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Brillouin instability in magnetoactive n-type piezoelectric semiconductors

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We consider the interaction of an intense electromagnetic wave (pump) with a low-frequency acoustic wave produced by the pump based on the mechanism of electrostriction in magnetoactive piezoelectric semiconductors. The general dispersion relation has been obtained and solved for both the cases of isotropic ($B_0=0$) and magnetoactive ($B_0\neq0$) media to study the threshold condition and the growth rate of the unstable Brillouin mode at a pump amplitude well above the threshold. Numerical estimates have been made for n-type InSb at 77 K. The crystal has been irradiated with a pulsed 10.6 μm CO_2 laser to obtain the necessary electric field. The threshold is appreciably lowered by the application of a large transverse magnetostatic field, and the growth rate of the unstable Brillouin mode is found in a magnetoactive semiconductor to be about 10^5 times that in an isotropic crystal.

I. INTRODUCTION

Stimulated Brillouin scattering (SBS) is caused by the nonlinear interaction of an intense pumping light beam, an initially weak scattered light beam, and the internal motion of a real medium. This phenomenon was discovered by Chiao *et al.*,[1] and it arises only when the intensity of the pumping light beam exceeds a certain value known as threshold. The physical origin of SBS is in electrostriction. The nonlinear interaction with the medium based on the mechanism of electrostriction simultaneously creates two waves: a frequency-shifted light wave and an intense sound wave whose frequency can be as high as 10^{11} sec^{-1} in a crystal.[2] Asam *et al.*[3] obtained SBS in Ge using a pulsed 10.6 μm CO_2 laser. The investigation of the SBS threshold in semiconductors when sound instability arises is of particular interest, as the sound-absorption coefficient not only decreases, but can even change in sign, and the threshold diminishes greatly or can even vanish.[4,5] Electromagnetic wave instability in the frequency range $\omega_0(\approx\omega_c)>\omega_p$ (where ω_0, ω_p, and ω_c represent the pump frequency, electron plasma, and cyclotron frequency, respectively) was studied by Andreev[6] in collisionless, fully ionized plasmas, and its experimental verification was made by Batanov and Sarksyan.[7] Recently, Guha and Sen have studied the phenomenon of parametric excitation of acoustic waves[8,9] and acoustohelicon waves[10] in magnetoactive piezoelectric semiconductors by irradiating the crystal with a spatially uniform pulsed 10.6 μm CO_2 laser where the usual coupled-mode theory[11] has been used.

The purpose of the present paper is to report the analytical results obtained while studying the phenomenon of SBS in a one-component (electron), homogeneous piezoelectric semiconductor subjected to a large transverse magnetostatic field when the semiconductor is irradiated with a spatially uniform laser beam. We have investigated the Brillouin instability in the semiconductor using the coupled-mode theory when $kl\ll1$ where k and l are the acoustic wave number and the electron mean-free path, respectively. As we have assumed the pump to be spatially uniform, the pump wave number k_0 is taken to be equal to zero, and the selection rules for the frequencies and wave numbers are

$$\omega_0=\omega_1+\omega \text{ and } 0=\vec{k}_1+\vec{k},$$

where ω_1 and $\vec{k}_1$ are the frequency and the wave vector of the scattered electromagnetic wave, respectively, and ω is the acoustic wave frequency such that $\omega_0\approx\omega_1\gg\omega$, and the selection rules yield $k_1=k$.

The effects of the nonlinear material parameters which play important roles in nonlinear interactions in strongly piezoelectric materials such as $LiNbO_3$ (Ref. 12) have been neglected in the present investigation, which deals with weakly piezoelectric materials (viz., n-InSb, for which the piezoelectric coefficient $\beta=0.054$ C m^{-2} and is ~0.027 times that of $LiNbO_3$ with $\beta=2.0$ C m^{-2}).[9] The nonlinear effects due to the band nonparabolicity being about 3% of that due to transport mechanisms[13] has also been neglected.[9]

Free-carrier absorption in an n-type semiconductor is the absorption of infrared energy by conduction electrons, and its magnitude α_{fc} depends upon the number (n_0) and mobility (μ) of free carriers available for absorption as well as the wavelength of the laser (λ) used, and is given by[14]

$$\alpha_{fc}=\frac{\lambda^2e^3}{4\pi^2c^3n^*\epsilon_0}\frac{n_0}{m^2\mu}$$

in cgs units, where n^* is the real part of the refractive index of the crystal and equals 3.96 for InSb. We have assumed that the electron mobility is independent of the laser field and have obtained the value of $\alpha_{tc} \sim 1.6\times10^{-4}$ m^{-1} with electron collision frequency $\nu = 3.5\times10^{11}$ sec^{-1} in n-InSb crystal at 77 K with $n_0 = 10^{22}$ m^{-3} irradiated with a pulsed 10.6 μm CO_2 laser. Thus the phenomenon of free-carrier absorption has been neglected in the present simplified treatment of Brillouin instability. Boiko *et al.*[15] have noted that the free carriers absorb only about 1% of the overall radiation when an InSb crystal with $n_0 = 2\times10^{22}$ m^{-3} is irradiated with a 10.6 μm CO_2 laser in the presence of a transverse magnetostatic field $B_0 = 5$ kG. Kobayashi and Otsuka[16] observed tunable far-infrared radiation from n-InSb at 4.2 K under crossed electric and magnetic fields. Kobayashi *et al.*[17] reported this phenomenon in the strong-magnetic-field ($B_0 \sim 20$ kG) region where a single crystal of n-InSb at 77 K with $n_0 \sim 10^{19}$ to 10^{20} m^{-3} was used.

The nonlinearities taken into account in the present analysis are the electronic nonlinearity due to nonlinear current and the nonlinear polarization, which is the cause of the nonlinear coupling between the produced acoustic wave and the scattered electromagnetic wave. The analysis considers the effect of the oscillatory Hall drift normal to both the pump and the magnetostatic field.

In Sec. II, we present the theoretical formulations and the general dispersion relation in the magnetoactive n-type piezoelectric semiconductor plasma. Section III deals with the isotropic plasma ($B_0 = 0$), for which the threshold condition for the onset of Brillouin instability and the growth rate of the unstable (Brillouin) mode well above the threshold have been obtained. Section IV considers the case of a magnetoactive plasma, in which the transverse magnetic field applied is taken to be so large that $\omega_c \sim \omega_0$. Section V has been devoted to the general discussion and numerical estimations of the threshold electric field amplitude of the pump and the growth rate of the unstable mode well above threshold in both isotropic and magnetoactive semiconductors.

II. THEORETICAL FORMULATIONS

We consider a one-component (electron) homogeneous piezoelectric semiconductor plasma subjected to a transverse magnetostatic field $\vec{B}_0$ (along the z axis), normal to the propagation vectors $\vec{k}_1$ and $\vec{k}$ as well as the spatially uniform pump wave $\vec{E}_0 \cos\omega_0 t$ (applied along the x axis). The pump field $\vec{E}_0$ gives rise to a time-varying electrostrictive strain and is thus capable of driving the acoustic waves in the medium, with the strain given by $\partial u/\partial x$, where the deviation of a point x from its equilibrium position is $u(x,t)$. The net electrostrictive force in the positive x direction acting on a unit volume is given by[18]

$$F = \frac{\gamma}{2}\frac{\partial}{\partial x}E^2$$

and consequently, the equation of motion for $u(x,t)$ in the piezoelectric semiconductor is given by

$$\rho\frac{\partial^2 u}{\partial t^2} = c\frac{\partial^2 u}{\partial x^2} - \beta\frac{\partial E}{\partial x} + \frac{\gamma}{2}\frac{\partial}{\partial x}E^2 \tag{2.1}$$

where γ is a phenomenological constant describing the change in the optical dielectric constant and typically of the order of 10^{-11} mks units.[18,19] ρ and c are the mass density and the elastic constant of the lattice. The other basic equations are

$$\frac{\partial \vec{v}_0}{\partial t} + \nu\vec{v}_0 = \frac{e}{m}(\vec{E}_0\cos\omega_0 t + \vec{v}_0\times\vec{B}_0)\,, \tag{2.2}$$

$$\frac{\partial E}{\partial x} = \frac{\eta e}{\epsilon} - \frac{\beta}{\epsilon}\frac{\partial^2 u}{\partial x^2}\,, \tag{2.3}$$

$$\frac{\partial n}{\partial t} + v_0\frac{\partial n}{\partial x} + n_0\frac{\partial v}{\partial x} = 0\,, \tag{2.4}$$

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v}_0\cdot\vec{\nabla})\vec{v} = \frac{e}{m}(\vec{E} + \vec{v}\times\vec{B}_0) - \frac{k_B T}{m\eta_0}\vec{\nabla}n - \nu\vec{v}\,, \tag{2.5}$$

$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}\,, \tag{2.6}$$

$$\vec{\nabla}\times\vec{H} = \vec{J} + \frac{\partial\vec{D}}{\partial t}\,, \tag{2.7}$$

$$\vec{D} = \epsilon\vec{E} + \vec{P}\,, \tag{2.8}$$

and

$$\vec{P} = -\gamma\vec{E}\frac{\partial u}{\partial x}\,. \tag{2.9}$$

Using Eqs. (2.6)–(2.9) we obtain the general wave equation[20]

$$\vec{\nabla}\times\vec{\nabla}\times\vec{E} = -\mu_0\frac{\partial\vec{J}}{\partial t} - \frac{1}{c_l^2}\frac{\partial^2\vec{E}}{\partial t^2} + \mu_0\gamma\frac{\partial^2}{\partial t^2}\left(\vec{E}\frac{\partial u}{\partial x}\right)\,, \tag{2.10}$$

where the perturbed current density $\vec{J}$ is given by

$$\vec{J} = n_0 e\vec{v} + ne\vec{v}_0 \tag{2.11}$$

with n and $\vec{v}$ being the perturbed carrier concentration and velocity, respectively. $c_l = 1/(\epsilon_0\epsilon_l\mu_0)^{1/2}$

is the electromagnetic wave velocity in the lattice. $\vec{v}_0$ is the oscillatory electron-fluid velocity in the presence of the pump and the transverse magnetostatic field $\vec{B}_0$. The components of $\vec{v}_0$ along the x and y axis are obtained from Eq. (2.2), assuming the proportionality of the pump as $\exp(i\omega_0 t)$ as

$$v_{0x}=\frac{i\omega_0+\nu}{(i\omega_0+\nu)^2+\omega_c^2}\frac{e}{m}E_0$$

and (2.12)

$$v_{0y}=\frac{\omega_c}{i\omega_0+\nu}v_{0x},$$

where $\omega_c = eB_0/m$.

In Eq. (2.11) we will remember that n and $\vec{v}$ will have two components, known as slow and fast, the slow being associated with the produced low-frequency acoustic wave, and the fast components associated with the high-frequency electromagnetic wave having two parts, one with sideband frequency $\omega_0+\omega$ and the other, with $\omega_0-\omega$ due to the beating of the low-frequency density perturbation at frequency ω with the pump at frequency ω_0. The higher-order terms with frequencies $p\omega_0\pm\omega$, where $p=1, 2, 3\ldots$, being off-resonant, are neglected. Following Guha and Sen,[21] one can obtain n_f in terms of n_s by using Eqs. (2.2)–(2.5) as

$$n_f=\frac{2i\delta k\bar{E}}{\omega_0(\nu^2+\delta^2)}n_s, \tag{2.13}$$

where suffixes f and s denote fast and slow components of the perturbed carrier concentration. In obtaining Eq. (2.13), we assume that the low-frequency perturbed quantities vary as $\exp[i(\omega t - kx)]$ and the side bands as $\exp\{i[(\omega\pm\omega_0)t-kx]\}$, as $k_0=0$ has been assumed in the present investigation. In Eq. (2.13)

$$\delta=\omega_0-\bar{\omega}_R,\quad \bar{\omega}_R^2=\omega_R^2\{1+\omega_c^2/(\nu^2+\omega_c^2)\},$$

$$\omega_R^2=\omega_p^2+k^2v_\theta^2,\quad \omega_p^2=n_0e^2/(m\epsilon_0\epsilon_l),$$

$$v_\theta=(k_BT/m)^{1/2},$$

and

$$\bar{E}=\frac{e}{m}E_0-\omega_c v_{0y}.$$

The notations used are similar to those used earlier.[8] The components of the velocity perturbation $\vec{v}$ are obtained by using Eq. (2.3) and (2.5)–(2.7) as

$$v_x=\frac{e}{m}\frac{\nu}{\nu^2+\omega_c^2}\left(bE_x+\frac{\omega_c}{\nu}E_y-i\frac{k^3v_\theta^2}{\omega_p^2}\frac{\beta}{\epsilon}u\right)$$

and (2.14)

$$v_y=\frac{e}{m}\frac{\nu}{\nu^2+\omega_c^2}\left(-\frac{\omega_c b}{\nu}E_x+E_y+i\frac{\omega_c}{\nu}\frac{k^3v_\theta^2}{\omega_p^2}\frac{\beta}{\epsilon}u\right),$$

where both v_x and v_y represent the sum of the slow and the fast components of $\vec{v}$ along the x and y axis, respectively.

The use of Eqs. (2.1) and (2.3) yields

$$n_s=\left\{\frac{e}{\rho\epsilon}\left(\beta-\frac{\gamma E_0}{2}\right)\Big/\left[\omega^2-k^2c_s^2-\frac{\beta k^2}{\rho\epsilon}\left(\beta-\frac{\gamma E_0}{2}\right)\right]\right\}u, \tag{2.15}$$

where $c_s=(c/\rho)^{1/2}$, the acoustic velocity in the crystal. Equation (2.7) can be employed to obtain u in terms of E_x as

$$u=\frac{J_x}{k\omega\beta}-\frac{i}{k\omega\beta}\left(\frac{k^2}{\mu_0\omega_1}-\epsilon\omega_1\right)E_x. \tag{2.16}$$

Using Eqs. (2.11) and (2.13)–(2.16), one obtains the components of $\vec{J}$ as

$$J_x=\frac{\epsilon}{1-Q}\left\{\left[\frac{\omega_p^2\nu b}{\nu^2+\omega_c^2}-i\frac{Q}{\omega_1}(k^2c_l^2-\omega_1^2)\right]E_x+\frac{\omega_p^2\omega_c}{\nu^2+\omega_c^2}E_y\right\}$$

and (2.17)

$$J_y=\frac{\omega_p^2\epsilon\nu}{\nu^2+\omega_c^2}\left(-\frac{\omega_c b}{\nu}E_x+E_y\right),$$

where $b=1+k^2v_\theta^2/\omega_p^2$,

$$Q=\frac{v_{0x}\rho\epsilon}{k\omega\beta}\left(1-\frac{2i\delta k\bar{E}}{\omega_0(\nu^2+\delta^2)}\right)\frac{\omega^2-k^2c_s^2-\beta k^2/\rho\epsilon(\beta-\gamma E_0/2)}{(\beta-\gamma E_0/2)},$$

and we have assumed

$$\frac{\nu k^2v_\theta^2\beta}{\nu^2+\omega_c^2}\ll v_{0x}\left(1-\frac{2i\delta k\bar{E}}{\omega_0(\nu^2+\delta^2)}\right)\times\frac{\omega^2-k^2c_s^2-\beta k^2/\rho\epsilon(\beta-\gamma E_0/2)}{e/\rho\epsilon(\beta-\gamma E_0/2)},$$

$$E_y\gg\frac{\omega_c}{\nu}\frac{k^3v_\theta^2}{\omega_p^2}\frac{\beta}{\epsilon}u,\quad\text{and}\quad E_x\gg\frac{k^3v_\theta^2}{\omega_p^2}\frac{\beta}{\epsilon}u.$$

Equation (2.10) is now considered, which represents the general wave equation for the scattered electromagnetic wave with frequency ω_1 ($\approx\omega_0$ as $\omega\ll\omega_0$) and wave number k_1 ($\approx k$ as $k_0=0$) and varying as $\exp[i(\omega_1 t-k_1x)]$. Under the chosen configuration this equation reduces to

$$-\hat{j}k^2E_y+i\omega_1\mu_0\vec{J}+\frac{\omega_1^2}{c_l^2}\vec{E}=-\mu_0\gamma\frac{\partial^2}{\partial t^2}\left(\vec{E}\frac{\partial u}{\partial x}\right), \tag{2.18}$$

where $\hat{j}$ represents a unit vector along the y axis.

Following Yariv,[14] and using Eqs. (2.3) and (2.15), one can write Eq. (2.1) as

$$u=-ik\frac{\beta-\gamma E_0/2}{(\omega^2-k^2c_s^2)\rho}E_1. \tag{2.19}$$

The uses of Eqs. (2.17) and (2.19) in Eq. (2.18) and mathematical simplifications yield the general dispersion relation for the phenomenon of SBS:

$$\begin{bmatrix} A_{xx} & A_{xy} \\ A_{yx} & A_{yy} \end{bmatrix} = 0\,, \tag{2.20}$$

where

$$A_{xx} = \omega_1^2\left[1 + \frac{k^2 c_l^2 \mu_0 \gamma E_0(\beta - \gamma E_0/2)}{\rho(\omega^2 - k^2 c_s^2)}\right] + \frac{\omega_1}{1-Q}\left[\frac{Q}{\omega_1}(k^2 c_l^2 - \omega_1^2) + i\frac{\omega_p^2 \nu b}{\nu^2 + \omega_c^2}\right],$$

$$A_{xy} = i\frac{\omega_1 \omega_p^2 \omega_c}{(1-Q)(\nu^2 + \omega_c^2)}\,, \quad A_{yx} = -i\frac{\omega_1 \omega_p^2 \omega_c b}{\nu^2 + \omega_c^2}\,,$$

and

$$A_{yy} = \frac{k^2 c_l^2 \omega_1^2 \mu_0 \gamma E_0(\beta - \gamma E_0/2)}{\rho(\omega^2 - k^2 c_s^2)} + i\frac{\omega_1 \omega_p^2 \nu}{\nu^2 + \omega_c^2}\,.$$

Equation (2.20) shows that the two modes represented by $A_{xx} = 0$ and $A_{yy} = 0$ are coupled to each other via the finite transverse magnetostatic field B_0 in absence of which we obtain the simple phenomenon of SBS under a one-dimensional configuration. The dispersion relation under such circumstances (i.e., $B_0 = 0$) can by analyzed to investigate the possibility of SBS and the corresponding Brillouin instability in an isotropic n-type piezoelectric semiconductor. One can also study the effect of transverse magnetostatic field on the one-dimensional SBS phenomenon by retaining $B_0 \neq 0$ in the equation represented by $A_{xx} = 0$. Although the finite value of B_0 makes the study much more complicated, one can obtain the possible information regarding the Brillouin instability in a strongly magnetoactive piezoelectric semiconductor by considering the general dispersion relation given by Eq. (2.20).

III. ISOTROPIC SEMICONDUCTOR PLASMA

When $B_0 = 0$, the plasma becomes isotropic, and we get the dispersion relation from Eq. (2.20) as

$$A_{xx} = 0\,,$$

which, on substitution of the value of A_{xx}, can be obtained on algebraic simplification in the form

$$(\omega^2 - k^2 c_s^2)\left[\frac{Q}{\omega_1}(k^2 c_l^2 - \omega_1^2) + i\frac{\omega_p^2 b}{\nu}\right] = \frac{\mu_0 \gamma k^2 c_l^2 \omega_1 E_0}{\rho}(\beta - \gamma E_0/2)(1-Q)\,. \tag{3.1}$$

In obtaining Eq. (3.1) we have assumed $\omega = kc_s$ in the second factor of A_{xx} with the further assumption

$$i\frac{\omega_p^2 b}{\nu} \gg \frac{Q}{\omega_1}(k^2 c_l^2 - \omega_1^2)\,,$$

which is possible in the present analysis with $\omega_1 \approx \omega_0 (\approx \omega_p) \gg \nu$. In order to explore the possibility of SBS and the Brillouin instability, we solve Eq. (3.1) with complex ω $(=\omega_r + i\omega_i)$ and real positive values of k such that $\omega_r = kc_s$ and $\omega_i \ll \omega_r$. It is also well known that a mode will be unstable only when $\omega_i < 0$, and the threshold value of the pump amplitude $(E_{0\mathrm{th}})_{B_0=0}$ is obtained at $\omega_i = 0$. Thus one obtains Eq. (3.1) in the much-simplified form

$$\omega_i = -(1-Q)(\beta - \gamma E_0/2)\frac{\nu k c_l^2 \omega_1 \mu_0 \gamma}{2\omega_p^2 \rho b c_s} E_0\,. \tag{3.2}$$

From Eq. (3.2) it can be seen that, in the absence of E_0, the growth rate disappears, and one can thus infer that the pump field amplitude must be finite in order to study SBS, which is also physically the basis of study of SBS phenomena. Thus $E_0 \neq 0$ is the precondition; from the knowledge of the values of β and γ we can assume $\beta \gg \gamma E_0/2$ for a value of E_0 even up to 10^8–10^9 V m^{-1}, and, under such circumstances, we observe from Eq. (3.2) that the Brillouin instability is possible only when $Q < 1$ yielding for the onset of Brillouin instability $(E_{0\mathrm{th}})_{B_0=0}$ at

$$1 - Q = 0\,. \tag{3.3}$$

Substituting the value of Q in Eq. (3.3), one obtains

$$(E_{0\mathrm{th}})_{B_0=0} = \frac{m}{e}\omega_0(\nu c_s/k)^{1/2}\,. \tag{3.4}$$

In obtaining Eq. (3.4), we have reasonably assumed that $2i\delta k\bar{E}/[\omega_0(\nu^2 + \delta^2)] \gg 1$ and seen that $\delta > 0$ or $\omega_0 > \omega_R$ at $B_0 = 0$ is another precondition to get Eq. (3.4), where it is also assumed that $\delta = \nu$, which is possible by optimizing ω_i with respect to δ.

In order to obtain the growth rate of the unstable Brillouin mode well above threshold, we take $Q \ll 1$, and one obtains from Eq. (3.2)

$$(\omega_i)_{B_0=0} = -(\beta - \gamma E_0/2)\frac{\nu k c_{l0}^2 \omega_0 \mu_0 \gamma}{2\omega_p^2 \rho c_s} E_0\,, \tag{3.5}$$

where we have taken $\omega_1 \approx \omega_0$ and $k^2 v_\theta^2/\omega_p^2 \gg 1$ so that $b = 1$.

Equations (3.4) and (3.5) can be employed suitably to the various isotropic n-type piezoelectric semiconductors (satisfying the approximations we have taken in obtaining the expressions) in studying the Brillouin instability.

IV. MAGNETOACTIVE SEMICONDUCTOR PLASMA

For this purpose, we consider the general dispersion relation given by Eq. (2.20) with $B_0 \neq 0$ and $\omega_c^2 \gg \nu^2$, yielding

$$\omega_i^2\left(1+k^2c_l^2\frac{\mu_0\gamma(\beta-\gamma E_0/2)}{\rho(\omega^2-k^2c_s^2)}E_0\right)+i\frac{\omega_1}{1-Q}\frac{\omega_p^2\nu b}{\omega_c^2}=\frac{\omega_p^4 b\rho(\omega^2-k^2c_s^2)}{(1-Q)\omega_c^2k^2c_l^2\mu_0\gamma(\beta-\gamma E_0/2)E_0} \quad (4.1)$$

It can be seen from Eq. (4.1) that for $\omega(=\omega_r+i\omega_i)\approx kc_s$ with $\omega_i\ll\omega_r$ one can assume

$$k^2c_l^2\frac{\mu_0\gamma(\beta-\gamma E_0/2)}{\rho(\omega^2-k^2c_s^2)}\gg 1,$$

and for $\omega_c^2\approx\omega_0^2(\sim\omega_1^2)>\omega_p^2\gg\nu^2$, we write Eq. (4.1) after algebraic simplifications as

$$\omega_i=-(Q-1)^{1/2}\frac{\omega_c k^3c_l^2\mu_0\gamma(\beta-\gamma E_0/2)\omega_0E_0}{2\omega_p^2\rho c_s}. \quad (4.2)$$

Remembering the discussion in Sec. III, here also we observe that the threshold condition ($\omega_i=0$) is obtained as $Q=1$, which, on substitution of the value of Q, yields the threshold value of the pump amplitude as

$$(E_{0\text{th}})_{B_0\neq0}=-\frac{m}{e}\frac{(i\omega_0+\nu)^2+\omega_c^2}{\omega_0-i\nu}\left(\frac{\nu c_s}{k}\right)^{1/2}. \quad (4.3a)$$

The threshold value of the pump field given by Eq. (4.3a) shows that, even at $\omega_c=\omega_0$, the value of $(E_{0\text{th}})_{B_0\neq0}$ remains finite due to the finite value of ν, and, under the assumption ω_0 $(\sim\omega_c)\gg\nu$, one obtains the approximate value of $(E_{0\text{th}})_{B_0\neq0}$, from Eq. (4.3a), as

$$(E_{0\text{th}})_{B_0\neq0}=\frac{m}{e}\frac{\omega_0^2-(\omega_c^2+\nu^2)}{\omega_0}\left(\frac{\nu c_s}{k}\right)^{1/2}. \quad (4.3b)$$

It can now be noticed from the comparison of Eq. (4.2) with Eq. (3.2) that the condition of instability ($\omega_i<0$) is just reversed, and for magnetoactive plasma the condition becomes $Q>1$. Thus, in order to obtain the growth rate of the unstable Brillouin mode well above the threshold in a strongly magnetoactive piezoelectric n-type semiconductor plasma, we will take $Q\gg1$, and consequently we obtain the growth rate as

$$(\omega_i)_{B_0\neq0}=-\left(\frac{k}{\nu c_s}\right)^{1/2}\times\frac{e}{m}\frac{\omega_0\omega_c^2kc_l^2\mu_0\gamma(\beta-\gamma E_0/2)}{2(\omega_0^2-\omega_c^2)\omega_p^2\rho c_s}E_0^2. \quad (4.4)$$

It is seen from Eq. (4.4) that in the strongly magnetoactive plasma the growth rate of the unstable Brillouin mode is proportional to the square of the applied pump amplitude, in contrast to the linear dependence of growth rate on E_0 in isotropic plasma. Equation (4.4) can be used for specific crystals for the quantitative determination of the growth rate of the unstable mode.

V. RESULTS AND DISCUSSION

Comparing Eqs. (3.4) and (4.3a), one can obtain

$$(E_{0\text{th}})_{B_0\neq0}/(E_{0\text{th}})_{B_0=0}=-\frac{(i\omega_0+\nu)^2+\omega_c^2}{\omega_0(\omega_0-i\nu)}. \quad (5.1a)$$

Equation (5.1a) can be obtained in a simplified form when one applies a large transverse magnetostatic field such that $\omega_c(\approx\omega_0)\gg\nu$ as

$$(E_{0\text{th}})_{B_0\neq0}/(E_{0\text{th}})_{B_0=0}=\frac{\omega_0^2-(\omega_c^2+\nu^2)}{\omega_0^2}. \quad (5.1b)$$

The ratio given by Eq. (5.1b) is the same as that obtained by Guha and Sen[9] while studying the modulational instability of a laser beam and the consequent parametric amplification of acoustic waves in transversely magnetoactive n-InSb crystal [Eq. (18) of Ref. 9], neglecting ν in comparison with $\omega_c(\approx\omega_0)$. If the crystal is irradiated with a pulsed 10.6 μm CO_2 laser, then at $B_0\sim11.3$ T we get $\omega_c=0.9\ \omega_0$ and Eq. (5.1b) yields a ratio ~0.19. Thus the threshold value of the electric-field amplitude of the pump can be reduced appreciably, and one can study the phenomenon of SBS and the corresponding Brillouin instability in a strongly magnetoactive semiconductor plasma at a comparatively lower value of the pump amplitude than that necessary in an isotropic semiconductor.

Comparison of Eq. (3.5) and (4.4) gives us

$$(\omega_i)_{B_0\neq0}/(\omega_i)_{B_0=0}=\left(\frac{k}{\nu c_s}\right)^{1/2}\frac{e}{m\nu}\frac{\omega_c^2}{\omega_0^2-\omega_c^2}E_0 \quad (5.2)$$

which shows that the growth rate of the unstable Brillouin mode can also be considerably increased by the application of a large transverse magnetostatic field.

We apply the above analysis to the specific case of n-InSb crystal at 77 K with the physical constants[8] $m=0.014\ m_0$, $\beta=0.054\ \text{C m}^{-2}$, $\rho=5.8\times10^3\ \text{Kg m}^{-3}$, $\epsilon_l=17.8$, $\nu=3.5\times10^{11}\ \text{sec}^{-1}$, $c_s=4\times10^3\ \text{m sec}^{-1}$. The carrier concentration $n_0=10^{22}\ \text{m}^{-3}$, and the crystal is assumed to be irradiated with a pulsed 10.6 μm CO_2 laser beam. Using these physical constants in Eq. (3.4), we obtain the threshold value of the pump amplitude necessary for the onset of Brillouin instability in the isotropic n-type Insb crystal as $(E_{0\text{th}})_{B_0=0}\sim1.7\times10^5\ \text{V m}^{-1}$ at $k=10^7\ \text{m}^{-1}$. The corresponding values of the power density of the laser beam $(P_{0\text{th}})_{B_0=0}$ is $3.26\times10^{12}\ \text{W m}^{-2}$.

A number of workers[8-10,22-23] have obtained the threshold electric field in the range of 10^5 to 10^6 V m^{-1} for the onset of parametric excitation of acoustic waves in n-InSb. Balkarei and Epshtein[24] proposed the irradiation of n-InSb with a 10.6 μm CO_2 laser to obtain an electric field amplitude $E_0\sim10^7\ \text{V m}^{-1}$ to study the breakdown of screening.

At high-power density corresponding to the large value of E_0, laser damage of the nonlinear crystal can be troublesome.[25] The main reason for such damage to transparent dielectrics is due to the light absorption by impurities or some other imperfections in the medium, and is referred to as one of the unsolved physical problems by Fabelinskii.[2] He studied the phenomenon of SBS, assuming that it takes place before the onset of the actual damage of the crystal. Ready[26] has discussed various damage mechanisms like thermal heating, induced absorption due to multiphoton absorption, which leads to heating or to breakdown, SBS, etc., and listed the semiconductor or damage thresholds as energy dependent near 5×10^4 to 10^5 J m^{-2}. The damage thresholds can be increased by reducing the pulse durations. Experimentally, damage threshold of the order of 10^{13} W m^{-2} was observed for GaAs for pulses near 1 nsec.[20] Keeping in view the above discussion, we have assumed, as did Fabelinskii,[2] that the power density $P_{0\text{th}}$ corresponding to the value of the threshold electric field $E_{0\text{th}}$ is smaller than the damage threshold for *n*-InSb crystal.

The growth rate $|\omega_i|$ of the unstable Brillouin mode in isotropic *n*-InSb is found as 8.3×10^3 sec^{-1} from Eq. (3.5) at $E_0 = 10^6$V m^{-1}. In obtaining the value of $|\omega_i|$ we have assumed $\beta \gg \gamma E_0/2$ which is quite reasonable at the value of $E_0 = 10^6$ V m^{-1}. It can further be seen from Eqs. (3.4) and (3.5) that an increase in the value of k can reduce the threshold value of the pump amplitude and increase the growth rate of the unstable mode. But k cannot be increased to a very large value, because in that case our assumption $kl \ll 1$ will break down, thus a quantum-mechanical approach should be followed.

To get a quantitative idea about the effect of the large transverse magnetostatic field on the Brillouin instability, we have already seen from Eq. (5.1b) that the threshold value of the pump can be greatly reduced by increasing the value of E_0. The effect of B_0 on $|\omega_i|$ can be seen from Eq. (5.2) and if found for the chosen physical constants as

$$(\omega_i)_{B_0\neq0}/(\omega_i)_{B_0=0} = 1.85\times10^5$$

at $k = 10^7$ m^{-1} and $E_0 = 10^6$ V m^{-1} with $\omega_c = 0.9\ \omega_0$, which is quite large.

Thus, in a magnetoactive semiconductor plasma one can obtain a considerable growth rate of the unstable Brillouin mode at a much smaller value of the pump amplitude E_0. If B_0 is taken so large that $\omega_c^2 > \omega_0^2$, then from Eq. (4.3b) one can note that the phenomenon of Brillouin instability does not take place.

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