Comments and Addenda

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Quadratic magnetoresistivity of closed-orbit, uncompensated metals

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(Received 22 January 1979)

This paper describes a mechanism capable of generating a quadratic field dependence in both the electrical and thermal magnetoresistivity of closed-orbit, uncompensated metals. This mechanism, consisting of a partial Corbino effect coupled to a linear magnetoresistance, may provide an explanation of the recently observed quadratic field behavior of the thermal magnetoresistivity of potassium.

Recently, several articles[1-4] have appeared, reporting, besides a linear field behavior, a quadratic field dependence of the thermal magnetoresistivity $W(H,T)T$ of potassium. Taking the influence of the lattice conductivity λ_g into account[5] one could explain but a small part of the quadratic field term, as attributing the whole quadratic effect to the influence of λ_g leads to anomalous values for the lattice conductivity[2] and the Righi-Leduc coefficient[4] of potassium. However, according to the authors of Refs. 1–4, no other mechanism could be found in the literature that is capable of producing an additional quadratic field dependence of the thermal magnetoresistivity. In this article we describe such an additional mechanism, which could induce a quadratic field behavior in both the thermal and the electrical magnetoresistivity of closed-orbit, uncompensated metals and which appears to have a considerably larger influence than the theoretical lattice conductivity[6] on the generation of a quadratic field term in the thermal magnetoresistance.

The quadratic field term seems to appear only in the transverse thermal magnetoresistivity[1-3] of potassium. The transverse electrical[7] magnetoresistivity as well as both the longitudinal electrical[8] and thermal[9] magnetoresisticity show but a linear field dependence. Although in contradiction to the Lifshitz-Azbel-Kaganov (LAK) theory,[10] which predicts a saturation of both the electrical and thermal magnetoresistivity in closed-orbit, uncompensated metals at high fields, the electrical and thermal linear field effect in these metals is well known experimentally and, besides in potassium, has also been observed in indium[11-14] and aluminum.[14-17] As for the quadratic field dependence, Fletcher[5] has shown that the thermal lattice conductivity may induce a quadratic field term in the thermal magnetoresistivity, i.e., for pure metals with small λ_g and provided the field is not too high, the measured transverse thermal magnetoresistivity $W^m(H)$ can be approximated by

$$W^m(H) = W(H) + \lambda_g A_{RL}^2 H^2 , \tag{1}$$

where $W(H)$ is the electronic transverse thermal magnetoresistivity and A_{RL} is the Righi-Leduc coefficient. Assuming the lattice conductivity to be the sole cause of the quadratic field term, however, leads to implausible and inconsistent results for potassium.[2-4] For instance, with that assumption, the temperature dependence of the extrapolated lattice conductivity $\lambda_g(T)$ displays a highly unexpected and unusual behavior. Besides having a magnitude which is about a factor 5~8 times the λ_g values as calculated theoretically by Ekin,[6] the resulting $\lambda_g(T)$ curve does not show the expected T^2 dependence and also exhibits a maximum at about 3 K (e.g., Fig. 3 of Ref. 2). Furthermore, Tausch and Newrock[4] have shown that the above assumption leads to a substantial decrease of the Righi-Leduc coefficient A_{RL} with increasing field, which is inconsistent with their measurements.[18] A more consistent explanation of the experimental results indicates that the lattice conductivity can account but for a part of the quadratic field dependence of the thermal magnetoresistivity. An additional quadratic field dependence can be found, however, as a result of a partial Corbino effect[19] coupled with the existence of a linear magnetoresistivity.

In the Corbino geometry the sample consists of a thin cylindrical disk, perforated by a central hole, while the magnetic field is applied perpendicular to the plane of the disk. The thermal current enters the disk in the middle and the thermal resistance between the inner and outer boundary contacts is measured as a function of the magnetic field. The field-dependent electronic thermal resistivity is given by

$$W(H) = W(0)(1 + \tan^2\theta_{RL}) , \tag{2}$$

where $W(0)$ is the thermal resistance in zero field and θ_{RL} is the Righi-Leduc angle, represented by

$$\tan\theta_{RL} = (A_{RL} H)/W(0) . \tag{3}$$

In the Corbino geometry the electronic thermal resistance is greatly enhanced and consequently the electronic thermal conduction considerably decreased by the application of a magnetic field, while the field-independent phononic conductivity remains unaffected. Indeed, following a suggestion by Miedema,[20] we have used the Corbino method to separate the phononic from the electronic thermal conductivity and to obtain the lattice conductivity of indium[12-14] and aluminum.[14,15] It appears from these measurements that the $W(H, T)T$ vs H curves of aluminum and indium show the same qualitative features as the equivalent curves of potassium, as measured by Fletcher[2] and Newrock and Maxfield,[1,3] i.e., a quadratic field term. In our case, however, the quadratic field dependence has a natural geometrical cause and finds its origin in the use of the Corbino configuration, as evidenced by Eqs. (2) and (3). When the Corbino effect is subtracted, the thermal magnetoresistivity reduces to a linear field dependence,[12-14] analogous to the behavior of the electrical magnetoresistivity. Hence, in our case, the quadratic field effect is not an intrinsic effect but arises from the application of the Corbino geometry. However, despite the fact that Fletcher[2] and Newrock and Maxfield[1,3] have used the standard geometry, consisting of a rectangular strip with the magnetic field applied perpendicular to it, the similarity in the qualitative feature of our and their results is striking indeed.

An explanation for the close parallel between the results of potassium, as obtained by Fletcher and Newrock and Maxfield without using the Corbino geometry, and our measurements on aluminum and indium, acquired with full use of the Corbino effect, can be found in the articles by Lippmann and Kuhrt.[21] (See also Ref. 22.)

For a rectangular plate the magnetoresistivity deviates strongly from the bulk behavior, when the length and width of the samples become comparable. This occurs because the (electrical or thermal) Hall field is reduced at the ends of the specimen, allowing the Lorentz force to remain partly uncompensated. Lippmann and Kuhrt have analyzed this phenomenon in detail and their results can be expressed in terms of the Corbino effect. Let $W(H, b/a)$ be the thermal resistivity of a rectangular plate with length a and width b in a field H perpendicular to the plate. The configuration where $b/a \to \infty$ corresponds to the complete Corbino geometry, for which the Corbino effect is maximal. The other limit, i.e., $b/a \to 0$ corresponds to the geometry of an infinitely long wire, for which the Corbino effect is zero. A partial Corbino effect occurs for the configuration $1 > b/a > 0$, which corresponds to the standard geometry.

Using the Lippmann-Kuhrt theory[23] one finds in this case for $W(H, b/a)$[24]

$$\frac{W(H, b/a)}{W(0, b/a)} = \frac{W(H, 0)}{W(0, 0)} \left[1 + \frac{b}{a} \left(\tan\theta_{RL} - \frac{4\ln 2}{\pi} \right) \right] ; \quad 1 > \frac{b}{a} > 0 . \tag{4}$$

Assuming $W(H, 0)$ to possess a linear field dependence, i.e.,

$$W(H, 0) = W(0, 0)(1 + \alpha H) \tag{5}$$

analogous to the electrical magnetoresistivity of a long wire of potassium,[7] it immediately follows from Eqs. (3)–(5) that, besides a linear field dependence, $W(H, b/a)$ would also display a quadratic field behavior. Here α represents the slope of the reduced thermal magnetoresistivity

$$\Delta W/W_0 = [W(H, 0) - W(0, 0)]/W(0, 0) ,$$

i.e., $\alpha = (\Delta W/W_0)/H$.

According to Newrock and Maxfield,[3] the quadratic field term has no analog in the electrical magnetoresistivity of potassium. However, the electrical magnetoresistivity measurements of potassium, as for instance discussed by Taub *et al.*,[7] have only been performed in one of the following ways: (i) four probe measurements on long wires (typically, $a = 1$ m, $b = 1\sim 2$ mm), (ii) helicon measurements on rectangular plates, and (iii) induced torque measurements on spherical samples.[8]

It follows from the above discussion that in all these cases the Corbino effect would be absent or, as in (i) immeasurably small. To see also a partial Corbino effect, i.e., a quadratic field term in the electrical magnetoresistivity, one has to measure on rectangular samples of the same size as were used for the thermal magnetoresistivity measurements. In fact, Lass[8] has made four probe electrical magnetoresistivity measurements on relatively "short" wires ($a = 30 \sim 60$ mm, $b = 2$ mm) and some of his results might be interpreted to indicate a quadratic term in $\rho(H)$.

As a check on the applied measuring techniques Fletcher[2] has also measured the thermal magnetoresistivity of an aluminum specimen, presumably of the same size as the potassium samples. For the aluminum specimen a quadratic field term could not be observed. This may be due, however, to the fact that the Righi-Leduc coefficient A_{RL} is considerably larger for K than for Al. According to the LAK theory,[10] in the high-field, low-temperature limit $A_{RL}T$ is constant and equal to the free-electron value R_H/L_0, i.e.,

$$A_{RL}T = R_H/L_0 , \tag{6}$$

where R_H is the Hall coefficient and the Lorenz constant $L_0 = 2.44 \times 10^{-8}$ WΩK^{-2}. Using the free-electron values[25] $R_H(\mathrm{K}) = 4.45 \times 10^{-8}$ V cm A^{-1}T^{-1} and $R_H(\mathrm{Al}) = 1.02 \times 10^{-8}$ V cm A^{-1}T^{-1}, we find for potassium: $A_{RL}T = 1.82$ cm K^{2}W^{-1}T^{-1} and for aluminum: $A_{RL}T = 0.42$ cm K^{2}W^{-1}T^{-1}. These results show that for potassium A_{RL} appears to be about 4.5 times larger than for Al. To obtain a quadratic field term for the Al sample, therefore, its b/a ratio should be proportionally larger than those of the K samples. Modifying Eq. (1) to take into account not only the effect of the lattice conductivity but also the effect of the partial Corbino geometry, i.e.,

$$W^m(H) = W(H, b/a) + \lambda_g A_{RL}^2 H^2 , \tag{7}$$

an estimate of the contribution of these effects to the quadratic field term is potassium can be made. It follows from Eq. (4) and (7) that the quadratic term can be written

$$a_2 H^2 = \{a(b/a)A_{RL}[W(0, b/a)/W(0,0)] + \lambda_g A_{RL}^2\} H^2 . \tag{8}$$

However, no experimental data exists for α, the slope of the reduced thermal magnetoresistivity of long wires, in which the Corbino effect is absent. Strictly speaking, these data cannot be obtained from the linear field term of the existing[2,3] thermal magnetoresistivity measurements[26] because, as can be seen from Eqs. (4) and (5), besides in the quadratic field term the Corbino effect manifests itself also in the linear field term of $W^m(H)$. Although realizing that $\Delta W/W_0$ generally have different values from $\Delta\rho/\rho_0$, for want of better data we have approximated

$$\alpha_{th} = (\Delta W/W_0)/H \text{ by } \alpha_{el} = (\Delta\rho/\rho_0)/H ,$$

the slope of the reduced electrical magnetoresistivity of long wires of potassium, as measured, e.g., by Taub *et al.*,[7] i.e., (i) $\alpha = \alpha_{th} \approx \alpha_{el}$. To evaluate Eq. (8) we have further used the approximations (ii) $W(0, b/a) \approx W(0,0)$ and (iii) $A_{RL}T = R_H/L_0 = 1.82$ cm K^{2}W^{-1}T^{-1}, the free-electron value. For samples with a residual resistance ratio (RRR) of approximately 3000, $\alpha \approx 0.07$ T^{-1} approximately,[7] while $b/a = 0.5$ typically.[3] Hence for $T = 1$ K the contribution of the partial Corbino geometry $a(\mathrm{Corb}) = \alpha(b/a)A_{RL} = 0.063$ cm K W^{-1}T^{-2}. This is in good agreement with the measured[2,3] coefficient of the total quadratic field term at $T = 1$ K, i.e., $a_2^m = 0.1 \sim 0.15$ cm K W^{-1}T^{-2}, considering all the approximations involved. This good agreement may be fortuitous, however, in view of the unknown value of α, in particular its unknown temperature and impurity dependence.

Taking for the lattice conductivity of potassium the measured[4] value $\lambda_g = 3.3 \times 10^{-4}$ T^{2}W cm^{-1}K^{-1} or the calculated[6] value $\lambda_g = 25 \times 10^{-4}$ T^{2}W cm^{-1}K^{-1} yields a much smaller lattice contribution $a(\mathrm{latt}) = \lambda_g A_{RL}^2 = 0.001 \sim 0.008$ cm K W^{-1}T^{-2}.

To summarize we conclude that a quadratic field term can be generated by a partial Corbino effect in conjunction with a linear magnetoresistance. This combination gives a quadratic contribution which is of the same order of magnitude as the measured quadratic field term. Hence the quadratic field behavior, as observed in the thermal magnetoresistivity of potassium, may be largely due to this combined partial Corbino-linear magnetoresistivity mechanism. In that case the quadratic field effect is not an intrinsic effect, but arises from the geometry of the experimental situation.

ACKNOWLEDGMENT

Part of this work has been supported by the "Stichting voor Fundamenteel Onderzoek der Materie" (FOM) with financial support of the "Nederlandse Organisatie voor Zuiver Wetenschappelijk Onderzoek" (ZWO).

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