case

The de Gennes-Werthamer theory[4,5] provides an excellent framework for understanding these results. Calculations previously were carried out for the dirty limit and temperatures near T_{cS} by de Gennes and coworkers,[4,19] and this was extended to lower temperatures by Clarke.[1] The data agree with the prediction rather well. Since then Svidzinskii *et al.*[20] as well as several other groups have pioneered work on various aspects of this problem in different limits. Our goal here is to apply these ideas to the clean *S-N-S* junction case studied here. Within dGW it is assumed that the only direction in which important spatial variations occur is in the direction of current flow. It is assumed that parameters which vary in the other directions can be replaced by average values and this same approach is taken here.

In the dGW theory, the characteristic quantity that determines whether a material is in the dirty or clean limit for the proximity system is the ratio of the mean free path l to the coherence distance ξ defined as

$$\xi(T) = \hbar v_F / 2\pi k_B T \ . \tag{4.1}$$

The decay lengths K_S^{-1} and K_N^{-1} of the condensation amplitude F in S and N are related (in the dirty limit) by the following relations

$$\ln(T_{cS}/T) = \sum_n [(n+\tfrac{1}{2})^{-1} - (n+\tfrac{1}{2}+\tfrac{1}{6}K_S^2\xi_S l_S)^{-1}] \ , \tag{4.2}$$

$$\ln(T_{cN}/T) = \sum_n [(n+\tfrac{1}{2})^{-1} - (n+\tfrac{1}{2}-\tfrac{1}{6}K_N^2\xi_N l_N)^{-1}] \ , \tag{4.3}$$

and

$$N_N D_N K_N \tanh K_N d_N = N_S D_S K_S \tanh K_S d_S \ . \tag{4.4}$$

Where N, D, and d are the density of states, diffusivity, and thickness of the respective material.

To find the critical current, one needs to know the detailed spatial variation of $F(\vec{x})$. Assuming that $d_N >> K_N^{-1}$ and $d_S >> K_S^{-1}$, near T_{cS}, $|F(\vec{x})|$ has been calculated to be,[21] for the S and N sides, respectively,

$$|F_S(\vec{x})| = |F_S(\infty)| \tanh K_S(x+c) \ , \quad x > 0 \tag{4.5}$$

and

$$|F_N(\vec{x})| = |F_N(0)| \exp(K_N x) \ , \quad x < 0 \ . \tag{4.6}$$

When the appropriate boundary conditions[4] are applied to F, we find that, near T_{cS},

$$|F_N(0)| = (D_N N_N / D_S N_S)(K_S/K_N)|F_S(\infty)| \quad . \tag{4.7}$$

We then determine J_c by using the relation $J_c \propto [F(d/dx)F^* - F^*(d/dx)F]$, assuming that the phase of F varies linearly in N. The maximum supercurrent is found to be

$$J_c = AF_S^2(\infty)(K_S^2/K_N)\exp(-K_N d_N) \quad , \tag{4.8}$$

where A is a proportionality constant. As $T \to T_{cS}$, K_S^2 and $F_S^2(\infty)$ both vary as $(1 - T/T_{cS})$, therefore, over a limited range near T_c where $\exp(-K_N d_N)$ is essentially independent of temperature,

$$J_c \propto (1 - T/T_{cS})^2 \quad (T \to T_{cS}) \quad . \tag{4.9}$$

To generalize the above results which were based on the original dGW Eqs. (4.2) and (4.3), the results of a recent calculation by Krähenbühl and Watts-Tobin[13] (KWT) have been used. They derived from the microscopic Eilenberger equations[22] the variation of the order parameter in a clean proximity system and found that, if the product of the mean free path and the energy gap is small ($l\Delta/\hbar v_F << 1$), the calculated results for dirty S-N-S system can be directly applied to a clean system by replacing the diffusivity $D = \frac{1}{3}v_F l$ with an effective $\tilde{D}$ defined as

$$\tilde{D} = \tfrac{1}{6}v_F^2(|\omega| + v_F/2l) \quad , \tag{4.10}$$

where $\omega = (2n+1)\pi k_B T/\hbar$. This replacement is equivalent to replacing $(\frac{1}{3}\xi l)^{1/2}$ in the dirty limit with $\xi[\frac{1}{3}(2n+1+\xi/l)]^{1/2}$ in Eqs. (4.2) and (4.3). We thus obtain the more general equations for the decay lengths

$$\ln(T_{cS}/T) = \sum_n \{(n+\tfrac{1}{2})^{-1} - [(n+\tfrac{1}{2}) + (\tfrac{1}{6}K_S^2\xi_S^2)/(2n+1+\xi_S/l_S)]^{-1}\} \quad , \tag{4.11}$$

$$\ln(T_{cN}/T) = \sum_n \{(n+\tfrac{1}{2})^{-1} - [(n+\tfrac{1}{2}) - (\tfrac{1}{6}K_N^2\xi_N^2)/(2n+1+\xi_N/l_N)]^{-1}\} \quad . \tag{4.12}$$

Near T_{cS}, the order parameter is small in both S and N, the KWT result is applicable throughout the S-N-S system. This explains the somewhat surprising experimental findings of Clarke[1] that, in an S-N-S sandwich with a *clean S* layer, J_c also followed the temperature dependence predicted in Eq. (4.9) near T_{cS}. Away from T_{cS}, the KWT theory is again applicable to the N side where $\Delta(\vec{x})$ is always small but applicable to the S side only if S is in the dirty limit. The full dGW theory, with the proper replacement of D, is thus applicable if S is dirty and N is dirty *or* clean. In our experimental system, the Cd barrier was always in the clean limit. We used numerical calculation to find the relation of $K_N\xi_N$ vs T/T_{cS}. The result is shown in Fig. 6.

With the substitution of the appropriate decay lengths, Eq. (4.8) is *in general* valid in the clean or dirty limit. Near T_{cS}, Eq. (4.9) predicted correctly our experimental results shown in Fig. 2. At $T << T_{cS}$, the dominating temperature dependence of J_c is the exponential term and

$$J_c \simeq B\exp(-K_N d_N) \quad , \tag{4.13a}$$

with B only weakly dependent on temperature. In the clean limit Eq. (4.13) can be written as

$$J_c = B\exp(-d_N T/\eta) \quad , \tag{4.13b}$$

where η is proportional to $\hbar v_F/k_B$ and has a weak temperature dependence as determined in Fig. 6. This exponential behavior is also confirmed in our experiments (Fig. 3). An interesting point here is that since η depends on T_{cN}, the experimentally measured value of η can be used to determine T_{cN} of a normal metal. In our experiments, where T_c of Cd is known, we can thus compare the experimental and calculated values of η. This is shown in Table II. The agreement is in general very good. [We have used $v_F = \pi^2 k_B^2/e^2\gamma(\rho l)$ (Ref. 23) to calculate $v_F = 7.7\times10^5\ \mathrm{m\,s^{-1}}$, where $\rho l = 1.8\times10^{-15}\ \Omega\,\mathrm{m}^2$ (Ref. 24) and γ is the coefficient of the electronic specific heat.] The implication here is that one can

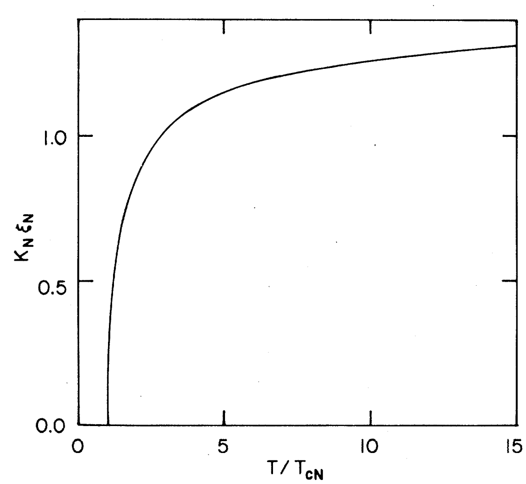

FIG. 6. Calculated value of $K_N\xi_N$ vs reduced temperature in the clean limit.

TABLE II. Comparison of experimentally obtained and calculated η [Eq. (4.15b)].

d_N (μm)	T (K)	η(expt.) (μm K)	η(calc.) (μm K)
101	0.62	2.24	2.11
38	0.7	1.60	1.60
33	0.7	1.60	1.58
15	1.1	0.98	1.06

make a *clean* S-N-S junction with N being a normal metal for determining T_{cN}. A similar conclusion was also made by Clarke[1] for a dirty normal metal.

The magnitude of J_c has been calculated by Svidzinskii *et al.*,[20] who used the microscopic theory to calculate $B(T\rightarrow 0)$ and found

$$B(T\rightarrow 0)=(\hbar en/md_N)\beta \ , \tag{4.14}$$

where m is the density of the electrons and β is a dimensionless constant of the order of and smaller than unity and is determined by the boundary conditions. For our Pb-Cd-Pb junction with $d_N=15$ μm, the calculated $B(T\rightarrow 0)$ is $\beta(5\times 10^{10})$ A m^{-2} while the experimentally extrapolated value is $10^{9\pm 1}$ A m^{-2}. The agreement is satisfactory.

For junctions at intermediate temperatures (e.g., ~ 4 K), our linearized derivation of Eq. (4.9) is not expected to be exactly valid. However, the implication of Eq. (4.8) is that J_c near T_{cS} will be further suppressed than the prediction of Eq. (4.13b). This is indicated in Fig. 4. The good linear behavior of ln $[J_c(1-T/T_{cS})^{-2}]$ vs T is somewhat fortuitous.

B. Effect of a magnetic field

As shown in Fig. 5, the critical current of a S-N-S junction in high magnetic field has an exponential behavior over a wide temperature range. This, along with the sharp structure of the I-V characteristics (Fig. 1) has led us to consider the possibility that the decay length K_N^{-1} is strongly magnetic-field dependent.

Paterson[8] has observed that K_N^{-1}, in S-N-S junctions, decrease with increasing magnetic impurity in N and has characterized this diminution with a pair-breaking parameter $\alpha=\hbar/2\tau_S$ where τ_S is the spin-flip scattering time. In many ways that experiment is analogous to the depairing effects of a magnetic field reported here. Theoretical work to describe the effects of a depairing on these junctions has been carried out by Hauser, Theuerer, and Werthamer[25] and by Entin-Wohlman[26] for the case of a dirty normal metal and this general approach is extended here to the case of a clean normal metal.

Following Hauser *et al.*,[25] one can derive a pair of equations to replace Eqs. (4.11) and (4.12) that are appropriate for *clean* N and *dirty* S with pair-breaking strengths of α and α', respectively:

$$\ln\frac{T}{T_{cN}}=\sum_n\left\{\left[\left(n+\frac{1}{2}+\frac{\alpha}{\pi k_BT}\right)-\frac{1}{6(2n+1)}K_N^2\xi_N^2\right]^{-1}-\left(n+\frac{1}{2}+\frac{\alpha}{\pi k_BT}\right)^{-1}\right\} , \tag{4.15}$$

$$\ln\frac{T}{T_{cS}}=\sum_n\left\{\left[\left(n+\frac{1}{2}+\frac{\alpha'}{\pi k_BT}\right)+\frac{1}{6}K_S^2\xi_Sl_S\right]^{-1}-\left(n+\frac{1}{2}+\frac{\alpha'}{\pi k_BT}\right)^{-1}\right\} . \tag{4.16}$$

Here T_{cN} and T_{cS} are the transition temperatures in the presence of the pair-breaking mechanisms.

As emphasized by Maki,[27] all pair-breaking ergodic perturbations are equivalent to magnetic impurities in their effect on T_c, and the effective pair-breaking parameters α and α' for type-II superconductors in their vortex states are

$$\alpha=D_NeH/c \ , \ \alpha'=D_SeH/c \ . \tag{4.17}$$

We make the ansatz that for the vortex formation in the barrier of an S-N-S sandwich, α is similarly given by Eq. (4.17) and that the *average* order-parameter variation in the N barrier can be characterized by Eqs. (4.15) and (4.16) with the substitution of Eq. (4.17). J_c can then be calculated.

To solve Eqs. (4.15) and (4.16), it is important to recall that several simplifying approximations can be made. The Pb-Bi is dirty so that D_S is small and α' is negligible. In addition the values of H involved in this experiment are far less than H_c of Pb-Bi so both T_{cS} and K_S do not change appreciably and attention can be focused on the normal side. Furthermore, T_{cN} is much less than T and can be set equal to zero. K_N is then found as the smallest root such that the right-hand side of Eq. (4.15) diverges,

$$K_N=\xi_N^{-1}\left[6\left(\frac{1}{2}+\frac{\alpha}{\pi k_BT}\right)\right]^{1/2} . \tag{4.18}$$

Substituting Eqs. (4.17) and (4.18) into Eq. (4.13) gives

$$J_c=B\exp(-K_Nd_N)=B\exp\left\{-2d_N\left[3\pi k_BT\left(\pi k_BT+2\frac{D_NeH}{c}\right)\right]^{1/2}(\hbar v_F)^{-1}\right\} . \tag{4.19}$$

In the low-field limit, $[H \ll (ck_BT/D_Ne)]$, J_c can be approximated by

$$J_c(H) = J_c(H=0)\exp[-2\sqrt{3}\,d_N D_N eH(c\hbar v_F)^{-1}] \tag{4.20}$$

and in the high-field limit $[H \gg (ck_BT/D_Ne)]$,

$$J_c(H) \propto \exp[-(6\pi k_B T D_N eH/c)^{1/2}\,2d_N(\hbar v_F)^{-1}] \ . \tag{4.21}$$

One consequence of Eq. (4.20) is that, at low field, $I_c(H)/I_c(H=0)$ should be a universal curve which is exponential in H and independent of T. The data, shown on Fig. 7, have considerable scatter but the general trend seems to obey Eq. (4.20) over an I_c change about a factor of 30. The spread in $I_c(H)/I_c(H=0)$ at each H is not monotonic in temperature and seems to be a genuine scatter rather than a temperature dependence. At the 10-Oe field the data lie above the straight line as predicted by Eq. (4.19). From the slope of Fig. 7 one can deduce a value of $D_N = 0.18 \pm 0.03\ \mathrm{m^2\,s^{-1}}$ in reasonable agreement with the D_N determined from resistivity measurements of $0.15 \pm 0.02\ \mathrm{m^2\,s^{-1}}$. This close agreement of D_N derived from the exponential decay of I_c and an independent measurement of resistivity lends strong support for the applicability of the approximations made in the analysis.

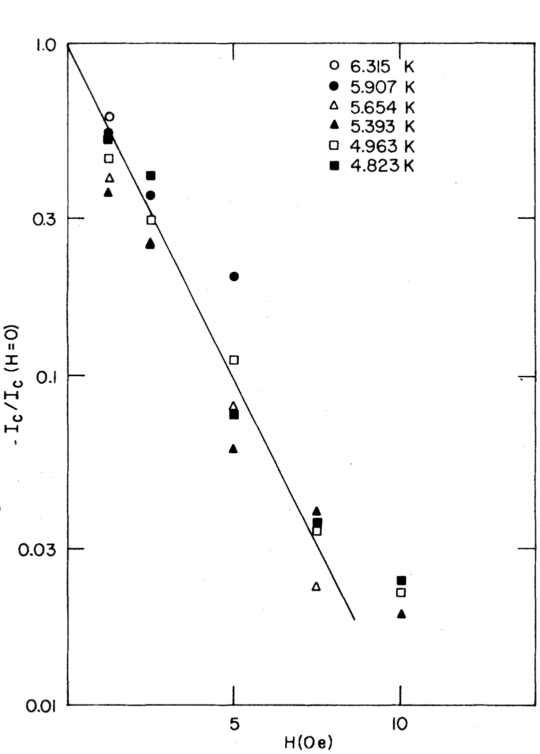

FIG. 7. $J_c(H)/J_c(0)$ vs H at several temperatures. Deviation from the universal solid line [Eq. (4.20)] is random below 7 Oe. Deviation from linearity is in agreement with Eq. (4.19).

Despite the good agreement, it is important to caution that the above analysis is phenomelogical. The use of the pair-breaking parameters Eq. (4.17) for the S-N-S junctions does not have a microscopic justification. In fact, one can argue that the vortex formation in the barrier of an S-N-S junction will produce more complicated spatial variation of the order parameter than the simple exponential behavior determined from Eqs. (4.15) and (4.16). Our analysis is thus at best a description in the sense of an order parameter averaging. But the fact that the experimental I-V curve shows a sharp structure near I_c and that theoretically it has been proved that local field generated by magnetic moments has the same effect as magnetic impurities in determining the order-parameter variation[26] (with a different α) adds confidence in the above analysis.

V. CONCLUSIONS

In zero applied magnetic field the critical currents obey $J_c = J_0(1-T/T_{cS})^2\exp(-K_N d_N)$ to rather good accuracy just as is found for dirty normal metal barriers. When a magnetic field is applied, J_c decreases exponentially with increasing H as might be expected if the decay length, K_N^{-1}, decreased linearly with H. If one treats the applied field as a pair-breaking parameter and modifies the dGW theory to account for both the clean limit case and the magnetic field, then one obtains a good fit to the J_c vs H curves. The diffusivity D_N obtained from a fit of J_c vs H to this model agrees well with D_N determined from resistivity. When a full theory for the temperature and magnetic field dependence of J_c is developed, it should reduce to this modified dGW model in the limit of thick, clean junctions and moderate magnetic fields. In these spatially inhomogeneous systems, the pair-breaking parameters enter additively just as for magnetic impurities and magnetic fields in homogeneous systems.[28]

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