Compound geometrical resonances in superconducting proximity-effect sandwiches

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In superconducting–normal-metal sandwich structures in which the pair potential has abrupt spatial changes, Andreev scattering occurs in which electronlike and holelike excitations are intermixed. The Tomasch effect, Rowell-McMillan oscillations, and de Gennes–Saint James bound states result from interference of these excitations *within a single film*. In this paper we discuss *compound resonances* which can occur in *both films* of a sandwich structure in which each film is clean and has a thickness on the order of its coherence length. Two of the compound-resonance effects have the following interpretations: (1) Below the energy gap, the de Gennes–Saint James bound states split into energy bands. (2) Above the energy gap, resonances occurring with different frequencies result in a beating of the oscillations of the density of states as a function of energy. A number of previously puzzling or poorly understood features of published experimental data are shown to be consistent with compound-resonance effects. Suggestions are given for systematic experiments.

Proximity effects in superconductors are now being used to study the vibrational[1] and the electronic[2] properties of normal metals and superconductors. Although these uses rely on generalizations of theories initially used to explain geometrical-resonance experiments,[2] aspects of these resonance experiments are still not understood.[3,4] In this paper we trace this lack of understanding to an often inappropriate assumption incorporated in the theories used to interpret the data. These theories[5] describe two-layer sandwiches with back layers assumed to be sufficiently thick and dirty to allow the neglect of reflections off their back surfaces. Experimentally the back films are usually thinner than the front films, and often these back reflections cannot be neglected. The theory we outline here treats all of the reflections occurring within two-layer sandwiches.[6] Qualitatively new effects are predicted, and these provide a natural interpretation of previously puzzling aspects of a number of experiments.

We consider a two-layer sandwich consisting of a superconductor a with a pair potential Δ_a between $x=-d_a$ and $x=0$ and a second superconductor b with pair potential Δ_b between $x=0$ and $x=d_b$. The Hamiltonian for such a sandwich is

$$H(x)=\left[-\left(\frac{\hbar^2}{2m}\right)\frac{\partial^2}{\partial x^2}-\mu_x\right]\tau_3+\Delta(x)\tau_1 \quad , \tag{1}$$

where $2\pi\hbar$ is Planck's constant, m is the electron effective mass, and τ_i are Pauli matrices in Nambu space. We have Fourier transformed the y and z dependence to $\vec{k}_\parallel=(0,k_y,k_z)$ and have defined μ_x as

$$\mu_x(k_\parallel)=\hbar^2(k_F^2-k_\parallel^2)/2m \equiv \hbar^2 k_{F_x}^2/2m \quad ,$$

where k_F is the Fermi wave vector. The zero-temperature Green's function satisfies

$$[E-H(x)]G(x,x';k_\parallel,E)=\delta(x-x') \quad . \tag{2}$$

We use the boundary conditions[7] $G(d_b,x')=G(x,d_b)=0$, $\partial G/\partial x=0$ at $x=-d_a$, and $\partial G/\partial x'=0$ at $x'=-d_a$.

Equation (2) can be solved by matching, at $x=0$, the Green's functions of two homogeneous thin-film superconductors. The local density of states for one spin projection $N(x,k_\parallel)$ normalized to its normal-state value of $m/(\hbar^2 k_{F_x})$ is then given by $-\hbar^2 k_{F_x}\times(\pi m)^{-1}\,\mathrm{Im}G_{11}(x,x,k_\parallel,E)$. To approximately account for impurity scattering and for the depairing influence of the proximity effect, we give E a *finite* imaginary component of magnitude $\hbar v_F/2l$, where v_F is the Fermi velocity and l is the mean free path. Neglecting corrections of order Δ_i/μ_x and averaging over spatial oscillations which occur on the scale of $1/k_F$, we find

$$N(-d_a,k_\parallel)=\mathrm{Re}\{D[\epsilon_a\sin\omega_a\cos\omega_b+\epsilon_b\sin\omega_b\cos\omega_a+\delta_a(\epsilon_b\delta_a-\epsilon_a\delta_b)\sin\omega_b(\cos\omega_a-1)]\} \quad , \tag{3}$$

where

$$D^{-2}=1-[\cos\omega_a\cos\omega_b-(\epsilon_a\epsilon_b-\delta_a\delta_b)\sin\omega_a\sin\omega_b]^2 \quad ,$$

$$\omega_i=2\Omega_i d_i/\hbar v_{F_x},\ v_{F_x}=\hbar k_{F_x}/m,\ \epsilon_i=E/\Omega_i,\ \delta_i=\Delta_i/\Omega_i,\ \text{and}\ \Omega_i^2=E^2-\Delta_i^2.$$

This result simplifies considerably at high energies where, neglecting terms of order $(\Delta/E)^4$ relative to 1 and to $\sinh^4[(d_a+d_b)/l]$, we find

$$N(-d_a,\vec{k}_\parallel)=\mathrm{Re}[1+\tfrac{1}{2}\delta_a^2+\delta_a(\delta_b-\delta_a)A+\tfrac{1}{2}(\delta_a-\delta_b)^2A^2] \ , \tag{4}$$

where $A=\sin\omega_b/\sin(\omega_a+\omega_b)$ and here $\delta_i\to\Delta_i/E$. The first two terms in Eq. (4) are the high-energy expansion of the BCS density of states. In sandwiches with thick back layers, the third term leads to Tomasch oscillations (periodicity $hv_{F_x}/2d_a$) and the fourth to McMillan-Rowell oscillations (periodicity $hv_{F_x}/4d_a$). Below we study the dependency on backing thickness predicted by Eq. (3).

Figure 1 shows the local density of states (for $k_\parallel=0$ as is approximately valid for tunneling experiments) at the surface of a superconductor ($a=s$) of normalized thickness $2d_s\Delta_s(\hbar v_F)^{-1}\equiv 2d_s(\pi\xi_s)^{-1}=20$ backed by normal-metal layers ($b=n$) of various thicknesses. The bottom plot shows Tomasch oscillations for a sandwich with a thick enough back layer $[2d_n(\pi\xi_s)^{-1}=100]$, so that reflections off its back surface are inconsequential. This is the limit described by earlier theories. For thinner normal-metal backings $[2d_n(\pi\xi_s)^{-1}=5.0,\ 2.5,\ 1.25]$, interference effects also occur in the back layer and the combination of the resonances in both layers results in a beating pattern in the oscillations of the density of states. As the thickness of the back layer is halved, the energy spacing of the oscillation cancellations can be seen to double. Eventually [as for $2d_n(\pi\xi_s)^{-1}=0.62$], the energy at which the first cancellation occurs becomes quite large and and only resonances occurring in the front film are evident. Finally, in the limit of an extremely thin back layer $[2d_n(\pi\xi_s)^{-1}=0.04]$, the amplitude of the Tomasch oscillations vanishes [linearly with d_n according to Eq. (4)], and the density of states becomes BCS-like.

In Fig. 2 we show the local density of states (for $k_\parallel=0$) at the surface of a normal metal ($a=n$) of normalized thickness $2d_n\Delta_s(\hbar v_F)^{-1}\equiv 2d_n(\pi\xi_s)^{-1}=5$ backed by superconductors ($b=s$) of various thickness. The bottom plot $[2d_s(\pi\xi_s)^{-1}=100]$ is the thick-backing limit described by earlier theories. Below Δ_s there are two de Gennes–Saint James "bound states," and above Δ_s McMillan-Rowell oscillations occur. As the thickness of the superconducting backing decreases $[2d_s(\pi\xi_s)^{-1}=20$ and 6], additional oscillations appear for $E>\Delta_s$ due to resonant effects occurring in the back film. The resulting compound oscillations are the normal-side analog of those we identified in Fig. 1 for the superconducting

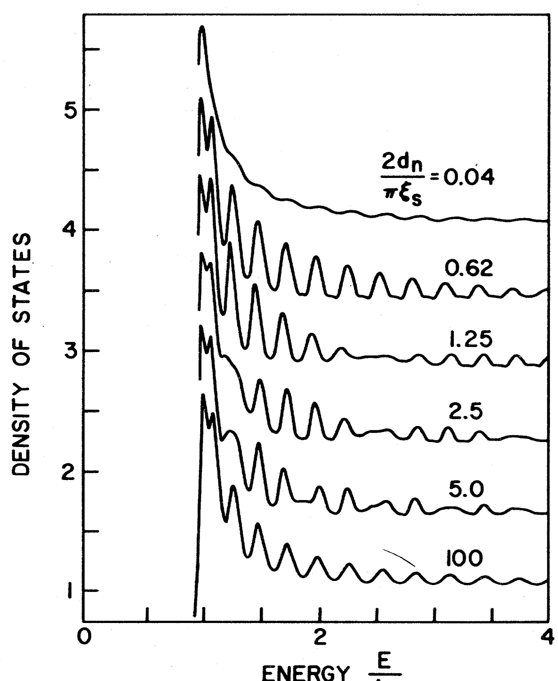

FIG. 1. Local density of states for $k_\parallel=0$ at the surface of a superconductor of normalized thickness $2d_s(\pi\xi_s)^{-1}=20$ backed by normal-metal layers of various thicknesses. The mean free path $l=d_s$. Successive curves are displaced vertically by 0.6 units.

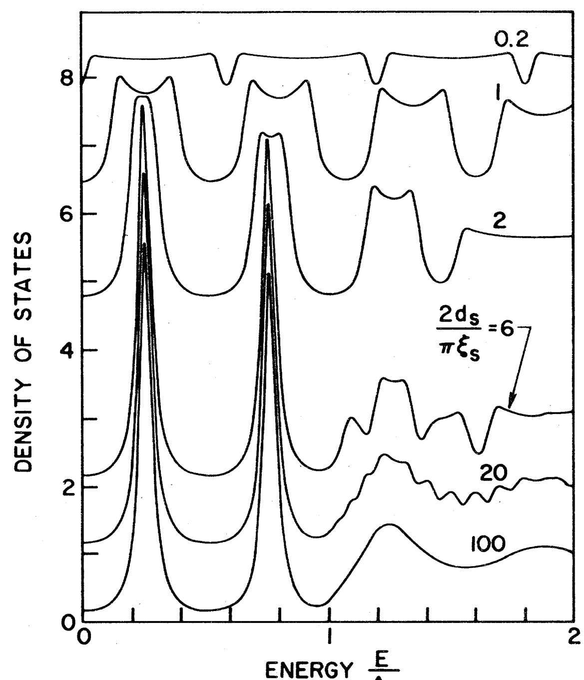

FIG. 2. Local density of states for $k_\parallel=0$ at the surface of a normal metal of normalized thickness $2d_n(\pi\xi_s)^{-1}=5$ backed by superconductors of various thicknesses. The mean free path $l=6.7d_n$. Successive curves are displaced vertically by 1.0, 1.0, 2.6, 1.6, and 1.0 units.

side. For still thinner back films $[2d_s(\pi\xi_s)^{-1}=2.0, 1.0, 0.2]$ a second compound-resonance effect occurs in these sandwiches. The bound-state peaks can be seen to spread into wider and wider *bands* of bound states as the backing thickness decreases. Eventually, the bands widen to the point where the band gaps become barely discernible, and we recover the flat density of states of an isolated normal-metal film. The band structure persists on the superconducting side as well, as incipient band gaps in the top plot of Fig. 3 indicate.

A simple analogy illuminates the nature of these bands. This two-layer problem with perfectly reflecting boundary conditions is equivalent to an infinite-medium problem with a pair potential having periodic up and down steps.[8] Narrow bound states localized in normal-metal regions separated by thick superconducting regions split into bands as the thickness of the intervening layers decreases. The bands are one dimensional with square-root singularities at the band edges.

The most convincing experimental evidence for these compound resonance effects would be an explicit demonstration of the predicted dependence on backing thickness. Unfortunately, the only experimental study of a dependency on backing thickness used sandwiches with very thin backings. Nedellec *et al.*[3] studied 1.65-μm Pb films[9] $[2d_{\rm Pb}\Delta_{\rm Pb}(\hbar v_F^{\rm Pb})^{-1}=7.15]$ backed by 1165 to 330 Å of Al $[2d_{\rm Al}\Delta_{\rm Pb}(\hbar v_F^{\rm Al})^{-1}=0.380$ to $0.108]$ and observed decreasing Tomasch-oscillation amplitudes with decreasing Al thickness. The amplitude decrease is consistent with the thin-backing limit of our theory.[10]

A number of experiments done at fixed-backing

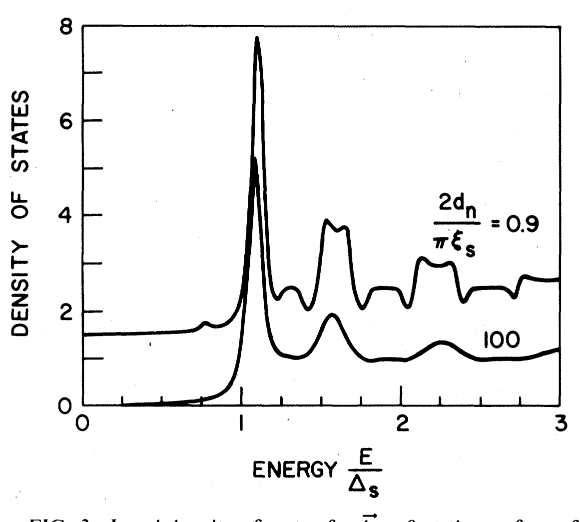

FIG. 3. Local density of states for $\vec{k}_{\parallel}=0$ at the surface of a superconductor of normalized thickness $2d_s(\pi\xi_s)^{-1}=7.75$ backed by thick and thin normal metals. The mean free path $l=4d_s$. The top curve is displaced vertically by 1.5 units.

thickness show features which are difficult to explain with the conventional theory but are easily explained using the present theory: (1) When tunneling from Pb into a 2000-Å-Sn−3000-Å-Pb sandwich, Romagnan and Guyon[11] observed a gradual increase in current between voltages (normalized to the Pb gap) of $eV/\Delta_{\rm Pb}=1.55$ and 2, no increase from 2 to 2.22, and a linear increase above 2.25. Using the stated film thickness, Sn and Pb Fermi velocities of 0.81 and 0.98×10^8 cm/sec, and Sn and Pb gaps of 0.72 and 1.4 MeV, we predict a broad band in the Sn surface density of states for normalized energies $E/\Delta_{\rm Pb}$ between 0.75 and 1.1 and a band gap between 1.1 and 1.45. Shifted by unity because of the Pb counterelectrode and allowing for some thermal smearing, these features in the density of states explain those in the experimental current-voltage characteristics. (2) Nedellec *et al.*[3] also observed larger-than-expected "subharmonic" (actually harmonic) oscillations in the dynamic resistance of junctions made on Pb with Al, Cu, and Fe backings, and pointed out that others[12,13] had seen similarly enhanced oscillations. The top plot in Fig. 3 is actually for the Nedellec *et al.* 1.79-μm − Pb−0.28-μm − Cu sandwich. It displays much stronger harmonic oscillations, due to incipient band gaps, than does a sandwich with a thick backing (the bottom plot) as described by the conventional theory. (3) Rowell[4,14] studied Pb-oxide-Zn-Pb junctions with Zn thickness of 1.3, 2.1, and 3.5 μm and Pb backings of 0.3 μm, and observed in conductance plots a number of bound-state peaks. Each of Rowell's two thinnest samples displayed a lowest-energy peak with a squarish shape compared to the more rounded shape of the higher-energy peaks. The structure of these lowest-energy peaks is the least smeared by finite-lifetime effects. Using Eq. (3) with a Pb gap and a Pb Fermi velocity as above, a Zn gap of 0.13 meV, and a Zn Fermi velocity of 1.5×10^8 cm/sec, we calculate bands of widths 0.37 and 0.26 meV for the lowest-energy peaks in the samples with 1.3 and 2.0 μm of Zn, respectively. These widths compare favorably with observed widths of about 0.40 and 0.28 meV, respectively. (4) Rowell[4] also observed, from about 4.0 to 5.5 mV, a drop in the amplitude of conductance oscillations relative to the amplitudes at both higher and lower energies. Our theory predicts the first interference minimum in the oscillation amplitude of Rowell's junctions to occur at 4.9 mV. (5) Finally, Rowell[4] noted a jump in the phase of the oscillations at voltages near 4.5 mV. In the present theory this phase jump is associated with a change in character of the oscillations from compound oscillations occurring in both layers to simple oscillations occurring in the front layer. The change results from an increase in damping within the Pb back layer due to phonon emission.

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