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Free-carrier absorption in quantizing magnetic fields

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The theory of free-carrier absorption in nondegenerate semiconductors is extended to take into account quantizing magnetic fields. In the presence of a magnetic field, the behavior of the free-carrier absorption depends not only upon the magnitude of the magnetic field but also upon the polarization of the electromagnetic field relative to the magnetic field direction. For electromagnetic radiation polarized transverse to the magnetic field, the phenomenon of free-carrier absorption merges into that of cyclotron resonance, which has been extensively reported in the literature. However, when the radiation is polarized parallel to the magnetic field, the absorption depends critically on the mechanisms by which the carriers are scattered in the semiconductor. When acoustic phonon scattering is dominant, we find that the free-carrier absorption is an oscillatory function of the magnetic field which increases in magnitude with the magnetic field. The oscillations only occur when the photon frequency Ω is greater than the cyclotron frequency of the carriers. When $\Omega < \omega_c$ the free-carrier absorption is predicted to increase linearly with the magnetic field. The magnetic-field dependence of the free-carrier absorption is explained in terms of phonon-assisted transitions between the various Landau levels of the carriers.

I. INTRODUCTION

Free-carrier absorption in semiconductors is of interest both because it is the dominant mechanism for the absorption of electromagnetic radiation of frequencies lower than those which give rise to interband transitions (i.e., $\hbar\Omega < E_g$) and because the dependence of the free-carrier absorption coefficient on photon frequency and temperature depends critically upon the scattering mechanism which is dominant in a particular semiconductor.[1] As is well known, the absorption of a photon by an electron in an intraband process is forbidden in a perfect crystal. The photon gives too little momentum to the electron for the energy it provides to leave the electron in an allowed state. However, such an intraband transition involving the absorption of a photon can occur if the free carrier gains the necessary momentum by scattering off phonons or other imperfections in the crystal. A quantum theory of the free-carrier absorption in nondegenerate semiconductors was worked out by Meyer[2] and by Rosenberg and Lax.[3] In quantizing magnetic fields, different scattering mechanisms give rise to different magnetic-field dependences of the scattering rates.[4-5] This has been used to try to determine the scattering mechanisms dominant in semiconductors by studying the magnetic-field dependence of the magnetoresistance[4-7] and the cyclotron-resonance linewidth[8-10] in quantizing magnetic fields. We are interested here in extending the theory of the free-carrier absorption developed previously[2,3] to take into account the presence of quantizing magnetic fields. For electromagnetic radiation polarized transverse to the direction of the applied magnetic field, cyclotron-resonance absorption of the radiation can occur when the photon frequency Ω equals the cyclotron frequency of the carrier $\omega_c = eB/m^*c$. In the presence of strong magnetic fields, the electronic energy levels in a band are split into subbands of discrete Landau levels. Cyclotron resonance occurs when a photon has enough energy to take an electron from one Landau level to the adjacent one. Transitions between other than neighboring Landau levels in semiconductors are forbidden by selection rules in semiconductors having parabolic energy bands,[11] although such transitions can take place in acoustic cyclotron resonance[12] and in the Azbel-Kaner resonances in metals.[13] Therefore, for transverse polarized electromagnetic radiation, the theory of free-carrier absorption merges into that of cyclotron resonance. Since the latter phenomenon has been extensively treated theoretically in the literature,[8-11] we will not consider it in detail in this paper.

When the radiation field is polarized parallel to the applied magnetic field, the dependence of the free-carrier absorption on the magnetic field only appears when the quantization of the electronic energy levels in the magnetic field becomes important. This occurs when the separation between adjacent Landau levels, $\hbar\omega_c$, is greater than either the collisional broadening, $\hbar/\tau$, or the thermal broadening, k_BT, of these Landau levels. It has been found that the calculation of transport coefficients in strong magnetic fields takes a particularly simple form[6,7] in the quantum limit where $\hbar\omega_c \gg k_BT$, where all the electrons are in the lowest Landau level. In this paper, we will present a calculation of the free-carrier absorption for

longitudinally polarized radiation in a nondegenerate semiconductor with parabolic energy bands and with acoustic phonon scattering via deformation potential coupling as the dominant scattering mechanism for the carriers. The assumption of parabolic energy bands is made so as not to obscure the essential features of the electron-photon interaction with band-structure effects. We limit our treatment to the case of acoustic phonon scattering via deformation potential coupling as work has been done by Mycielski *et al.*[14,15] for the case of impurity scattering and Bass and Levinson,[16] Enck *et al.*,[17] and Zawadski *et al.*[18] for the case of optical phonon scattering. Similar structures in the free-carrier absorption should be expected for all the different scattering mechanisms.

In Sec. II, we present the theory of free-carrier absorption in the presence of a quantizing magnetic field. The results are then specialized to the quantum limit. In Sec. III, the results are presented in the form of the ratio of the absorption coefficient in the presence of the magnetic field to its value in zero magnetic field. Finally, the magnetic-field dependence of the absorption coefficient is explained in terms of the field dependence of the scattering rates and the possibility of phonon-assisted transitions between various Landau levels when $\Omega > \omega_c$.

II. THEORY OF FREE-CARRIER ABSORPTION

The free-carrier-absorption coefficient can be related to the transition probability for an electron to make an intraband transition absorbing a photon and simultaneously absorbing or emitting a phonon. This transition probability, W_i, is given by the Born, second-order golden rule approximation.

$$W_i = \frac{2\pi}{\hbar} \sum_f |\langle f|M|i\rangle|^2 \delta(E_f - E_i - \hbar\Omega - \hbar\omega_q), \quad (1)$$

where $\langle f|M|i\rangle$ are the transition matrix elements for this interaction

$$\langle f|M|i\rangle = \sum_\alpha \left(\frac{\langle f|H_{\rm rad}|\alpha\rangle\langle\alpha|V_s|i\rangle}{E_i - E_\alpha - \hbar\omega_q} + \frac{\langle\alpha|H_{\rm rad}|i\rangle\langle f|V_s|\alpha\rangle}{E_i - E_\alpha - \hbar\Omega} \right). \quad (2)$$

Here E_i and E_f are the initial and final energies of the electron, and the photon and phonon energies are $\hbar\Omega$ and $\hbar\omega_q$, respectively. In Eq. (2), $H_{\rm rad}$ is the Hamiltonian for the interaction between the electrons and radiation field, and V_s is the scattering potential due to the electron-phonon interaction. The sum is over all intermediate states, α, of the carrier. The free-carrier-absorption coefficient is given by

$$K = \frac{\epsilon^{1/2}}{nc} \sum_i W_i f_i, \quad (3)$$

where ϵ and n are the dielectric constant and index of refraction, respectively, of our semiconductor, and f_i is the free-carrier distribution function. The sum is over all the initial states of the free carrier. The above expressions have to be evaluated using the energy eigenvalues and eigenfunctions of the free carriers. For electrons in a parabolic energy band, these eigenvalues and eigenfunctions are

$$E_{k,n} = (n + \tfrac{1}{2})\hbar\omega_c + \frac{(\hbar k_z)^2}{2m^*}, \quad (4)$$

$$|k,n\rangle = (L_y L_z)^{-1/2} \exp[i(k_y y + k_z z)] \times \phi_n(x - \lambda^2 k_y), \quad (5)$$

$$\lambda = \left(\frac{\hbar c}{eB}\right)^{1/2},$$

respectively. Here ϕ_n are the harmonic-oscillator wave functions, k_y and k_z are the components of the electron wave vector in the y and z directions, and m^* is the effective mass of the free carriers.

The electron-phonon scattering potential when deformation potential coupling is the dominant interaction mechanism is[19]

$$V_s = \left(\frac{k_B T}{2\rho u^2 V}\right)^{1/2} C \exp(i\vec{q}\cdot\vec{r}). \quad (6)$$

Here q is the phonon wave vector, C is the deformation potential constant, ρ is the density of the semiconductor, T is the absolute temperature, u is the sound velocity, and V is the volume of the crystal. In Eq. (6) we have used the fact that, except at the lowest temperatures, the electrons will interact mainly with long-wavelength phonons whose energies are much less than the thermal energy, i.e., $\hbar\omega_q \ll k_B T$. The Hamiltonian which gives the interaction between the radiation field and the electrons is

$$H_{\rm rad} = -\frac{e}{m^*} \left(\frac{2\pi n\hbar}{\Omega\epsilon V}\right)^{1/2} \hat{\epsilon}\cdot\vec{p}, \quad (7)$$

where $\hat{\epsilon}$ is the polarization vector of the radiation field and $\vec{P}$ is the kinetic momentum operator. Using the Landau gauge for the vector potential, i.e., $\vec{A} = (0, Bx, 0)$, we find the following for the matrix elements of the scattering potential and the electron-radiation field-interaction Hamiltonian between different electronic states in the same band in the presence of the magnetic field.

$$\langle k',n'|V_s|k,n\rangle = \left(\frac{k_B T}{2\rho u^2 V}\right)^{1/2} C J_{n',n}(q_x, q_y) \times \delta_{k_y', k_y + q_y} \delta_{k_z', k_z + q_z}, \quad (8)$$

$$\langle k',n'|H_{\rm rad}|k,n\rangle = -\frac{e}{m^*}\left(\frac{2\pi n\hbar}{\epsilon\Omega V}\right)^{1/2}\hbar k_z \times \delta_{n',n}\delta_{k_z',k_z}\delta_{k_y',k_y}. \quad (9)$$

We have only taken the matrix elements of the electron-photon interaction Hamiltonian between electronic states for the case where the radiation field is polarized parallel to a dc magnetic field. The case of a transverse polarized radiation field has been treated in papers concerned with cyclotron resonance[8-11] and is somewhat more complicated[15] to treat when the photon frequency is greater than the cyclotron frequency. The function $J_{n',n}(q_x,q_y)$ which appears in Eqs. (8) and (9) is just the overlap integral of the harmonic-oscillator wave functions

$$J_{n',n}(q_x,q_y) = \int_{-\infty}^{\infty} dx \exp(iq_x x)\phi_{n'}(x-\lambda^2 k_y-\lambda^2 q_y) \times \phi_n(x-\lambda^2 k_y). \quad (10)$$

Using the expressions in (3) and (9) in the relation (1) for the transition probability, in which we have used the energy-conserving delta function to do the integrals over q_z (Ref. 20) and the normalization integral for the harmonic-oscillator wave functions[21]

$$\int_0^{\infty} dq_\perp q_\perp |J_{n',n}(q_x,q_y)|^2 = \frac{1}{\lambda^2} \quad (11)$$

to do the integral over $q_\perp$, we have

$$W_i = \left(\frac{2ne^2C^2k_BT}{\epsilon^{1/2}\rho u^2\hbar^3 V}\right)\frac{\omega_c}{\Omega^3}\sum_{n_f}\frac{k_{z_i}^2 - \frac{m^*}{\hbar}[(n_f-n_i)\omega_c-\Omega]}{\left\{k_{z_i}^2-\frac{2m^*}{\hbar}[(n_f-n_i)\omega_c-\Omega]\right\}^{1/2}}. \quad (12)$$

Here the sum over n_f goes over only those values for which the term under square-root sign is positive for all k_{z_i}, i.e., $n_f \leq n_i + \Omega/\omega_c$. With this restriction, we find a maximum value of n_f, such that $n_f^{\max}$ is the largest integer satisfying the relation

$$n_f^{\max} \leq n_i + \Omega/\omega_c. \quad (13)$$

The absorption coefficient is, from Eqs. (3) and (12),

$$K(B) = \frac{2n_0}{\rho\mu^2}\left(\frac{eC}{\hbar}\right)^2\left(\frac{2k_BT}{\pi m^*\epsilon}\right)^{1/2}\left(\frac{\omega_c}{\Omega^3}\right)\sinh\left(\frac{\hbar\omega_c}{2k_BT}\right) \times \sum_{n_i=0}^{\infty}\exp\left[-(n_i+\tfrac{1}{2})\frac{\hbar\omega_c}{k_BT}\right] \times \sum_{n_f=0}^{n_f^{\max}}\int_{-\infty}^{\infty} dk_{z_i}\frac{\left\{k_{z_i}^2-\frac{m^*}{\hbar}[(n_f-n_i)\omega_c-\Omega]\right\}}{\left\{k_{z_i}^2-\frac{2m^*}{\hbar}[(n_f-n_i)\omega_c-\Omega]\right\}^{1/2}}\exp\left(-\frac{\hbar^2k_{z_i}^2}{2m^*k_BT}\right), \quad (14)$$

where we have used the electron-distribution function in the presence of the magnetic field.

$$f_{k,n} = 2n_0\left(\frac{2\pi\hbar^2}{m^*k_BT}\right)^{1/2}\sinh\left(\frac{\hbar\omega_c}{2k_BT}\right)\exp-\left((n+\tfrac{1}{2})\frac{\hbar\omega_c}{k_BT}+\frac{(\hbar k_z)^2}{2m^*k_BT}\right) \quad (15)$$

and n_0 is the density of free carriers. The integrals over k_{z_i} can be evaluated using an integral relation for the modified Bessel functions of the first kind,[22]

$$K_0(z) = \int_0^{\infty} d\theta \exp(-z\cosh\theta) \quad (16a)$$

and

$$K_1(z) = -\frac{dK_0(z)}{dz} = \int_0^{\infty} d\theta \cosh\theta \exp(-z\cosh\theta) \quad (16b)$$

to obtain our final expression for the absorption coefficient for the longitudinal configuration (i.e., radiation field polarized parallel to the external magnetic field):

$$K(B)=\frac{2n_0e^2C^2}{\rho u^2\hbar^3 c}\left(\frac{2m^*k_BT}{\pi\epsilon}\right)^{1/2}\frac{\omega_c}{\Omega^3}\sum_{n_i=0}^{\infty}\exp\left(-(n_i+\tfrac{1}{2})\frac{\hbar\omega_c}{k_BT}\right)$$
$$\times\sum_{n_f=0}^{n_f^{\max}}[\Omega-(n_f-n_i)\omega_c]\exp\left(\frac{\hbar}{2k_BT}[\Omega-(n_f-n_i)\omega_c]\right)$$
$$\times\sinh\left(\frac{\hbar\omega_c}{2k_BT}\right)K_1\left(\frac{\hbar}{2k_BT}[\Omega-(n_f-n_i)\omega_c]\right). \quad (17)$$

In the quantum limit, $\hbar\omega_c \gg k_BT$, all the terms in the sum over n_i except for $n_i=0$, are negligible, and the free-carrier-absorption coefficient takes the particularly simple form

$$K(B)=\frac{n_0e^2C^2}{\rho u^2\hbar^3 c}\left(\frac{2m^*k_BT}{\pi\epsilon}\right)^{1/2}\left(\frac{\omega_c}{\Omega^2}\right)\sum_{n_f=0}^{n_f^{\max}}\left(1-\frac{n_f\omega_c}{\Omega}\right)\exp\left[\frac{\hbar\Omega}{2k_BT}\left(1-\frac{n_f\omega_c}{\Omega}\right)\right]K_1\left[\frac{\hbar\Omega}{2k_BT}\left(1-\frac{n_f\omega_c}{\Omega}\right)\right], \quad (18)$$

where $n_f^{\max}\leq\Omega/\omega_c$. Using the zero-field expression for the free-carrier-absorption coefficient[2]

$$K(0)=\frac{2}{3}\left(\frac{n_0e^2C^2}{\rho u^2\hbar^3 c\Omega}\right)\left(\frac{2m^*k_BT}{\pi\epsilon}\right)^{1/2}$$
$$\times\exp\left(\frac{\hbar\Omega}{2k_BT}\right)K_2\left(\frac{\hbar\Omega}{2k_BT}\right), \quad (19)$$

we can express our results in terms of the ratio of the absorption coefficient in the presence of the magnetic field to its value in zero magnetic field:

$$\frac{K(B)}{K(0)}=\frac{3}{2}\left(\frac{\omega_c}{\Omega}\right)\sum_{n_f=0}^{n_f^{\max}}\left(1-\frac{n_f\omega_c}{\Omega}\right)\exp\left(-\frac{n_f\hbar\omega_c}{2k_BT}\right)$$
$$\times\frac{K_1\left[\frac{\hbar\Omega}{2k_BT}\left(1-\frac{n_f\omega_c}{\Omega}\right)\right]}{K_2\left(\frac{\hbar\Omega}{2k_BT}\right)}. \quad (20)$$

In this form, the ratio depends only upon the magnetic field, absolute temperature, and photon frequency and does not depend upon such material parameters as the values of the deformation potential, sound velocity, or density of the material, although, of course, the absolute value of the absorption coefficient does depend upon the numerical values of these parameters. This ratio takes a particularly simple form in high fields where the cyclotron frequency exceeds the photon frequency, and the photon energy is greater than the thermal energy of the carriers.

$$\frac{K(B)}{K(0)}=\frac{3}{2}\frac{\omega_c}{\Omega}. \quad (21)$$

In this limit, the free-carrier absorption is a linearly increasing function of the magnetic field. This indicates that, in such high fields, the free-carrier absorption for the longitudinal configuration can be considerably enhanced by the presence of quantizing magnetic fields.

III. RESULTS

The ratio of the free-carrier-absorption coefficient in the presence of a *dc* magnetic field to its zero-field value is shown in Fig. 1 for a radiation field polarized parallel to the direction of the magnetic field. The parameters which have been used in the calculations leading to Fig. 1 are the carrier effective mass, which has been taken to be that characteristic of *n*-InSb, $m^*=0.013\ m_0$, a photon frequency corresponding to that of the 10.6 μm radiation characteristic of a CO_2 laser and liquid-nitrogen temperature ($T=77$ K). We have used the parameters characteristic of InSb in this calculation, even though the nonparabolicity of the energy bands, which can be important in high magnetic fields, has been neglected here. Since Eq. (20), from which Fig. 1 is plotted, is a universal function of Ω, ω_c, and T, these results can be used for other nondegenerate semiconduc-

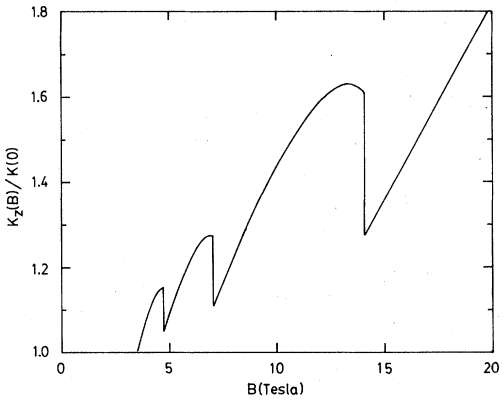

FIG. 1. The ratio of the absorption coefficient in the presence of the magnetic field to the zero-field coefficient is shown as a function of the magnetic field for the longitudinal configuration. The parameters used are those of InSb at 77 K in a radiation field of 10.6 μm wavelength.

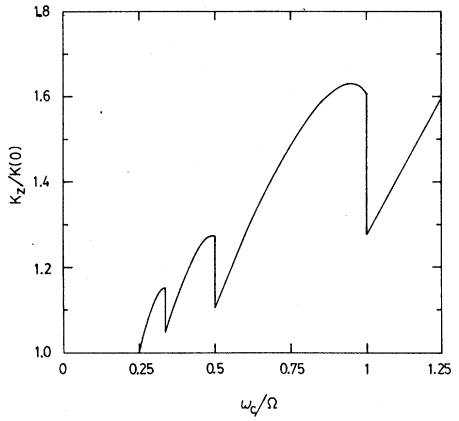

FIG. 2. The absorption in a magnetic field normalized to its zero-field value is shown as a function of ω_c/Ω for a fixed value of $\hbar\Omega/k_BT = 17.6$.

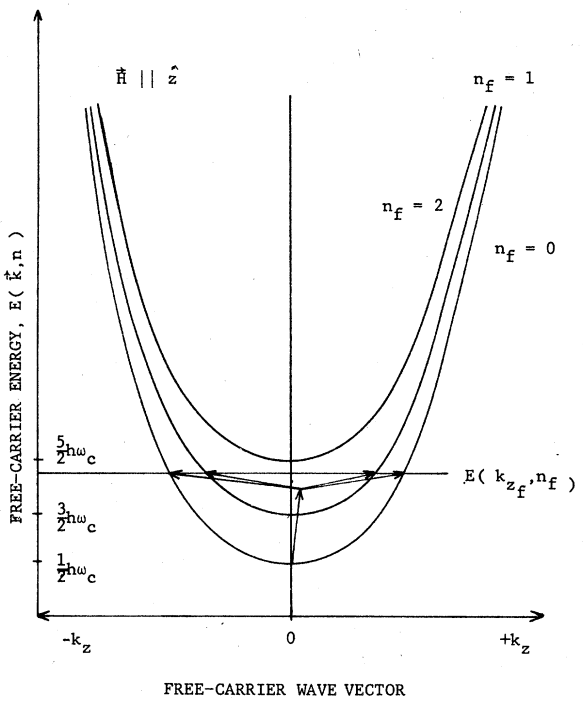

FIG. 3. The energy subband structure for electrons in quantizing magnetic fields showing the various transitions leading to the free-carrier absorption.

tors and for other photon frequencies and temperatures by an appropriate scaling of the magnetic field. Actually, Eq. (20) can be written as a function of two dimensionless parameters, $\hbar\Omega/k_BT$, the ratio of the photon energy to the thermal energy and ω_c/Ω, the ratio of the cyclotron frequency to the photon frequency. In Fig. 2, the free-carrier absorption for the longitudinal configuration is shown as a function of ω_c/Ω for a fixed value of the ratio $\hbar\Omega/k_BT$. From Figs. 1 and 2, we see that the absorption coefficient is an oscillatory function of the magnetic field which increases with increasing magnetic field. We note that there are inflection points in the curve which occur when the ratio Ω/ω_c has an integer value. Approximately midway between these inflection points, there are local maxima. When the cyclotron frequency exceeds the photon frequency, the absorption coefficient increases monotonically as a linear function of the magnetic field and there are no further oscillations in this coefficient.

The oscillatory dependence of the absorption on magnetic field can be understood in terms of the Landau subband structure of the electronic energy levels in quantizing magnetic fields. When $\Omega > \omega_c$, the absorption of a photon can occur with the simultaneous emission or absorption of phonons in an electronic transition to the same or other Landau subbands. As the magnetic field, and therefore ω_c, increases, there are fewer and fewer subbands to which the transition can place until finally, when $\omega_c > \Omega$, the transition can only take place to a final state in the same subband. Every time that the ratio Ω/ω_c equals an integer value, the transition can take place with an additional subband ending as a final state. This is illustrated in Fig. 3.

For magnetic fields such that $\omega_c > \Omega$, the absorption process just depends upon the rate at which the free carriers are scattered by the emission or absorption of phonons. The linear increase in the absorption coefficient with increasing magnetic fields in this limit reflects just the linear increase in the electron-phonon scattering rate with field predicted for the longitudinal magnetoresistance in the quantum limit.[6]

In conclusion, we predict that when acoustic-phonon scattering is dominant, the free-carrier absorption should increase with magnetic field with an oscillatory dependence on the field when $\Omega > \omega_c$ for the longitudinal configuration. Although the results for the transverse configuration have not been analyzed in detail on the low-field side of the cyclotron resonance, there is reason[15] to expect that similar structures will show up in this configuration as well. In addition, although the linear increase of the absorption with field is a result of considering only acoustic-phonon scattering, the subband structure which gives rise to the oscillatory dependence of the absorption on field should also give rise to similar dependences when other scattering mechanisms are dominant.

We should also mention that similar structure to that predicted here for the free-carrier-absorp-

tion coefficient has been observed in the photoconductivity of InSb in strong magnetic fields in the presence of 10 μm laser radiation.[23] Although the photoconductivity does not directly reflect the behavior of the absorption, since a transport process is also involved, the magnetic-field dependence of the photoconductivity is suggestive of the results presented here, showing not only an oscillatory behavior with field, but also showing an increase in the photoconductivity with increasing magnetic field.

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