

Effect of electron-hole scattering on ambipolar diffusion in semiconductors

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The ambipolar diffusion coefficient of a semiconductor in the absence of an external electric field is calculated from the Boltzmann transport equation using Kohler's variational principle. For the regime where electron-hole scattering is important, it is found that the ambipolar Einstein relation is not valid unless a newly defined free-carrier "diffusion mobility" is employed rather than the usual "conductivity mobility." The distinction between the two mobilities is due to the different relative velocities of the electron and hole systems which characterize the two cases.

I. INTRODUCTION

Recent experiments have shown that when laser excitation is employed to inject high densities of electrons and holes into a semiconductor, the mobilities of the carriers can differ significantly from the values measured in the absence of optical excitation.¹⁻³ This is due largely to electron-hole scattering, since the excess carriers represent a high concentration of additional scattering centers. In a dc electric field, e-h scattering has the effect of decreasing the conductivity mobility of both types of carriers,⁴ since the plasma of electrons with an average velocity in one direction exerts a drag on the plasma of holes which is flowing in the opposite direction (and vice versa). However, for ambipolar diffusion the situation is quite different, since here the net flow of electrons is in the same direction as that of the holes. For example, if the electrons have a higher average velocity than the holes, then collisions between the two tend to speed up the holes and in some sense the hole "mobility" is actually increased by the e-h scattering. These considerations suggest that in cases where e-h scattering is important, it may be necessary to modify the usual form of the generalized Einstein relation⁵ for the ambipolar diffusion coefficient⁶

$$D = \frac{k_B T}{e} \left[n_n \frac{\mathcal{F}_{1/2}(\eta_e)}{\mathcal{F}_{-1/2}(\eta_e)} + n_e \frac{\mathcal{F}_{1/2}(\eta_h)}{\mathcal{F}_{-1/2}(\eta_h)} \right] \frac{\mu_e \mu_h}{n_e \mu_e + n_h \mu_h}, \quad (1)$$

where n_m and μ_m are the carrier concentration and mobility, respectively, of carriers of the type m , $\eta_m \equiv E_{Fm}/k_B T$ is the reduced quasi-Fermi energy, and $\mathcal{F}_k(\eta_m)$ is the Fermi integral⁷ of order k .

In the following sections, the ambipolar diffusion coefficient is calculated using Kohler's variational principle for the general case where e-h scattering is included. It will be seen that Eq. (1) remains valid as long as μ_m is taken to be a newly

defined "diffusion mobility" μ_m^D , rather than the more familiar conductivity (or drift) mobility μ_m^C . The effects of e-h scattering on both μ_m^D and μ_m^C will be discussed through consideration of the zero-order approximation to the exact result. (The μ_m^D defined below should not be confused with the "ambipolar drift mobility,"⁶ which characterizes the response of an electron-hole system to both an external field and a carrier density gradient.)

II. THE VARIATIONAL METHOD

Kohler's variational principle⁸ provides a method for solving the Boltzmann transport equation to any desired degree of accuracy. The outline presented in this section will be relatively brief, since the formalism has been discussed in detail elsewhere.^{9,10} McLean and Paige were the first to extend the variational method to include electron-hole scattering.¹¹ Appel^{12,13} also considered this problem employing an e-h scattering potential which is more physical than that used by McLean and Paige. Meyer¹⁴ generalized this work to incorporate the effects of interband hole scattering, electron mass anisotropy, and wave-vector-dependent screening using the random-phase approximation (RPA)¹⁵ dielectric constant. The electron and hole mobilities obtained experimentally by Meyer and Glicksman¹ for a broad range of photo-excited carrier densities and temperatures agree well with the results of the latter theory.

In the present treatment, isotropic and parabolic energy bands will be assumed for both electrons and holes. For brevity, multiple hole bands and scattering between carriers of the same type will not be considered, although these effects can be added straightforwardly.^{12,13} The magnetic field and temperature gradient are assumed to be zero. Under these conditions, the Boltzmann equation for the net change with time of the distribution function $f_m(\vec{v}_1)$ can be written⁹

$$\left[\frac{\partial f_m(\vec{v}_1)}{\partial t} \right]_{\text{net}} = \left[\frac{\partial f_m(\vec{v}_1)}{\partial t} \right]_{\text{field}} + \left[\frac{\partial f_m(\vec{v}_1)}{\partial t} \right]_{\text{grad}} + \left[\frac{\partial f_m(\vec{v}_1)}{\partial t} \right]_{\text{coll}}, \quad (2)$$

where \vec{v}_1 is the velocity of particle 1 in band m . The first two terms on the right side of Eq. (2) represent the tendency of an electric field or carrier density gradient to perturb the distribution function $f_m(\vec{v}_1)$ from its quasiequilibrium value $f_{0m}(\vec{v}_1)$. The third term represents the tendency of collisions with phonons, impurities, other carriers, etc. to restore the quasiequilibrium distribution. For a dc electric field and quasiequilibrium, $(\partial f_m / \partial t)_{\text{net}} \rightarrow 0$.

If the electric field and carrier density gradient are both assumed to be in the z direction, one has in the small field limit⁹

$$\left[\frac{\partial f_m(\vec{v}_1)}{\partial t} \right]_{\text{field}} = -\frac{q_m e F v_{1z}}{k_B T} \frac{\partial f_{0m}(\vec{v}_1)}{\partial \epsilon_1} \quad (3)$$

and

$$\left[\frac{\partial f_m(\vec{v}_1)}{\partial t} \right]_{\text{grad}} = v_{1z} \frac{\partial \eta_m}{\partial z} \frac{\partial f_{0m}(\vec{v}_1)}{\partial \epsilon_1}, \quad (4)$$

$$L_m^{mn}(\vec{v}_1) = \frac{2}{k_B T} \left(\frac{m_m \hbar}{2\pi \hbar} \right)^3 \int \alpha_{\text{eh}}((\vec{v}_1 \rightarrow \vec{v}_2), (\vec{v}_3 \rightarrow \vec{v}_4)) \exp(\epsilon_1 + \epsilon_3 - \eta_m - \eta_n) \times f_{0m}(\vec{v}_1) f_{0m}(\vec{v}_2) f_{0n}(\vec{v}_3) f_{0n}(\vec{v}_4) [\Phi_m(\vec{v}_1) - \Phi_m(\vec{v}_2) + \Phi_n(\vec{v}_3) - \Phi_n(\vec{v}_4)] \sin \chi d\chi d\gamma d^3 v_2. \quad (7)$$

The quantity $\alpha_{\text{eh}}((\vec{v}_1 \rightarrow \vec{v}_2), (\vec{v}_3 \rightarrow \vec{v}_4)) = \alpha_{\text{eh}}(\vec{v}_1, \vec{v}_2, \chi, \gamma)$ is proportional to the electron-hole scattering cross section (see Ref. 12), where χ and γ are the polar and azimuthal angles between the vectors $\vec{v}_1 - \vec{v}_3$ and $\vec{v}_2 - \vec{v}_4$. The subscripts 1 and 2 represent states for particles of type m while 3 and 4 are states for type n . We will use the convention that if m represents carriers of one type, then n must refer to carriers of the opposite type. That is, $(mn) \rightarrow (eh)$ or (he) , but not (ee) or (hh) .

The $\Phi_m(\vec{v}_i)$ which appear in Eqs. (6) and (7) are related to the effect of the electric field and carrier density gradient on the distribution function $f_m(\vec{v}_i)$:

$$f_{1m}(\vec{v}_i) \equiv f_m(\vec{v}_i) - f_{0m}(\vec{v}_i) = -\frac{\Phi_m(\vec{v}_i)}{k_B T} \frac{\partial f_{0m}(\vec{v}_i)}{\partial \epsilon_i}. \quad (8)$$

If the perturbation $f_{1m}(\vec{v}_i)$ is sufficiently small, one may assume it proportional to the terms of the Boltzmann equation which bring it about, $(\partial f_m / \partial t)_{\text{field}} + (\partial f_m / \partial t)_{\text{grad}}$. From Eqs. (3) and (4), it follows¹² that Φ_m must have the form

$$\Phi_m(\vec{v}_i) = \left(eF - q_m k_B T \frac{\partial \eta_m}{\partial z} \right) v_{iz} \tau(\epsilon_i), \quad (9)$$

where F is the electric field, $\epsilon_1 (\equiv \frac{1}{2} m_m v_1^2 / k_B T)$ is the reduced energy of the particle, m_m is its effective mass, q_m is the sign of its charge (+1 for holes and -1 for electrons), and $f_{0m}(\vec{v}_1) (= [1 + \exp(\epsilon_1 - \eta_m)]^{-1})$ is the Fermi distribution function.

The effect of collisions may be written

$$\left[\frac{\partial f_m(\vec{v}_1)}{\partial t} \right]_{\text{coll}} = -[L_m^{\text{ph}}(\vec{v}_1) + L_m^{\text{imp}}(\vec{v}_1) + L_m^{\text{eh}}(\vec{v}_1) + \dots], \quad (5)$$

where L_m^{ph} , L_m^{imp} , L_m^{eh} represent scattering operators for phonon, impurity, and electron-hole scattering, respectively. For mechanisms involving only a single carrier such as simple phonon or impurity scattering, L_m has the form¹²

$$L_m^i(\vec{v}_1) = \frac{1}{k_B T} \left(\frac{m_m}{2\pi \hbar} \right)^3 \int W_m^i(\vec{v}_1 \rightarrow \vec{v}_2) f_{0m}(\vec{v}_1) [1 - f_{0m}(\vec{v}_2)] \times [\Phi_m(\vec{v}_1) - \Phi_m(\vec{v}_2)] d^3 v_2, \quad (6)$$

where the significance of $\Phi_m(\vec{v}_i)$ is given below and $W_m^i(\vec{v}_1 \rightarrow \vec{v}_2)$ is the transition probability for a carrier of type m to be scattered from the state \vec{v}_1 to the state \vec{v}_2 by the mechanism i . Similarly, for processes such as electron-hole scattering which involve two particles, L_m can be written¹³

where $\tau_m(\epsilon_i)$ may be thought of as an energy-dependent relaxation time,¹⁶ whose functional form must be determined through solution of the Boltzmann equation. If τ_m is expanded

$$\tau_m(\epsilon_i) = \sum_{s=0}^{s_1} c_{ms} \epsilon_i^s, \quad (10)$$

and the sum is terminated at finite s_1 , then the c_{ms} are variational parameters which can be adjusted to give the best solution to Eq. (2).⁹

Operating on Eq. (2) with

$$-2\nu_m \left(\frac{m_m}{2\pi \hbar} \right)^3 \int d^3 v_1 v_{1z} \epsilon_1^i$$

and setting $(\partial f_m / \partial t)_{\text{net}} = 0$, one obtains the set of $2 \times s_1$ simultaneous linear equations:

$$\sum_{s=0}^{s_1} \sum_{j=\text{eh}} H_j c_{js} \Gamma_{rs}^{mj} = q_m H_m \beta_{mr}, \quad (11)$$

where

$$H_m = q_m (eF - q_m k_B T \partial \eta_m / \partial z), \quad (12)$$

$$\beta_{m\tau} \equiv \frac{-2q_m}{k_B T} \left(\frac{m_m}{2\pi\hbar} \right)^3 \int \epsilon_1^r v_{1z}^2 \frac{\partial f_{0m}(\vec{v}_1)}{\partial \epsilon_1} d^3 v_1$$

$$\rightarrow \frac{(\nu + \frac{3}{2})!}{(\frac{3}{2})!} \left[\frac{\mathcal{F}_{\nu+1/2}(\eta_m)}{\mathcal{F}_{1/2}(\eta_m)} \right] \frac{q_m \nu_m}{m_m}, \quad (13)$$

$$\Gamma_{rs}^{mn} = d_{rs}^{mn} + g_{rs}^{mn}, \quad (14)$$

$$\Gamma_{rs}^{mn} = h_{rs}^{mn}, \quad (15)$$

$$g_{rs}^{mn} = {}^{12}V_{rs}^{mn}, \quad (16)$$

$$h_{rs}^{mn} = {}^{34}V_{rs}^{mn}, \quad (17)$$

$$d_{rs}^{mn} \equiv \frac{2}{k_B T} \left(\frac{m_m}{2\pi\hbar} \right)^6$$

$$\times \int (W_m^{ph} + W_m^{mp} + \dots) f_{0m}(\vec{v}_1) [1 - f_{0m}(\vec{v}_2)]$$

$$\times v_{1z} \epsilon_1^r (v_{1z} \epsilon_1^s - v_{2z} \epsilon_2^s) d^3 v_1 d^3 v_2, \quad (18)$$

and

$${}^{ij}V_{rs}^{mn} \equiv \frac{4}{k_B T} \left(\frac{m_m}{2\pi\hbar} \right)^3 \left(\frac{m_n}{2\pi\hbar} \right)^3 \int \alpha_{eh}(\vec{v}_1, \vec{v}_2, \chi, \gamma) \exp(\epsilon_1 + \epsilon_3 - \eta_m - \eta_n) f_{0m}(\vec{v}_1) f_{0m}(\vec{v}_2) f_{0n}(\vec{v}_3) f_{0n}(\vec{v}_4)$$

$$\times v_{1z} \epsilon_1^r (v_{1z} \epsilon_1^s - v_{2z} \epsilon_2^s) \sin \chi d\chi d\gamma d^3 v_1 d^3 v_2. \quad (19)$$

In order to evaluate the integrals which appear in Eqs. (18) and (19), one must characterize the cross sections for the various types of scattering events. Since this depends on a number of considerations which are outside the scope of the present paper, these terms will be left in their general form. The reader should see Refs. 11, 13, and 14 for evaluations of the electron-hole scattering integral, Eq. (19), in various limits.

The Ritz method can be applied to Eq. (11) to yield the c_{ns} . It is then apparent from Eqs. (8)–(10) that this is equivalent to having solved the Boltzmann equation for $f_{1m}(\vec{v}_1)$. To illustrate, we note that the current density for particles of type m due to the combined electric field and carrier density gradient is equivalent to the average over the distribution $f_m(\vec{v}_1)$ of the quantity $eq_m v_{1z}$:

$$j_{mz} = 2 \left(\frac{m_m}{2\pi\hbar} \right)^3 \int [f_{0m}(\vec{v}_1) + f_{1m}(\vec{v}_1)] (eq_m v_{1z}) d^3 v_1. \quad (20)$$

From Eqs. (8), (9), and (13) and the result that the integral over the quasiequilibrium distribution function $f_{0m}(\vec{v}_1)$ vanishes, this becomes

$$j_{mz} = eq_m H_m \sum_{r=0}^{s_1} c_{mr} \beta_{mr} \quad (21)$$

or

$$j_{mz} = H_m q_m \mu_m, \quad (22)$$

where

$$\mu_m \equiv \frac{e}{n_m} \sum_{r=0}^{s_1} c_{mr} \beta_{mr}. \quad (23)$$

Although the physical interpretation of the quantity μ_m has not yet been established, it will be seen below that it corresponds to either the conductivity or diffusion mobility, depending on the relation between the electric field and the carrier density gradient.

Combining Eq. (23) with the Ritz method solution

to Eqs. (11), one obtains

$$\mu_m = \frac{q_m e H_e}{n_m H_m} \frac{\det \begin{vmatrix} 0 & b'_m \\ b & G \end{vmatrix}}{\det |G|}, \quad (24)$$

where

$$b'_e \equiv (\beta_{e0} \ 0 \ \beta_{e1} \ 0 \ \dots \ \beta_{es_1} \ 0),$$

$$b'_h \equiv (0 \ \beta_{h0} \ 0 \ \beta_{h1} \ \dots \ 0 \ \beta_{hs_1}),$$

$$b \equiv \begin{pmatrix} \beta_{e0} \\ -R\beta_{h0} \\ \beta_{e1} \\ -R\beta_{h1} \\ \vdots \\ \beta_{es_1} \\ -R\beta_{hs_1} \end{pmatrix},$$

$$G \equiv \begin{pmatrix} G_{00} & G_{01} & \dots & G_{0s_1} \\ G_{10} & G_{11} & & G_{1s_1} \\ \vdots & & & \vdots \\ G_{s_1 0} & G_{s_1 1} & \dots & G_{s_1 s_1} \end{pmatrix},$$

$$G_{rs} \equiv \begin{pmatrix} \Gamma_{rs}^{ee} & \Gamma_{rs}^{eh} \\ \Gamma_{rs}^{he} & \Gamma_{rs}^{hh} \end{pmatrix},$$

and $R \equiv H_h/H_e$.

III. CONDUCTIVITY AND DIFFUSION MOBILITIES

Equation (24) is equivalent to an exact solution of the Boltzmann equation if $s_1 \rightarrow \infty$. In practice, successive approximations usually converge quickly, so that excellent results are obtained when only a few low-order rows and columns are retained. In the analysis presented below, the effects of

electron-hole scattering on the mobility of each will be illustrated through consideration of the $s_1 = 0$ approximation (which typically gives an error of no greater than 20%). In this limit, Eq. (24) becomes

$$\mu_m = \frac{(e/n_m)\beta_{m0}[\beta_{m0}(d_{00}^n + g_{00}^{nm}) + (H_n/H_m)\beta_{n0}h_{00}^{mn}]}{(d_{00}^n + g_{00}^{nm})(d_{00}^n + g_{00}^{nm}) - h_{00}^{nm}h_{00}^{mn}}. \quad (25)$$

Equation (25) is more easily interpreted if several substitutions are made. From conservation of momentum, it can be shown that $\vec{v}_3 - \vec{v}_4 = (m_m/m_n)(\vec{v}_1 - \vec{v}_2)$, which with Eqs. (16), (17), and (19) gives $h_{00}^{nm} = h_{00}^{mn} = -(m_m/m_n)g_{00}^{mn}$. Equation (13) can then be used to give

$$\mu_m = \mu_{m0} \frac{1 + (1 + n_n H_n / n_m H_m) n_m \mu_n^0 J^{eh}}{1 + (n_n \mu_m^0 + n_m \mu_n^0) J^{eh}}, \quad (26)$$

where $J^{eh} \equiv -(m_e m_h / n_e n_h e) h_{00}^{eh}$ and

$$\mu_m^0 = \frac{e\beta_{m0}^2}{n_m d_{00}^m} \quad (27)$$

is the zero-order mobility one obtains if electron-hole scattering is ignored (that is, if $J^{eh} \rightarrow 0$). Until $R = H_h/H_e$ is specified, the μ_m cannot be explicitly determined from Eq. (26).

We now calculate the conductivity mobility, which is defined

$$\mu_m^C = \frac{j_{mz}}{n_m e F} \Big|_{\partial \eta_m / \partial z = 0}. \quad (28)$$

From Eqs. (12) and (22) one finds that μ_m^C is given by Eq. (23), which has been shown to reduce to Eq. (26) in zero order. It is evident from Eq. (12) that $\partial \eta_m / \partial z = 0$ implies $H_h = -H_e$ or $R = -1$. Thus, one obtains for the zero-order conductivity mobility

$$\mu_m^C = \mu_m^0 \frac{1 + (n_m - n_n) \mu_n^0 J^{eh}}{1 + (n_n \mu_m^0 + n_m \mu_n^0) J^{eh}}. \quad (29)$$

Since J^{eh} is positive definite, we see that electron-hole scattering always has the effect of decreasing the conductivity mobility for both electrons and holes. Equation (29) is equivalent to Eq. (26) of Appel.¹³

We now turn to the case of carrier diffusion in the absence of an external field. If, for example, $n_e = n_h$ and $\mu_e > \mu_h$, the electrons will have a tendency to diffuse more rapidly than the holes. The resulting charge separation then causes an induced electric field which holds back the electrons and pulls the holes forward. In fact, it can be shown⁶ that for most cases of interest it is reasonable to assume that the electron density gradient at a particular point in space is equal to the hole gradient

$$\frac{\partial n_e}{\partial z} = \frac{\partial n_h}{\partial z}, \quad (30)$$

and that the electron current density is equal and opposite to that of the holes

$$j_{ez} = -j_{hz}. \quad (31)$$

In other words, the electrons and holes diffuse at a mutual "ambipolar" rate due to the presence of the induced electric field. To determine the magnitude of eF , we note that Eqs. (12), (22), and (30) yield

$$R = -\frac{eF - k_B T \partial \eta_h / \partial z}{eF + k_B T \partial \eta_e / \partial z} = \frac{n_e \mu_e^D}{n_h \mu_h^D}, \quad (32)$$

where μ_e^D and μ_h^D are given by Eq. (23). (Because $R \neq -1$, μ_e^D and μ_h^D are different from μ_e^C and μ_h^C obtained above.) Equation (32) can be rewritten to give the induced field in terms of μ_e^D and μ_h^D ,

$$eF = \frac{k_B T \left(\frac{\partial \eta_h}{\partial z} - \frac{n_e \mu_e^D}{n_h \mu_h^D} \frac{\partial \eta_e}{\partial z} \right)}{1 + \frac{n_e \mu_e^D}{n_h \mu_h^D}}. \quad (33)$$

With Eq. (33), the electron current density can now be evaluated in terms of μ_e^D and μ_h^D . With

$$\frac{\partial \eta_m}{\partial z} = \frac{1}{n_m} \frac{\partial n_m}{\partial z} \frac{\mathcal{F}_{1/2}(\eta_m)}{\mathcal{F}_{-1/2}(\eta_m)}, \quad (34)$$

one obtains

$$j_{ez} = k_B T \frac{\partial n_e}{\partial z} \left[n_h \frac{\mathcal{F}_{1/2}(\eta_e)}{\mathcal{F}_{-1/2}(\eta_e)} + n_e \frac{\mathcal{F}_{1/2}(\eta_h)}{\mathcal{F}_{-1/2}(\eta_h)} \right] \frac{\mu_e^D \mu_h^D}{n_e \mu_e^D + n_h \mu_h^D}. \quad (35)$$

If the ambipolar diffusion coefficient is defined in the usual way,¹⁷ $D = j_{ez} / (e \partial n / \partial z)$, one finally obtains

$$D = \frac{k_B T}{e} \left[n_h \frac{\mathcal{F}_{1/2}(\eta_e)}{\mathcal{F}_{-1/2}(\eta_e)} + n_e \frac{\mathcal{F}_{1/2}(\eta_h)}{\mathcal{F}_{-1/2}(\eta_h)} \right] \frac{\mu_e^D \mu_h^D}{n_e \mu_e^D + n_h \mu_h^D}, \quad (36)$$

which is equivalent to the conventional form, Eq. (1), except that the "diffusion mobility" μ_m^D must be employed rather than the conductivity mobility.

Evaluation of μ_m^D involves solving three equations [Eq. (32), Eq. (24) with $m = e$, and Eq. (24) with $m = h$] for three unknowns (R , μ_e^D , and μ_h^D). To illustrate, we again approximate Eq. (24) with the lowest-order result, Eq. (26). Solution of the equations then yields

$$R = \frac{n_e \mu_e^0 [1 + (n_e - n_h) \mu_n^0 J^{eh}]}{n_h \mu_h^0 [1 + (n_h - n_e) \mu_e^0 J^{eh}]} \quad (37)$$

and

$$\mu_m^D = \frac{\mu_m^0}{1 + (n_n - n_m) \mu_m^0 J^{eh}}. \quad (38)$$

First we note that although the diffusion and conductivity mobilities are equivalent when electron-hole scattering is unimportant (since they both reduce to μ_m^0), the two are distinct for any finite J^{eh} . It is also apparent from Eq. (38) that if $n_e = n_h$, then μ_m^D reduces to μ_m^0 in zero order¹⁸ even for large J^{eh} . This follows from the assumption that $j_{ez} = -j_{hz}$, which implies that when $n_e = n_h$ the average velocity of the electrons is equal to that of the holes. In lowest order, no current is gained or lost due to scattering by a system which is traveling in the same direction at the same velocity.

Now consider the effect of electron-hole scattering on the diffusion mobility when $n_e \neq n_h$. For definiteness, we consider Eq. (38) for electrons,

$$\mu_e^D = \frac{\mu_e^0}{1 + (n_h - n_e)\mu_e^0 J^{\text{eh}}} \quad (39)$$

If $n_h > n_e$, the electron mobility is decreased by electron-hole scattering. Since the electron and hole currents are equal but opposite, the average velocity of the holes must be smaller than that of the electrons. The drag due to scattering from a slower system causes a decrease in the electron mobility. On the other hand, if $n_e > n_h$, the holes have a higher average velocity and the scattering will actually increase the electron mobility.

It has been demonstrated that when the conduc-

tivity mobility is significantly affected by electron-hole scattering, μ_m^D rather than μ_m^C should be used in calculating the ambipolar diffusion coefficient from the generalized Einstein relation. If $n_e = n_h$, it is often reasonable to employ the conductivity mobility appropriate for low carrier densities, since e-h scattering does not affect the diffusion mobility in zero order. It should be remembered, however, that μ_m^0 from Eq. (27) is not always equivalent to μ_m^C at low carrier concentrations, since the effects of free-carrier screening or degeneracy can significantly alter the phonon and impurity interactions which determine μ_m^0 .

IV. CONCLUSIONS

Kohler's variational principle has been employed to solve the Boltzmann equation for the ambipolar diffusion coefficient of electrons and holes in a semiconductor. It has been shown that in the regime where electron-hole scattering significantly affects the transport properties, the ambipolar Einstein relation is inappropriate unless a newly defined "diffusion mobility" is employed rather than the more familiar conductivity mobility. The difference between the two mobilities has been analyzed in lowest order of the variational method. Physically, the two cases are characterized by differing relative velocities of the electrons and holes.

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¹⁸In higher order, a change in mobility will occur due to the manner in which the populations of the various energy states are rearranged by the scattering. A similar situation is that of electron-electron scattering in the presence of a dc electric field. Although conservation of momentum implies that no immediate change in the current will occur due to events between carriers of the same charge and mass, a decrease in mobility results when the variational calculation is carried to higher order (see Ref. 11).