Geometry-induced splitting of third sound into fifth and sixth sounds in two facing He-II films

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Splitting of the third-sound mode propagating in the two parallel superfluid films interacting by the evaporation and condensation processes were theoretically studied by calculating superfluid hydrodynamic equations. It was found that not only the suppressed mode (temperature wave) of the phase-change processes, called fifth sound, but also the enhanced mode (thickness wave) of such processes propagate according to the phase difference between the variations in the two films.

It is well known that the velocity of third sound in superfluid film is greatly modified because of mass and heat exchanges between liquid and vapor resulting from evaporation and condensation. These processes act as a dissipation in the hydrodynamic equations for a superfluid film.[1] An essential effect of the phase-change processes through the liquid-gas interface is the isothermal effect, which decreases the temperature variation of film occurring by the propagation of third sound. Therefore, if the evaporation and condensation effects are enhanced or suppressed by a geometrically suitable waveguide for the third sound, a new mode different from the normal third sound must be observed. In the present paper, the physical nature of the new modes propagating in the two parallel films separated by a small gap of vapor is studied. These modes correspond to the enhanced mode and the suppressed mode of the evaporation and condensation according to the phase difference between the variations of two films.

As this manuscript was being prepared, two papers connected with this theme appeared by Jelatis *et al.*[2] and by Williams *et al.*[3] They have experimentally found the fifth-sound mode with the velocity $C_5 = (\rho_n/\rho)^{1/2} C_2$, where ρ_n/ρ is the normal-fluid fraction and C_2 is the velocity of second sound. Fifth sound is a propagating wave in the superfluid film under the condition that the evaporation and condensation processes can be geometrically neglected. In order to achieve this purpose, Williams *et al.* have used a superleak made of powder particles, and they have utilized the small bubbles of helium vapor localized in the middle of powder particles. Jelatis *et al.* have utilized the small spacing of helium vapor constructed by the two parallel quartz substrates with the surface of optical flat. As shown in the following discussion, the structure of the waveguide considered by us is the same as that used by Jelatis *et al.*

It is to be noted that the coupled modes by the evaporation and condensation processes are interesting not only to study the dynamics of sounds propagating in superfluid film but also to study the evaporation and condensation efficiency at the liquid-gas interface. In addition the geometrical coupling of waves is an interesting system with different properties from the coupling between the internal freedoms of the substances such as spin-phonon coupling,[4] helicon–spin-wave coupling,[5] etc. In fact, in the experiment of Williams *et al.*[3] fifth-sound waves couple with surface tension waves because the surface shape of liquid is spherical.

For waveguide conditions we assume that the spacing of helium vapor between two parallel superfluid films (named film 1 and film 2) is smaller than the mean free path of gaseous atoms, and also we assume that the wavelength of sound is much larger than the distance of this spacing. When the temperature of film 1 and film 2, respectively, change by δT_1 and δT_2 from the equilibrium value T due to the sound disturbance, we can write a following expression for the mass flux from film 1 to vapor by evaporation and condensation[6–10]:

$$J_1 = \beta K \delta T_1 - \gamma K \delta T_2 \ , \tag{1}$$

where K is defined by the expression $J = K\delta T$ for the mass flux due to the evaporating atoms from liquid to perfect vacuum. The coefficients β and γ are the net evaporating rate and the net condensing rate, taking into account the recondensation effect that the evaporating atoms from one film condense to the original film by the reflection at the surface of opposite film. In the case of no reflection, β and γ are both equal to unity.

In the same manner as the third sound,[1] we write the superfluid hydrodynamic equations for the two films coupled by evaporation and condensation,

$$\frac{\partial v_{s1}}{\partial t} - \bar{S}\frac{\partial \delta T_1}{\partial z} + \frac{\partial \Omega}{\partial d}\frac{\partial \delta \xi_1}{\partial z} = 0 \ , \tag{2}$$

$$\rho\frac{\partial \delta \xi_1}{\partial t} + \bar{\rho}_s d\frac{\partial v_{s1}}{\partial z} = -J_1 \ , \tag{3}$$

$$S\frac{\partial \delta \xi_1}{\partial t} + d\frac{\partial \bar{S}}{\partial T}\frac{\partial \delta T_1}{\partial t} = -\frac{L+ST}{\rho T}J_1 \ , \tag{4}$$

and the equations for film 2 are given by replacing subscript 1 by 2. The variables ν_s and $\delta\xi$ are the superfluid velocity and the deviation from the equilibrium value d in the film thickness due to the sound disturbance, Ω is the van der Waals potential, L is the latent heat of evaporation, ρ and S are the density and the entropy of bulk liquid, and $\bar{\rho}_s$ and $\bar{S}$ are the superfluid density and the entropy averaged over the film thickness. In these equations, it is assumed that the conditions of both films are equal to each other such as $d_1=d_2=d$, $\bar{S}_1=\bar{S}_2=\bar{S}$ and so on, except for the variables ν_s, δT, and $\delta\xi$. Also we have defined the z axis as parallel to the films. As indicated in Eq. (2), two forces due to the temperature and the van der Waals potential operate on the motion of superfluid. Therefore, it is obvious that the sound wave can propagate in the superfluid film even in such a condition that the temperature variation of the film is completely canceled out.

Assuming solutions of a traveling-wave form with frequency ω and wave number k, we obtain the dispersion law;

$$\frac{\omega^2}{k^2}=\frac{\bar{\rho}_s d}{\rho}\frac{\partial\Omega}{\partial d}+\frac{\bar{\rho}_s S T}{\rho}\left[\bar{S}-\frac{i(\beta\pm\gamma)K}{\rho\omega}\frac{\partial\Omega}{\partial d}\right]\Bigg/\left[T\frac{\partial\bar{S}}{\partial T}-\frac{i(\beta\pm\gamma)KL}{\rho\omega d}\right] , \tag{5}$$

where it is clear that the third sound separates into the evaporation enhanced mode (+) with $\beta+\gamma$ and evaporation suppressed mode (−) with $\beta-\gamma$. Replacing γ by zero, Eq. (5) is the well-known dispersion equation for third sound. The velocities and attenuations of plus and minus modes normalized by those of third sound are shown in Fig. 1 as a function of γ/β. In that calculation we used $d=500$ Å, $\omega=2\pi\times10^3\ \mathrm{sec}^{-1}$, $T=2.1$ K, and

$$\beta K=(m_4/2\pi k_B T)^{1/2}(dP/dT)_{\mathrm{co}} ,$$

where m_4 is the mass of ^{4}He atoms and $(dP/dT)_{\mathrm{co}}$ is the temperature derivative of saturate vapor pressure for liquid ^{4}He taken along the coexistence curve. The difference between the velocity C_+ for plus mode and third-sound velocity C_3 is small as shown in Fig. 1(a), which depends on the values of d, T, and ω. The attenuation of plus mode α_+ and $|C_+-C_3|/C_3$ decrease with decreasing d, T, and ω. Even in such a situation that C_+-C_3 is unobservably small, α_+ has a clear difference from the attenuation of third sound α_3, which is equal to half of α_3 in $\gamma/\beta=1$. The velocity C_- and attenuation α_- for the minus mode have very different values in comparison with those of third sound. For the case of $\gamma/\beta=1$, C_-^2 is equal to $C_3^2+\bar{C}_5^2$ and α_- is zero, where $\bar{C}_5^2=(\bar{\rho}_s/\rho)S\bar{S}/(\partial\bar{S}/\partial T)$. The velocity of the minus mode which increases with decreasing film thickness has been observed by Jelatis *et al.*[2] The splitting of the third sound into the evaporation enhanced and suppressed modes is analogous to the splitting of sound into first and second sounds. Therefore, strictly speaking these two split modes should be called fifth and sixth sounds, though the previous authors[2,3] called the fifth modes as the deviation of the sound frequency from the third sound by the geometric feature.

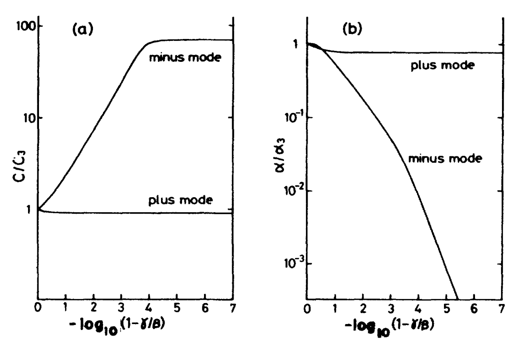

FIG. 1. (a) Velocity C and (b) attenuation α for two modes normalized by those for third sound as a function of the ratio of the net condensing rate γ to the net evaporating rate β, which are calculated from the dispersion law by using the values shown in text. The plus and minus modes correspond to the evaporation enhanced and suppressed modes of phase-change processes.

From the normal mode analysis we get

$$\xi_1'=-\left[\frac{d}{S}\frac{\partial\bar{S}}{\partial T}-\frac{i(\beta\pm\gamma)K(L+ST)}{\rho\omega ST}\right]T_1' , \tag{6}$$

$$\nu_{s1}'=\frac{k\pm}{\omega}\left[\frac{\partial\Omega}{\partial d}\xi_1'-\bar{S}T_1'\right] , \tag{7}$$

$$T_1'=\mp T_2' , \tag{8}$$

where ξ', T', and ν_s' are the amplitudes of $\delta\xi$, δT, and ν_s. The upper and lower signs correspond to the plus and minus modes, respectively. From Eq. (6) we can know the intensity and phase relations between ξ' and T' for two modes, which are shown in Fig. 2 as a function of γ/β, where we used the same values for d, T, and ω as in Fig. 1. The ratio of temperature variation to thickness variation for the minus mode is much greater than that for third sound or plus mode. However, for the plus mode the thickness variation is dominant rather than the temperature variation. Therefore, we might also call the plus mode as a thickness mode and the minus mode as a temperature mode. As shown in Fig. 2(b), the phase difference between the variations of temperature and of film thickness is π for minus mode and $\frac{1}{2}\pi$ for the plus mode as well as third sound. As regards the phase relation between the

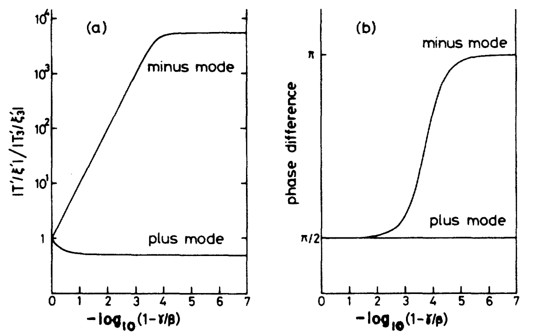

FIG. 2. (a) Ratio of temperature variation to thickness variation $|T'/\xi'|$ for two modes normalized by the value for third sound as a function of the ratio of the net condensing rate γ to the net evaporating rate β. (b) Phase difference between the temperature variation and thickness variation for two modes as a function of γ/β. These curves are calculated from the results of normal mode analysis by using the values shown in text. The plus and minus modes correspond to the evaporation enhanced and suppressed modes of phase change processes.

variations of film 1 and film 2, it is easily understood that out of phase and in phase for plus and minus modes from Eq. (8). Schematic drawing of amplitude relations and of phase relations for plus and minus modes are shown in Fig. 3 in the case of $\gamma/\beta=1$.

For the generation of the plus mode, one needs to provide a boundary condition for the out-of-phase component between the variations of the two films. For the generation of a minus mode, a boundary condition having an in-phase component is needed. Such circumstances are quite similar to the boundary condition for the generation of first and second sounds, which depends on the phase relation between

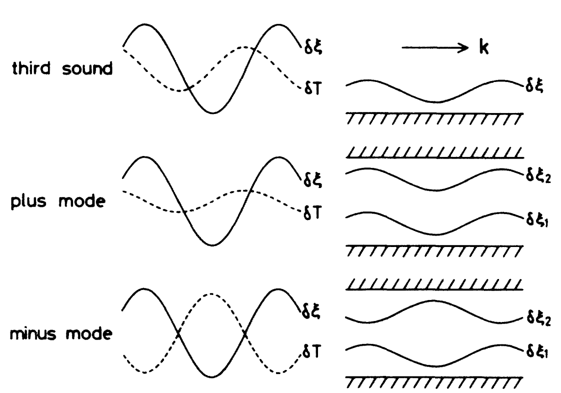

FIG. 3. Schematic drawing of the amplitude and phase relations for two modes in comparison with the third sound. The plus and minus modes correspond to the evaporation enhanced and suppressed modes of phase change processes. $\delta\xi$ and δT are the variations of film thickness and of temperature due to the sound propagation, and k is the wave vector.

the velocities of superfluid and normal fluid components. In the experiment of Jelatis *et al.*,[2] the boundary condition which is produced by the heating of the resistance wire is not so clear, namely, whether it has the component out of phase or not. In order to generate the plus mode and also to observe the splitting of plus and minus modes at the same time, we propose the following experiment of the conversion from third sound to plus and minus modes. The experimental setup considered here is the same configuration as the previous one except that the upper substrate (S_2) is placed in the space of $z \geq 0$, in other words, the lower substrate (S_1) is opened so as to be able to propagate third sound in the range of $z<0$. The third sound propagating on S_1 of $z<0$ is divided into two parts of reflection and transmission by the boundary located in $z=0$, and the transmission part propagates on S_1 and S_2 of $z>0$ as plus and minus modes. By using the continuity conditions of v_s and δT at the boundary, it was found that the temperature variation of the minus mode is three times as great as that of the plus mode. The calculated value of the transmission coefficient for the incidence of third sound was $\frac{4}{9}$. Therefore, using a bolometer as the detector of sounds, both signals of plus and minus modes can be observed. In addition, if we perform this experiment by using time of flight at $T=1.4$ K, $d=150$ Å, and $\omega=4\pi\times10^2$ sec^{-1}, we should be able to observe separately the signals of plus and minus modes with the time delay of about $2\pi/\omega$ by the bolometer placed at the distance of 1 cm from the boundary.

We also investigated the splitting of third sound in the case of $d_1 \neq d_2$ arising from the condition that the two substrates consist of different materials. The dispersion law for this system is different from Eq. (5) only on the terms including $\beta \pm \gamma$ because of the equilibrium condition $\Omega_1(d_1)=\Omega_2(d_2)$. Therefore, it is obvious that the minus mode is not entirely affected by the difference of thickness between two films. This fact suggests that the minus mode is a well-defined mode even in the waveguide with inhomogeneous film thickness such as in the superleak used by Williams *et al.*[3]

Several authors[6–10] have proposed some models for the boundary conditions between liquid and gas phase including phase-change processes. However, in the present condition that the spacing is smaller than the mean free path of gaseous atoms, it seems that the problem is unrelated to these models. Assuming the specular reflection of atoms at the liquid surface, by taking into account the recondensation effect, we get $\beta=\gamma=(2-f)^{-1}$, where f is the probability of condensation for the incidence atoms. When the spacing of vapor is longer than the mean free path of atoms, it becomes important to consider the existence of a gas phase between the two films on the condensation process. The condensing atoms to the film is not the evaporating atoms directly coming from the opposite film such as previous case, but the atoms coming from the gas phase with the pressure defined by the thermodynamics. However, this pressure is fundamentally decided by the temperatures of the two films. According to the Hunter and Osborne model,[6] the pressure variation of gas phase is obtained as $(dP/dT)_{co}\frac{1}{2}(\delta T_1+\delta T_2)$, and we get same value for β and γ as $\beta=\gamma=\frac{8}{9}/(2-f)$. Finally, even in such a case that the spacing of vapor is smaller than the viscous penetration depth of gas phase, the two modes can be completely decoupled.

To conclude, we have theoretically studied the geometrical coupling modes of third sound propagating in the two facing superfluid films interacting by the evaporation and condensation processes. The observation of the evaporation enhanced mode and also the observation of the splitting of third sound into evaporation enhanced and suppressed modes would be interesting.

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