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Anomalous Hall effect in nickel and nickel-rich nickel-copper alloys

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Hall-effect measurements on nickel and its alloys containing 9.38, 18.77, 28.32, and 38.13 at.% of copper were performed from 77° K to their ferromagnetic Curie temperatures in fields up to 15 kOe. An appraisal of the theories proposed to understand the anomalous Hall effect (AHE) in ferromagnetic compounds in the light of the present experimental data showed that (i) one (or more) of the scattering processes (impurity, phonon, and spin-disorder scattering) dominantly contributes to the AHE in different temperature ranges which, in turn, are different for different samples, (ii) spin-disorder scattering surpasses the contribution to the AHE arising from all other scattering processes for temperatures in the vicinity of the ferromagnetic Curie temperature for all the specimens, and (iii) *d*-spin–*s*-orbit interaction plays a vital role so far as the AHE in pure nickel is concerned, whereas the intrinsic spin-orbit interaction is important for understanding the AHE behavior in nickel-copper alloys.

I. INTRODUCTION

It is now well known both from theoretical as well as experimental standpoints that the Hall resistivity ρ_H in ferromagnetic compounds as a function of applied magnetic induction B,[1] can be represented by the general expression

$$\rho_H(B) = R_0 B + 4\pi M_s R_s \ , \tag{1}$$

in which the first term, characterized by the ordinary Hall coefficient R_0, is the contribution from the ordinary Hall effect with a behavior expected for a nonmagnetic metal of the same band structure from a simple consideration of the Lorentz force. The second term, characterized by the extraordinary or anomalous Hall coefficient R_s, represents a strongly temperature-dependent contribution (peculiar to ferromagnetic compounds) connected with the spontaneous magnetization of these materials. Equation (1) easily permits the separation of the ordinary and anomalous contributions to the Hall resistivity once it is recognized that the high-field saturated portion of the ρ_H vs B curve has a slope R_0 and an extrapolation of the saturated portion to $B = 0$ yields the quantity $4\pi M_s R_s$ ($= \rho_{Hs}$, anomalous Hall resistivity), from which R_s can be calculated provided the value of $4\pi M_s$ is known. Considerable interest in the anomalous Hall effect (AHE), in particular, stems from the fact that it is intimately connected with the electron scattering processes[2–19] in ferromagnetic metals and alloys.

The potential of AHE to provide a great amount of information concerning various scattering mechanisms prevalent in ferromagnetic materials, though acknowledged for the last two decades, has not been fully exploited. The above statement is true even for the nickel-copper alloy system, which is supposed to be one of the most thoroughly studied systems amongst the 3*d*-transition-metal alloys. The experimental data on the Hall effect in nickel and nickel-copper alloys already existing in the literature[20–26] cannot be considered exhaustive and the conclusions drawn from there are not complete for three reasons. First, the early measurements neither cover the complete temperature range nor the solute-concentration range over which the ferromagnetism exists for the alloy system in question. Moreover, it was too premature to draw any fruitful conclusions from these measurements because the physical mechanism underlying the AHE phenomenon just began to reveal itself in 1954 when the Karplus and Luttinger theory was published.[2] Second, most of the recent experiments were carried out in the paramagnetic region where the distinction between contributions to the AHE arising from spin-disorder scattering and phonon scattering can be made with ease. The contradictory conclusions regarding the dominant scattering mechanism in the ferromagentic region[18,19] therefore remain unresolved. Third, no systematic attempt has been made to evaluate the influence of impurities (solute) on the AHE of a pure host.

With the above-mentioned observations in mind, Hall-effect measurements on nickel and nickel-copper alloys were carried out from 77° K to their ferromagnetic Curie temperatures in fields up to 15 kOe. In the present work, the contributions arising from various scattering processes in various temperature ranges have been identified without ambiguity and the conclusions concerning the dominant scattering mechanism for temperatures in the vicinity of the Curie temperature are found to be in perfect agreement with those arrived at by various workers[23,24,26] in the paramagnetic region. The present work, in addition clearly brings out the importance of various types of spin-orbit interaction in connection with the AHE.

II. THEORETICAL BACKGROUND

Numerous theoretical attempts[2–18] have been made to understand the anomalous-Hall-effect phenomenon in ferromagnetic metals and alloys. These theories, however, fall into two main categories depending upon whether they treat the magnetic electrons as itinerant or localized.

A. Theories with itinerant magnetic carriers

Karplus and Luttinger[2] were the first to propose a model for the anomalous Hall effect in a ferromagnet consisting of magnetic free-charge carriers (responsible both for the electric and magnetic properties of the material and distributed unequally between spin-up and spin-down states to account for the spontaneous magnetization) which under the influence of the external electric field move through the periodic potential of the nonmagnetic ions and experience an intrinsic spin-orbit coupling between their spin and their orbital angular momentum. Since the spin-up and spin-down populations are unequal, this spin-orbit coupling gives rise to a transverse current which is of the correct order of magnitude and symmetry to account for the anomalous effect. It follows from this theory that the anomalous Hall coefficient R_s should vary as ρ^2.

The basic concept introduced by Karplus and Luttinger for the AHE triggered an immense theoretical activity in this field. Such an intense theoretical effort, in turn, led the Karplus-Luttinger theory to undergo many refinements at the hands of various workers[3–6,13,14] from time to time. The physical mechanism underlying the AHE phenomenon, which has now received general recognition, can be summarized as follows. The anomalous-Hall-effect is a result of two contributions, one arising from the skew scattering[4,27] which can be explained on the basis of the Boltzmann equation and is important for pure metals and dilute alloys at low temperatures,[28,29] and the other arising from the nonclassical anomalous velocity[3,6,9,13,30–32] or "side-jump"[14,33] which finds no interpretation in terms of the classical Boltzmann equation as it requires new terms in the Boltzmann equation which are expected to become important[3,34] as soon as the dimensionless quantity $\hbar/\epsilon_F\tau$ is no longer very small (ϵ_F is the Fermi energy and τ the relaxation time), namely, for high temperatures or concentrated alloys or both (small τ). Both these contributions put together lead to the following relation between the anomalous Hall coefficient and the total electrical resistivity:

$$R_s = a\rho + b\rho^2 \ , \tag{2}$$

where the coefficients a and b are independent of temperature but depend on the chemical composition of the magnetic substance.

B. Theories with localized magnetic carriers

The fact that the theories with itinerant magnetic electrons cannot, in particular, be applicable to the rare-earth metals (which like other ferromagnetic metals exhibit a large AHE) for which it is clear beyond doubt that the majority consist of trivalent ions with highly localized magnetic $4f$ electrons, led Kondo[7] to propose a model where the magnetic electrons (either d or f) are localized at the ions, as contrasted with the conduction electrons (s) which are free and equally distributed between states of opposite spin, in order to account for the magnetic properties of the material. The s electrons get scattered by the nonperiodic spin potential, generated by the temperature-induced disorder in the localized spin system (d or f spins), through the direct spin-spin interaction (better known as s-d or s-f exchange interaction). Based on the above model, Kondo showed that in the case of a degenerate orbital ground state of d electrons, the anomalous Hall effect is a consequence of the combined effect of anisotropic s-d (or s-f) interaction and the intrinsic spin-orbit interaction of d electrons within the magnetic ions. He arrived at an expression for the anomalous Hall resistivity ρ_{Hs} which is given by

$$\rho_{Hs} = \text{const.}\ \langle (M - \langle M \rangle)^3 \rangle \ , \tag{3}$$

where $\langle (M - \langle M \rangle)^3 \rangle$ is a three-spin correlation function which primarily contains all the temperature dependence of the effect. In view of the fact that the anisotropy of s-d interaction and the intrinsic spin-orbit interaction within an ion can exist only when the orbital angular momentum of the d electrons within the ion is not quenched, Kondo's theory predicts that the anomalous effect should not be observed for metals in which the orbital angular momentum is quenched. It is, however, well known that the orbital angular momentum in Gd is completely quenched and yet it shows a particularly large AHE. This limitation of Kondo's theory led subsequent workers[8,10,11] to invoke another type of spin-orbit interaction, now known as d-spin–s-orbit interaction, appropriate to the AHE.

Irkhin and Abel'skii[8] and later Kagan and Maksimov[10] evaluated the three-spin correlation functions in the molecular-field approximation and found that for a spin $S=\frac{1}{2}$, the d-spin–s-orbit interaction and/or the intrinsic spin-orbit interaction together with spin-disorder scattering lead to the following relations[35] for the anomalous Hall effect:

$$\rho_{Hs} = \text{const. } M_s(T)[M_s^2(0) - M_s^2(T)] \ ,$$

or (4)

$$R_s = \text{const. } [M_s^2(0) - M_s^2(T)] \ ,$$

where $M_s(T)$ is the spontaneous magnetization at temperature T. Maranzana[11] following essentially the calculations of Kondo[7] and using the d-spin–s-orbit interaction in place of the intrinsic spin-orbit interaction, found the anomalous Hall resistivity to be again represented by Eq. (3). In the molecular-field approximation, when the reduced magnetization σ (which is negative, ranging from 0 to -1) is given by $\sigma = B_s(y)$, (Brillouin function), where

$$y = [3S/(S+1)](T_f/T)\sigma + g\mu SH/k_BT \ ,$$

the three-spin correlation function in Eq. (3) equals $S^3B_s''(y)$ which when expressed in terms of spontaneous and induced magnetizations becomes

$$S^3\left[\frac{B_s''(y_0)}{B_s(y_0)}\sigma_0 + \frac{B_s'''(y_0)}{B_s'(y_0)}\delta\sigma\right] \ , \tag{5}$$

where

$$y_0 = \left[\frac{3S}{(S+1)}\right]\frac{T_f}{T}\sigma_0$$

and $\sigma = \sigma_0 + \delta\sigma$. For $S = \frac{1}{2}$ and $H = 0$, so that the computed values correspond to the anomalous Hall resistivity, Eq. (5) shows that ρ_{Hs} as a function of temperature passes through a local maximum at 0.88 T_f and thereafter drops to zero at $T = T_f$, the ferromagnetic Curie temperature.

To summarize, it is generally agreed that, in addition to the scattering mechanism appropriate to a particular temperature range (impurity scattering at absolute zero; spin-disorder and phonon scattering for temperatures above absolute zero), some spin-orbit interaction is required to account for the AHE.

III. EXPERIMENTAL DETAILS

Samples of dimensions $25 \times 5 \times 0.25$ mm^3 were cut from spec-pure (99.999%) sheets supplied by Johnson Matthey and annealed in a vacuum of 10^{-5} Torr for 24 hours at 950° C. Since the sample dimensions were measured to an accuracy of 0.001 cm, uncertainities in the geometrical factors are believed to be less than 0.2% and those in electrical resistivity less than 0.5%.

The Hall-effect measurements on pure nickel and nickel-copper alloys containing 9.38, 18.77, 28.32, and 38.13 at.% copper were performed from 77° K to their ferromagnetic Curie temperatures in fields up to 15 kOe by the standard dc technique employing a combination of a K-3 universal potentiometer (Leeds and Northrup Co.) and a 148 dc ninovoltmeter (Keithley Instruments), which provided a merit figure of ±2.0 nV. The details of the measurement procedure are given elsewhere.[36,37] The well-known four-probe method was used to measure the electrical resistivity of the Ni–38.13-at.%-Cu alloy from 77 to 700° K. Electrical-resistivity results obtained previously[38] by the same method on the remaining samples have also been used in the present investigation.

The ultimate results reported in this paper are the averages of results obtained in two or three different experimental runs on the same sample. A procedure of this type brings out the experimental scatter, if any.

It was found that the scatter in Hall-voltage data never exceeds 1%.

IV. RESULTS AND DISCUSSION

The Hall-effect measurements on nickel and nickel-copper alloys containing 9.38, 18.77, 28.32, and 38.13 at.% copper were performed from 77° K to their ferromagnetic Curie temperatures in fields upto 15 kOe. The procedure outlined in the introduction was adopted to derive the values of the anomalous Hall resistivity ρ_{Hs} at different temperatures from the field-dependence curves for ρ_H taken at various temperatures. It was found that (i) for all the samples, ρ_{Hs} as a function of temperature passes through a peak before dropping to zero at the ferromagnetic Curie temperature T_f and (ii) both the peak height and the peak position change with increasing copper concentration in nickel. In the text that follows the above findings are discussed in the light of theories developed in Sec. II. Section II A demonstrates that $R_s \propto \rho^2$ for pure ferromagnetic metals at high temperatures and for concentrated alloys at moderate temperatures. Without distinguishing between various theories under this category, we henceforth attribute this prediction to the theory due to Karplus and Luttinger[2] since they arrived at it for the first time. Similarly, the theories with localized magnetic carriers (Sec. II B) boil down basically to only two theories, viz., Maranzana's theory [Eqs. (3) and (5) of Sec. II B] and Kagan and Maksimov's theory [Eq. (4)]. In order to bring out the temperature and solute concentration dependence of ρ_{Hs} clearly, we rewrite the predictions based on the Karplus and Luttinger and Kagan and Makisimov theories in the form

$$\rho_{Hs} = AM_s(c,0)\,\sigma(c,T)\,\rho^2(c,T) \ , \tag{6}$$

and

$$\rho_{Hs} = BM_s^3(c,0)\,\sigma(c,T)\,[1-\sigma^2(c,T)] \ , \tag{7}$$

respectively, where $\sigma(c,T) = M_s(c,T)/M_s(c,0)$ A and B are constants, and c is the solute concentration.

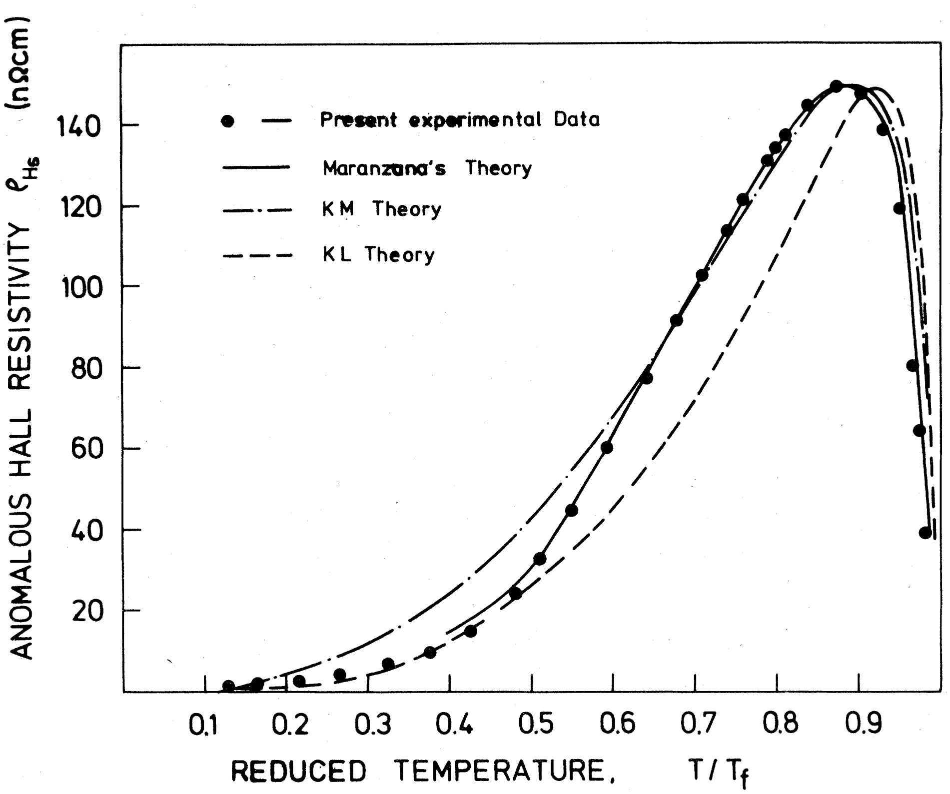

FIG. 1. Experimental (solid circles) and theoretical variation of the anomalous Hall resistivity with reduced temperature; Maranzana's theory, solid curve; Kagan-Maksimov (KM) theory, dash-dot curve; and Karplus-Luttinger (KL) theory, dashed curve.

The theoretical variations, which result when the peak values predicted by Eqs. (5)–(7) are normalized[39] to the experimental peak value, are shown in Fig. 1 together with the experimentally observed temperature dependence of ρ_{Hs} for pure nickel. Figure 1 reveals that Maranzana's theory provides an excellent fit to the experimental data above $0.6T_f$ whereas Kagan and Maksimov's theory is in much closer agreement with the observed variation above $0.65T_f$ than Karplus and Luttinger's theory. Two points that at once emerge from this exercise are: (i) The anomalous Hall effect arises mainly from spin-disorder scattering above $0.65T_f$ for pure nickel, and (ii) *d*-spin–*s*-orbit interaction plays a vital role so far as the anomalous Hall effect in pure nickel is concerned.

Next, we focus our attention on the observed change in peak height and peak position when copper

TABLE I. Experimental and theoretical values of the peak height ρ^*_{Hs} and peak position γ^* for the anomalous Hall resistivity of nickel and nickel-copper alloys.

Copper concentration (at. %)	ρ^*_{Hs} (nΩ cm) (Experiment)	ρ^*_{Hs} (Theory) KM	ρ^*_{Hs} (Theory) KL	γ^* (Experiment)	γ^* (Theory) KM	γ^* (Theory) KL
0.00	148.39	148.39	148.39	0.881	0.900	0.920
9.38	170.00	82.09	166.65	0.827	0.8675	0.866
18.77	180.00	38.78	194.42	0.773	0.835	0.812
28.32	118.38	14.03	178.23	0.718	0.8025	0.757
38.13	55.20	2.92	130.94	0.640	0.766	0.675

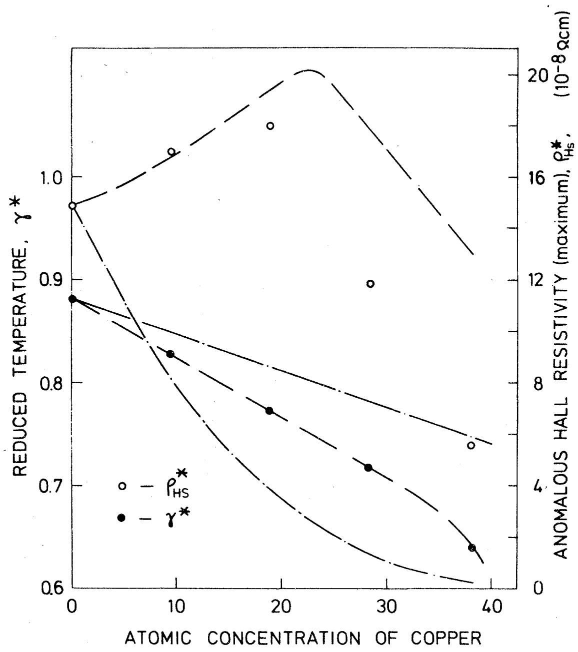

FIG. 2. Observed peak value of the anomalous Hall resistivity ρ^*_{Hs} (open circles) and the peak position in terms of reduced temperature γ^* (solid circles) as a function of copper concentration; peak-height and peak-position variation with solute concentration due to Kagan-Maksimov (KM) theory, dash-dot curve, and Karplus-Luttinger (KL) theory, dashed curve.

is added to nickel. This observation is in direct contradiction with the prediction of Maranzana's theory that both peak height and peak position should remain the same for all ferromagnetic materials for which $S=\frac{1}{2}$. Failure of Maranzana's theory to explain the behavior observed in alloy samples possibly points to the fact that no attempt has been made in this theory to account for the spin inhomogeneities caused by solute spins in an alloy, and consequently limits the discussion of experimental results to the theories proposed by Karplus and Luttinger (KL) and Kagan and Maksinov (KM). The values for the peak height ρ^*_{Hs} and peak position in terms of the reduced temperature γ^* for all the samples, computed from Eqs. (6) and (7) using the same values for the constants A and B as obtained by the normalization procedure mentioned earlier for pure nickel, are given in Table I. In order to arrive at a normalized variation of peak position γ^* with copper concentration, like the peak height, the experimental and theoretical values of the peak position have been matched for nickel by subtracting 0.019 and 0.039 from the values predicted by KM and KL theory, respectively, and the same values subtracted from the corresponding values given in Table I for alloys with different copper concentrations. Figure 2 shows the experimental data on the change in peak height and peak position with copper concentration along with the normalized variations of peak height and peak posi-

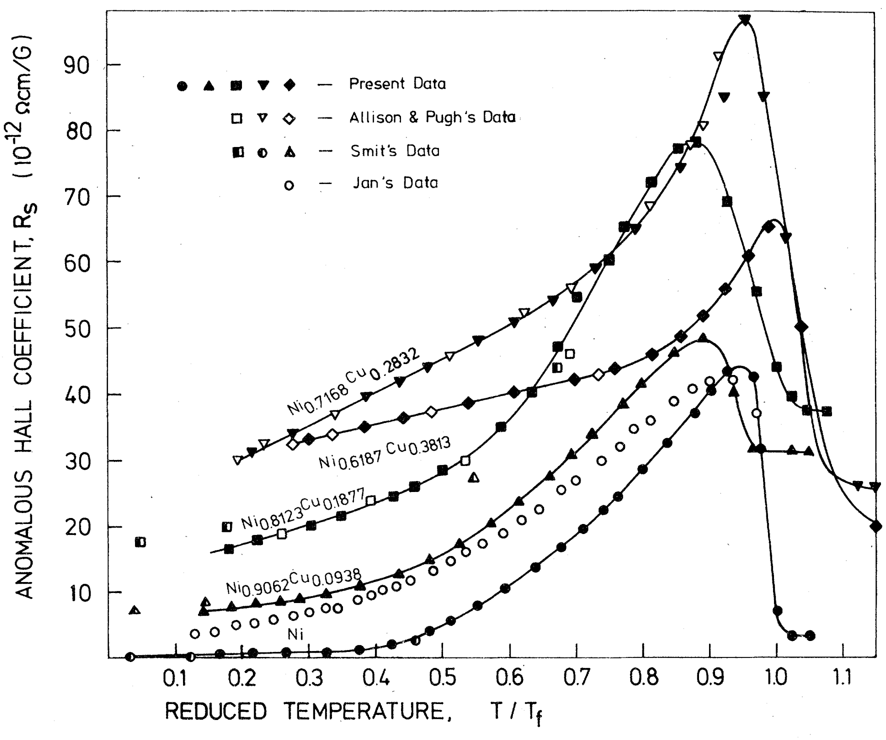

FIG. 3. Anomalous Hall coefficient as a function of reduced temperature for nickel and nickel-copper alloys.

TABLE II. Values of the ferromagnetic Curie temperature T_f, saturation magnetization at 0° K, $M_s(0)$, electrical resistivity ρ at 80 and 300° K, anomalous Hall coefficient R_s at 80° K, peak in R_s versus reduced temperature curve R_s^*, and the reduced temperature corresponding to this peak, γ^*, for nickel and nickel-copper alloys.

Copper (at. %)	T_f (° K)	$M_s(0)$ (Gauss)	ρ ($\mu\Omega$ cm) 80° K	300° K	$-R_s$ (10^{-12} Ω cm/G) 80° K	$-R_s^*$ (10^{-12} Ω cm/G)	γ^*
0.00	631	510.0	0.75	7.55	0.22	44.50	0.950
9.38	531	418.7	8.40	18.00	6.60	48.25	0.900
18.77	431	325.7	16.82	30.40	16.50	77.50	0.880
28.32	330	232.5	26.02	43.00	32.40	96.25	0.955
38.13	230	138.0	35.55	47.00	34.25	66.00	1.000

tion predicted by KM and KL theories. It is seen from Fig. 2 that the theory due to Karplus and Luttinger gives an excellent qualitative description of the experimentally observed variation in peak height and peak position with solute concentration. This observation suggests that intrinsic spin-orbit interaction is important particularly for understanding the influence of impurities (solute) on the anomalous Hall effect of a pure host.

With the intention of obtaining further information regarding the dominant scattering processes in the samples of the present investigation, the data on the temperature dependence of R_s are discussed in terms of theories leading to Eqs. (2) and (4) of Sec. II. The values of the spontaneous magnetization at different temperatures, $M_s(T)$, needed to evaluate the temperature dependence of R_s from that of ρ_{Hs}, were calculated using the spontaneous-magnetization values at 0° K, $M_s(0)$, (derived from the atomic-magnetic-moment versus composition curve for nickel-copper alloys reported by Ahern *et al.*[40] and the density values for these alloys given by Bozorth[41]) and the universal curves for $M_s(T)/M_s(0)$ as a function of T/T_f were deduced from the work of Ahern *et al.*[40] In order to arrive at reliable values of $M_s(T)$ below and above the Curie temperature for the alloys containing 28.32 and 38.13 at.% copper, for which magnetic order is known to exist for temperatures even above their ferromagnetic Curie temperature, the reduced magnetization curves appropriate to the anomalous Hall effect given by Allison and Pugh[22] were normalized to the curves deduced from the data of Ahern *et al.*[40] at $0.96T_f$, where the last experimental point exists for the reported results of Ahern *et al.* The temperature dependence of R_s, computed employing values of $M_s(T)$ at different temperatures, is shown for all the samples of the present investigation in Fig. 3 which also includes data of Jan,[20] Smit,[21] and Allison and Pugh[22] on similar samples, for comparison. The values of R_s in the paramagnetic region have been taken from the work of Dutta Roy and Subrahmanyam[24] on the same alloy system. Figure 3 shows that both the peak height and the peak position change from sample to sample.[42] Table II gives, besides the observed values for the peak height and peak position, the values for $M_s(0)$, the ferromagnetic Curie temperature T_f, the total electrical resistivity at 80 and 300° K, and R_s at 80° K for all the samples of present interest.

For analyzing the data on the basis of Eq. (2), a plot of R_s/ρ vs ρ has been made and such plots for all the present samples are shown in Fig. 4. A close scrutiny of these plots reveals two important features, namely, (i) two straight-line fits, widely differing in slope, represent all the observed data except for

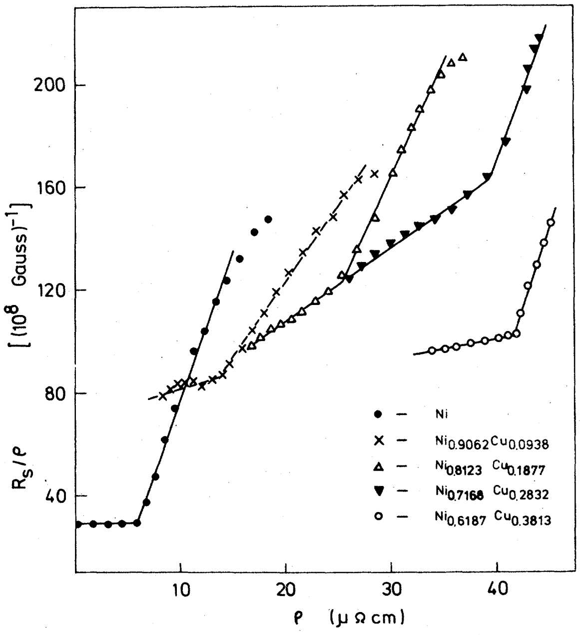

FIG. 4. R_s/ρ vs ρ plot for nickel and nickel-copper alloys.

TABLE III. Values of the coefficients a and b and the powers m and n in the expressions $R_s = a\rho + b\rho^2$, $R_s = \text{const.}\rho^m$, and $R_s = \text{const.}[M_s^2(0) - M_s^2(T)]^n$ for nickel and nickel-copper alloys in the specified temperature ranges.

Specimens	Temperature range (°K)	a $[(10^7\ \text{G})^{-1}]$	b $[(10^2\ \Omega\ \text{cm G})^{-1}]$	m	n
Pure Ni	77 ··· 260 $(0.21T_f)$ $(0.41T_f)$	2.96	0.00	1.00	···
	260 ··· 420 $(0.41T_f)$ $(0.66T_f)$	−3.90	11.60	2.05	
	420 ··· 600 $(0.66T_f)$ $(0.90T_f)$	···	···	1.40	1.00
Ni−9.38-at.% Cu	77 ··· 220 $(0.15T_f)$ $(0.41T_f)$	7.00	1.20	1.13	···
	220 ··· 425 $(0.41T_f)$ $(0.80T_f)$	0.45	5.90	2.03	···
	425 ··· 480 $(0.80T_f)$ $(0.90T_f)$	···	···	1.35	0.98
Ni−18.77-at.% Cu	77 ··· 240 $(0.18T_f)$ $(0.55T_f)$	5.30	2.70	1.64	···
	240 ··· 360 $(0.55T_f)$ $(0.835T_f)$	−10.10	8.80	2.50	1.00
	360 ··· 380 $(0.835T_f)$ $(0.88T_f)$	···	···	1.50	1.00
Ni−28.32-at.% Cu	64 ··· 265 $(0.19T_f)$ $(0.80T_f)$	5.25	2.80	1.64	···
	265 ··· 315 $(0.80T_f)$ $(0.955T_f)$	−28.45	11.30	4.31	1.03
Ni−38.13-at.% Cu	64 ··· 185 $(0.28T_f)$ $(0.80T_f)$	6.90	0.78	1.05	···
	185 ··· 230 $(0.80T_f)$ $(1.00T_f)$	−41.68	12.40	4.50	1.05

those in the immediate vicinity of the peak in R_s expecially for Ni, Ni−9.38-at.%-Cu and Ni−18.77-at.%-Cu and (ii) for pure nickel the contribution to R_s up to 0.41 T_f arises from impurity or skew scattering [first term in Eq. (2)] alone. The temperature ranges where Eq. (2) holds and the corresponding values of the coefficients a and b are given in Table III. The observation that two straight-line fits represent the data very closely suggests that (i) Eq. (2) by itself is inadequate to explain the data in the sense that it does not take into account the contribution to R_s arising from spin-disorder scattering [Eq. (4)], which is expected to become important in a certain temperature range depending upon the nature of magnetic substance, and (ii) different scattering processes become important in different temperature ranges and, as such, in a particular temperature range two scattering processes simultaneously (or one of them dominantly) could contribute to the anomalous Hall coefficient. In view of the above remarks, it was imperative to make double logarithmic plots of R_s vs ρ and R_s vs $[M_s^2(0) - M_s^2(T)]$ in order to establish the predominant scattering process and the temperature range in question from the exponent values of ρ and $[M_s^2(0) - M_s^2(T)]$.[43] The results of such an effort are summarized in Table III where the exponent values for $[M_s^2(0) - M_s^2(T)]$ less the unity, observed in certain temperature ranges, are not included for the simple reason that they lack physical basis. The salient features that emerge from the exponent values tabulated in Table III are: (i) impurity scattering gives the sole or major contribution to R_s for pure nickel and alloys containing 9.38 and 38.13 at.% copper whereas both impurity and phonon scattering are comparable in magnitude (see values of coefficients a and b) for alloys containing 18.77- and 28.32 at.% copper, within the temperature ranges where the straight-line fit with lesser slope (among the two straight-line fits, see Fig. 4) represents the data on these samples; (ii) for temperature ranges where the straight-line fit with greater slope holds, the dominant contribution to the anomalous Hall coefficient arises from phonon scattering for pure nickel and the alloy with 9.38 at.% copper and from spin-disorder scattering for the alloys containing 18.77, 28.32, and 38.13 at.% copper; (iii) spin-disorder scattering surpasses the contribution to the anomalous Hall effect arising from all other scattering processes for temperatures in the vicinity of the ferromagnetic Curie temperature for all the samples of present investigation. Now that the anomalous Hall coefficient in the paramagnetic region is known[23,24,26] to arise solely from spin-disorder scattering for nickel and nickel-copper alloys, the argument that the scattering mechanism responsible for the anomalous Hall effect above and below the Curie temperature should be the same, lends a firm support to observation (iii). It should be mentioned in the passing that in view of the above findings the negative values of the coefficient a found in certain temperature ranges (Table III) lose meaning and hence should not be given any importance whatsoever.

V. CONCLUSION

An appraisal of the theories proposed in the literature to understand the anomalous-Hall-effect phenomenon on the basis of the present experimental data on the Hall effect in nickel and nickel-copper alloys leads to the following conclusions: (i) No single theory is able to account for the diversity of the present experimental results. (ii) One or more of the scattering processes (impurity, phonon, and spin-disorder scattering) give dominant contribution to the anomalous Hall effect in different temperature ranges which, in turn, are different for different samples. (iii) Spin-disorder scattering surpasses the contribution to the anomalous Hall effect arising from all other scatterings for temperatures close to the ferromagnetic Curie temperature for all the samples of present investigation. (iv) d-spin–s-orbit interaction plays a vital role so far as the anomalous Hall effect in pure nickel is concerned. (v) The intrinsic spin-orbit interaction is of paramount importance for understanding AHE behavior in nickel-copper alloys.

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[1] Note that in ferromagnetic materials $B = H + (1 - N)4\pi M$, where N is the demagnetization factor and for Hall geometry, i.e., for external field direction perpendicular to the plane of the Hall plate sample, N approaches unity and B approaches H.

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[43] At this stage it is worth mentioning that contrary to the claim of various workers (see Refs. 18 and 19) that for *pure ferromagnetic* materials ρ^2 and $[(M_s^2(0) - M_s^2(T)]$ have a *similar* dependence on temperature, we observe (Table III) their temperature dependences to be widely different for pure nickel and more so for the alloys. Such an observation, in turn, permits identification of the contributions to the AHE arising from various scattering mechanisms to be made unambiguously.