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Two-dimensional electrical transport in GaAs-$Al_xGa_{1-x}As$ multilayers at high magnetic fields

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We have studied the magnetotransport properties of two-dimensional electrons in thin GaAs-$Al_xGa_{1-x}As$ multilayers with fields up to 210 kG from 4.2 to 2.4 K. We observed that ρ_{xx} decreases with decreasing T in the high-field limit $\nu = 2\pi n l_0^2 \lesssim 1$, but ρ_{xy} shows no such changes. We show that this result cannot be explained by models based on independent electrons and suggest as possible explanations the formation of a highly correlated state, such as a charge-density wave or Wigner solid previously suggested for the Si inversion layer, and the enhancement of g factor at $\nu \lesssim 1$.

I. INTRODUCTION

Magnetotransport of quasi-two-dimensional (2-D) electrons has been investigated most thoroughly in the case of the electron inversion layer at the Si-SiO_2 interface.[1] Of particular current interest is the observation that at low temperatures ($T \lesssim$ 10 K) the magnetoconductivity (σ_{xx}) is thermally activated in the low-density and high-magnetic-field limit $2\pi n l_0^2 \lesssim 1$.[2-4] Here $l_0 = (\hbar c/eB)^{1/2}$ is the radius of the lowest-energy Landau orbit, n is the electron density per unit area, and $(2\pi l_0^2)^{-1}$ is the Landau orbital degeneracy per unit area. The quantity $2\pi n l_0^2$, which is the average number of electrons in each Landau orbit, is usually denoted by ν. In this limit of $\nu \lesssim 1$ it has also been observed that the far-infrared cyclotron resonance is shifted to higher frequencies and exhibits anomalously narrow linewidth.[5-7] These observations are incompatible with independent-electron models based on trapping by conventional bound states or localization by potential fluctuations at the Si-SiO_2 interface. On the other hand, in this high-field limit, electrons in such 2-D systems have no kinetic energy and the formation of a highly correlated state may be favored. Although the Wigner solid[2,3,5-17] and the charge-density-wave states[18-21] have been proposed, it has been difficult to relate these concepts to the experimental observations made in that particular system. One major difficulty is that the potential fluctuations at the Si-SiO_2 interface are not negligible in the range of n at which the experiments can be carried out[2-4] (e.g., at $B = 200$ kG, $\nu \lesssim 1$ for $n \lesssim 5 \times 10^{11}/\mathrm{cm}^2$). This is evident from the fact that thermally activated conduction was observed even in the absence of magnetic fields.[22]

In this paper we report an investigation of the dc magnetotransport properties of the 2-D electrons in thin multilayers of GaAs-$Al_xGa_{1-x}As$, in magnetic fields B up to 210 kG, and in the temperature range from 4.2 to 2.4 K. In this case the electrons are confined to the GaAs layer of the heterojunction structure, which is made of lattice-matched single crystals, and its interfacial properties are known to be excellent.[23] In contrast to the inversion layer in Si MOSFET's (metal-oxide-semiconductor field-effect transistors), which is capacitively induced by the gate voltage, the 2-D electrons in GaAs arise from the donor impurities placed inside the $Al_xGa_{1-x}As$ crystal,[24] and therefore their density cannot be varied easily. However, electrons in GaAs have a smaller effective mass ($m^* = 0.067m$) and a single conduction valley.[25] As a result, the density-of-states of the 2-D electrons is low ($dn/dE \simeq 2.8 \times 10^{10}/\mathrm{cm}^2\,\mathrm{meV}$), compared to that in the inversion layer on (100) Si ($dn/dE \simeq 1.7 \times 10^{11}/\mathrm{cm}^2$ meV). This fact has made it possible to realize the high-field limit, $\nu \lesssim 1$, with conventional magnets at the National Magnet Laboratory, in samples whose electron Fermi energy (~13 meV) is much larger than potential fluctuations (a few meV) in the system. Consequently, potential fluctuations are negligible in this 2-D electron system and we deemed it desirable to further our studies of the high-field electronic phenomena, previously carried out only in the Si inversion layer.

In this experiment, the electrical conductivity of the 2-D electrons, in the absence of a magnetic field, B, is independent of temperature from 4.2 to 1.0 K. In the presence of a perpendicular B and in the temperature range of this experiment (4.2 to 2.4 K), the transverse magnetoresistivity ρ_{xx} decreases with decreasing T in the $\nu \lesssim 1$ limit, when the electron is localized by B to a distance smaller than the average interelectron separation. The Hall resistivity ρ_{xy} shows no such changes in this range of T, indicating the absence of conventional electron trapping. In the rest of this paper, we shall describe the experimental details in Sec. II and present the data in Sec. III. In Sec. IV we compare the data with Ando's free-electron theory of high-field magnetotransport of 2-D electrons and point out that the anomaly, reported in this paper, cannot be understood by introducing electron trapping by conventional bound states or localization by interfacial inhomogeneities. We suggest as possible explanations the formation of a highly correlated electronic state, such as charge-density wave or Wigner solid, previously suggested for the 2-D electrons at the Si-SiO_2 interface, and the enhancement of the electron effective g factor at $\nu \lesssim 1$. Section V gives a summary.

II. EXPERIMENTAL DETAILS

The samples are from a film of modulation doped n-type GaAs-$Al_xGa_{1-x}As$ ($x=0.18$) superlattice, grown by MBE (molecular-beam epitaxy).[24,26] The superlattice consists of 150 layers of 200-Å-thick GaAs separated by 200-Å-thick $Al_xGa_{1-x}As$ layers, with Si donors intentionally placed only inside the $Al_xGa_{1-x}As$ layers, ~60 Å from the interfaces, to reduce interfacial scattering. The multilayer structure was grown on top of an ~2-μm-thick GaAs buffer layer, which in turn was grown epitaxially on the Cr-doped, semi-insulating GaAs substrate.

The energy gap at $T = 4.2$ K is 1.519 eV in GaAs and 1.743 eV in $Al_{0.18}Ga_{.72}As$.[25] In going from one material to the other, approximately 85% of this change in energy gap occurs in the conduction band edge and 15% in the valence band edge.[23] As a result, electrons from the donor impurities in the $Al_xGa_{1-x}As$ layer are confined to the undoped GaAs layer by a one-dimensional potential well, illustrated in Fig. 1(a). The electronic motion perpendicular to the layer is size quantized and the electrons constitute a 2-D system.[27,28] We have studied the B orientation dependence of the Shubnikov–de Haas (SdH) effect and observed that the period of the SdH oscillations, $\Delta(1/B)$, depends strictly on the perpendicular component of B. This observation confirms the 2-D nature of the electrons and allows direct determination of the electron density in each layer through[29]

$$n = 2e/hc\,\Delta(1/B) \qquad (1)$$

where hc/e is the flux quantum, equal to 4.14 $\times 10^{-7}$ G cm^2, and the factor of 2 takes into account the spin degeneracy. We have observed only one set of SdH oscillations, indicative of one subband of 2-D electrons, and obtained $n = 3.4 \times 10^{11}/\mathrm{cm}^2$. We made variational calculations[30] and obtained the potential profile in GaAs, shown in Fig. 1(a), and the two lowest quantum levels at 19 and 46 meV, measured from the bottom of the potential well at the GaAs-$Al_xGa_{1-x}As$ interface. The calculation is not self-consistent and the energy levels

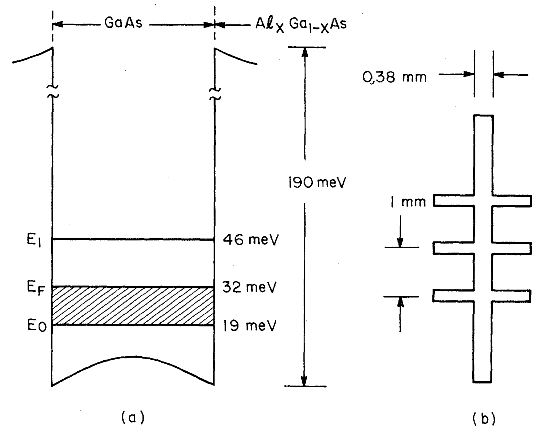

FIG. 1. (a) Energy diagram of electrons confined to the GaAs layer (200 Å thick) of a GaAs-$Al_{0.18}Ga_{0.82}As$ supperlattice. (b) Geometry of a sample.

are accurate to better than a few meV. We recall that the density-of-states is $dn/dE = 2.8 \times 10^{10}/\mathrm{cm}^2$ meV in this system. Thus, in the temperature range of this experiment, only one subband is occupied with $E_F = 13$ meV, the second one being ~14 meV above the Fermi level. Moreover, owing to the 200-Å-thick $Al_xGa_{1-x}As$ barriers, there is no coupling between electrons in neighboring GaAs layers and the sample is indeed made of 150 2-D electron layers.

The magnetotransport measurements were made using conventional Hall bridges, as illustrated in Fig. 1(b). The dimensions of the bridges are so chosen that corrections due to sample geometry are negligible. Ohmic contacts to the 2-D electrons were made by alloying In metal into the GaAs-$Al_xGa_{1-x}As$ multilayers in H_2 atmosphere at 400 °C. We have found that contacts prepared by following this simple procedure are ohmic to 1 K (which is the lowest temperature we have checked), but they do not contact the entire 6 μm thickness of the superlattice. Consequently, we have no direct confirmation of the number of layers that were contacted and direct measurement of the sheet conductance, $\sigma_\square$ per 2-D electron layer, was not possible. Instead, we use $\sigma_\square = ne\mu$, where n is determined from the SdH effect, e the electronic charge, and μ the Hall mobility measured at 4.2 K with $B = 287$ G. We obtain $n = 3.4 \times 10^{11}/\mathrm{cm}^2$, $\mu = 13,000$ cm^2/V sec, and $\sigma_\square = 7.07 \times 10^{-4}\Omega^{-1}$.

This experiment was carried out at the Francis Bitter National Magnet Laboratory, using the high-field facilities there. All the data presented in this paper were obtained by applying the magnetic field perpendicular to the plane of the 2-D electron layers. The samples were immersed in the liquid helium bath and the sample temperature was determined from the vapor pressure of the bath. The magnetic field calibration is accurate to better than 5%.

III. DATA

Figures 2 and 3 show the transverse magnetoresistivity (ρ_{xx}) and the Hall resistivity (ρ_{xy}), respectively, of a sample at 4.2 K, normalized to the resistivity at $B=0$ ($\rho_0 = 1.41\times10^3 \Omega/\square$). The SdH oscillations, observable in both ρ_{xx} and ρ_{xy}, give $n = 3.4\times10^{11}/\mathrm{cm}^2$. ν is calculated from $\nu = n(hc/e)/B$ and marked in the top frame of each figure. It should be noted that the effective g factor of electrons in GaAs is small ($g^* = 0.522$) (Ref. 31) and spin splitting is not resolved. The dip in ρ_{xx} at $B\sim 70$ kG, corresponding to $\nu = 2$, reflects the complete filling of the lowest Landau level, which has a twofold spin degeneracy, at $E_F = \hbar\omega_c$ (ω_c is the cyclotron frequency given by eB/m^*c).

When T is reduced from 4.2 to 2.4 K, ρ_{xx} decreases with decreasing T for $B \gtrsim 120$ kG, corresponding to the high-field limit $\nu \lesssim 1$ (Fig. 4). ρ_{xy}, on the other hand, shows no such change, indicating that this change in ρ_{xx} is not due to changes in the carrier density. We recall that σ_{xx} and σ_{xy} are related to ρ_{xx} and ρ_{xy} through

$$\sigma_{xx} = \frac{\rho_{xx}}{\rho_{xx}^2 + \rho_{xy}^2}$$

and (2)

$$\sigma_{xy} = \frac{-\rho_{xy}}{\rho_{xx}^2 + \rho_{xy}^2}.$$

At high magnetic fields $\rho_{xy} \gg \rho_{xx}$. Consequently, σ_{xy} is also independent of T and σ_{xx}, just like ρ_{xx}, decreases with decreasing T for $\nu \lesssim 1$. This temperature dependence of σ_{xx} is shown in Fig. 5,

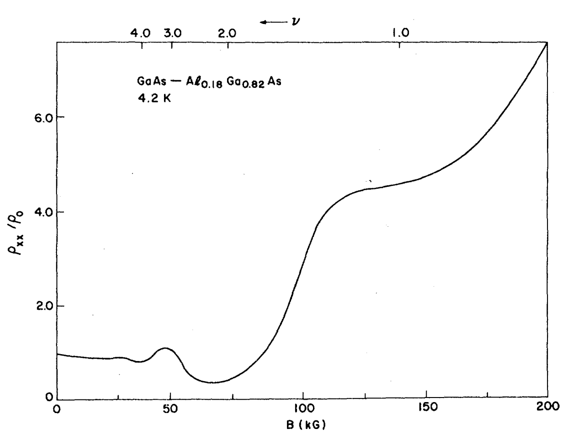

FIG. 2. ρ_{xx}/ρ_0 vs B. $\rho_0 = 1.41\times10^3\ \Omega/\square$. $\nu \equiv 2\pi n l_0^2 = n(h_c/e)/B$, with $n = 3.4\times10^{11}/\mathrm{cm}^2$.

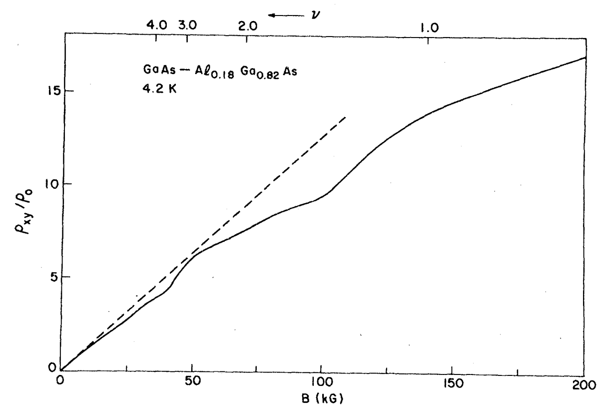

FIG. 3. ρ_{xy}/ρ_0 vs B. The dashed line is the classical value $\rho_{xy} = B/nec$.

where $\log(\sigma_{xx})$ is plotted versus $1/T$ at fixed values of ν. In this limited temperature range, σ_{xx} shows no dependence on T for $\nu \gtrsim 1.2$. At $\nu\sim 1.2$, σ_{xx} begins to decrease with decreasing T for $T \lesssim 3$ K. This dependence on T becomes stronger and occurs at higher T as ν decreases to ~0.9. For $\nu \lesssim 0.9$, σ_{xx} shows one dependence on T and the data for different ν, if plotted in Fig. 5, are indistinguishable from one set to another. Our data extend to $\nu = 0.68$.

IV. DISCUSSIONS

Magnetic quantization of a 2-D electronic system splits the subband into discrete Landau levels, and as a result electrical conduction becomes impossible in the absence of level broadening. In real physical systems scattering, which also impedes electrical conduction, broadens each Landau level into a band and gives rise to magnetotransport. Theoretical results on the Landau level broadening and the magnetoconductivity tensors of 2-D electrons have been obtained by Ando

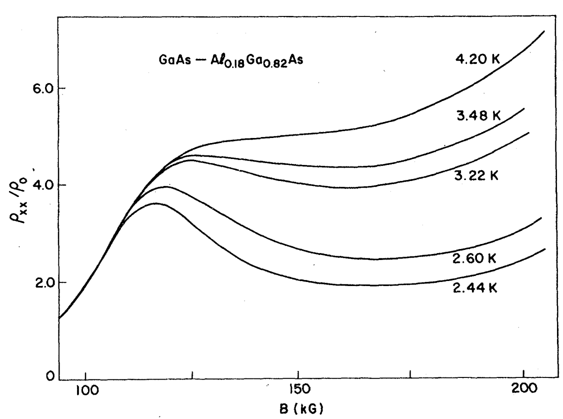

FIG. 4. ρ_{xx}/ρ_0 versus B for T from 4.20 to 2.44 K.

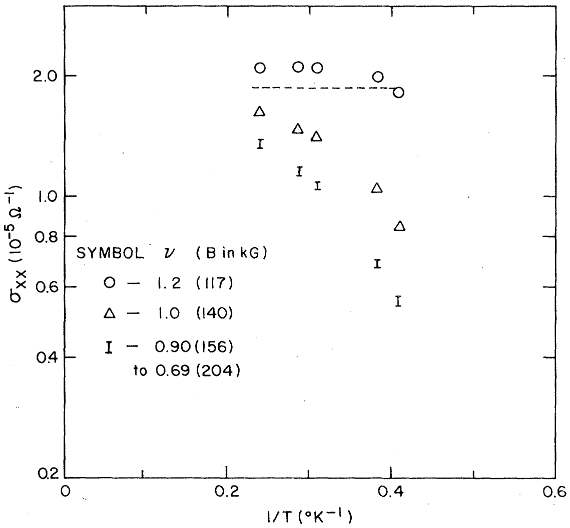

FIG. 5. σ_{xx} versus $1/T$ for several values of ν [$\nu \equiv 2\pi n l_0^2 = n(h_c/e)/B$]. The dashed line is drawn for $\nu = 1.25$, above which σ_{xx} remains independent of T.

and Uemura, assuming a short-range scattering mechanism and using the self-consistent Born approximation to the independent electron theory of quantum transport.[32-34] In order to facilitate the discussion of our data, we summarize briefly these results. As shown below, however, this theory fails to account for all features of the data.

When scattering between Landau levels is neglected, the density-of-states in the broadened Nth Landau level, in the case of short-range scattering, is given by

$$\frac{dn}{dE} = \frac{1}{2\pi l_0^2}\,\frac{2}{\pi\Gamma}\left[1-\left(\frac{E-E_N}{\Gamma}\right)^2\right]^{1/2}, \tag{3a}$$

where $E_N = \hbar\omega_c(N+\frac{1}{2})$ and Γ is related to the electron relaxation time τ through

$$\Gamma = \left(\frac{2}{\pi}\hbar\omega_c\frac{\hbar}{\tau}\right)^{1/2}. \tag{3b}$$

The conductivity tensors are then given by

$$\sigma_{xx} = \frac{e^2}{\pi^2\hbar}(N+\tfrac{1}{2})\int dE\left(\frac{-\partial f}{\partial E}\right)\left[1-\left(\frac{E-E_N}{\Gamma}\right)^2\right] \tag{4a}$$

and

$$\sigma_{xy} = \frac{-nec}{B} + \Delta\sigma_{xy}, \tag{4b}$$

with

$$\Delta\sigma_{xy} = e^2/\pi^2\hbar(N+\tfrac{1}{2})\int dE\left(\frac{\partial f}{\partial E}\right)\frac{\Gamma}{\hbar\omega_c}\left[1-\left(\frac{E-E_N}{\Gamma}\right)^2\right]^{3/2}. \tag{4c}$$

Here $f(E)$ is the Fermi distribution function. We calculate σ_{xx} and σ_{xy} at finite T, for a sample with fixed electron density ($n = 3.4\times10^{11}/\mathrm{cm}^2$) as a function of B. We used $\tau = 5\times10^{-13}$ sec (approximately equal to that deduced from the Hall mobility of our sample), and restricted the calculation to fields where there is no overlap between neighboring Landau levels. Figure 6 shows the 4.2 K magnetoresistivity tensors (ρ_{xx} and ρ_{xy}) obtained by inverting σ_{xx} and σ_{xy}. These calculated results (independent of T from 0 to ~10 K) are shown in Fig. 6 as a function of B/B_0, where B_0 is the field at which $\nu = 1$ (i.e., $B_0 = nhc/e$).

The theory is not expected to account for our data at low fields, when the density-of-states (dn/dE) between neighboring Landau levels overlaps. In particular, Eq. (3) gives unphysical dn/dE near the band edges, where it vanishes abruptly. In our calculation, when $B = B_0/2(N+1)$, E_F is halfway between the Nth and the $(N+1)$th Landau levels and the dn/dE at E_F vanishes. Consequently, ρ_{xx} vanishes at $B = B_0/2(N+1)$, and ρ_{xy} approaches its classical value, B/nec. In the experimental data ρ_{xx} does not vanish at $B = B_0/2(N+1)$, suggesting that dn/dE from neighboring Landau levels indeed overlaps. Moreover, ρ_{xy} does not approach its classical value, which is the dashed line in Fig. 3, at the ρ_{xx} minima, and the SdH oscillations in ρ_{xy} vs B are not 180° out of phase with those in ρ_{xx} vs B, as predicted by theory. In fact, ρ_{xy} appears as step increases at magnetic fields where ρ_{xx} reaches maxima. We also note that a negative magnetoresistivity, $\Delta\rho/\rho_0 \sim 4\%$ at 10 kG, was observed at low fields. Such low-field magnetoresistance, common in degenerate semiconductors,[35] has also been seen in the inversion layer at the Si-SiO_2 interface,[36] and at present it lacks definite explanation.

At higher fields, when $B > B_0/2$, the theory predicts a monotonic increase in the magnetoresistivity tensors and predicts no observable depen-

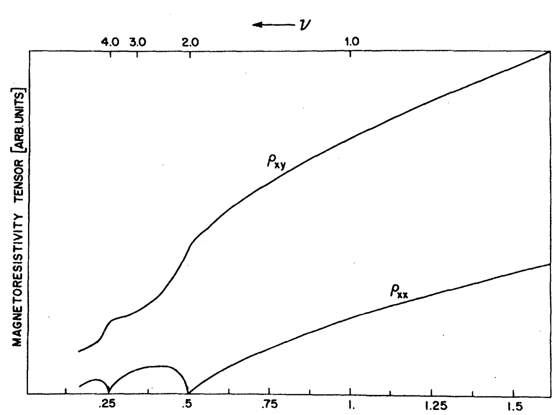

FIG. 6. ρ_{xy} and ρ_{xx} as calculated from the free-electron theory of Ando and Uemura (Refs. 32–34).

dence on T in the temperature range of our experiment. The data, on the other hand, show oscillatory effects in ρ_{xx} and in ρ_{xy} up to $B \sim B_0$. For $B \gtrsim B_0$ (i.e., $\nu \gtrsim 1$), ρ_{xx} decreases with decreasing T from 4.2 to 2.4 K, but ρ_{xy} shows no such changes. This temperature dependence, shown in Fig. 4 for ρ_{xx} and in Fig. 5 for σ_{xx}, cannot be explained by quantum transport of 2-D electrons moving independently of each other in the presence of scattering centers. Trapping by conventional bound states, as well as magnetic freeze out, can be ruled out by noting that the Hall effect shows no such temperature dependence. In fact, the data suggest thermal activation of electron mobility. The apparent activation energy is ~8 K at $T \sim 2.5$ K, although determination of the detailed functional dependences of the mobility must await further measurements over wider ranges of temperature and Landau degeneracy.

Our data are also incompatible with Anderson localization of band tail states in the $N = 0$ Landau level.[37,38] They give no evidence for transport by electrons activated above a mobility edge or for transport by nearest-neighbor hopping in case the entire band is localized. On the contrary, the temperature dependence was not observed for $B/B_0 \simeq 0.5$ to 1, when E_F lies in the upper half of the $N = 0$ Landau level. These considerations are consistent with the fact that, in our samples, E_F (= 13 meV) is much larger than the potential fluctuations. It is interesting to note that, if we regard the T dependence of σ_{xx} as indicative of electron localization, this localization occurs when $\sigma_{xx} \lesssim 2 \times 10^{-5} \Omega^{-1}$, ~30% lower than $\sigma_m \sim 0.1\ e^2/\hbar$, which is the lowest minimum metallic conductivity in two-dimensional transport.[39] In addition, it should be pointed out that, for $\nu \gtrsim 1.2$, σ_{xx} can be lower than $2 \times 10^{-5} \Omega^{-1}$ and still shows no localization in our data. For example, no temperature dependence was observed at $B = 94$ kG in Fig. 4. This corresponds to $\nu = 1.5$ and $\sigma_{xx} = 1 \times 10^{-5} \Omega^{-1}$, much lower than σ_m in the absence of magnetic quantization. Thus, we believe that the localization phenomenon seen here is not determined by the sheet conductivity of the system, as is believed for the case of independent particles in the absence of magnetic fields. Instead, the high-field parameter ν appears to be crucial to this localization phenomenon.

A more likely explanation of our observations is the formation of a highly correlated state in the limit $\nu \lesssim 1$, when the 2-D electron is localized by B to a region smaller than the average interelectron separation, as previously suggested for the 2-D electrons at the Si-SiO_2 interface. More recent model calculations, based on the Hartree-Fock approximation, have shown that in the $\nu < 1$ limit, the ground state of a 2-D electron system is a Wigner solid at $T = 0$ and a charge-density-wave state for T above the Wigner solidification temperature. Electrical transport in such a highly correlated state is of great current interest. Kawaji and Wakabayashi[2] suggested conduction by thermally activated vacancies and interstitials to explain their observations in the case of Si inversion layer. Here however, such a model cannot explain the lack of temperature dependence in the Hall data. By the same token, conduction by thermally activated depinning of charge-density waves[21] does not apply. In any case, the lack of appreciable temperature dependence in ρ_{xy} appears to be crucial to an understanding of electrical transport in this system. In this regard, it should be noted that in the limit $\nu \to 0$, a classical 2-D electron system is expected to crystallize at $\Gamma = (\pi n)^{1/2} e^2/\epsilon KT \gtrsim 137$.[40] (Here, Γ is the ratio of the Coulomb potential energy to the kinetic energy per electron). For $32 \lesssim \Gamma \lesssim 55$, corresponding to the density and temperature range of our experiment, the electrons can be expected to have highly correlated motions.

An alternative but related explanation is that the effective electron g factor is enhanced at $\nu \lesssim 1$. Without enhancement, $g^* = 0.522$ in GaAs and, at $B \approx 140$ kG, the spin splitting $\Delta E \sim 0.4$ meV is comparable with lifetime broadening of the Landau levels. An enhancement of g^* will resolve the spin splitting[41] of the $N = 0$ Landau level and produce a density-of-states minimum at $\nu \approx 1$. The T dependence of this enhancement gives rise to the observed T dependence in ρ_{xx}.

V. SUMMARY

We have presented our data on the electrical transport of 2-D electrons in a GaAs-$Al_xGa_{1-x}As$ superlattice at high magnetic field and compared them with the independent electron theory of quantum transport of Ando and Uemura. The data were taken using conventional four-terminal measurements in magnetic fields up to 210 kG at temperatures from 4.2 to 2.4 K. In terms of the Landau occupation parameter $\nu \equiv 2\pi n l_0^2$, they cover the range $\nu \gtrsim 0.7$. We found that the SdH oscillations in ρ_{xy} vs B are not 180° out of phase with those in ρ_{xx} vs B, as predicted by theory, and instead they appear as step increases at B where ρ_{xx} reaches maxima. In the high-field limit $\nu \lesssim 1$, ρ_{xx} decreases with decreasing T, but ρ_{xy} shows no such changes in this range of T. We showed that this anomaly is not due to magnetic freeze out, trapping by conventional bound states, or Anderson localization of the $N = 0$ Landau level. Rather, it suggests that formation of a highly correlated state or alternatively, an enhancement of the effective electron g factor at $\nu \lesssim 1$.

Previously, high-field magnetotransport of 2-D electrons was studied using the electron inversion layer in Si MOSFETs. Both dc transport and the far-infrared cyclotron resonance have been measured in the $\nu \lesssim 1$ limit. In the dc transport experiments, standard two-terminal, Corbino geometry samples were employed and, consequently, σ_{xx} was measured directly, but no information on the Hall effect (i.e., σ_{xy} or ρ_{xy}) could be obtained. Thus thermal activation of mobility, as observed in our experiment, was not established. However, normal Hall effect has already been observed at low magnetic fields[42] and in comparable density range, where σ_{xx} was thermally activated in the absence of magnetic fields. More recently, the far-infrared cyclotron resonance was studied in great detail, and it was established unequivocally that the frequency shift and the line narrowing, observed previously, occur for $\nu \lesssim 1$ and $T \lesssim 10$ K.[7] All these experiments were carried out at electron densities where the potential fluctuations at the Si-SiO_2 interface have observable effect at $B=0$. Our present experiment, on the other hand, was carried out with a 2-D electron system, which has no observable potential fluctuation effect at $B=0$. Clearly, our measurements should be extended to a wider temperature range and to smaller values of ν, by reducing n or by increasing B. In addition, the electric field dependence of dc transport, as well as the far-infrared cyclotron resonance, should be investigated to further clarify the nature of the electronic state in this high-field limit and to elucidate its 2-D electrical transport properties.

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[1]See, for example, G. Landwehr, *Festkörperprobleme Vol. XV*, *Advances in Solid State Physics* (Pergamon-Vieweg, Braunschweig, 1975), p. 49.
[2]S. Kawaji and J. Wakabayashi, Surf. Sci. 58, 238 (1976), and Solid State Commun. 22, 87 (1977).
[3]D. C. Tsui, Solid State Commun. 21, 675 (1977).
[4]R. J. Nicholas, R. A. Stradling, S. Askenazy, P. Perrier, and J. J. C. Portal, Surf. Sci. 73, 106 (1978).
[5]T. A. Kennedy, R. J. Wagner, B. D. McCombe, and D. C. Tsui, Solid State Commun. 21, 459 (1977).
[6]R. J. Wagner and D. C. Tsui, J. Magn. Magn. Mater. 11, 26 (1979).
[7]B. A. Wilson, D. C. Tsui and S. J. Allen, Jr., Bull. Am. Phys. Soc. 24, 436 (1979), and (unpublished).
[8]A. V. Chaplik, Pis'ma Zh. Eksp. Teor. Fiz. 62, 746 (1972) [JETP Lett. 35, 395 (1972)].
[9]Y. E. Lozovik and V. J. Yudson, Pis'ma Zh. Eksp. Teor. Fiz. 22, 16 (1975) [JETP Lett. 22, 11 (1975)].
[10]W. G. Kleppmann and R. J. Elliott, J. Phys. C 8, 2729 (1975).
[11]H. Fukuyama, Solid State Commun. 19, 551 (1976).
[12]M. Tsukadai, J. Phys. Soc. Jpn. 40, 1515 (1976), and 42, 391 (1977).
[13]H. Aoki, Surf. Sci. 73, 281 (1978).
[14]Y. Kuramoto, J. Phys. Soc. Jpn. 45, 390 (1978).
[15]M. Jonson and G. Srinivasan, Solid State Commun. 24, 61 (1977).
[16]E. Canel, Surf. Sci. 73, 350 (1978).
[17]G. Meissner, H. Namaizawa, and M. Voss, Phys. Rev. B 13, 1370 (1976).
[18]H. Fukuyama, P. Platzman, and P. W. Anderson, Surf. Sci. 73, 374 (1978).
[19]Yu. P. Monarkha and V. B. Shikin, Zh. Eksp. Teor. Fiz. 68, 1423 (1975) [Sov. Phys.—JETP 41, 1710 (1976)].
[20]Y. Kuramoto, J. Phys. Soc. Jpn. 44, 1035 (1978).
[21]H. Fukuyama and P. A. Lee, Phys. Rev. B 18, 6245 (1978).
[22]See, for example, D. C. Tsui and S. J. Allen, Phys. Rev. Lett. 34, 1293 (1975).
[23]R. Dingle in *Festkörperprobleme*, *Advances in Solid State Physics*, *Vol. XV* (Pergamon-Vieweg, Braunschweig, 1975), p. 21.
[24]R. Dingle, H. L. Stormer, A. C. Gossard, and W. Wiegmann, Appl. Phys. Lett. 37, 665 (1978).
[25]H. C. Casey, Jr. and M. Panish, *Heterostructure Lasers* (Academic, New York, 1978), Chap. 4.
[26]A. Y. Cho and J. R. Arthur, in *Progress in Solid State Chemistry* (Pergamon, New York, 1975), Vol. 10, p. 157.
[27]L. L. Chang, H. Sakaki, C. A. Chang, and L. Esaki, Phys. Rev. Lett. 38, 1489 (1977).
[28]H. L. Stormer, R. Dingle, A. C. Gossard, W. Wiegmann, and M. D. Sturge, Solid State Commun. 29, 705 (1979).
[29]D. C. Tsui, Phys. Rev. B 4, 4438 (1971).
[30]F. Stern, Phys. Rev. B 5, 4891 (1972).
[31]W. Duncan and E. E. Schneider, Phys. Lett. 7, 23 (1963).
[32]T. Ando and Y. Uemura, J. Phys. Soc. Jpn. 36, 959 (1974).
[33]T. Ando, J. Phys. Soc. Jpn. 37, 959 (1974), and 37, 1233 (1974).
[34]T. Ando, Y. Matsumoto, and Y. Uemura, J. Phys. Soc. Jpn. 39, 279 (1975).
[35]D. N. Nasiedov, J. Appl. Phys. 32, 2140 (1961).
[36]G. Dorda and I. Eisele, Phys. Rev. Lett. 32, 1360 (1974).
[37]M. Tsukada, J. Phys. Soc. Jpn. 41, 1466 (1976).

[38]H. Aoki and H. Kamimura, Solid State Commun. 21, 45 (1977); H. Aoki, J. Phys. C 10, 2583 (1977), and 11, 3823 (1978).
[39]D. C. Licciardello and D. J. Thouless, J. Phys. C 8, 4157 (1975).
[40]C. C. Grimes and G. Adams, Phys. Rev. Lett. 42, 795 (1979).
[41]F. F. Fang and P. J. Stiles, Phys. Rev. 174, 823 (1968).
[42]E. Arnold, Appl. Phys. Lett. 25, 705 (1974).