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Effect of energy-band structures on transverse magnetoresistance of degenerate semiconductors in strong magnetic fields

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The effect of energy-band structures on the transverse magnetoresistance in degenerate semiconductors has been studied for the case where acoustical phonons are the dominant scattering mechanism. The calculation has been performed taking into account the inelasticities in the electron-phonon scattering due to the finite energy of the phonons involved. Results show that the transverse magnetoresistance for the nonparabolic band structure is enhanced much more considerbly than that for the parabolic band structure. We also found that the transverse magnetoresistance for both parabolic and nonparabolic band structures oscillates with the dc magnetic field owing to the degeneracy of the electron gas. However, the number of oscillations for the nonparabolic band structure is larger than that for the parabolic band structure.

I. INTRODUCTION

Transverse and longitudinal magnetoresistances are the two most commonly investigated properties of semiconductors in which the effect of the dc magnetic field on electronic transport properties is exhibited. Arora[1] found that the transverse magnetoresistance changes dramatically with inelasticity, while the longitudinal magnetoresistance remains essentially unchanged. Consequently, inelasticity may be expected to play an active role and should be included for the electronic transport in the transverse configuration. Some experimental results for the inelastic scattering mechanism show that the transverse magnetoresistance depends strongly on the dc magnetic field.[2] The transverse magnetoresistance for nondegenerate semiconductors with the isotropic parabolic energy bands has been investigated for the case where acoustic phonons are the dominant scattering mechanism.[3] It was shown that the transverse magnetoresistance increases with the dc magnetic field in the quantum limit. Arora *et al.*[4] also discussed the behavior of the strong-field magnetoresistance under conditions where the acoustic phonon scattering in the high-temperature limit is considered to be the dominant mechanism of scattering. They found that the transverse magnetoresistance increases linearly with the dc magnetic field in the quantum limit. In our previous works,[5,6] the effect of nonparabolicity on transverse magnetoresistance in nondegenerate semiconductors has been studied for the inelastic scattering of acoustic phonons. We found that the nonparabolicity of the energy-band structure changes the effect of the temperature on the transverse magnetoresistance besides the enhancement of its magnitude. Askerov *et al.*[7–9] have pointed out that in the case of degenerate semiconductors the band nonparabolicity and the scattering inelasticity have a strong influence on the field dependence of the magnetoresistance. However, they considered only the ultraquantum limit by assuming that all electrons are in the lowest state. For the nonparabolic band structure one has to consider the effect of the band shape not only in the density of states but also in the scattering probability. It is the purpose of our present paper to study the effect of energy-band structure on the transverse magnetoresistance in degenerate semiconductors. From our previous paper,[6] it was shown that the deformation-potential coupling mechanism plays the dominant role for the transverse magnetoresistance in strong magnetic fields in nondegenerate semiconductors. Therefore, we shall take into account the inelasticity in the electron-phonon scattering from the deformation-potential coupling only in our present work. The scattering is treated in the Born approximation for strong magnetic fields. For a degenerate semiconductor, the distribution function of electrons is represented by the Fermi-Dirac statistics. In Sec. II, we perform the calculation of the transverse magnetoresistance of degenerate semiconductors with the nonparabolic band structure throughout the strong-field region. It is assumed here that the inelasticity is the dominant mechanism in resolving the divergence which occurs for the strong-field transverse magnetoresistance. In Sec. III, we present numerical results and give a brief discussion.

II. FORMULATION

In the nonparabolic model, the energy eigenvalue equation for electrons in a uniform dc magnetic field

$\vec{B}$ directed along the z axis is[10]

$$H_0(1+H_0/E_g)\Psi_{\vec{k}n}=(1/2m^*)\times[p_x^2+(p_y-eBx/c)^2+p_z^2]\Psi_{\vec{k}n}=E_{\vec{k}n}(1+E_{\vec{k}n}/E_g)\Psi_{\vec{k}n}\ , \tag{1}$$

where E_g is the energy gap between the conduction and valence bands, m^* is the effective mass of electrons at the minimum of the conduction band, and $E_{\vec{k}n}$ is the true energy of the system, defined by $H_0\Psi_{\vec{k}n}=E_{\vec{k}n}\Psi_{\vec{k}n}$. The eigenfunctions and eigenvalues for Eq. (1) are given by

$$\Psi_{\vec{k}n}=\Phi_n[x-(\hbar c/eB)k_y]\exp(ik_yy+ik_zz) \tag{2}$$

and

$$E_{\vec{k}n}=-\tfrac{1}{2}E_g(1-\{1+(4/E_g)[(n+\tfrac{1}{2})\hbar\omega_c+\hbar^2k_z^2/2m^*]\}^{1/2})\ , \tag{3}$$

where k_y and k_z are the y and z components of the electron wave vector $\vec{k}$, $\Phi_n(x)$ is the harmonic-oscillator wave function, and $\omega_c=|e|B/m^*c$ is the cyclotron frequency of electrons. Since $(\hbar k_{z\,\max})^2/2m^*<E_g$ and $k_BT\ll E_g$ in strong magnetic fields and at the low temperatures in which we are interested ($T<10$ K), Eq. (3) can be expanded as

$$E_{\vec{k}n}=\tfrac{1}{4}E_g(b_n-b_n^{-1})+\hbar^2k_z^2/2m^*b_n\ , \tag{4}$$

where $b_n=1+2(n+\frac{1}{2})\hbar\omega_c/E_g$. When $(n+\frac{1}{2})\hbar\omega_c+\hbar^2k_z^2/2m^*\ll E_g$, the energy eigenvalues reduce to those obtained using the parabolic model for the band structure

$$E_{\vec{k}n}=(n+\tfrac{1}{2})\hbar\omega_c+\hbar^2k_z^2/2m^*\ . \tag{5}$$

However, when the dc magnetic fields come into the high-field region, the energy levels of electrons are quite different from those predicted by using the parabolic model.

For the scattering due to acoustic phonons, the dissipative current lying in the direction of the total electric field is given by[3,11-13]

$$J_d=\frac{|e|L^2v_H\hbar}{k_BT}\sum_{\vec{k}n,\vec{k}'n'}\tfrac{1}{2}(k_y-k_y')^2\times f_{\vec{k}n}(1-f_{\vec{k}'n'})W_{\vec{k}n,\vec{k}'n'}\ , \tag{6}$$

where $L=(\hbar/m^*\omega_c)^{1/2}$ is the classical radius of the lowest Landau level, $\vec{v}_H=c(\vec{E}\times\vec{B})/B^2$ is the Hall velocity with the applied electric field $\vec{E}$, $f_{\vec{k}n}$ is the distribution function of electrons, and $W_{\vec{k}n,\vec{k}'n'}$ is the transition probability in the Born approximation between the Landau states $\vec{k}n$ and $\vec{k}'n'$. Following the same method as our previous papers,[5,6] Eq. (6) becomes

$$J_d=\frac{\pi|e|L^2v_H}{k_BT}\sum_{\vec{k}n,\vec{k}'n',q}|C(q)|^2q_y^2N_q(N_q+1)\Bigg[(f_{\vec{k}'n'}-f_{\vec{k}n})\left|\frac{n!}{n'!}\right|(\tfrac{1}{2}L^2q_\perp^2)^{n'-n}\exp(-\tfrac{1}{2}L^2q_\perp^2)$$
$$\times[L_n^{n'-n}(\tfrac{1}{2}L^2q_\perp^2)]^2\delta(E_{\vec{k}n}-E_{\vec{k}'n'}-\hbar\omega_q)\delta(k_z-k_z'-q_z)$$
$$-(f_{\vec{k}'n'}-f_{\vec{k}n})\left|\frac{n'!}{n!}\right|(\tfrac{1}{2}L^2q_\perp^2)^{n-n'}\exp(-\tfrac{1}{2}L^2q_\perp^2)$$
$$\times[L_{n'}^{n-n'}(\tfrac{1}{2}L^2q_\perp^2)]^2\delta(E_{\vec{k}n}-E_{\vec{k}'n'}+\hbar\omega_q)\delta(k_z-k_z'+q_z)\Bigg]\ , \tag{7}$$

where $q(i,\vec{q})$ denotes collectively the branch and wave vector for the phonon mode with the energy $\hbar\omega_q$, $C(q)$ is the electron-phonon coupling constant, $q_\perp$ and q_y are the components of the phonon wave vector normal to the dc magnetic field and in the $\vec{B}\times\vec{E}$ direction, respectively, $N_q=[\exp(\hbar\omega_q/k_BT)-1]^{-1}$ is the Planck distribution function for the phonons in thermal equilibrium, $\delta(x)$ is the Dirac δ function, and $L_n^m(x)$ is the associated Laguerre polynomial.[14] $\delta(E_{\vec{k}n}-E_{\vec{k}'n'}-\hbar\omega_q)$ and $\delta(E_{\vec{k}n}-E_{\vec{k}'n'}+\hbar\omega_q)$ in Eq. (7) are given as follows:

$$\delta(E_{\vec{k}n}-E_{\vec{k}'n'}-\hbar\omega_q)=\begin{cases}(m^*/\hbar^2q_{n'n})b_nb_{n'}(b_{n'}-b_n)^{-1}\{\delta[k_z+q_zb_n/(b_{n'}-b_n)+q_{n'n}]+\delta[k_z+q_zb_n/(b_{n'}-b_n)-q_{n'n}]\} & \text{for } n'>n\ , \quad (8a)\\ (m^*/\hbar^2q_{nn'})b_nb_{n'}(b_n-b_{n'})^{-1}\{\delta[k_z+q_zb_n/(b_n-b_n')+q_{nn'}]+\delta[k_z+q_zb_n/(b_n-b_{n'})-q_{nn'}]\} & \text{for } n'<n\ , \quad (8b)\end{cases}$$

with

$$q_{n'n}=(m^*E_g/2\hbar^2)^{1/2}[1+b_{n'}b_n+4\omega_q\hbar b_{n'}b_n/E_g(b_{n'}-b_n)+2q_z^2\hbar^2b_{n'}b_n/m^*E_g(b_{n'}-b_n)^2]^{1/2}\ .$$

For acoustical-phonon scattering in semiconductors via the deformation-potential-coupling mechanism, the electron-phonon coupling constant is given by[11,12]

$$|C(q)|^2 = E_1^2 q\hbar/2\rho v_s \ , \tag{9}$$

where E_1 is the deformation-potential constant, ρ is the mass density of the crystal, and v_s is the sound velocity. We employ the high-temperature approximation which is generally satisfied at the temperature where the acoustical-phonon scattering dominates,[3,11,12] then we have $N_q \simeq k_B T/\hbar\omega_q = k_B T/\hbar v_s|\vec{q}|$. Therefore, we obtain the expression for the dissipative current via the deformation-potential-coupling mechanism using the Fermi-Dirac statistics

$$\begin{aligned} J_d = {} & \frac{|e|L^2 v_H m^{*2}\omega_c E_1^2 k_B T}{8\pi^3 \rho v_s^3 \hbar^4} \\ & \times \sum_{p=-P}^{P}\sum_{m=0}^{M}\int_0^\infty q_\perp^2\, dq_\perp \frac{m!}{(p+m)!}\left(\tfrac{1}{2}L^2 q_\perp^2\right)^p \exp\left(-\tfrac{1}{2}L^2 q_\perp^2\right)\left[L_m^p\left(\tfrac{1}{2}L^2 q_\perp^2\right)\right]^2 (b_{p+m}b_p)^{1/2} \\ & \qquad \times \ln\left\{\left\{P_{p,m}^{1/2} + \left[P_{0,m} + \frac{v_s q_\perp \hbar b_m}{E_g}\right]^{1/2} + \left[Q_{p,m} + 2\left[P_{0,m} + \frac{v_s q_\perp \hbar b_m}{E_g}\right]^{1/2} P_{p,m}^{1/2} + \frac{v_s q_\perp \hbar b_{p+m}}{E_g}\right]^{1/2}\right\}\right. \\ & \qquad\qquad \left.\times \left\{P_{0,m}^{1/2} + \left[P_{p,m} - \frac{v_s q_\perp \hbar b_{p+m}}{E_g}\right]^{1/2} + \left[Q_{p,m} + 2\left[P_{p,m} - \frac{v_s q_\perp \hbar b_{p+m}}{E_g}\right]^{1/2} P_{0,m}^{1/2} - \frac{v_s q_\perp \hbar b_m}{E_g}\right]^{1/2}\right\}^{-1}\right\} , \end{aligned} \tag{10}$$

with the condition $(M+P+\frac{1}{2})\hbar\omega_c < E_F(1+E_F/E_g)$, where E_F is the Fermi energy at $B=0$, $P_{p,m} = (E_F/E_g)b_{p+m} - \frac{1}{4}(b_{p+m}^2 - 1)$, and $Q_{p,m} = (E_F/E_g)(b_m + b_{p+m}) + \frac{1}{4}[(b_m/b_{p+m}) + (b_{p+m}/b_m) - 2b_{p+m}b_m]$. The quantum number m indicates the Landau level n or n' in Eq. (6) or (7), and p is the transition quantum number between n and n'. The corresponding maximum values of m and p are M and P, respectively, which should satisfy the condition in Eq. (10).

In strong magnetic fields, the transverse magnetoresistance, $\rho_\perp$ can be approximated by

$$\rho_\perp = EJ_d/(n_0 e v_H)^2 \ , \tag{11}$$

where $n_0 = (2m^* E_F)^{3/2}/(3\pi^2\hbar^3)$ is the electron density of semiconductors. The expression for the resistivity in the absence of a dc magnetic field due to the deformation-potential coupling in the degenerate case is[12]

$$\rho_0 = (m^*/n_0 e^2)(E_1^2 k_B T/\rho v_s^2)(E_F^{1/2}/2\pi\hbar)(2m^*/\hbar^2)^{3/2} \ . \tag{12}$$

From Eqs. (10)–(12), the transverse magnetoresistance due to the deformation-potential coupling for the non-parabolic band structure can be obtained as

$$\begin{aligned} \left[\frac{\rho_\perp}{\rho_0}\right] = {} & \frac{3\hbar^4\omega_c}{32 v_s (m^* E_F)^2} \\ & \times \sum_{p=-P}^{P}\sum_{m=0}^{M}\int_0^\infty q_\perp^2\, dq_\perp \frac{m!}{(p+m)!}\left(\tfrac{1}{2}L^2 q_\perp^2\right)^p \exp\left(-\tfrac{1}{2}L^2 q_\perp^2\right)\left[L_m^p\left(\tfrac{1}{2}L^2 q_\perp^2\right)\right]^2 (b_{p+m}b_m)^{1/2} \\ & \qquad \times \ln\left\{\left\{P_{p,m}^{1/2} + \left[P_{0,m} + \frac{v_s q_\perp \hbar b_m}{E_g}\right]^{1/2} + \left[Q_{p,m} + 2\left[P_{0,m} + \frac{v_s q_\perp \hbar b_m}{E_g}\right]^{1/2} P_{p,m}^{1/2} + \frac{v_s q_\perp \hbar b_{p+m}}{E_g}\right]^{1/2}\right\}\right. \\ & \qquad\qquad \left.\times \left\{P_{0,m}^{1/2} + \left[P_{p,m} - \frac{v_s q_\perp \hbar b_{p+m}}{E_g}\right]^{1/2} + \left[Q_{p,m} + 2\left[P_{p,m} - \frac{v_s q_\perp \hbar b_{p+m}}{E_g}\right]^{1/2} P_{0,m}^{1/2} - \frac{v_s q_\perp \hbar b_m}{E_g}\right]^{1/2}\right\}^{-1}\right\} , \end{aligned} \tag{13}$$

with the condition $(M+P+\frac{1}{2})\hbar\omega_c < E_F(1+E_F/E_g)$. Similarly, the transverse magnetoresistance due to the deformation-potential coupling for the parabolic band structure is obtained as

$$\left(\frac{\rho_\perp}{\rho_0}\right) = \frac{3\hbar^4\omega_c}{32 v_s (m^* E_F)^2}\sum_{p=-P}^{P}\sum_{m=0}^{M}\int_0^\infty q_\perp^2\, dq_\perp \frac{m!}{(p+m)!}\left(\tfrac{1}{2}L^2 q_\perp^2\right)^p \exp\left(-\tfrac{1}{2}L^2 q_\perp^2\right)\left[L_m^p\left(\tfrac{1}{2}L^2 q_\perp^2\right)\right]^2$$

$$\times \ln\left(\frac{[E_F + v_s q_\perp \hbar - (m+\frac{1}{2})\hbar\omega_c]^{1/2} + [E_F - (p+m+\frac{1}{2})\hbar\omega_c]^{1/2}}{[E_F - (m+\frac{1}{2})\hbar\omega_c]^{1/2} + [E_F - v_s q_\perp \hbar - (p+m+\frac{1}{2})\hbar\omega_c]^{1/2}}\right), \quad (14)$$

with the condition $(M+P+\frac{1}{2})\hbar\omega_c < E_F$.

III. NUMERICAL RESULTS AND DISCUSSION

The expressions in Eqs. (13) and (14) can be approximated by making use of the conditions for the strong magnetic field region,[1,3,6,11,12] $\hbar\omega_q \simeq \hbar v_s q_\perp >> m^* v_s^2$ and $\hbar\omega_c = \hbar^2/m^* L^2 >> m^* v_s^2$. Using the integral representation[15]

$$\int_0^\infty \exp(-x)x^{n+\beta}[L_k^n(x)]^2\,dx = \frac{\Gamma(1+n+\beta)\Gamma(1+n+k)}{(k!)^2\Gamma(1+n)} \times \left[\frac{d^k}{dh^k}\left(\frac{(1-h)^\beta F(\frac{1}{2}(1+n+\beta), 1+\frac{1}{2}(n+\beta); 1+n; 4h/(1+h)^2)}{(1+h)^{1+n+\beta}}\right)\right]_{h=0},$$

$$\mathrm{Re}(n+\beta) > -1\ , \quad (15)$$

where $\Gamma(z)$ is the gamma function and $F(a,b;c;z)$ is a hypergeometric function, we obtain the transverse magnetoresistance for the nonparabolic and parabolic band structures in the Appendix.

As a numerical example in highly doped n-type InSb, the relevant values of physical parameters are $v_s = 4\times10^5$ cm/sec and those in Table I.[16] Our numerical results are shown in Figs. 1–3. It can be seen that the transverse magnetoresistance for degenerate semiconductors oscillates with the dc magnetic field for both parabolic and nonparabolic band structures owing to the degeneracy of the electron gas. It can also be seen that the amplitudes of these oscillations will increase with the dc magnetic field. These results for degenerate semiconductors are quite different from those for nondegenerate semiconductors in which the transverse magnetoresistance increases monotonously with the dc magnetic field.[3,5,6] However, the transverse magnetoresistance for the nonparabolic band structure oscillates more strongly than that for the parabolic band structure. Askerov *et al.*[7–9] discussed the effect of the inelasticity of scattering and nonparabolicity on the magnetoresistance in n-type InSb for the ultraquantum limit with the quantum number $n=0$ in strong magnetic fields. But this could not be a good approximation for degenerate semiconductors in which the distribution function of electrons is represented by the Fermi-Dirac statistics. Since the effective mass for electrons in an energy level of the nonparabolic band structure with the quantum number n is $m^* b_n$, the effective mass of electrons defined by $m^* b_n$ will depend strongly upon the dc magnetic field. Therefore, the transverse magnetoresistance for the nonparabolic band structure is enhanced much more than that for

TABLE I. Physical parameters for n-type InSb.[a]

n_0 (cm^{-3})	m^* (m_0)	E_g (eV)	E_F (eV)
3×10^{18}	0.029	0.38	0.2616
10^{19}	0.039	0.50	0.4340
3×10^{19}	0.054	0.85	0.6520

[a]Note that n_0 is the electron density of n-type InSb at $B=0$; m_0 is the mass of a free electron.

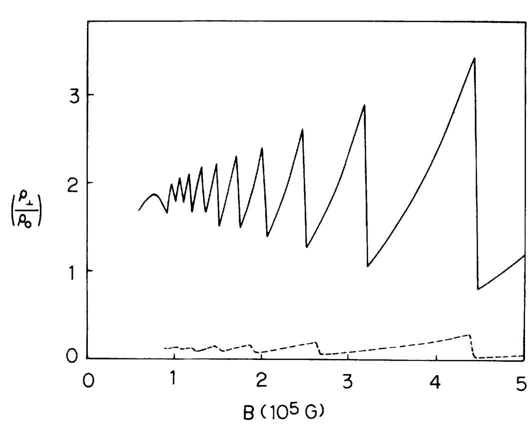

FIG. 1. Transverse magnetoresistance ($\rho_\perp/\rho_0$) as a function of dc magnetic field B in degenerate n-type InSb ($n_0 = 3\times10^{18}$ cm^{-3}) for parabolic band structure (dashed curve) and nonparabolic band structure (solid curve).

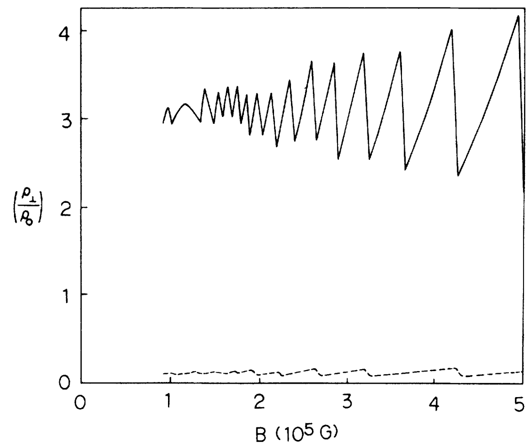

FIG. 2. Transverse magnetoresistance ($\rho_\perp/\rho_0$) as a function of dc magnetic field B in degenerate n-type InSb ($n_0 = 10^{19}$ cm^{-3}) for parabolic band structure (dashed curve) and nonparabolic band structure (solid curve).

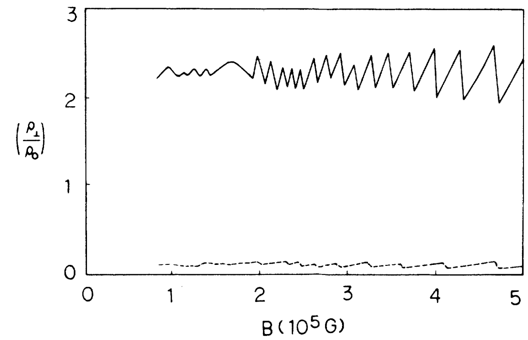

FIG. 3. Transverse magnetoresistance ($\rho_\perp/\rho_0$) as a function of dc magnetic field B in degenerate n-type InSb ($n_0 = 3\times 10^{19}$ cm^{-3}) for parabolic band structure (dashed curve) and nonparabolic band structure (solid curve).

the parabolic band structure. Moreover, the number of oscillations in the transverse magnetoresistance for the nonparabolic band structure is larger than that for the parabolic band structure. The electron wave vector, which is defined by $\vec{k}_n = \hat{z}[(2m^*)^{1/2}/\hbar][E_F(1+E_F/E_g) - \hbar\omega_c(n+\frac{1}{2})]^{1/2}$ for the nonparabolic band structure and $\vec{k}_n = \hat{z}[(2m^*)^{1/2}/\hbar][E_F - \hbar\omega_c(n+\frac{1}{2})]^{1/2}$ for the parabolic band structure owing to the degeneracy of the electron gas, gives a significant contribution to the scattering rate of phonon emission and absorption processes. As the magnetic field increases, the quantum numbers M and P in Eqs. (13) and (14), or Eqs. (A1) and (A2), decrease, so the number of oscillations in the transverse magnetoresistance for both band structures will decrease. From our expressions for the transverse magnetoresistance in Eqs. (13) and (14), or Eqs. (A1) and (A2), and Table I, we can see that the quantum numbers M and P increase with the product of m^* and E_F, the transition quantum number P will thus increase with the electron density. Therefore, the number of oscillations with the magnetic field increase with the electron density. However, the amplitudes of oscillations in the transverse magnetoresistance for the nonparabolic band structure are enhanced as the electron density decreases. This kind of quantum oscillation can be interpreted as the "giant quantum oscillations,"[17,18] which occur in a degenerate electron gas in the case when the electron level is near the Fermi surface and the sound wave vector $\vec{q}$ has a component along the dc magnetic field. These oscillations arise because the electrons in semiconductors interact with the acoustical phonons.

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APPENDIX

For the nonparabolic band structure, ($\rho_\perp/\rho_0$) is given by

$$\left(\frac{\rho_\perp}{\rho_0}\right) = \frac{3}{32}\left(\frac{\hbar\omega_c}{E_F}\right)^3 \sum_{m=0}^{M}(2m+1)\left[1+(2m+1)\left(\frac{7\hbar\omega_c}{4E_g}+\frac{5\hbar\omega_c}{16E_F}\right)\right] - \frac{9\sqrt{\pi}}{2048\Gamma(\frac{5}{4})\Gamma(\frac{7}{4})}\left(\frac{\hbar\omega_c}{E_F}\right)^{9/2}\left(\frac{m^*v_s^2}{2E_F}\right)^{1/2}\left(\frac{10E_F}{E_g}-1\right)$$
$$\times\sum_{m=0}^{M}(2m+1)\left[\sum_{k=0}^{m}\sum_{j=0}^{m-k}\frac{(-1)^{m-k-j}4^k\Gamma(k+\frac{5}{4})\Gamma(k+\frac{7}{4})}{(k!)^2}\begin{pmatrix}\frac{3}{2}\\ m-k-j\end{pmatrix}\begin{pmatrix}-2k-\frac{5}{2}\\ j\end{pmatrix}\right]$$
$$+\frac{3}{16}\left(\frac{\hbar\omega_c}{E_F}\right)^3\sum_{p=1}^{P}\sum_{m=0}^{M}(p+2m+1)\left[1+(p+2m+1)\left(\frac{7\hbar\omega_c}{4E_g}+\frac{5\hbar\omega_c}{16E_F}\right)\right] - \frac{3}{256}\left(\frac{\hbar\omega_c}{E_F}\right)^{9/2}\left(\frac{m^*v_s^2}{2E_F}\right)^{1/2}\left(\frac{10E_F}{E_g}-1\right)$$
$$\times\sum_{p=1}^{P}\sum_{m=0}^{M}\frac{(p+2m+1)\Gamma(p+\frac{5}{2})}{\Gamma(\frac{1}{2}p+\frac{5}{4})\Gamma(\frac{1}{2}p+\frac{7}{4})}\left[\sum_{k=0}^{m}\sum_{j=0}^{m-k}\frac{(-1)^{m-k-j}4^k\Gamma(k+\frac{1}{2}p+\frac{5}{4})\Gamma(k+\frac{1}{2}p+\frac{7}{4})}{(k+p)!k!}\right.$$
$$\left.\times\begin{pmatrix}\frac{3}{2}\\ m-k-j\end{pmatrix}\begin{pmatrix}-2k-p-\frac{5}{2}\\ j\end{pmatrix}\right], \quad \text{(A1)}$$

with the condition $(M+P+\frac{1}{2})\hbar\omega_c < E_F(1+E_F/E_g)$. For the parabolic band structure, $(\rho_\perp/\rho_0)$ is given by

$$\left[\frac{\rho_\perp}{\rho_0}\right]=\frac{3}{32}\left[\frac{\hbar\omega_c}{E_F}\right]^3\sum_{m=0}^{M}(2m+1)\left[1+\frac{(2m+1)\hbar\omega_c}{4E_F}\right]+\frac{3}{16}\left[\frac{\hbar\omega_c}{E_F}\right]^3\sum_{p=1}^{P}\sum_{m=0}^{M}(p+2m+1)\left[1+\frac{(p+2m+1)\hbar\omega_c}{4E_F}\right], \tag{A2}$$

with the condition $(M+P+\frac{1}{2})\hbar\omega_c < E_F$. Here

$$\binom{n}{k}=\frac{n!}{(n-k)!k!}.$$

[1]V. K. Arora, Phys. Rev. B 13, 2532 (1976).

[2]M. I. Aliev, B. M. Askerov, R. G. Agaeva, A. Z. Daibov, and I. A. Ismailov, Fiz. Tekh. Poluprovodn. 9, 570 (1975) [Sov. Phys. Semicond. 9, 377 (1975)].

[3]D. R. Cassiday and H. N. Spector, Phys. Rev. B 9, 2618 (1974).

[4]V. K. Arora, D. R. Cassiday, and H. N. Spector, Phys. Rev. B 15, 5996 (1977).

[5]Chhi-Chong Wu and Anna Chen, Appl. Phys. Lett. 30, 434 (1977).

[6]Chhi-Chong Wu and Anna Chen, Phys. Rev. B 18, 1916 (1978).

[7]B. M. Askerov and F. M. Gashimzade, Phys. Status Solidi 28, 783 (1968).

[8]R. G. Agaeva, B. M. Askerov, and F. M. Gashimzade, Phys. Status Solidi B 59, K43 (1973).

[9]R. G. Agaeva, B. M. Askerov, and F. M. Gashimzade, Fiz. Tekh. Poluprovodn. 7, 1625 (1973) [Sov. Phys. Semicond. 7, 1085 (1974)].

[10]Chhi-Chong Wu and H. N. Spector, Phys. Rev. B 3, 3979 (1971).

[11]R. Kubo, S. J. Miyake, and N. Hashitsume, in *Solid State Physics*, edited by F. Seitz and D. Turnbull (Academic, New York, 1965), Vol. 17, p. 269.

[12]L. M. Roth and P. N. Argyres, in *Semiconductors and Semimetals*, edited by R. K. Willardson and A. C. Beer (Academic, New York, 1966), Vol. 1, p. 159.

[13]H. F. Budd, Phys. Rev. 175, 241 (1968).

[14]A. Erde'lyi, W. Magnus, F. Oberhettinger, and F. G. Tricomi, *Higher Transcendental Functions* (McGraw-Hill, New York, 1953), Vol. 2, p. 188.

[15]I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products* (Academic, New York, 1965), p. 845.

[16]O. Madelung, *Physics of III-V Compounds* (Wiley, New York, 1964); H. J. Hrostowski, G. H. Wheatley, and W. F. Flood, Jr., Phys. Rev. 95, 1683 (1954).

[17]V. L. Gurevich, V. G. Skobov, and Yu. A. Firsov, Zh. Eksp. Teor. Fiz. 40, 786 (1961) [Sov. Phys. JETP 13, 552 (1961)].

[18]S. H. Liu and A. M. Toxen, Phys. Rev. 138, A487 (1965).