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Reversible stage of the magnetization and migration field of pure lead superconducting slabs

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Measurements of the magnetic moment of pure lead superconducting slabs in the first reversible stage and of the migration field are given and compared with two-dimensional models. The agreement is particularly good with a model recently worked out which takes into account the macroscopic properties of the intermediate state in the corners of the slab.

I. INTRODUCTION

It has been shown by direct observation [1–5] that the flux penetration into type-I superconducting samples proceeds in two stages. The first reversible stage is characterized by flux penetration into the sample corners which are in the intermediate state with a complicated domain structure. The second stage is characterized by the migration of flux tubes from the corner structure towards the median region of the sample. A few models exist for the first stage and the calculation of the migration field. A cylindrical-symmetry model based on the assumption of Baird[2] that the corner may be considered as normal metal and limited by a curved critical wall was proposed by Olafsson and Allen[3] and was in good agreement with experiments on cylinders and discs in a field parallel to the axis. These experiments are also in agreement with a model which completely ignores the corner structure.[6] We have also given the two-dimensional version of the model of Olafsson and Allen, in agreement with the experimental study of the magnetization of bars.[7] However, all these models do not take into account the properties of the intermediate state in the corners where the field has a constant strength, and its lines of force are straight lines. This problem was considered in a foregoing theoretical paper[8] with a thermodynamic and electromagnetic treatment. In this paper a detailed analysis of the outside field distribution was carried out and permitted by means of a computational procedure to calculate the equilibrium configuration of the field for a rectangular type-I superconducting slab. In particular, inside the edges, the equilibrium configuration of the lines of force look like a fan. The main results of this model were the first magnetization curves up to migration field H_m for some values of the dimension ratio l/a (l along the applied field H_a) and the variation of H_m with l/a (Figs. 7 and 9 of Ref. 8). This model will be referred to as the "fanlike model". To our knowledge, it is the first one which takes into account the structure and the properties of the intermediate state of the edges without any unrealistic assumptions. The choice of the geometry (infinite slabs) allowed us to complete the calculations which we were not able to do in the case of discs and cylinders. In the present paper a first explicit experimental proof of the magnetization process involved in the model is given, by measuring the first-stage magnetization curve and the migration field of lead slabs. A difficulty appeared in the comparison of the experimental results with the theoretical ones obtained with a two-dimensional model. We have been able to overcome this difficulty by imagining an extrapolation procedure of the measurements carried out on slabs of finite lengths, which permitted us to achieve a detailed quantitative test of the theoretical results.

II. EXPERIMENTAL DETAILS

A. Samples

The slabs used were prepared from Hobboken 99.9999%-pure lead. They were spark cut from long, nearly monocrystalline cylinders obtained by a method described elsewhere.[9] The length was of the order of a few cm while the transverse dimensions were of the order of one or a few mm. Before using them, the samples were chemically polished with a mixture of pure acetic acid and hydrogen peroxide, and rinsed with acetic acid and then with distilled water, and then dried on paper, and finally coated with GE 7031 varnish to prevent surface oxidation.

B. Magnetic-moment-measurement device

The slabs were placed in the liquid-helium bath and subjected to a uniform magnetic field produced by a superconductive solenoid. The solenoid had a 56-mm inside diameter.

The magnetic moment M of a slab was measured as a function of the applied field H_a by usual method, integrating electronically the voltage induced

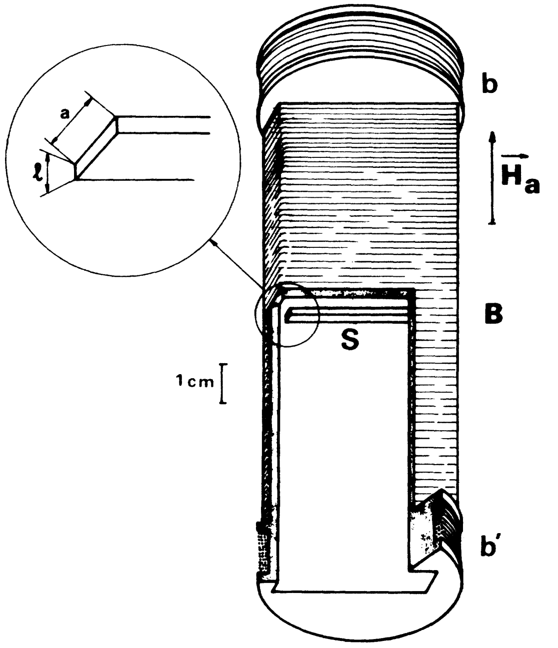

FIG. 1. Schematized view of magnetic-moment-measurement apparatus; B: rectangular cross-section-measuring coil with 8000 turns of 0.1-mm diameter copper wire, b and b': compensating circular coils, and s: sample.

in a coils set. The measuring set gives an accurate result if it creates a uniform field over the slab when supplied by a current.[10] We have chosen a maximum uniformity defect of 1%: This condition led us to build a set allowing to use slabs with maximum practicable dimensions 4 mm for the width a and 4 cm for the length L (Fig. 1). From the known characteristics of the apparatus, it has been possible to calibrate the magnetic-moment measurements.

III. EXPERIMENTAL RESULTS

We give the reduced migration field value $h_m = H_m/H_c$ and the reduced magnetic moment per unit volume m as a function of the reduced applied field $h = H_a/H_c$ measured on some slabs of different l/a ratios.

As previously mentioned, the maximum length of the slabs was 4 cm. This length cannot be seen as infinite as compared with the transverse dimensions.

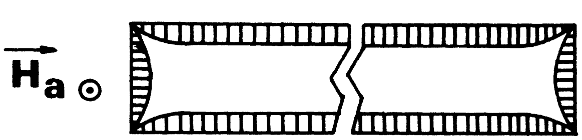

FIG. 2. Very simplified representation of the peripheral flux penetration in a slab as in de Sorbo and Healy (Ref. 1).

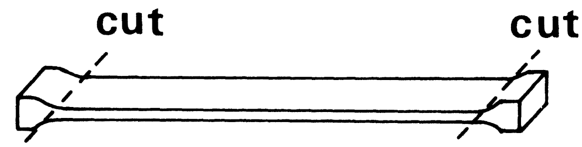

FIG. 3. Particular shape of sample used to test the end effect on the migration field H_m.

There was a flux penetration in the ends[1] (Fig. 2) which contributed to the total moment of the slabs. We have verified that this penetration did not affect the H_m value. First, for a given l/a, H_m did not depend on L in the limits given further on. Second, we have verified that for H_m, the flux migration toward the median region did not occur from the slab ends; indeed we have measured H_m on a slab with thickened ends, then on the same slab with ends cut (Fig. 3), and we have found the same value of H_m.

In order to eliminate end-effect contribution to the magnetic moment of the unit length of a slab, the virgin magnetization curve of the slab was first recorded for $L = 4$ cm and then for some smaller values of L. H_m value remained constant as long as $L \geq 2.5$ cm. For a given value of H_a, we have plotted $M(H_a)$ vs L and we have found possible to take a linear law

$$M(H_a) = M_\infty(H_a)L + K(H_a) \ .$$

This law may be interpreted as follows. $M_\infty(H_a)$ may be seen as the magnetic moment per unit length of an infinite slab and the exprapolated constant $K(H_a)$ is the ends contribution to the magnetic moment of the slab. Taking into account the ends penetration schematized in Fig. 2, it may be understood that its contribution remains nearly constant for a given H_a when L is changed but there is no reason for this contribution to be independent of H_a. This linear law and its interpretation provided the way for handling data.

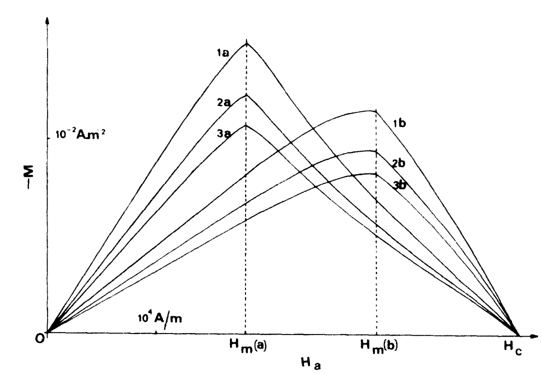

FIG. 4. Magnetization curves of a sample for three different lengths [$L = 4.04$ cm (1), 3.33 cm (2), and 2.91 cm (3)] and two orientations a ($l/a = 0.477$) and b ($l/a = 2.10$).

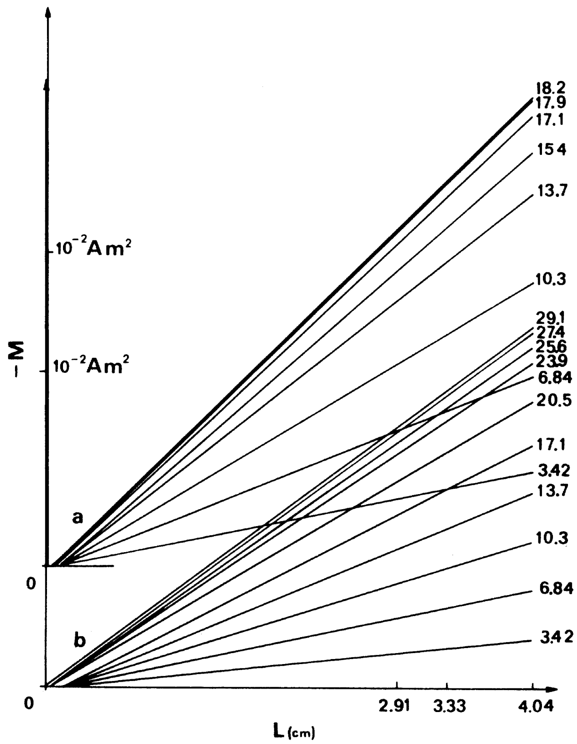

FIG. 5. Plot of $-M$ vs L with H_a as a parameter for the two orientations. H_a is given in 10^3 A/m. The slopes of the straight lines give us $M_\infty(H_a)$. Note the shift of the origin on the vertical axis.

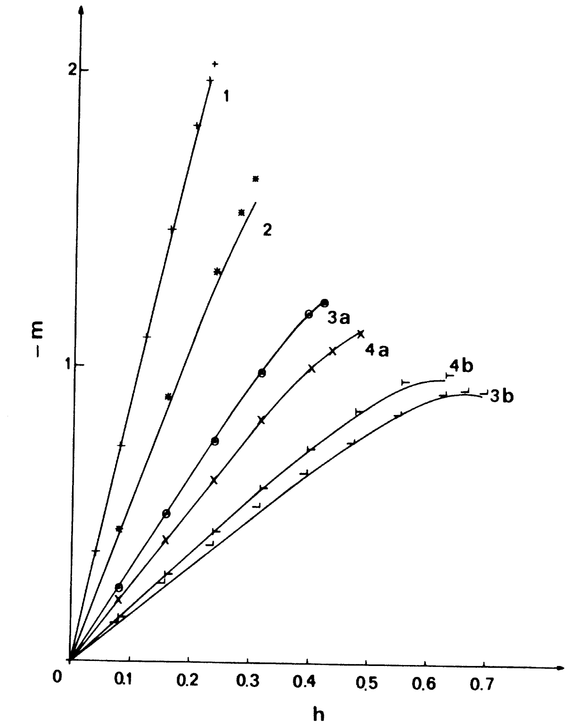

FIG. 6. Plot of the reduced magnetic moment m vs the reduced applied field h up to h_m. Theoretical solid curves are given by the fanlike model. The curves are labeled as in the table. Experimental points are as follows: 1: +, 2: *, 3a: ⊙, 4a: ×, 4b: ⊢, and 3b: ⊣.

We now give an example of a detailed treatment of the data obtained on a typical sample (sample 3). The transverse dimensions of this sample were 1.85 ± 0.02 and 3.88 ± 0.02 mm, the lengths were $L = 4.04$, 3.33, and 2.91 cm. For a first orientation of H_a ($a = 3.88$ mm and $l = 1.85$ mm) we have obtained the magnetization curves 1a, 2a, and 3a in Fig. 4. For the second orientation of H_a perpendicular to the first one, we have obtained the curves 1b, 2b, and 3b. For each orientation, H_m values were determined by a discontinuity of the slope of the curves. Moreover we have verified the reversibility of the magnetization for $H_a \leq H_m$ and the lack of reversibility beyond H_m.

In Fig. 5, $M(H_a)$ is plotted versus the length L for various values of H_a. $K(H_a)$ is a decreasing function of H_a and the slope of the straight lines gives $M_\infty(H_a)$. The reduced magnetic moment per unit volume m defined by

$$m = M_\infty(H_a)/alH_c ,$$

is plotted as a function of the reduced applied field h up to the migration field in Fig. 6. More experimental results for other l/a values are also plotted in the same figure. These other values correspond to samples 1, 2, and 4 (see Table I).

TABLE I. Transverse dimensions, l/a ratios and corresponding h_m of the samples used.

Sample	a (mm)	l (mm)	l/a	h_m
1	4.00	0.45	0.112	0.228
2	4.06	0.87	0.214	0.303
3a	3.88	1.85	0.477	0.420
3b	1.85	3.88	2.10	0.697
4a	4.04	2.76	0.683	0.482
4b	2.76	4.04	1.46	0.627

IV. DISCUSSION

In Fig. 6, we have also plotted the theoretical magnetization curves computed from the fanlike model for each sample. Furthermore, the curve a in Fig. 7 gives $h_m(a/l)^{1/2}$ vs l/a from the results of Ref. 8. Figures 6 and 7 show the agreement between experimental points and theoretical results both for the magnetization curves and for the variation of h_m with l/a.

Let us recall the definition of h_m. It is the applied field value such that the points 7′ and 7 in Fig. 7 meet one each other with the middle point ω. For this field the scalar potential difference is just

$$\phi_{5'} - \phi_5 = l_{5'\omega 5} H_c ,$$

in which $l_{5'\omega 5}$ is the length of the wall $5'\omega 5$. The fanlike model takes into account the intermediate structure in the corners. Before this model, more simple ones have been imagined, neglecting this structure either by replacing the intermediate state regions by vacuum or by completely ignoring them. In the first case, we obtained a model inspired by several authors[2,3] and already used by us.[7] In this model the field along the curved walls 5′7′ and 57 is taken as critical and h_m is reached when 7′ and 7 meet each other with the point ω. This leads to the curve b. In the second case, h_m is defined as the applied field such that the scalar potential difference

$$\phi_{6'} - \phi_6 = l H_c$$

and this leads to curve c.

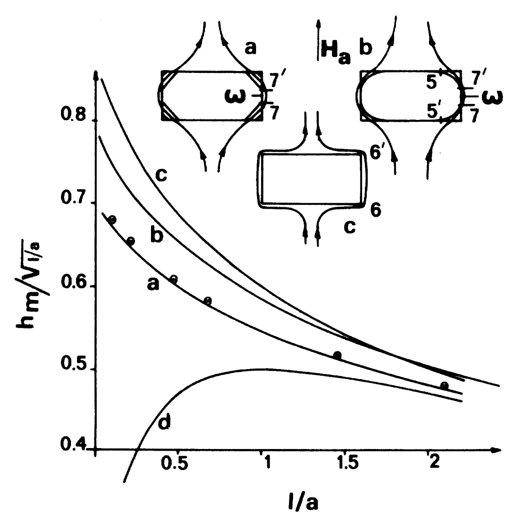

FIG. 7. Plot of $h_m(a/l)^{1/2}$ vs l/a. Solid curves correspond to different models; a: fanlike model, b: model used in Ref. 7, c: model ignoring intermediate state in corners, d: cylinder with elliptic cross section, ⊙: experimental points. The choice of $h_m(a/l)^{1/2}$ rather than h_m emphasizes the differences between the models for small l/a values.

In Fig. 7 we have also plotted curve d defined by

$$h_m(a/l)^{1/2} = (l/a)^{1/2}/(1 + l/a)$$

obtained for cylinders with elliptic cross section inscribed in the slabs. Curve d is very far away from experimental results. It is well known that curve d can not be related to the migration process into the slabs as the migration field in a disc or in a cylinder is not given by demagnetizing factor of the inscribed ellipsoid.[6,11,12] Curves b and c are not far away from experimental results. In reality, the differences between the three curves a, b, and c are rather weak and this may be interpreted with the following argument. The common physical feature of the three models is the way of writing the migration condition; i.e., the length of the limiting wall is chosen to be equal to $\Delta\phi/H_c$, $\Delta\phi$ denoting the potential difference along the wall. The differences between the three models come from differences between the field distributions. In the model leading to curve b, it is assumed that the metal in the corners may be in normal state in a field lower than the critical field H_c. On the contrary, in the model leading to curve c it is supposed that the metal in the corners may be superconducting in a field greater than H_c, the field being infinite at the corner. Thus, these two models contain a physically unrealistic assumption, and their agreement with experimental h_m values might seem rather surprising. With the present study, we may appreciate to what extent they may be used as rough approximations of the fanlike model. They are much more tractable in a mathematical point of view and have proved very useful for understanding the migration process. Moreover, it may be recalled that, if the model leading to curve b may give first-stage magnetization curve in qualitative agreement with experiment,[7] the model leading to curve c is unable to do that. The agreement between fanlike model and experiment shown by Figs. 6 and 7(a) confirms the validity of the thermodynamic and electromagnetic treatment on which the model is based.

V. CONCLUSION

These experimental results of the first magnetization of lead slabs at 4.2 K are in agreement with the so called fanlike model previously published. The good fit between theoretical and experimental magnetization curves and h_m values allows us to claim that the fanlike model which takes into account the intermediate state in the corners is probably the real one at least for the studied sample dimensions. In order to verify this model completely other experimental studies may be undertaken: (a) Magnetization curves of the first stage at other temperatures and of other type-I superconductors. That should allow us to see that the h_m vs (l/a) curve is or is not dependent on the temperature and on the nature of the metal. We may think this curve can slightly depend on the *NS* wall energy of the intermediate state, and this effect is likely to be negligible for lead in which the walls energy is weak. (b) Magneto-optical observations like those of Kirchner on discs.[5]

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