Studies of the decay of persistent currents in unsaturated films of superfluid ^{4}He

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We report the results of systematic measurements of the decay of persistent currents of superfluid ^{4}He films as a function of both temperature and film thickness. For film currents where the fractional decay per decade is less than 20%, the data can be represented by $v = v_0[1 - \xi \times \ln(t/t_0)]$. For stronger decays we document deviations from the $\ln t$ behavior and observe that the empirical rule $v = A(B+t)^{-n}$ reasonably represents the data over the full range of our studies. The Iordanskii-Langer-Fisher thermal fluctuation theory is examined in the vortex pair model, and inadequacies of the theory are discussed. A comparison is also made to the predictions of Donnelly, Hills, and Roberts.

I. INTRODUCTION

A striking demonstration of superfluidity in ^{4}He is the observation of the persistent flow of macroscopic mass currents. Such flow has been observed in both bulk helium in a restricted geometry[1,2] and in saturated[3] and unsaturated[4] superfluid films. Studies of the stability of these currents and information on the conditions under which they decay provides the potential for a more complete understanding of the nature of superfluidity in these films. Previous studies[1,2,4] of the decay of persistent currents in both bulk and films have yielded results consistent with

$$v = A - B \ln t \quad . \tag{1}$$

Here A and B are empirical constants, t the time, and v the superfluid velocity. We report here the observations of persistent current decays for which the final supefluid velocity is in some cases consistent with zero and for which the description provided by Eq. (1) is entirely inadequate. We present measurements[5] as a function of both film thickness and temperature which show the evolution of departures from Eq. (1). The data are analyzed in terms of the Iordanskii[6]-Langer-Fisher[7] fluctuation theory under the assumption that the excitations are pairs of quantized vortex lines oriented perpendicular to the substrate and plane of the superfluid flow. We show that this model is not in quantitative agreement with the data over the full range of film thickness values and temperatures studied. We also compare our experimental results with predictions based on the competing barrier model of Donnelly and Roberts[8] and Donnelly, Hills, and Roberts.[9]

II. BACKGROUND

Studies of the stability of persistent currents in ^{4}He have been rare. Kukich *et al.*[10] were the first to carry out such studies. In their work bulk helium in the presence of filter material or Vycor was set into motion using rotational techniques. When the rotation was stopped after cooling the ^{4}He below T_λ the decay of the resulting mass currents was studied through measurments of the angular momentum. The velocity was observed in all cases to decay according to the rule given by Eq. (1) in spite of the fact that velocity changes from the starting velocity of as much as 10% were seen. In these measurements the characteristic dimension of the filter material and Vycor ranged from 2000 Å for the filter material down to 40 Å for the Vycor.

The data obtained from these measurements were analyzed in terms of the Iordanskii[6]-Langer-Fisher[7] model in which the deceleration of the superfluid flow is given by

$$\frac{dv}{dt} = -\frac{h}{m} A \nu_0^* e^{-E_a/kT} \tag{2}$$

Here h is Planck's constant, m the mass of a helium atom, A the cross-sectional area available to the flowing film, ν_0^* the attempt frequency, k the Boltzmann constant, T the temperature, and E_a the activation energy of the excitation. Since expansion of E_a about a critical velocity yields an expression of the form of Eq. (1) it was argued that the results provided a confirmation of the theory. Studies of the temperature dependence of the critical velocity were used to provide an indication of the velocity dependence of E_a and although the numerical agreement was not good the velocity dependence for the larger pore sizes was consistent with a vortex ring model for E_a and for the Vycor was consistent with a model for E_a which consists of pairs of quantized vortex lines. Additional studies on the filter material substrate were carried out for helium films[11] but no detailed measurements on the decay of persistent film currents were made.

The techniques of Doppler-shifted fourth sound were used by Kojima *et al.*[2] to study the decay of persistent bulk ^{4}He currents in a compressed powder geometry. In this work it was observed that dv/dt was not a function of the velocity alone, but rather the decay rate was dependent on the state of the persistent current. The decay rule given by Eq. (1) was seen in these studies and the fractional decay per decade was observed to decrease with a decrease in the initial velocity of the persistent current. This work was the first to suggest that vortex flow might be important in a complete understanding of the decay of these persistent currents. This was significant since the Iordanskii[6]-Langer-Fisher[7] theory does not include the notion of vortex flow.

Until recently[5] the only measurements of the decay of persistent currents in helium films were those of Telschow and Hallock.[4] Their experiments were carried out on a Pyrex substrate and persistent currents were generated thermally without rotation. The velocity was measured using the techniques of Doppler-shifted third sound.[12] Studies of this type offer a distinct advantage over studies of bulk helium in a constrained geometry since the characteristic dimension (in this case the film thickness) can be readily changed during the course of an experiment. In the work of Telschow and Hallock[4] the decay rule, Eq. (1), was again observed, in some cases for velocity changes in excess of 60% although slight deviations from Eq. (1) at small times were reported. It was also observed that the fractional decay per decade was independent of the initial persistent current velocity. In this respect the results differed from those of Kojima *et al.*[2]

The present work includes a systematic study of the decay of persistent film currents as a function of both film thickness and temperature. We find,[5] in agreement with Telschow and Hallock,[4] that even for moderately strong decays which involve relatively large changes in the film flow velocity, the data obey Eq. (1) remarkably well. For the strong decays, some of which are observed to decay completely, substantial deviations from Eq. (1) are observed and a more general decay rule is found to represent the data.

III. APPARATUS AND PROCEDURE

The apparatus for the present measurements is an improved version of that used previously by Telschow and Hallock.[4] The film flow path consists of a Pyrex annular ring, Fig. 1, with a diameter of 10 cm. Persistent currents are induced on the ring by means of the heater Q. Application of current to heater Q results in a flow of superfluid film from the reservoir R to the heater. The flow velocity on path A is related to Q the rate of heating by the expression

$$\dot{Q} = \langle \rho_s \rangle v_A P_A (L + TS) d \ , \tag{3}$$

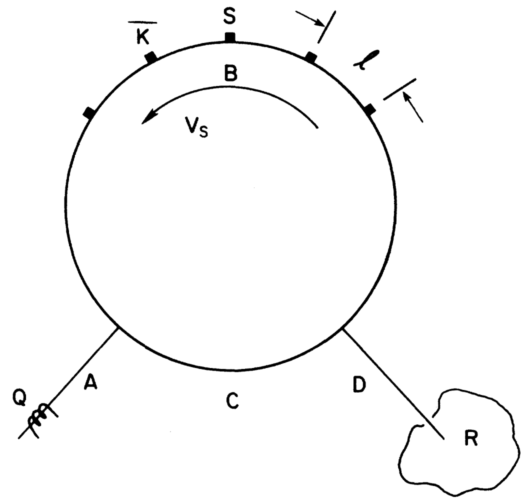

FIG. 1. Schematic representation of the persistent current apparatus. The reservoir R consists of 8 g of 0.05-μm Al_2O_3 powder to provide a large surface area for the film. The flow surfaces are fire-polished Pyrex with a perimeter of ≈ 2 cm. The diameter of the ring is 10 cm. The source S and detectors of third sound are thin evaporated Al films with a separation l equal to 1.0 cm. Q is a resistive heater used to induce superfluid film flow from the reservior around the ring. $\bar{K}$ is an additional heater.

where $\langle \rho_s \rangle$ is the effective superfluid density in the film, P_A the perimeter of flow path available to the film of thickness d, L the latent heat, T the temperature, and S the entropy of the film. For low values of $\dot{Q}$ the circulation $K = \oint \vec{v} \cdot d\vec{l} = v_B l_B - v_c l_c$ (see Fig. 1) around the ring remains zero. Thus, since $l_B \gg l_C$ the velocity $|v_C| \gg |v_B|$ although so long as $\dot{Q}$ is kept small, neither velocity becomes critical. A reduction in $\dot{Q}$ to zero results in a return to zero of v_B and v_C.

Larger values of $\dot{Q}$ cause the velocity v_C to reach a critical value. Further increases in $\dot{Q}$ then result in an increase in v_B while v_C remains at the critical value. Thus, for $\dot{Q}$ above a certain value the circulation increases. Subsequent reduction of $\dot{Q}$ results in a finite circulation around the ring. In this manner persistent currents are induced around the ring without using rotational techniques. Small persistent currents may be increased by an increase in the $\dot{Q}$ value. At any time a persistent current can be destroyed by operation of the heater $\bar{K}$ located symmetrically with respect to the film reservoir R.

The flow velocity v is measured by means of the technique of Doppler-shifted third sound.[12] On a film of thickenss d the third-sound velocity C_{30} is given by[13]

$$C_{30}^2 = \frac{\langle \rho_s \rangle}{\rho} f d \left(1 + \frac{TS}{L} \right)^2 , \tag{4}$$

where f is the Van der Walls force which attracts the helium to the substrate. We have taken[14,15] $f=\alpha\beta(3\beta+4d)/[d^4(d+\beta)^2]$ with $\alpha=2.58\times10^{-14}$ $\mathrm{erg\,cm^3g^{-1}}$ and $\beta=41.7$ layers, and $\langle\rho_s\rangle/\rho=(\rho_s/\rho)\times(1-D/d)$ with[16] $D=0.5+1.13T\rho/\rho_s$. Accurate measurements of C_{30} require an accurate knowledge of the effective distance between the source and detectors of third sound. We determine this under the operating conditions used for the third-sound measurements from measurements of the sound velocity in the vapor above the film at known temperatures. We are able to do this since our third-sound detectors are sensitive to sound in the vapor even though they are covered with superfluid film. Sound pulses of width 31 μsec were used. Our measurements of C_{30} are consistent with Eq. (4) to within a few percent when d is obtained[17] from

$$\beta^{-1}d^4+d^3+\frac{27}{T\ln(P/P_0)}=0 \quad . \tag{5}$$

Here P_0 is the saturated vapor pressure at the temperature T and P is the pressure in the experimental chamber where the film thickness is d. A comparison between thickness values obtained from Eqs. (4) and (5) using observed values of C_{30}, and P for a temperature of 1.602 K is shown in Table I. Given the uncertainties in the various constants used in Eqs. (4) and (5) we view this agreement as satisfactory. We have recently observed[18] that a film in persistent flow is thinner than a static film under the same conditions by an amount consistent with predictions based on the work of Kontorovich.[19,20] This effect is small and can be safely neglected in the context of the measurements we describe here. For the remainder of the work we describe here, all thicknesses have been determined from third-sound velocity measurements and the use of Eq. (4).

If the superfluid film moves with velocity v the velocity of third sound is Doppler shifted according to

$$C_{\pm}=C_{30}\pm(\langle\rho_s\rangle/\rho)v \quad . \tag{6}$$

For our measurements the upstream and downstream arrival times for a third-sound pulse emitted at source S are t_1 and t_2, respectively. We have then

$$\tfrac{1}{2}\Delta C=\tfrac{1}{2}(C_+-C_-)=\tfrac{1}{2}l(t_2^{-1}-t_1^{-1})=(\langle\rho_s\rangle/\rho)v \quad , \tag{7}$$

where l is the separation between the generator and detector of third sound. Thus, through measurements of the arrival times we are able to make measurements of $\frac{1}{2}\Delta C$ and using $\langle\rho_s\rangle/\rho$ can deduce the flow velocity v.

The third-sound generator and detectors consist of Al strips 1 cm $\times$ 0.25 mm $\times$ 50 nm evaporated onto a Pyrex substrate. Under typical operating conditions the generator S is driven with a single-cycle sine voltage pulse of frequency 1.6 kHz at a repetition rate of 30–100 Hz. We have carefully ensured that the observed persistent current decay rates are not modified due to changes in either the third-sound drive voltage or the bias current supplied to the third-sound detectors. We are thus convinced that our measurement technique has no observable effect on the persistent current decays we discuss here.

The Pyrex ring is enclosed in a brass chamber which can be isolated by means of a superfluid value. The brass chamber has a dead volume of 195 $\mathrm{cm^3}$ and incorporates Al_2O_3 powder which enhances the surface area to 920 $\mathrm{m^2}$. This large surface area provides a ^{4}He reservoir which serves to stabilize the film thickness on the Pyrex ring during the measurements and which acts as a film source during the dynamical process which creates the persistent current. Each of the numerous persistent current decay measurements we report here was conducted with the superfluid valve sealed and hence was conducted in the presence of stable conditions within the chamber. This brass chamber was mounted at the end of a standard cryostat. Temperature was measured by means of Allen-Bradley carbon resistors calibrated against the vapor pressure of ^{4}He during the various experimental runs. Manostat techniques were used to control the temperature during the measurements. The temperature stability during a given measurement was ±1 mK.

TABLE I. Thickness values in atomic layers determined from vapor-pressure and third-sound velocity measurements. For these data $T=1.602$ K and $P_0=5.746$ Torr. The values obtained here were not collected in the order presented but rather interleaved during a thickening and subsequent thinning of the film. The quantity $\delta(\%)=100\times[d(P)-d(C_{30})]/d(P)$ shows that a small systematic difference between the values of d deduced from Eqs. (4) and (5) exists. The origin of this small effect is not clear; it may be due to our choice of $\langle\rho_s\rangle/\rho$.

C_{30} (cm/sec)	P_0-P(Torr)	$d(P)$	$d(C_{30})$	δ(%)
1969	0.4207	5.80	5.68	+2.07
1699	0.2799	6.63	6.45	+2.71
1423	0.1934	7.47	7.45	+0.27
1257	0.1471	8.16	8.19	−0.37
1084	0.1072	9.03	9.15	−1.33
1042	0.0986	9.27	9.41	−1.51
958	0.0824	9.81	10.00	−1.94
846	0.0630	10.68	10.93	−2.34

IV. RESULTS AND DISCUSSION

Under certain conditions of thickness and temperature we observe persistent current decays which resemble Eq. (1) in their form. An example of a

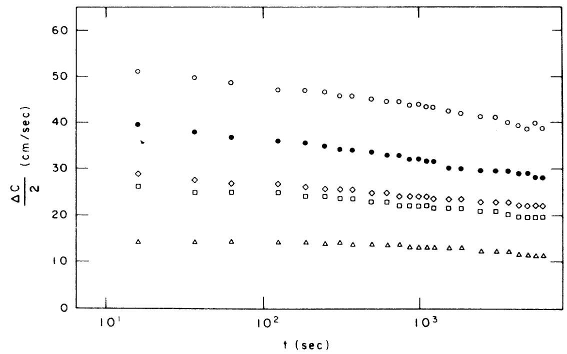

FIG. 2. An example of several persistent current decays where the fractional decay per decade is relatively small. $\frac{1}{2}\Delta C$ is proportional to the superfluid velocity around the ring [Eq. (7)]. The data shown here were taken at a film thickness of 7.6 atomic layers with $T=1.30$ K and are consistent with the empirical rule given by Eq. (9).

series of such decays is shown in Fig. 2 for various different initial velocities. If we write the empirical expression

$$\tfrac{1}{2}\Delta C=(\tfrac{1}{2}\Delta C)_0-A\ln(t/t_0) \tag{8}$$

we observe that the quantity $\xi=A/(\Delta C/2)_0$ is independent of $(\Delta C/2)_0$ over a wide range of observed $(\Delta C/2)_0$ values. This is shown (including the case of the data of Fig. 2) in Fig. 3. We thus choose to write

$$\tfrac{1}{2}\Delta C=(\tfrac{1}{2}\Delta C)_0[1-\xi\ln(t/t_0)]\ , \tag{9}$$

where the parameter ξ is independent of $(\Delta C/2)_0$

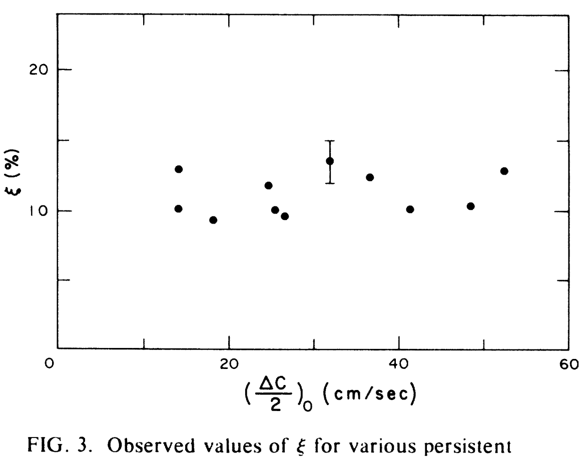

FIG. 3. Observed values of ξ for various persistent current decays with different initial velocities ($t_0=60$ sec). The values shown here are obtained from fits of the data (including that shown in Fig. 2) to Eq. (9). We conclude that ξ is essentially independent of starting velocity v_0 over the range of values studied. Note that $v_0=(\langle\rho_s\rangle/\rho)^{-1}\times(\Delta C/2)_0$.

and hence is independent of v_0.[4,5] The independence of ξ from v_0 for persistent current decays of the sort shown in Fig. 2 suggests that ξ is a useful quantity to characterize decays of this general type.

Given this we have made a systematic study of ξ as a function of film thickness and temperature for persistent current decays which have the general form given by Eq. (9). In all cases we have taken $t_0=60$ sec and carried out computer fits for the time domain $t_0\leq t\leq 6000$ sec to facilitate the determination of ξ. In Fig. 4 we display the temperature dependence of ξ for several of the film thicknesses studied. We find that due to the limited temperature range available for the data at any one thickness, no specific functional dependence of ξ on T is clearly superior. Figure 5 shows the thickness dependence of ξ at one particular temperature. A two-parameter functional form which fits the data is $\xi(\%)=100\exp[A(1-d/d_0)]$ with $A=4.7\pm0.4$ and $d_0=5.8\pm0.1$. This is shown on the figure as the solid line. The data of Fig. 5 are in qualitative agreement with the earlier measurements of Telschow and Hallock.[4] The differences which do exist are possibly attributable to differences in the substrate or perhaps the thickness scale.[21]

Not all of the persistent current decays we have observed can be well described by Eq. (9) although in

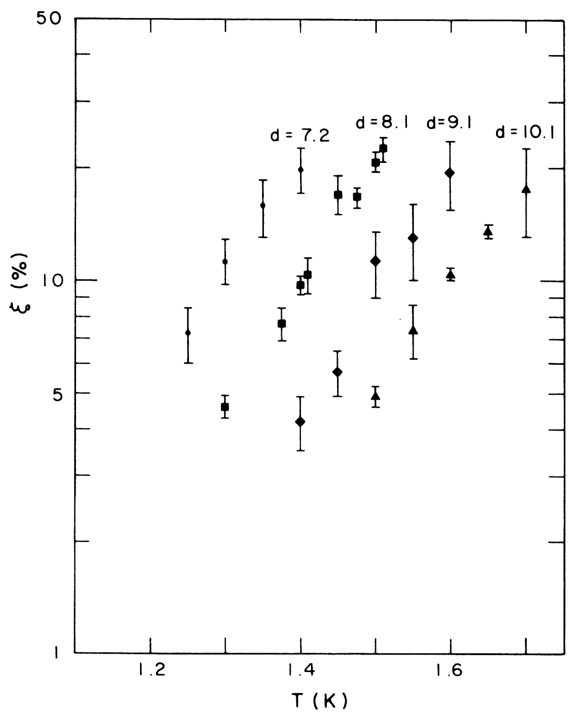

FIG. 4. Observed values of ξ vs temperature for several values of the film thickness. The data shown at 8.1 atomic layers is a superposition of data taken in two experimental runs several months apart.

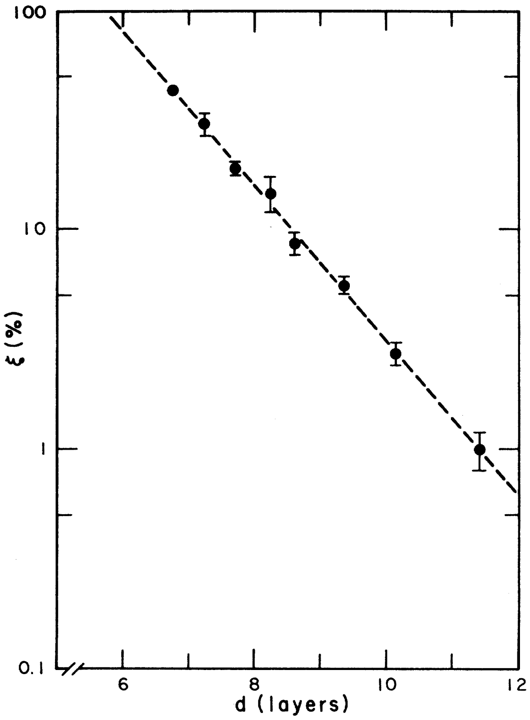

FIG. 5. Observed dependence of ξ on the film thickness at $T=1.45$ K. The data are empirically described by $\xi(\%)=100\exp[4.7(1-d/5.8)]$ and extrapolate to $\xi=100\%$ at $d=5.8$ atomic layers. The fitted expression is shown as the dashed line.

most cases the data can be approximated by it over a part of the complete decay. This is illustrated by Fig. 6 in which we display an observation of the *complete* decay of a persistent current. Deviations from the decay rule given by Eq. (9) are clearly evident. The transition from the linear behavior illustrated by Fig. 2 to the behavior illustrated by Fig. 6 is not abrupt

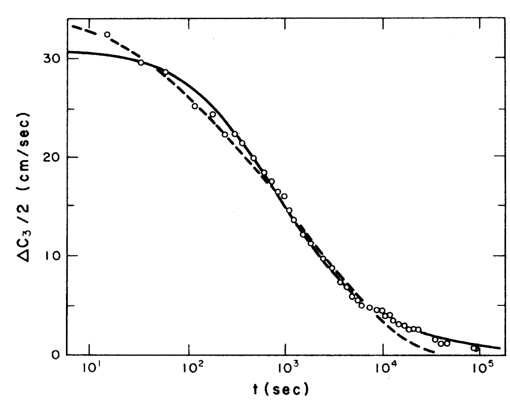

FIG. 6. Example of a nearly-complete decay of a persistent current. The solid and broken curves result from fits to the data using the functional forms as discussed in the text.

but gradual. An example of this evolution at fixed temperature as a function of film thickness is shown in Fig. 7. For all of these data except for the case of the thinnest film the initial velocity for each decay was within 20% of 25 cm/sec. To facilitate the comparison of the evolution of the flow velocity for the various film thicknesses studied we have normalized the data in each case to the velocity observed at $t_0=15$ sec.

Before the observation of decays which deviated from Eq. (9) it was possible to draw several conclusions from the observed decay rule.[4,22] If $v=v_0[1-\xi\ln(t/t_0)]$ then

$$\frac{dv_s}{dt}=\frac{\xi v_0}{t_0}\exp\left[-\frac{2.3}{\xi}\left(1-\frac{v}{v_0}\right)\right] \tag{10}$$

and comparison to Eq. (2) allowed the conclusion that $E_a \approx (1-v_s/v_0)$, a result apparently inconsistent with either vortex rings or paired vortex lines. Note here that this velocity dependence of the activation energy is a direct conclusion which results from the differentiation of Eq. (9) and is not motivated by an expansion[1] about a critical velocity. In fact since the rule given by Eq. (9) is under certain conditions observed over *substantial* (60%) changes in velocity, no such expansion can be trusted.

With the advent of observations[23,5] of decays which deviate strongly from Eq. (9) it is clear that a more general approach must be taken. It is found that the empirical relation

$$v=A_1(1+B_1t)^{-n} \tag{11}$$

can be reasonably (solid line) fit to the data of Fig. 6.

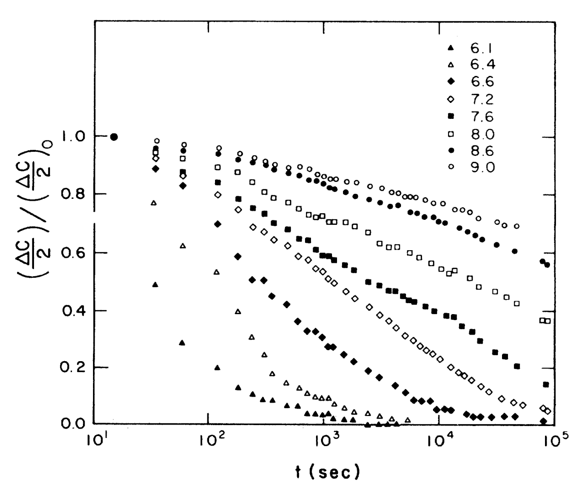

FIG. 7. Observed decay of persistent current velocity for several cases of different film thickness at $T=1.45$ K. The film thickness values are given in atomic layers. For the most dramatic decays the flow velocity became consistent with zero; i.e., the decays appear to be complete.

This leads to a superfluid deceleration of the form

$$dv/dt = -A_0 v^{\alpha} \ , \tag{12}$$

where $\alpha = 1 + n^{-1}$. This empirical decay rule is also consistent with data which appears linear in $\ln t$ if large enough values of α are used. The quantity α is observed to have a strong dependence on both thickness and temperature and examples of this dependence are shown in Figs. 8 and 9.

One can ask whether or not there is any physical motivation for a deceleration of the form given by Eq. (12). If we consider the energy of a pair of vortex lines for the activation energy in the Iordanskii[6]-Langer-Fisher[7] deceleration, Eq. (2), we have

$$E_a = \frac{d\langle \rho_s \rangle K^2}{2\pi} \left[\ln\left(\frac{K}{2\pi v a} \right) - 1 \right] \ , \tag{13}$$

where K is the quantum of circulation and a is the radius of the vortex core. If this expression is substituted into Eq. (2) we find a result for the deceleration of the form given by Eq. (12) with

$$\alpha = \frac{d\langle \rho_s \rangle K^2}{2\pi k T} \tag{14}$$

and

$$A_0 = \frac{h}{m} A v_0^* e^{\alpha} \left(\frac{K}{2\pi a} \right)^{-\alpha} \ . \tag{15}$$

One can now compare the expected values of α as a function of thickness and temperature with values obtained from our observations of n and this is done by means of the solid curves in Figs. 8 and 9. We should point out here that effective values of ρ_s (i.e., $\langle \rho_s \rangle$) for the film have been used to obtain the solid lines shown in the figures.

It is interesting to note here that the value of α obtained from fits of Eq. (11) to the data are independent of the starting velocity of the persistent current as would be expected from an examination of Eq. (14). This is shown in Fig. 10. It is possible to show for decays of the shape shown in Fig. 2 that $\xi \simeq -(\alpha - 1)^{-1}$ and hence from this point of view it is not surprising that ξ would be found to be independent of v_0. Thus, the features of the data well represented by $v = v_0[1 - \xi \ln(t/t_0)]$ can be encompassed in the more general deceleration rule given by Eq. (12) for appropriate values of α.

An objection[8] to this sort of analysis exists howev-

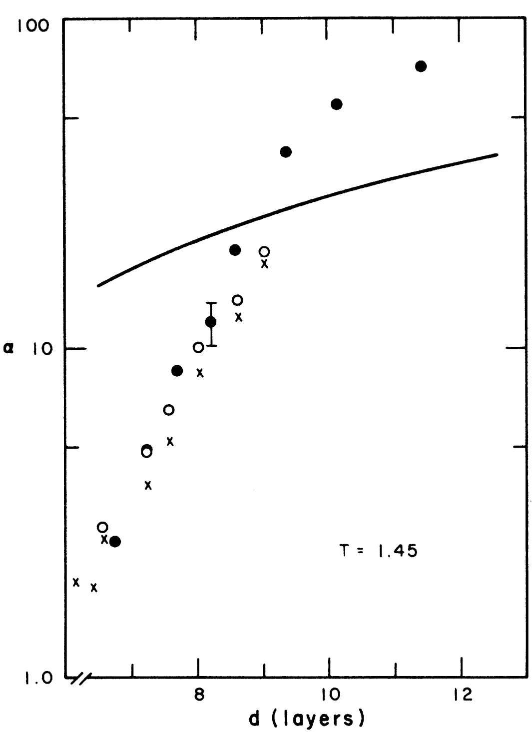

FIG. 8. The thickness dependence of α at fixed temperature. The crosses represent fits of Eq. (11) to the data shown in Fig. 7 and the circles represent the same data fit over the restricted time interval $t < 6000$ sec. The solid dots represent restricted time interval data from a separate experimental run over a wider range of film thicknesses. For the thickest films the decays became extremely weak and the appropriateness of Eq. (11) may be questionable. The observed thickness dependence is much stronger than predicted by the ILF theory in the vortex line pair model (solid line).

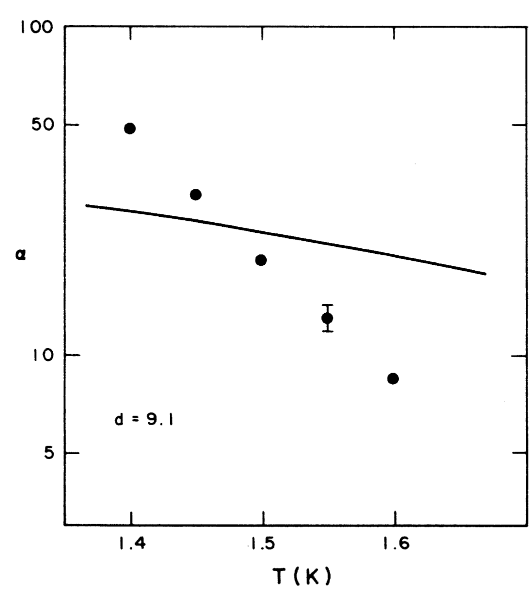

FIG. 9. The temperature dependence of α at fixed film thickness. The solid line represents the ILF theory in the vortex pair model.

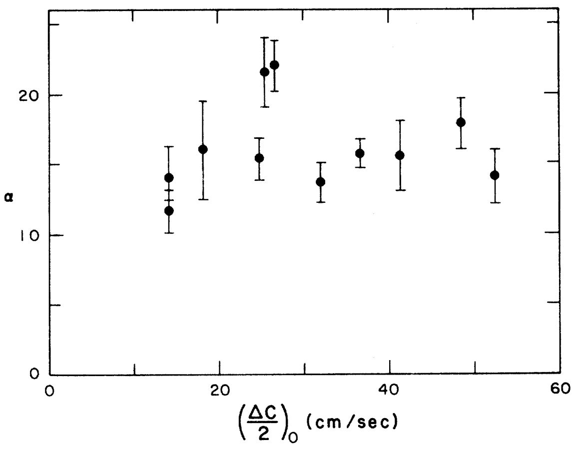

FIG. 10. The parameter α as a function of $(\Delta C/2)_0$ at $t = 60$ sec as deduced from fits of the data to Eq. (11). The data used for these fits was the same as that used to produce Fig. 3 and had the general character of that shown in Fig. 2.

er. That objection stems from the observation that for the Iordanskii[6]-Langer-Fisher[7] process to have meaning an energy barrier must be present. The presence of the barrier in the Iordanskii[6]-Langer-Fisher[7] theory is brought about by the presence of the $\vec{p}\cdot\vec{v}$ term in the energy at finite v. However, it can be shown[8] that the experimental velocities necessary to observe persistent current decays are as much as two or three orders of magnitude smaller than the velocities needed to form an energy barrier. The experimental velocities are macroscopically observed velocities and it is not clear whether or not much larger velocities might exist on a microscopic scale.

One way around this conceptual obstacle has been introduced by Donnelly and Roberts[8] and emphasized recently by Donnelly, Hills, and Roberts.[9] In their work the effects of boundaries are included and the boundary naturally gives rise to an energy barrier even in the absence of finite v. If a vortex surmounts this energy barrier and annihilates itself at the walls, then the system will gain or lose a quantum of circulation depending upon the sign of the vortex. When $v = 0$ the size of the energy barrier is independent of the sign of the vortex. However, when superflow is present, the energy barrier is modified by the $\vec{p}\cdot\vec{v}$ interaction and becomes sign dependent. For very large velocities the difference between the barrier heights for the two types of vortex is so large that only one type of vortex has a significant effect on the motion of the superfluid. When the superfluid moves with a smaller velocity the difference between the two barriers is not as pronounced so the weaker process will compete and modify the decay of the superfluid. Using this "competing barrier" model DHR obtain a general result for a superfluid moving in a toroidal geometry. They find that

$$v = B^* \ln[(1 + B_2 C^{-\nu_0 t})/(1 - B_2 C^{-\nu_0 t})] \ ,$$

where $B^* = kT/P_c$, P_c the momentum of the critical excitation, $B_2 = \tanh\lambda/2$ with $\lambda = P_c v_0/kT$ where v_0 is the initial persistent current velocity and $\nu_0 = 2n\kappa f P_c/kT \exp(-\Delta E/kT)$. Here n is the number of candidates for nucleation per unit length, κ is the quantum of circulation, f is an adjustable parameter (a fundamental attempt frequency), and ΔE is the energy barrier when $v = 0$. For our purposes we have taken

$$\frac{\Delta C}{2} = F \ln\left|\frac{1 + Ge^{-\nu_0 t}}{1 - Ge^{-\nu_0 t}}\right| \ , \tag{16}$$

and hence expect $F = \langle\rho_s\rangle kT/\rho P_c$. This functional form, Eq. (16), is in reasonable accord with the data (dashed curve on Fig. 6). The product $F\lambda$ where $\lambda = \tanh^{-1}(2G)$ is predicted to be equal to the initial value of $\frac{1}{2}\Delta C$.

Computer fits to data taken at several film thickness values confirm this prediction (Fig. 11). Data from other experimental runs show the same general behavior. A separate examination of the dependence of F and λ on the quantity $(\Delta C/2)_0$ reveals that it is the quantity F which contains the strong dependence on initial velocity. This is shown for a typical case in Fig. 12. When data for various film thickness values are compared, the slope of the linear dependence of F on $(\Delta C/2)_0$ reveals a substantial dependence on the film thickness. This is shown for data taken at

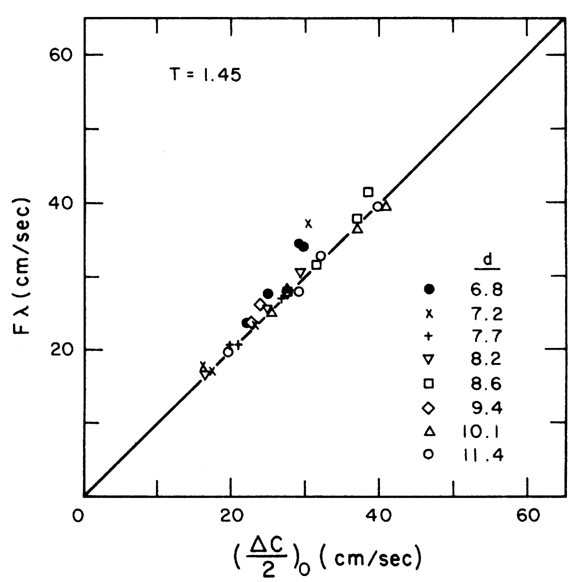

FIG. 11. Deduced parameter product $F\lambda$ as a functions of initial $\Delta C/2$ value at $T = 1.45$ K for several values of the film thickness.

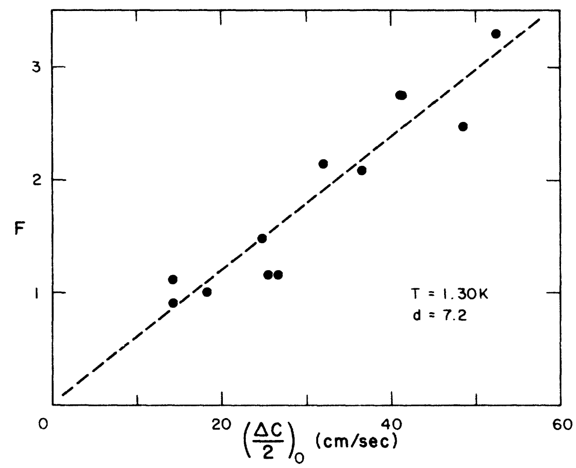

FIG. 12. Fitted parameter F vs $(\Delta C/2)_0$. Since $F = kT\langle\rho_s\rangle/P_c\rho$, we conclude that P_c, depends inversely on the initial value of the persistent current velocity.

1.45 K in Fig. 13. Thus, we find that $F \sim f(d) \times (\Delta C/2)_0$ where $f(d)$ is independent of $(\Delta C/2)_0$. We thus conclude that P_c depends inversely both on the film thickness and on the magnitude of the initial persistent current.

Within the context of the DHR theory, if we assume $P_c = \langle\rho_s\rangle d\kappa\phi$ as appropriate for vortex pairs oriented perpendicular to the substrate and separated by the distance ϕ we can establish the dependence of ϕ on thickness and initial persistent current velocity. The data can be presented in several ways. For example, in Fig. 14, we display the quantity $\kappa/2\pi\phi$ for

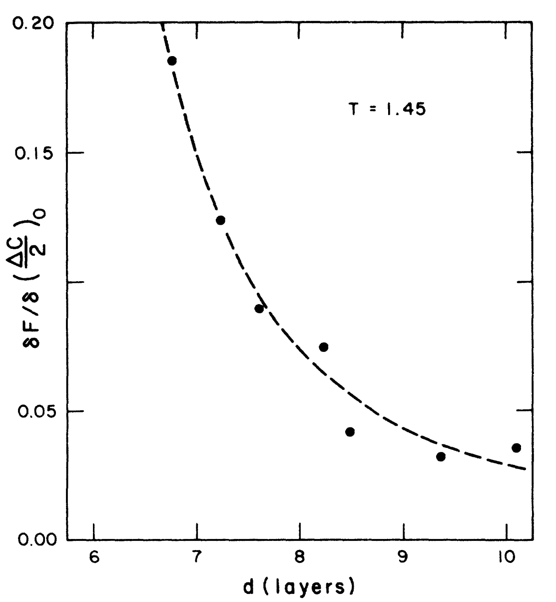

FIG. 13. The slope $\partial F/\partial(\Delta C/2)_0$ as a function of the film thickness. These data along with those of Fig. 12 allow the conclusion that $F \sim f(d)(\Delta C/2)_0$.

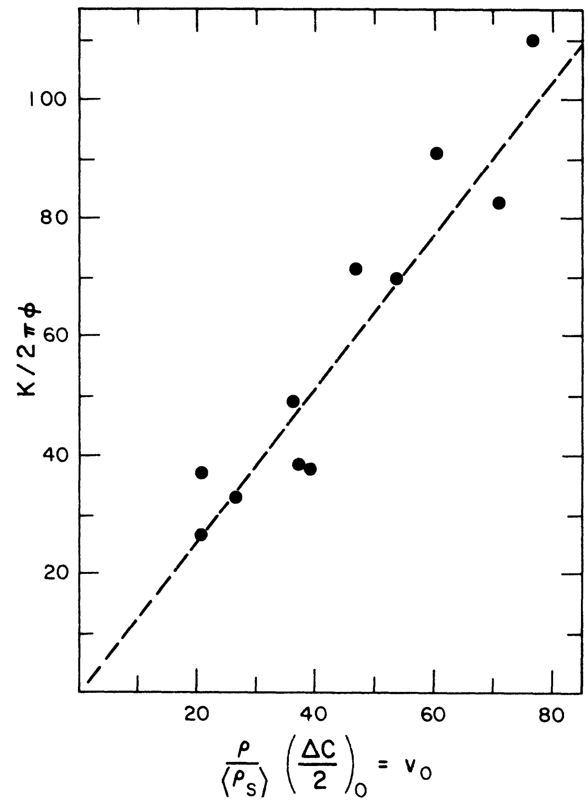

FIG. 14. The quantity $\kappa/2\pi\phi$ vs the initial persistent current velocity $v_0 = (\rho/\langle\rho_s\rangle)(\Delta C/2)_0$.

the data of Fig. 12 as a function of the initial persistent current velocity v_0. A fit to the data reveals $\kappa/2\pi\phi = 1.99(\Delta C/2)_0$. Alternatively ϕ vs $(\Delta C/2)_0$ is shown in Fig. 15. In this case the solid curve is a fit of the form $\phi = a_1(\Delta C/2)^{a_2}$ where $a_1 = 737$ nm and $a_2 = -0.963$. The separation ϕ (or P_c since we have chosen to write $P_c = \langle\rho_s\rangle d\kappa\phi$) also depends on the thickness of the film. Since the maximum available initial persistent current also depends on film thickness, it is not obvious what the best procedure to remove the velocity dependence is. We have arbitrarily normalized to $(\Delta C/2)_0 = 27$ cm/sec and display the results of a number of determinations of ϕ vs d at fixed $(\Delta C/2)_0$ in Fig. 16.

The parameter ν_0 has also been determined from fits to the data and we find ν_0 to be independent of $(\Delta C/2)_0$. We do observe substantial dependence of ν_0 on both film thickness and temperature and this is shown in Figs. 17 and 18. A quantity of fundamental interest central to any such theory is the nucleation probability $P = f\exp(-\Delta E/kT)$. In the DHR theory we can write the dependence

$$\frac{\nu_0 F}{2\kappa}\frac{\rho}{\langle\rho_s\rangle} = nf\exp\left[\frac{-\Delta E}{kT}\right] . \qquad (17)$$

Experimental values for ν_0 and F allow this quantity

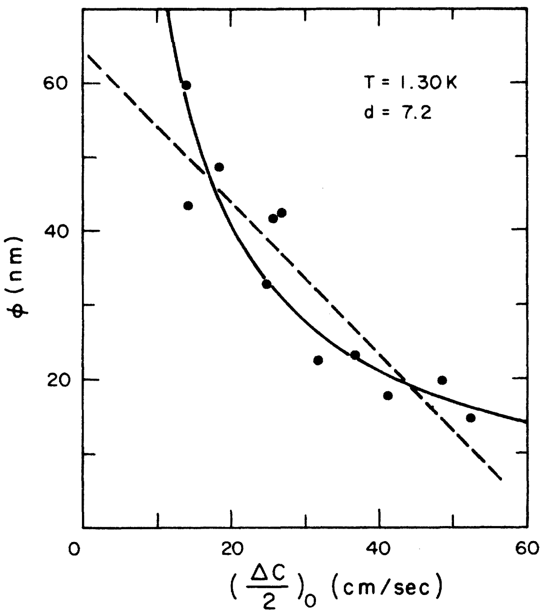

FIG. 15. The vortex separation distance ϕ as a function of the initial $\frac{1}{2}\Delta C$ value. The solid curve through the data is a least-squares fit $\phi \sim (\Delta C/2)_0^{-0.96}$. The dashed line is a linear fit.

to be determined as a function of film thickness. The results of this, neglecting the small dependence of F on velocity, are shown in Fig. 19. Attempts to extract the specific dependence of the attempt frequency f on the film thickness depend on the explicit choice for ΔE in a crucial way; in particular, the

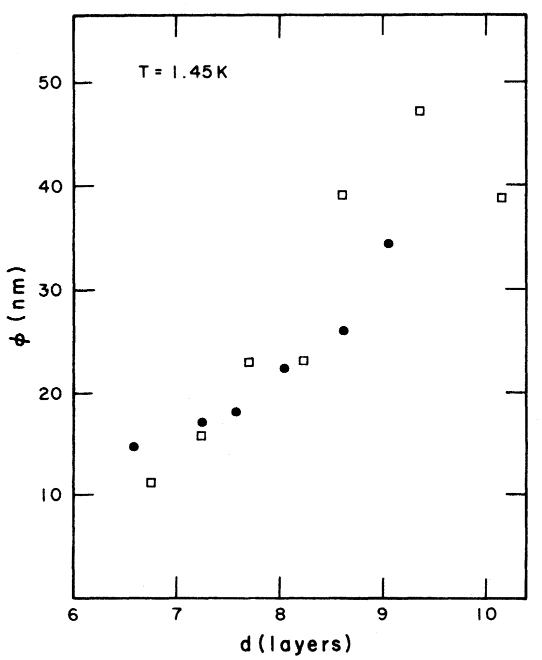

FIG. 16. The vortex separation ϕ vs film thickness for data normalized to constant $(\Delta C/2)_0$.

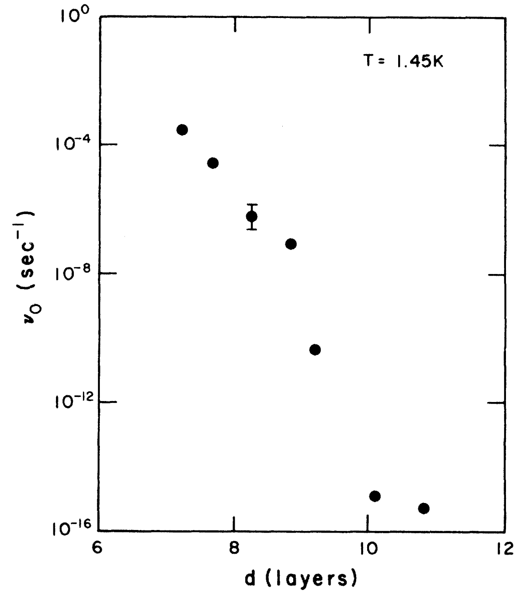

FIG. 17. ν_0 values as a function of thickness. The solid symbols represent ν_0 values which result from fits of Eq. (16) the data shown in Fig. 7.

choice $\delta E = (1/2\pi)\langle \rho_s \rangle d\kappa^2 \ln(d/a)$ results in a dependence of f on d given by $f = \epsilon \exp(gd)$ where $\epsilon = e^{3.08}$ and $g = 4.83$. If f is to be independent of film thickness a correction to δE linear in d is required. Since f may not in fact be independent of the film thickness, in the absence of an explicit expression for ΔE, we depart the discussion with the data presented in Fig. 19.

Additional theoretical work based on the ideas of Kosterlitz and Thouless[24] as it can be related to dissi-

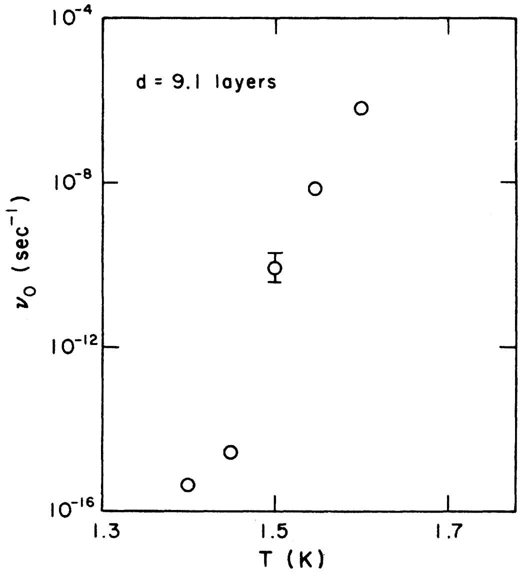

FIG. 18. The parameter ν_0 as a function of temperature.

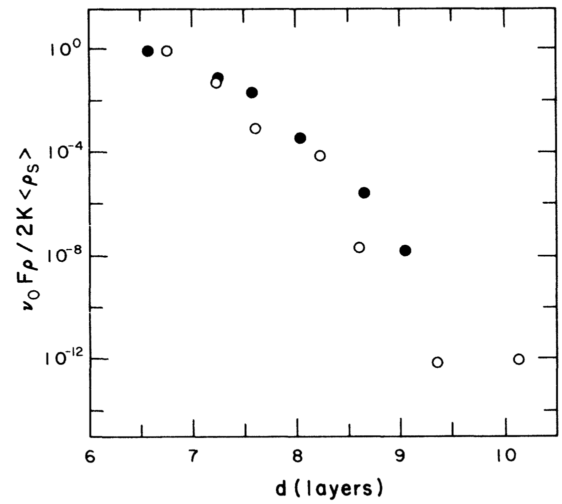

FIG. 19. $\nu_0 F\rho/2\kappa\langle\rho_s\rangle$ vs film thickness. In the DHR theory the abscissa should be closely related to $f\exp(-\Delta E/kT)$.

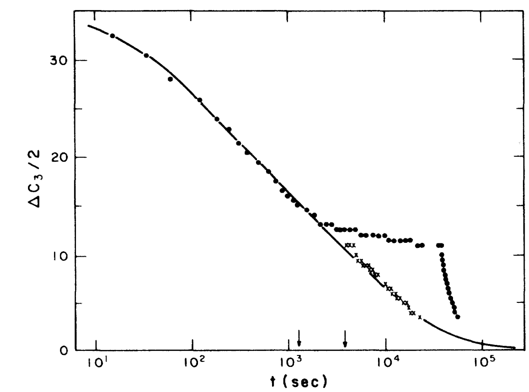

FIG. 20. Persistent current decay accompanied by a change in the temperature. The solid curve represents the result of observations taken at a constant temperature of 1.50 K. The solid circles represent subsequent observations during which the temperature was first lowered to 1.40 K (during the interval between the arrows) and then raised back to 1.50 K. An interval was used for the warming, but it is compressed by the time axis. The crosses are explained in the text.

pation in superfluid films has been carried out[25,26] recently. In this work, which incorporates vortex line pairs, the deceleration of the flow of a superfluid film is again given by an expression of the form of Eq. (12). In this case, however, the exponent α is given by $\alpha \simeq 3 + (1 - T/T_c)^{1/2}$ for temperatures within a fraction of a percent of T_c. For our strongest decays, and hence for conditions closest to T_c, the experimental values of α approach three, but we are unable to confirm the expected temperature dependence due to the rapidity of the decays near T_c. However, it is quite possible that the rapidly increasing dissipation we observe in the thinnest films and at the highest temperature may well have a contribution due to the reduction in the energy necessary for the creation of vortex pairs as a result of screening.

In this context it is important to emphasize the manner in which experiments of this type differ from those described, for example, by Bishop and Reppy[27] of Chester and Yang.[28] In the present work the dissipation observed via the deceleration of the metastable superfluid flow represents an extremely sensitive probe of the system: the flow destroys itself with no external stimulation. To the extent that other perturbations can be accounted for (see below) experiments of this type may offer the possibility of exploring the appearance of Kosterlitz-Thouless behavior more generally as T_c is approached from below.

In none of these theories has there been an effort made to incorporate dissipative processes in addition to those brought about by the thermal nucleation of excitations. For example, it is quite possible that the persistent current decays we observe are due in part to the thermally activated motion of pre-existing vortex line pairs. An example of data at least consistent with this point of view is shown in Fig. 20. During the decay the temperature was reduced from 1.50 and at 1.40 K. While at the lower temperature the decay rate was greatly reduced as might be expected. Upon returning the temperature to its original value the decay rate again increased. The solid circles represent the actual data as collected. The crosses plotted here represent the coordinates $(v, t - t_0^*)$ where t is the clock time and t_0^* the interval of time during which the film was at the lower temperature. The solid curve is representative of data taken at a constant temperature of 1.50 K. We observe that a given temperature and thickness velocity alone do not specify the decay. In this sense the films can be considered to have a memory. An interpretation consistent with the data is that existing vorticity is frozen in site at the lower temperature and only released under the stronger thermally activated motion brought about by higher temperature. From this point of view the superfluid deceleration might be expected to be a strong function of the details of vortex pinning. The idea of dissipation due to vortex hopping is not new and is a well-established phenomena in superconductivity. It would be most useful if such a model were developed so as to be applied to the case of persistent current decays in helium. The experimental situation may well be a combination of both thermal nucleation processes and dissipation induced by the motion of both thermally activated and pre-existing vortices.

Within the general context of vortex pinning, the empirical rule [Eq. (11)] fails at the smallest times but the functional dependence proposed by Donnelly *et al.*[9] does not. This suggests that what is needed at large velocity is in fact a permanent barrier of *some* kind. Vortex pinning could provide this. At large times and small velocities a deceleration of the form v^α rather than the exponential dependence predicted by Donnelly *et al.*[9] is observed. Thus, the work of Donnelly *et al.*[9] is not entirely consistent with the data.

[After this manuscript was completed we learned of work due to Browne and Doniach[29] which predicts that $dv/dt = -v(R_1 e^{\gamma v} + R_2 v^{\beta})^{1/2}$. This work modified the earlier work of Huberman *et al.*[25] so as to include an additional energy barrier due to vortex-substrate interactions.]

IV. CONCLUSION

We have reported on detailed measurements of the decay of persistent currents of thin superfluid ^{4}He films as a function of both film thickness and temperature. These measurements include observations of the complete decay of persistent currents to zero flow velocity. Various theoretical models have been applied to the data and discrepancies have been noted where appropriate. A more detailed treatment of the effect of the motion of vortices as applied to thin helium films is called for in the hope that it may more completely describe the data.

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