Observation of nonresonant vortex motion in a long Josephson tunnel junction

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We have observed resistive branches in the *I-V* characteristics of long Josephson junctions which can be simply understood in terms of the motion of individual Josephson fluxoids with reflection as antifluxoids at the junction edges. The characteristics of these resistive branches differ qualitatively from those of the current singularities previously reported by Chen *et al.* and by Fulton and Dynes. Our results indicate that the current singularities are not simply related to the motion of individual fluxoids.

I. INTRODUCTION

In the presence of an external magnetic field, the current-voltage (I-V) characteristic of a Josephson tunnel junction may display the well-known Fiske steps[1–5] which have been interpreted in terms of excitation of resonant electromagnetic modes in the junction. Later, Chen *et al.*[6] reported the observation of "anomalous dc current singularities" resembling the even-mode Fiske steps (FS) in one-dimensional junctions in the absence of any external magnetic field. Fulton and Dynes[7] independently observed similar current singularities and attributed them to the resonant propagation of isolated vortices in the junction.

It is expected that the vortex motion of isolated vortices in a long Josephson junction will lead to the appearance of resistive branches in the I-V characteristic of the junction, since the Lorentz force driving the vortex is proportional to the current and the time-average voltage across the junction is proportional to the vortex velocity.[7–9] (By a "long" junction is meant one with $L/\lambda_J > 2\pi$, where L is the junction length and λ_J is the Josephson penetration depth.) Because the vortex velocity may take on any value up to the velocity of electromagnetic waves in the junction $\bar{c}$ the expected resistive branch should be approximately linear and extend over a *wide* range of voltage. In contrast with this expected vortex behavior, the current singularities observed by Chen *et al.*[6] and by Fulton and Dynes[7] were near a set of discrete voltages corresponding to the cavity modes.[1–5] Furthermore, they were observed mainly in "short" junctions with nearly uniform current density (i.e., with $L/\lambda_J \lesssim 4$).[6,7,10] In this article, we report the observation of several resistive branches shown in Figs. 1 and 2 for long Josephson tunnel junctions with large L/λ_J ratios.[11] We believe that these resistive branches are the manifestation of nonresonant vortex motion with reflection at the junction ends since they have the expected properties of such vortex motion.[7,12,13] We also reexamine the basic

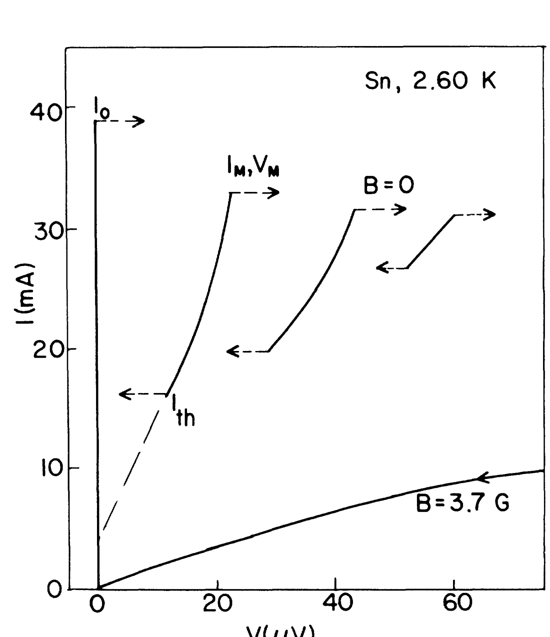

FIG. 1. The I-V characteristic of a long Sn-Sn oxide-Sn junction showing three resistive branches. Junction parameters are: length $L = 1.0$ mm, width $W = 0.1$ mm, and $L/\lambda_J = 31$ at 2.60 K. [λ_J, 0.032 mm, was calculated from $I_0 = 4\lambda_J W j_0$ and $\lambda_J = (\Phi_0/2\pi\mu_0 j_0 d)^{1/2}$, where j_0 is the maximum Josephson current density and d is twice the London penetration depth, about 850 Å.]

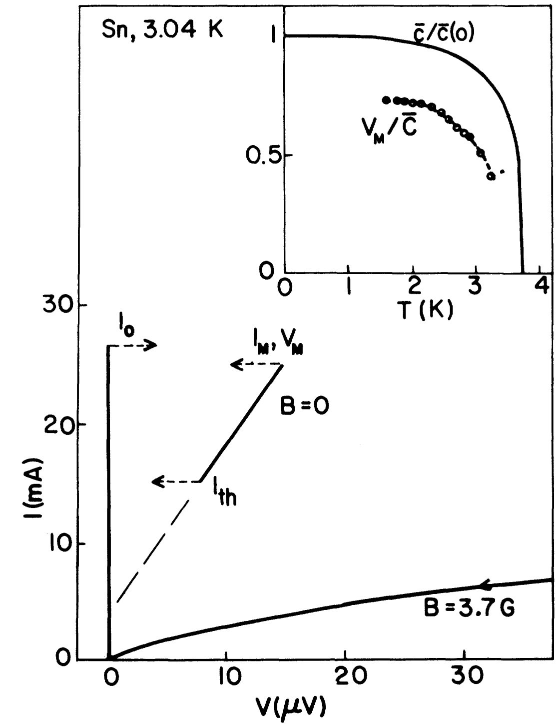

FIG. 2. The I-V characteristic of the same junction as in Fig. 1 at a higher temperature, 3.04 K. $L/\lambda_J = 24$. Inset shows the temperature dependence (Ref. 3) of $\bar{c}(T)/\bar{c}(0) = [1-(T/T_c)^4]^{1/4}$ and $v_{max}(T)/\bar{c}(T)$, where $\bar{c}(0) = 1.7 \times 10^7$ m/s. The dashed line is a visual fit to the experimental data.

features of the current singularities[6] for junctions with small L/λ_J and demonstrate several major differences between the resistive branches (RB) observed here and the current singularities (CS).[6]

II. EXPERIMENT

The samples studied were Sn-Sn oxide-Sn and Pb-Pb oxide-Pb tunnel junctions prepared by conventional techniques of thermal evaporation and oxidation. The background quasiparticle currents were consistent with the absence of any metallic short in the junction. The junctions had the in-line geometry shown in the inset of Fig. 3. Because of the symmetry, the self-field effect was negligible. We have studied numerous tunnel junctions with Josephson current densities differing by more than two orders of magnitude, ranging from 2 to 400 A/cm^2, corresponding to L/λ_J ratio of 1 to 33 at 1.20 K. In addition, the junctions with large current densities allowed us to study the heavy damping case because of their large quasiparticle current densities at higher temperatures.

As shown in Fig. 1, the I-V characteristic of a long

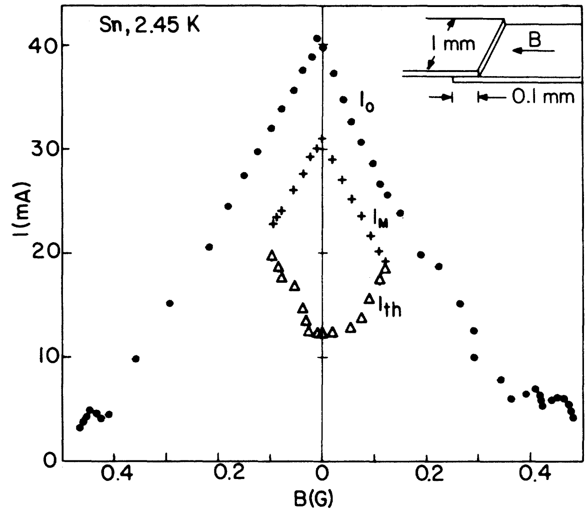

FIG. 3. The magnetic field dependences of I_0, I_{max}, and I_{th} as defined in Fig. 1. Inset shows the geometry of the sample and the direction of the applied magnetic field.

junction in the absence of an external field contains several resistive branches. Each RB (e.g., the first one) is bounded below by a threshold current I_{th} and above by a maximum current I_{max}. The voltage range on each RB is about 50% of the maximum voltage V_{max}. The shapes of the RB's shown in Fig. 1 are consistent with the numerical result obtained by Nakajima *et al.*[13] for the low-damping case. According to their analysis, the shape of the I-V characteristic associated with the vortex motion is nonlinear for low-damping situations (at low temperatures) and linear for large-damping cases (at higher temperatures). This is indeed what we have observed. As shown in Fig. 2, at higher temperature the RB's become linear at about 3.0 K. We also note that, in most cases, if the background quasiparticle current is subtracted, the resistive branch extrapolates close to zero current at zero voltage (see the thin dashed lines in Figs. 1 and 2). This linear portion above the background quasiparticle characteristic represents the *additional* current due to the moving vortices.

III. DISCUSSION

For the heavy-damping case, we can describe qualitatively the I-V characteristic of the vortex motion by equating the input power to the rate of dissipation

$$IV = n\eta v^2 , \tag{1}$$

where η is the viscosity coefficient, v the speed of vortex, and n the number of isolated vortices in the junction. Since the voltage across the junction due to vortices undergoing reflection at the edges is given by[7] $V = n\Phi_0 v/L$, where $\Phi_0 = h/2e$ is the flux quantum, Eq. (1) reduces to

$$I = (\eta L^2/n\Phi_0^2)V \quad . \tag{2}$$

It should be noted that since η is velocity and voltage dependent, the voltage and the dynamic resistance are only approximately quantized as n. Also, there will be a relativistic effect when v approaches $\bar{c}$.[7,9]

The maximum vortex speed v_{max} can be compared with $\bar{c}$ by using V_{max} and the voltage positions of the Fiske steps, $V_n = n\Phi_0\bar{c}/2L$, where n is an integer. Since the Fiske steps for a junction with large L/λ_J are usually smeared, we have used a separate junction with small L/λ_J in order to get an accurate value of $\bar{c}$. For example, V_2 was 37 μV at $T = 3$ K for the second FS of another Sn-Sn oxide-Sn junction of $L = 0.8$ mm. This gives us $\bar{c} = 1.5\times10^7$ m/s $= c/20$ for that temperature. The cutoff voltage of the RB shown in Fig. 2 is only 15 μV which gives $v_{max} = 7.5\times10^6$ m/s. Thus, the maximum speed of the vortex in this case is only $v_{max} = 0.5\bar{c}$. Maximum vortex velocities less than $\bar{c}$ were observed over the entire experimental temperature range as shown by the $v_{max}/\bar{c}$ curve in the inset of Fig. 2.

From Fig. 2, the maximum current I_{max} minus the background quasiparticle current at V_{max} is about 0.8 I_0, where I_0 is the maximum zero-voltage current. This is approximately what would be expected for $v \simeq 0.5\bar{c}$ from the numerical results of Nakajima *et al.* (see Fig. 3 of Ref. 13. For the large-damping case, $I_{max}/I_0 \simeq 0.7$ for $v/\bar{c} = 0.5$.) For lower temperatures where the damping is smaller, the change in shape with temperature also agrees with the numerical results, considering that the loss factor decreases with decreasing temperature. The nonlinear shapes of the observed RB's at low temperature are consistent with the picture of Lorentz contraction of a vortex.[7] For example, at 2.6 K, V_{max} for the first RB is 22 μV and the voltage for the second FS is 44 μV (from the previously mentioned separate junction with $L = 0.8$ mm). This gives $v_{max} = 0.625\bar{c}$. For the first RB in Fig. 1, I_{max} minus the background quasiparticle current is 29.3 mA and the corresponding current obtained from the linear extrapolation of the lower portion of the RB is 23.5 mA. They give a ratio of 1.25. The expected value from Lorentz contraction, $(1 - v_{max}^2/\bar{c}^2)^{-1/2}$, is 1.28 for $v_{max} = 0.625\bar{c}$. The ratio $v_{max}/\bar{c}$ is larger at lower temperatures, as shown in the inset of Fig. 2.

One may ask why the observed RB's terminate before v_{max} reaches $\bar{c}$. To understand this instability, we have studied experimentally the magnetic field dependences of various critical currents as shown in Fig. 3. It can be seen that I_{max} has a magnetic field dependence similar to that of I_0, except that the magnitude of I_{max} is smaller. Therefore, we suspect that I_{max} and I_0 are related to the same instability criterion. According to Owen and Scalapino,[11] the instability of a stationary vortex occurs when the total magnetic field (the external plus that induced by the current I_0) reaches the critical field B_{c1} at the edges of the junction. For the geometry of our samples (shown in the inset of Fig. 3), the magnetic field in the junction area due to the current in the films is negligible because of the symmetry and direction of the applied magnetic field. Thus, we have to consider the magnetic field associated with the tunneling current at both edges. We assume that the dc current is confined to each edge over a distance of $2\pi\lambda_J$. To simplify the calculation, we replace the rectangular cross section of area $2\pi\lambda_J W$ by a circular cross section of radius R. The maximum magnetic field associated with each edge current is then

$$B_{l,r} = (\mu_0/2\pi R)I_{l,r} \quad , \tag{3}$$

where $I_{l,r}$ is the edge current at the l or r side, W is the junction width, and $R = (2\lambda_J W)^{1/2}$. The maximum current can be obtained by setting the total magnetic field to be equal to B_{c1}; i.e., $|B| + B_{l,r} = B_{c1}$. Thus the total current $I_0 = I_r + I_r$ can be obtained as

$$I_0 = (4\pi R/\mu_0)(B_{c1} - |B|) \quad . \tag{4}$$

Experimentally, I_0 decreases linearly with B with a slope of 90 mA/G. From Eq. (4), the expected slope is $\Delta I_0/\Delta B = 4\pi(2\lambda_J W)^{1/2}/\mu_0$. For our sample with $\lambda_J = 3.17\times10^{-2}$ mm and $W = 0.1$ mm, we estimate this slope to be 80 mA/G. We have also estimated $B_{c1} \simeq 0.5$ by using $I_0 = 40$ mA. The experimental value is about 0.45 G. The agreement is good, considering the simple approximation we have used to obtain Eqs. (3) and (4).

Since the critical field $B_{c1} \propto \lambda_J j_0$, where j_0 is the maximum Josephson current density, we assume that B_{c1} for a moving vortex is reduced by a factor $(1 - v_{max}^2/\bar{c}^2)^{1/2}$ due to the Lorentz contraction. With this assumption, we expect $I_{max}^*/I_0 = (1 - v_{max}^2/\bar{c}^2)^{1/2}$, where I_{max}^* is I_{max} minus the background quasiparticle current at the same voltage. The experimental values of I_{max}^*/I_0 are shown in Fig. 4, where the solid line is $(1 - v_{max}^2/\bar{c}^2)^{1/2}$ using the $v_{max}/\bar{c}$ shown in Fig. 2. Although no adjustable parameters are involved here, the qualitative agreement between experiment and theory is excellent and the quantitative agreement is quite good. A 10% adjustment of v_{max} gives the theoretical curve shown by a dashed line.

Also shown in Fig. 4 are the temperature dependences of the maximum currents for current singularities ($B = 0$) and Fiske steps ($B \neq 0$) observed in a short junction. Those dependences are qualitatively different from the dependence for the resistive branch, and are discussed further below. We believe that the instability at I_{max} for the resistive branch is due to the destruction of the vortex at the edge of the junction, preventing it from being reflected.

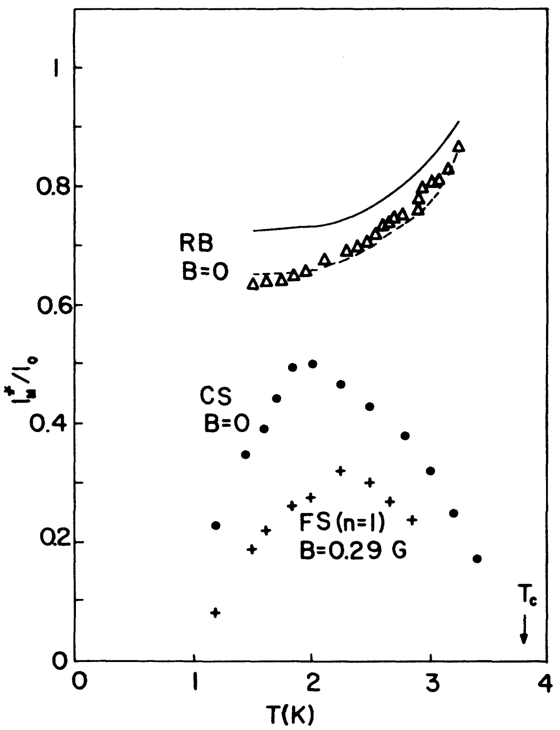

FIG. 4. I^*_{max} vs T, where triangles are for the first RB, dots for the first CS (at the voltage of the second FS) and the crosses for the first FS. Solid and dashed lines are $(1-v^2_{max}/\bar{c}^2)^{1/2}$ as explained in the text. I^*_{max} is I_{max} minus the quasiparticle current.

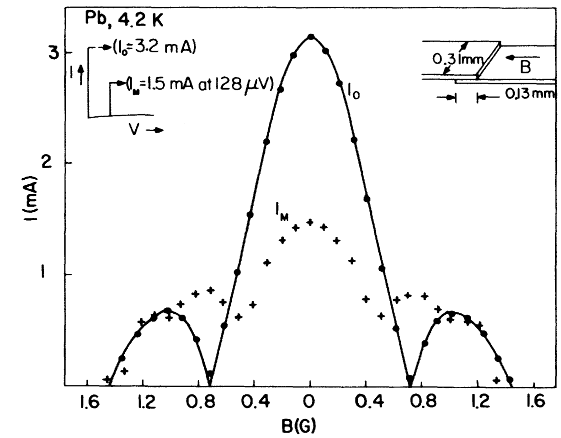

FIG. 5. Magnetic field dependences of zero-voltage current (dots) and first current singularity (crosses) of a small Pb junction. Solid line is the theoretical I_0 vs B. The left inset shows the I-V curve with one CS, and the right inset shows the junction geometry. At 4.2 K, $\bar{c}=1.92\times10^7$ m/s, $j_0=8\times10^4$ A/m^2, and $\lambda_J=0.185$ mm.

Consequently, the vortex motion responsible for RB below I_{max} is that involving reflection at the edges.

Without showing it, we point out that the experimental V_{max} of each RB decreases linearly with B at higher temperatures, where the I-V curve is linear in agreement with Eq. (2). At lower temperatures, it decreases with increasing B monotonically, but does not have a simple functional form. This indicates that V_{max} is a secondary condition depending upon I_{max}.

In regard to the threshold current I_{th}, we can only make some qualitative statements. It is probably a consequence of the fact that a vortex and an antivortex must have a minimum energy in order to pass through each other without annihilation.[13,14] The higher order RB's have larger threshold currents, as can be seen from Fig. 1. This is consistent with the theoretical calculations of Ref. 13. The magnitudes of the threshold currents, typically a large fraction (tenths) of I_0, are in qualitative agreement with their predictions.

As mentioned in the Introduction, a Josephson junction with small L/λ_J may display current singularities at discrete voltages.[6] An example of a CS is shown in the inset of Fig. 5. Not only the sample conditions (such as L/λ_J and j) are quite different for obtaining RB's and CS's, almost all the observed properties are also in sharp contrast. The differences include the characteristic shapes, voltage positions, the magnetic field and the temperature dependences of various critical parameters, such as I_0, I_{max}, V_{max}, and I_{th}. The voltage spread of the CS shown in Fig. 5 is less than 3% over the entire current range and it has no lower threshold current. The Josephson current density of this junction is spatially uniform since its L/λ_J is about 1.7. (Theoretically, vortex can be generated in zero applied field if $L/\lambda_J>2\pi$.) The field dependence of I_0 agrees completely with the theory for a junction with uniform current density as shown in Fig. 5. Thus, it is fair to conclude that for this junction the self-field associated with Josephson current is not sufficient to produce a complete vortex.[11,15] In addition, at low magnetic field, except for a constant factor, I_{max} of the CS has exactly the same magnetic field dependence as I_0, indicating that the current amplitude is probably spatially quite uniform even when the junction is biased on its CS state. In an increasing magnetic field the CS makes a smooth transition to Fiske step, which has the correct periodic B dependence of a junction with uniform current density. Again, this is very different from the behavior of an RB.

To further demonstrate that the CS is a phenomenon related to cavity modes, not necessarily involving motion of isolated vortices, we compare the temperature dependences of an RB, a CS, and an FS in Fig. 4. There is an obvious similarity between CS and FS, namely, they both have a maximum near $0.5T_c$. No such maximum exists for the RB, where I^*_{max}/I_0 increased monotonically with T until it suddenly disappeared at 3.3 K, in agreement with the theoretically predicted behavior.[13] Another similarity between CS and FS is that their voltage positions are independent of magnetic field, while $V_{\max}$ of RB decreases with increasing B. This was observed but is not shown here.

Recently, Costabile *et al.*[16] have obtained a theoretical I-V curve for resonant-fluxoid motion similar to the vortex-motion picture described by Fulton and Dynes.[7] Since no obvious magnetic field or temperature dependences have been given by them, it is not possible to make a definite comparison with our results. It seems to us that their theoretical treatments are more relevant to RB of a long junction with small loss, rather than a CS in a small junction.

It should be noted that although the shape of a CS appears to resemble a RB of vortex motion for low-damping case, they are qualitatively quite different. First of all, the numerical results of Nakajima *et al.*[13] and Costabile *et al.*[16] have both shown that, even for the low-damping case, the I-V branch of the vortex motion extends over a *wide* voltage range whereas the CS (as shown in Fig. 5) is limited to a narrow region near the voltage expected for an even-mode Fiske step. The current in a CS drops rapidly and continuously to the background quasiparticle current indicating that no additional current other than the usual quasiparticle current is flowing in the junction. We like to emphasize that the quasiparticle characteristic should *not* be mistaken as the low-voltage portion of the vortex motion as the former is insensitive to a small change of magnetic field. Secondly, the shape of the CS does *not* strongly depend upon the temperature as expected for the vortex motion. Even at a higher temperature, e.g., $T \approx 0.8\,T_c$, where the quasiparticle current is as large as the magnitude of the current step, it still maintains the shape of a current step with nearly constant voltage. This is in sharp contrast with the temperature sensitive RB of the vortex motion. Thirdly, the maximum voltage of each RB is much less than the expected Fiske-step voltage indicating that it is far away from resonance, whereas the CS occurs exactly at the cavity-mode-related voltage. In our opinion, there is no apparent theoretical ground for requiring that the vortex motion should be in resonance with the cavity mode. In fact, because of the relativistic contraction of vortex, the current density as well as the associated magnetic field increase drastically when $v \approx \bar{c}$. Thus it seems unlikely that the vortex can be stable when it approaches the edge current with a speed near $\bar{c}$. Finally, the resistive branch has a threshold current and voltage which, we believe, is associated with the minimum energy required for the vortex-antivortex pair to pass each other. No such threshold current exist for the constant-voltage current steps.

IV. CONCLUSION

We have observed resistive branches in the I-V characteristics of long tunnel junctions which can be simply understood in terms of the motion of individual Josephson fluxoids with reflection as antifluxoids at the junction edges. The characteristics of the current singularities reported previously by Chen *et al.* and by Fulton and Dynes, and interpreted in terms of individual fluxoid motion by the latter authors, differ qualitatively from those of the resistance branches. Our data suggest that these current singularities are related to the electromagnetic cavity modes of the junction and are not simply related to the motion of individual fluxoids. This conclusion is supported by another fact that our RB's were observed in the voltage range below the overall cutoff voltage[7] corresponding to the Josephson plasma frequency,[17–20] while the cavity-mode-related CS's were observed above such overall cutoff voltages.[21]

Recently, Takanaka[22] has shown that the basic features of the zero-field current steps can indeed be obtained by extending the existing theory of Fiske steps using the boundary conditions appropriate for the standing electromagnetic modes in the junction. Our conclusion based upon the experimental comparisons of CS and RB and FS is in agreement with this latest theoretical work.

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[21]The Josephson plasma frequency is given by $\omega_J = \bar{c}/\lambda_J$ or for a small junction also by $(2eI/\hbar C)^{1/2}$, where C is the junction capacitance. The corresponding voltage is $V_J = \Phi_0\omega_J/2\pi$. For Fig. 1, $\bar{c} = 1.6 \times 10^{17}$ m/s and $V_J \simeq 165$ μV which is above the voltage range of RB's. For Fig. 5, $V_J \simeq 34$ μV, which is below the voltage of the observed CS. Note that the cutoff voltages estimated in Ref. 7 were larger by a factor of 2π, consequently, led to a different conclusion. It can be easily seen that with the correction of 2π, if $L < 2\pi\lambda_J$, all the even-mode Fiske-step voltages, $n\Phi_0\bar{c}/L$, are greater than this cutoff voltage $\Phi_0\bar{c}/2\pi\lambda_J$. Hence, according to the argument of Ref. 7, the observed current singularities or steps cannot be due to the moving fluxoids.
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