PHYSICAL REVIEW B VOLUME 20, NUMBER 11 1 DECEMBER 1979

Enhancement of the energy gap in superconducting aluminum by tunneling extraction of quasiparticles

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(Received 23 July 1979)

Al - Al_2O_3 - Al - Al_2O_3 - Al tunnel junctions have been used to induce and detect enhancements of up to 40% in the energy gap of superconducting aluminum. Quasiparticles are extracted from the middle aluminum film through the first tunnel junction into an aluminum film with a larger energy gap, and the gap enhancement in the middle film is measured from the characteristics of the second tunnel junction. A theory developed for the enhancement is in good agreement with the experimental results for bias voltages below the difference of the energy gaps, but overestimates the enhancement at higher voltages. This discrepancy is believed to arise from the effects of nonequilibrium phonons, which are neglected in the calculation. The low-voltage data are consistent with a value of about 80 ns for the characteristic electron-phonon scattering time, τ_0, in aluminum films with an average transition temperature of 1.34 K.

I. INTRODUCTION

Wyatt *et al.*,[1] Dayem and Wiegand,[2] and (later) Latyshev and Nad,[3] observed that the critical current of Dayem bridges could be enhanced by microwave irradiation. This enhancement was first successfully explained by the work of Eliashberg and co-workers.[4] In their model, photons of frequency less than $2\Delta/h$, where Δ is the energy gap, excite quasiparticles from low-lying states to states of higher energy, thereby making additional pair states near the Fermi wave vector, k_F, available for occupancy. Since pairs near k_F contribute most strongly to the pairing interaction, this redistribution of pair-state occupancy increases the condensation energy, and leads to an enhancement of Δ and hence of the critical current. In the Eliashberg model,[4] the quasiparticle recombination rate is assumed to be independent of energy, so that the number of quasiparticles remains constant. Furthermore, the phonons are assumed to be in thermal equilibrium. More recently, Chang and Scalapino[5] used the coupled kinetic equations for quasiparticles and phonons to compute the gap enhancement using a realistic energy-dependent recombination rate, and taking into account the nonequilibrium phonon population. They find that a substantial contribution to the gap enhancement arises from the reduction in the quasiparticle population as the average quasiparticle energy is increased by the microwave pumping.

Several further experiments have by now strongly supported the concept of gap enhancement. These include the phonon-induced enhancement of the critical current of superconducting microbridges and point contacts,[6] the microwave-induced enhancement of the critical currents and transition temperatures of aluminum strips,[7] the microwave-induced enhancement of the voltage at which gap structure occurs in point contacts,[8] and the direct tunneling measurement of microwave-induced gap enhancement in aluminum films.[9] More recently, Gray [10] observed energy-gap enhancement by quasiparticle tunneling between two identical superconductors. In this case, the average energy of the quasiparticles is increased, and the enhancement is again a result of the Eliashberg mechanism. Gray presented results showing an enhancement of about 0.5%, and reported enhancements of up to 10%. Peskovatskii and Seminozhenko[11] have calculated the expected enhancement assuming an equilibrium phonon population, while Chang[12] has computed the enhancement taking into account the energy-dependent scattering rate and the nonequilibrium phonons. His calculation predicts an enhancement that exceeds Gray's observations by a factor of about 4.

Long before these experiments were performed or these theories developed, Parmenter[13] proposed that Δ could be enhanced by the extraction of quasiparticles through a tunnel barrier into a second superconductor with a larger gap. This process is illustrated in Fig. 1 for $\Delta_1 > \Delta_2$, using the semiconductor tunneling model.[14] Because of the difference between the thermal populations of the two superconductors at voltages below $(\Delta_1 + \Delta_2)/e$ "electrons" tunnel from superconductor 2 to superconductor 1 (a process that reduces the quasiparticle population in 2) with a higher probability than "holes" tunnel from 1 into 2 (a process that increases the quasiparticle population in 2). Thus, a first-order effect of the tunneling process is the net reduction in the quasiparticle popula-

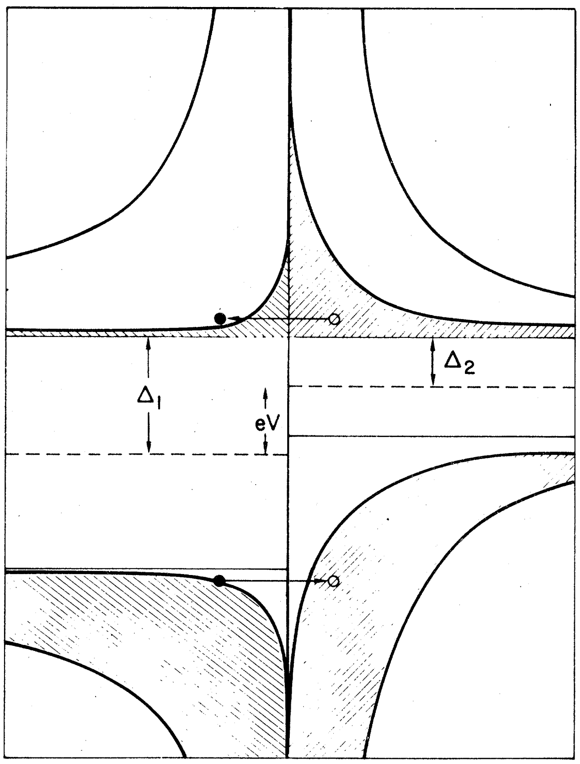

FIG. 1. Semiconductor model of tunneling between two superconducting films with $\Delta_1 > \Delta_2$, biased at a voltage $V = (\Delta_1 - \Delta_2)/e$. The upper arrow indicates "electron" tunneling, and the lower "hole" tunneling.

tion of 2, and a subsequent enhancement of the pair population. This enhancement will be reduced to some extent by the diffusion of recombination phonons with energies $\geqslant 2\Delta_1 > 2\Delta_2$ from superconductor 1 into superconductor 2.

In Sec. II we report experiments demonstrating gap enhancement in aluminum by the tunneling extraction of quasiparticles. Enhancements of up to 40% have been observed. In Sec. III, we develop a simple model of the enhancement that assumes that the phonons remain in thermal equilibrium. At voltages below $(\Delta_1 - \Delta_2)/e$, the model can be fitted to the experimental data with a characteristic electron-phonon scattering time[15] τ_0 of about 80 ns for an average transition temperature of 1.34 K. This value is in reasonable agreement with the value obtained by Chi and Clarke[16] using charge-relaxation measurements. At voltages above $(\Delta_1 - \Delta_2)/e$, however, the theory predicts a much greater enhancement than is observed experimentally. We believe that this discrepancy is the result of neglecting the effects of nonequilibrium phonons. Section IV contains our discussion and conclusions.

II. EXPERIMENT

The sample consisted of three superimposed films evaporated onto a glass slide to form two tunnel junctions (Fig. 2). Two insulating layers, one of SiO and one of Duco cement, masked all the edges of the films from the junctions. The lower (extraction) junction between films 1 and 2 was of low resistance to produce a high current density at the voltages below $(\Delta_1 + \Delta_2)/e$ necessary for extraction from film 2. The upper (detection) junction between films 2 and 3 was of relatively high resistance so that its current-voltage characteristic could be used to determine Δ_2 without significantly perturbing its value. Since the degree of gap enhancement was expected to be proportional to τ_0, we chose Al for film 2, as this material has the longest τ_0 of any superconductor with a transition temperature in the liquid-^{4}He temperature range. It was necessary to make Δ_1 greater than Δ_2 in order to achieve quasiparticle extraction from film 2, and Δ_3 greater than Δ_2 so that Δ_2 could be determined with reasonable accuracy close to the transition temperature, T_{c2}, of film 2. Al was also used for films 1 and 3, with the transition temperatures enhanced by oxygen doping. Thus, films 1 and 3 were evaporated at a pressure of typically 10^{-4} Torr, while film 2 was evaporated at 10^{-6} Torr.

The sample was immersed directly in liquid helium. The I_d - V_d characteristics of the detection junction were studied as a function of the extraction current I_e. Gap enhancement was always observed in Al(2) provided that the quality of the extraction junction was such that its zero-voltage quasiparticle conductance was no more than 5% greater than the value calculated using the measured values of Δ_1, Δ_2, and the junction resistance. Of the 11 samples in which we observed enhancement, we present results on the two which showed the greatest enhancement, A and B. Table I shows the film thicknesses, d_1, d_2, and d_3, and the transition temperatures T_{c1}, T_{c2}, and T_{c3} (defined as the temperature at which Δ extrapolates to zero[17]) of the three Al films, and the extraction and detection junction resistances, R_e and R_d, measured at voltages much greater than the sum of

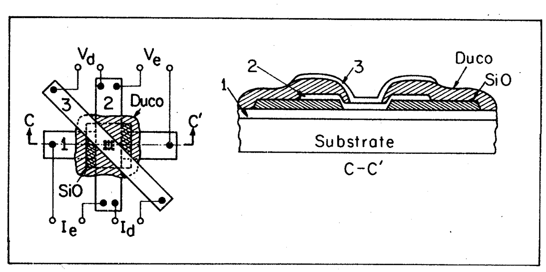

FIG. 2. (Left) plan view of sample configuration; (right) section cc' of sample (film thicknesses greatly exaggerated). Width of films 1 and 3 is 3 mm.

TABLE I. Parameters of samples A and B.

	First Al film		Second Al film		Third Al film		Extraction junction resistance	Detection junction resistance
	d_1 (nm)	T_{c1} (K)	d_2 (nm)	T_{c2} (K)	d_3 (nm)	T_{c3} (K)	R_e (Ω)	R_d (Ω)
A	56	1.49	37	1.353	28	1.40	0.021	3.5
B	79	1.36	45	1.321	40	1.38	0.037	12

the gaps. Figure 3 shows I_d vs V_d and dV_d/dI_d vs V_d for sample A at $T/T_{c2}=0.986$, where T is the bath temperature. The labels indicate the bias points on the extraction junction characteristic (inset of Fig. 3) at which the various detection curves were obtained. As I_e is increased from zero to a point just below the cusp at $(\Delta_1-\Delta_2)/e$, the I_d-V_d curves show clearly that the sharp rise in current at $(\Delta_3+\Delta_2)/e$ moves to a higher voltage while the cusp at $(\Delta_3-\Delta_2)/e$ moves to a lower voltage. Thus, Δ_2 is enhanced. We associate the higher- and lower-voltage minima in dV_d/dI_d with $(\Delta_3+\Delta_2)/e$ and $(\Delta_3-\Delta_2)/e$, respectively. The derivatives in Fig. 3 show that, as I_e is increased from zero, there is no significant enhancement at b, while there is substantial enhancement at c and d. At e, an extraction voltage greater than $(\Delta_1-\Delta_2)/e$, the enhancement of Δ_2 is much less than at d, indicating that the extraction rate is greatly reduced. Identical results were obtained when the extraction current was reversed.

A dc Josephson current was always observed in the extraction junction. The dc supercurrent was quenched by briefly raising I_e to a large value, thereby trapping flux in the junction. It should be noted that the observed enhancement could not have been caused by photons[4] generated by ac Josephson currents since V_e $[\approx(\Delta_1-\Delta_2)/e]$ was greater than Δ_2/e at temperatures near T_{c2} and the photon energy ($>2\Delta_2$) was high enough to break pairs in Al(2).

From dV_d/dI_d vs V_d, we obtained $\delta\Delta_2(V_e)\equiv\Delta_2(V_e)-\Delta_2(0)$ and $\delta\Delta_3(V_e)\equiv\Delta_3(V_e)-\Delta_3(0)$,

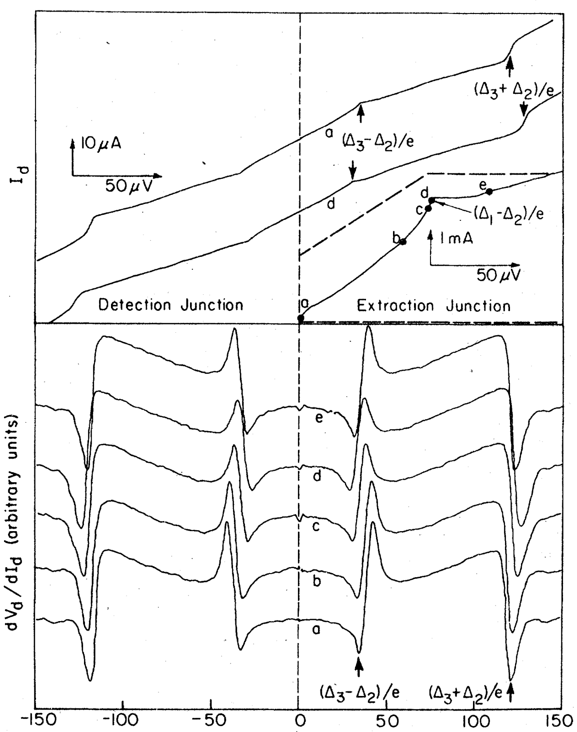

FIG. 3. (Upper) I_d vs V_d and (lower) dV_d/dI_d vs V_d for sample A for the various extraction bias points a, b, c, d, and e on the I_e - V_e characteristic in the inset. The bath temperature was 0.986 T_{c2}.

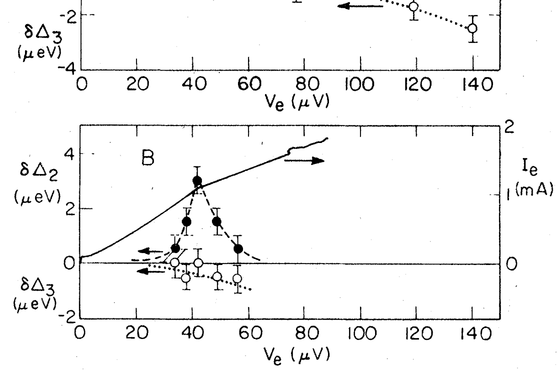

FIG. 4. $\delta\Delta_2$ and $\delta\Delta_3$ vs V_e, and I_e vs V_e for sample A at $T=0.986T_{c2}$ (upper), and sample B at $T=0.995T_{c2}$ (lower). The dashed and dotted lines are drawn through the data.

where $\Delta_2(V_e)$ and $\Delta_3(V_e)$ are the steady-state gaps at an extraction voltage V_e. Figure 4 shows $\delta\Delta_2$ and $\delta\Delta_3$ vs V_e for samples A and B, together with the characteristics of the extraction junctions. In both cases, $\delta\Delta_2$ is positive and sharply peaked near $(\Delta_1-\Delta_2)/e$, reflecting the high rate of extraction near $(\Delta_1-\Delta_2)/e$. For sample A, $\delta\Delta_3$ is negative and increases smoothly with increasing V_e, while for B, $\delta\Delta_3$ is essentially zero for $V_e \lesssim (\Delta_1-\Delta_2)/e$ and negative for $V_e > (\Delta_1-\Delta_2)/e$. The absence of data for sample A at voltages between the peak in $\delta\Delta_2$ and about 120 μV was the result of hysteresis in the extraction junction that made it impossible to current bias the junction in this range. We note that, at high currents, the voltage appears to switch on the I_e-V_e curves, suggesting that part of Al(2) was driven normal by the high-current density in the extraction region. When the extraction junction was biased near the switching point, the dV_d/dI_d vs V_d curve became very noisy, and its origin shifted abruptly along the V_d axis. Therefore, we could obtain useful data only when V_e was below the switching voltage. Unfortunately, the switching voltage decreased as the temperature approached T_{c2}, thus preventing us from obtaining data very close to T_{c2}, and removing the possibility of our observing an enhancement in T_{c2}.

Figure 5(a) shows the maximum gap enhancement $\delta\Delta_2^{\max}$ and $\delta\Delta_2^{\max}/\Delta_{2T}$ (Δ_{2T} is the equilibrium gap) at $V_e=(\Delta_1-\Delta_2)/e$ as functions of T/T_{c2} for samples A and B. The absolute magnitude of the gap enhancement, $\delta\Delta_2^{\max}$, increases sharply as T approaches T_{c2}. For sample B, the gap is enhanced by over 40% at $T/T_{c2}=0.998$, and there is no indication that this enhancement is leveling off. Therefore, we suspect that an enhancement of T_{c2} would be possible if it were not for the switching induced by I_e. Figure 5(b) shows $\delta\Delta_3$ at $V_e=(\Delta_1-\Delta_2)/e$ vs T/T_{c2}. For A, $\delta\Delta_3$ is always zero or negative, while for B, $\delta\Delta_3$ is zero at low temperatures and becomes positive at temperatures close to T_{c2}. We believe that the changes in Δ_3 shown in Figs. 4 and 5(b) are induced by nonequilibrium phonons. To a first approximation, the steady-state phonon distribution is uniform across all three films because the total thickness is less than the phonon mean free path, and the phonon transmission coefficient between the Al films is close to unity. When $V_e=(\Delta_1-\Delta_2)/e$, there is an excess of phonons with energies $\gtrsim 2\Delta_1$ generated by the recombination of excess quasiparticles in Al(1). For sample A, $\Delta_1 > \Delta_3$, so that the $2\Delta_1$ phonons can break pairs in Al(3), thereby reducing Δ_3 at all temperatures and extraction voltages. On the other hand, for sample B_1, Δ_3 is slightly greater than Δ_1, the difference increasing as the temperature is raised towards T_{c1}. As the temperature is increased, a growing fraction of the recombination phonons from Al(1) have energies between $2\Delta_1$ and $2\Delta_3$, and are unable to break pairs in Al(3). In fact, for $T>0.995T_{c2}$, it appears that the predominant action of the phonons is to excite quasiparticles in Al(3) to higher energy states in such a way that Δ_3 is enhanced.[4,6]

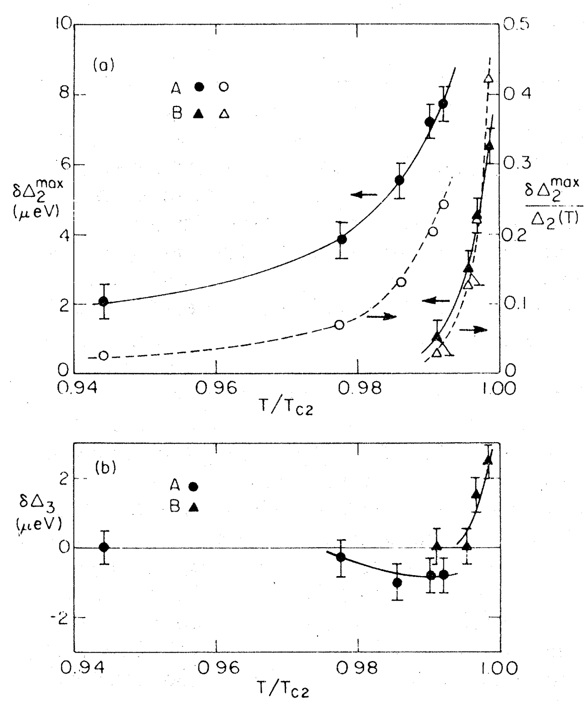

FIG. 5. (a) $\delta\Delta_2^{\max}$ (solid line) and $\delta\Delta_2^{\max}$ (dashed line) vs T/T_{c2}, and (b) $\delta\Delta_3$ vs T/T_{c2} for samples A and B, at $V_e=(\Delta_1-\Delta_2)/e$. The lines are drawn through the data.

III. THEORY

In order to interpret the experimental results reported in Sec. II, we present a simplified theory of gap enhancement by the tunneling extraction of quasiparticles. We consider a tunnel junction consisting of two superconducting films with $\Delta_1 > \Delta_2$, and calculate the enhancement in Δ_2 under the simplifying assumption that Δ_1 is unperturbed by the tunneling process. This assumption is valid first because $d_1 > d_2$ (typically $d_1 \approx 2d_2$), and, second, more importantly, because Δ_1 is substantially higher than Δ_2 at the temperature of the experiment so that Δ_1 is relatively insensitive to changes in the quasiparticle population.

For a junction biased at a voltage V (V is defined to be positive when film 2 is positive with respect to film 1) the generation rate of quasiparticles of energy E in film 2 can be written

$$G_\epsilon = \Gamma(g_{e\epsilon}+g_{h\epsilon}) \ . \tag{3.1}$$

In Eq. (3.1), $\epsilon \equiv \pm(E^2-\Delta_2^2)^{1/2}$, where the plus and minus signs refer to the $k>k_F$ and $k<k_F$ branches,

respectively, Δ_2 is the steady-state gap, and

$$\Gamma \equiv G_{NN}/2N(0)\,\Omega e^2 \ . \tag{3.2}$$

Here, G_{NN} is the junction tunneling conductance when both films are in the normal state, $N(0)$ is the density of states for electrons of single spin at the Fermi energy, Ω is the tunneling volume (the tunneling area multiplied by d_2), and e is the electronic charge. Note that Γ has the dimensions of sec^{-1}. The two dimensionless functions $g_{e\epsilon}$ and $g_{e\epsilon}$ in Eq. (3.1) are given by

$$g_{e\epsilon}=\frac{1}{2}\left[1+\frac{\epsilon}{E}\right]\rho_{\Delta_1}(|E-eV|)[f_T(E-eV)-f_\epsilon] \tag{3.3}$$

and

$$g_{h\epsilon}=\frac{1}{2}\left[1-\frac{\epsilon}{E}\right]\rho_{\Delta_1}(|E+eV|)[f_T(E+eV)-f_\epsilon] \ , \tag{3.4}$$

where $\rho_{\Delta_1}(E)=\Theta(E-\Delta_1)E/(E^2-\Delta_1^2)^{1/2}$ (Θ being the Heaviside function), f_ϵ is the steady-state distribution of quasiparticles of energy $E=(\epsilon^2+\Delta_2^2)^{1/2}$, and $f_T(E)$ is the Fermi-Dirac function. The presence of f_T in Eqs. (3.3) and (3.4) assumes that the quasiparticles in film 1 are unperturbed. It is easy to see that $g_{e\epsilon}$ ($g_{h\epsilon}$) is the dominant term for extremely electronlike, $\epsilon \gg \Delta$ (holelike, $\epsilon \ll -\Delta$) quasiparticles.

From Eq. (3.1) one can obtain the total quasiparticle generation rate G_{tot}

$$G_{\text{tot}}=2N(0)\,\Omega \int_{\Delta}^{\infty} dE\,\rho_\Delta(E)(G_\epsilon+G_{-\epsilon})$$
$$=2N(0)\,\Omega\Gamma \int_{-\infty}^{\infty} d\epsilon\,(g_{e\epsilon}+g_{h\epsilon}) \ . \tag{3.5}$$

The tunneling current I can also be expressed in terms of $g_{e\epsilon}$ and $g_{h\epsilon}$ as follows:

$$I=2eN(0)\,\Omega\Gamma \int_{-\infty}^{\infty} d\epsilon\,(g_{e\epsilon}-g_{h\epsilon}) \ . \tag{3.6}$$

In the weak-perturbation limit, Eq. (3.6) reduces to a more familiar form[14] because both f_ϵ and $f_{-\epsilon}$ can be replaced with $f_T(E)$, and Δ_2 with Δ_{2T} to yield

$$I=\frac{G_{NN}}{e}\int_{\Delta_{2T}}^{\infty} dE\,\rho_{\Delta_{2T}}(E)\{\rho_{\Delta_{1T}}(|E-eV|)[f_T(E-eV)-f_T(E)]+\rho_{\Delta_{1T}}(|E+eV|)[f_T(E)-f_T(E+eV)]\} \ . \tag{3.7}$$

Similarly, in this limit, G_{tot} reduces to

$$G_{\text{tot}}=\frac{G_{NN}}{e^2}\int_{\Delta_{2T}}^{\infty} dE\,\rho_{\Delta_{2T}}(E)\{\rho_{\Delta_{1T}}(|E-eV|)[f_T(E-eV)-f_T(E)]-\rho_{\Delta_{1T}}(|E+eV|)[f_T(E)-f_T(E+eV)]\} \ . \tag{3.8}$$

Since the current I is an experimentally measurable quantity whereas G_{tot} is not, it is convenient to define a quasiparticle generation factor,

$$F_q=G_{\text{tot}}/|I/e| \ . \tag{3.9}$$

It should be noted that I and G_{tot} diverge at $V=(\Delta_{1T}-\Delta_{2T})/e$ if the BCS density of states is used for both films. This divergence is not observed because in real metals the gap is smeared. To simulate this smearing in our calculation, we used the following form of the density of states for both films:

$$\rho_\Delta(E)=\Theta(E-\Delta)\ \mathrm{Re}\{E/[E^2-(\Delta-i\gamma)^2]^{1/2}\} \ . \tag{3.10}$$

We chose $\gamma=0.01\Delta$ to give good agreement between the experimental and theoretical I-V characteristics at $V\approx(\Delta_1-\Delta_2)/e$, where the shape of the curves is very sensitive to the degree of smearing. We note that the total number of states described by Eq. (3.10) is smaller than that given by the BCS density of states by $4N(0)(\Delta\gamma)^{1/2}$ because of the low-energy cutoff in Eq. (3.10) which we adopted for the convenience of our numerical calculations. Since we are interested only in temperatures near T_{c2}, we neglect this error which is about $4N(0)(\Delta\gamma)^{1/2}/4N(0)k_BT_c \approx 0.1\Delta/k_BT_c$, no more than 4% at $T\gtrsim 0.98\,T_{c2}$.

Figures 6(a) and 7(a) show the normalized I-V curves calculated from Eqs. (3.7) and (3.10) for the two experimental samples referred to in Fig. 4. The solid circles in these two figures correspond to the experimental I_e - V_e curves in Fig. 4 for samples A and B, respectively. There is good agreement between the theoretical curve and the experimental data, especially at low bias voltage. The small discrepancy at higher voltages is presumably due to the fact that Δ_{2T} and $f_T(E)$ were used in the calculation rather than the steady-state values of Δ_2 and f_ϵ.

Figures 6(b) and 7(b) show the normalized calculated values of G_{tot} versus normalized bias voltage. For $\Delta_1>\Delta_2$, G_{tot} is always negative for $V<(\Delta_1+\Delta_2)/e$, corresponding to quasiparticle extraction from film 2, and becomes positive at higher voltages. The quasiparticle extraction rate, G_{tot}, peaks sharply at $V=(\Delta_1-\Delta_2)/e$. Figures 6(c) and 7(c) show the quasiparticle extraction factor, F_q, as a function of the normalized bias voltage. Note that F_q is approximately linear in V for small bias voltages and also peaks sharply at $V=(\Delta_1-\Delta_2)/e$. This behavior can be understood qualitatively from the semiconductor model[14] (see Fig. 1). For small bias voltages, the difference between the number of "holes" tunneling into film 2 and the number of "electrons" tunneling out of it increases linearly with the bias voltage. At $V=(\Delta_1-\Delta_2)/e$, the "electron"

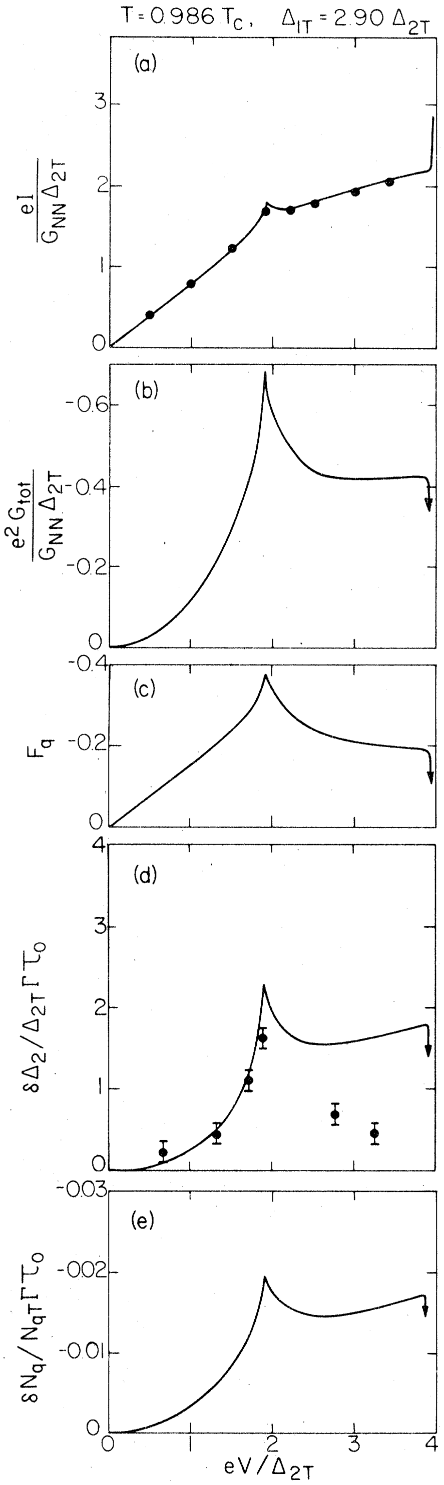

FIG. 6. For the parameters given for sample *A* in Fig. 4, calculations of : (a) $eI/G_{NN}\Delta_{2T}$ vs eV/Δ_{2T}; (b) $e^2G_{\mathrm{tot}}/G_{NN}\Delta_{2T}$ vs eV/Δ_{2T}; (c) F_q vs eV/Δ_{2T}; (d) $\delta\Delta_2/\Delta_{2T}\Gamma\tau_0$ vs eV/Δ_{2T}; and (e) $\delta N_q/N_{qT}\Gamma\tau_0$ vs eV/Δ_{2T}.

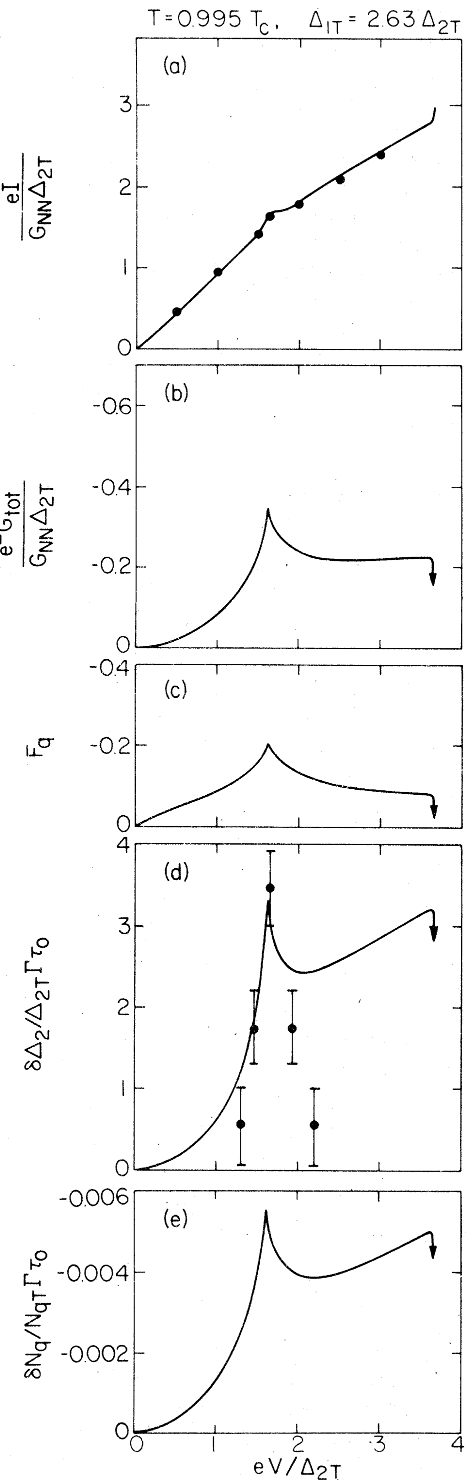

FIG. 7. For the parameters given for sample *B* in Fig. 4, calculations of (a) $eI/G_{NN}\Delta_{2T}$ vs eV/Δ_{2T}; (b) $e^2G_{\mathrm{tot}}/G_{NN}\Delta_{2T}$ vs eV/Δ_{2T}; (c) F_q vs eV/Δ_{2T}; (d) $\delta\Delta_2/\Delta_{2T}\Gamma\tau_0$ vs eV/Δ_{2T}; and (e) $\delta N_q/N_{qT}\Gamma\tau_0$ vs eV/Δ_{2T}.

extraction becomes extremely efficient because the two sharply peaked densities of states are aligned, while the "hole" injection rate does not become correspondingly large. For $V > (\Delta_1 - \Delta_2)/e$, the magnitude of F_q drops initially because the "electron" extraction becomes less efficient and then gradually increases again because the back flow of the thermal "electrons" from film 1 is gradually blocked off as the bias voltage increases.

To demonstrate that the gap enhancement produced by this quasiparticle tunneling-extraction mechanism accounts adequately for the experimental results in Sec. II, we performed a first-order calculation of the steady-state distribution of quasiparticles, and hence calculated the gap enhancement. The calculation was carried out as follows: (i) The quasiparticle extraction rate, G_ϵ, was calculated from Eqs. (3.1), (3.3), and (3.4) with the thermal equilibrium energy gaps and quasiparticle distributions. (ii) The inelastic collision integral for quasiparticles at energy E (labeled with ϵ) was calculated with the thermal equilibrium distribution of phonons, $n_T(\hbar\omega) = [\exp(\hbar\omega/k_BT)-1]^{-1}$, and the thermal equilibrium gap, Δ_{2T}, in the following expression[15]:

$$\mathcal{G}_\epsilon = \tau_0^{-1}(k_BT_{c2})^{-3}\int_{-\infty}^{\infty} d\epsilon' \left[\frac{1}{2}\left(1-\frac{\epsilon\epsilon'}{EE'}+\frac{\Delta_{2T}^2}{EE'}\right)(E+E')^2[f_\epsilon f_{\epsilon'} - n_T(E+E')(1-f_\epsilon - f_{\epsilon'})] \right.$$
$$+\frac{1}{2}\left(1-\frac{\epsilon\epsilon'}{EE'}-\frac{\Delta_{2T}^2}{EE'}\right)(E-E')^2\{\theta(E-E')[f_\epsilon - f_\epsilon f_{\epsilon'} - n_T(E-E')(f_{\epsilon'}-f_\epsilon)]$$
$$\left. +\,\Theta(E'-E)[-f_{\epsilon'}+f_\epsilon f_{\epsilon'}+n_T(E'-E)(f_\epsilon - f_{\epsilon'})]\}\right] . \tag{3.11}$$

In Eq. (3.11) $\tau_0 \equiv 2\pi k_BT_{c2}\alpha^2F(k_BT_{c2})/\hbar$ is the characteristic electron-phonon interaction time,[15] and $\alpha^2F(\hbar\omega)$ is the product of the square of the electron-phonon interaction matrix element and the phonon density of states; $\alpha^2F(\hbar\omega)$ was assumed to be quadratic in $\hbar\omega$. (iii) The steady-state solution of the change in the distribution, $\delta f_\epsilon \equiv f_\epsilon - f_T(E)$, was calculated from

$$\dot{f}_\epsilon = G_\epsilon - \mathcal{G}_\epsilon = 0 \tag{3.12}$$

using a simple iteration scheme.[18] (iv) Finally, the first-order change in the gap was calculated from the steady-state values of δf_ϵ by using the BCS gap equation in the following form[19]:

$$\frac{\delta\Delta_2}{\Delta_{2T}} = \int_{-\infty}^{\infty}\frac{d\epsilon}{E}(-\delta f_\epsilon)\Big/\int_0^{\infty}\frac{\Delta_{2T}^2\,d\epsilon}{2k_BTE^2}\left(\frac{\tanh(E/2k_BT)}{E/2k_BT}-\frac{1}{\cosh^2(E/2k_BT)}\right) . \tag{3.13}$$

In addition the change in the total quasiparticle density, $\delta N_q = N_q - N_{qT}$, was calculated from

$$\frac{\delta N_q}{N_{qT}} = \int_{-\infty}^{\infty} d\epsilon\,\delta f_\epsilon \Big/ \int_{-\infty}^{\infty} d\epsilon\, f_T(E) . \tag{3.14}$$

If $\delta f_\epsilon << f_T(E)$, the collision integral, Eq. (3.11), can be linearized in δf_ϵ. In this limit, it is easy to see that the steady-state solution of δf_ϵ scales with $\Gamma\tau_0$, as do $\delta\Delta_2/\Delta_{2T}$ and $\delta N_q/N_{qT}$. The calculated curves of $\delta\Delta_2/\Delta_{2T}\Gamma\tau_0$ vs eV/Δ_{2T} for samples A and B are shown in Figs. 6(d) and 7(d), respectively. The solid circles are the experimental data reproduced from Fig. 4, with τ_0 chosen to give the best fit to the data at voltages below $(\Delta_1-\Delta_2)/e$. The value of τ_0 was chosen to be 77 and 83 ns for samples A and B, respectively, to fit the theoretical curve in the low extraction voltage region, i.e., $V_e \leq (\Delta_1-\Delta_2)/e$. The average value of τ_0 was 80 ns, and the average value of the transition temperature was 1.34 K. Although one should be cautious in applying the theory, given the approximations involved, it is noteworthy that this value of τ_0 is in reasonable agreement with the value obtained by Chi and Clarke[15] in their charge relaxation measurements on Al films with comparable transition temperatures, about 60 ns.

Although the theoretical curves fit the experimental data reasonably well at bias voltages $V \lesssim (\Delta_1-\Delta_2)/e$, at higher voltages the experimental data lie significantly below the calculated curve. For $V > (\Delta_1-\Delta_2)/e$, we believe that the experimental values of $\delta\Delta_2$ are substantially depressed by recombination phonons from film 1, which are of course able to break pairs in film 2 and suppress the gap enhancement substantially at the higher power levels. This effect was completely neglected in our calculation. In addition, we have not taken into account the magnetic fields generated by the currents flowing in the thin films.

Figures 6(e) and 7(e) show the calculated values of $\delta N_q/N_{qT}\Gamma\tau_0$ vs eV/Δ_{2T}. It is interesting to note that these curves are similar to the corresponding plots of $\delta\Delta_2/\Delta_{2T}\Gamma\tau_0$ vs eV/Δ_{2T} in shape, but are roughly two orders of magnitude smaller in amplitude. This is because the quasiparticles are extracted at energies close to the energy gap where they have a major effect on the energy gap. Furthermore, the temperatures are rather close to T_{c2} so that Δ_2 is very sensitive to small changes in the quasiparticle density.

To conclude the section, we would like to make the following remarks:

(i) The calculation is not self-consistent, and the results are supposedly accurate to order $\delta\Delta_2/\Delta_{2T}$. As a check, we have performed a self-consistent calculation for sample A at $V=(\Delta_1-\Delta_2)/e$ by allowing Δ_2 to deviate from Δ_{2T} in both the extraction rate G_ϵ [Eq. (3.1)] and the collision integral $\mathcal{G}_\epsilon$ [Eq. (3.11)]. The result showed a 5% decrease in $\delta\Delta_2/\Delta_{2T}$ compared with the first-order calculation. Therefore, we believe that our first-order calculation is everywhere within 5% of the result of a self-consistent calculation.

(ii) The more serious approximation made in our calculation is the assumption of a phonon distribution in thermal equilibrium at the bath temperature. This assumption is valid only if the phonon escape time from the sample, τ_{es}^{ph}, is extremely short compared with the phonon pair-breaking time, τ_B^{ph}, which is of the order of 10^{-10} sec for Al.[15] For our samples, τ_{es}^{ph} was at least the same order of magnitude as τ_B^{ph} even if we assume that the phonon transmission coefficient to the helium bath was unity. Although we have not undertaken the much more complicated calculations taking into account the effects of the nonequilibrium phonon distribution, we suspect that the results would be in much better agreement with the experimental data, particularly for $V>(\Delta_1-\Delta_2)/e$.

(iii) The fact that gap enhancement was not observed at temperatures above 0.998 T_c does not imply that quasiparticle extraction vanishes as T approaches T_{c2}. As a matter of fact, our calculation indicated a nonzero extraction rate even at $T=T_{c2}$, where the equilibrium value of Δ_2 is zero. Therefore we believe that T_c enhancement would have been possible, had the extraction current not exceeded the critical current of the extracted film.

IV. DISCUSSION AND CONCLUSIONS

We have demonstrated that the tunneling extraction of quasiparticles can produce substantial enhancements of the energy gap of superconducting aluminum films. There is good agreement between the experimental results and the simple theoretical model of Sec. IV for $V\leq(\Delta_1-\Delta_2)/e$. However, for $V>(\Delta_1-\Delta_2)/e$, the experimentally observed gap enhancement decreases as V increases, while the model calculation predicts an approximately constant enhancement. We suspect that this discrepancy is caused by the nonequilibrium phonons generated in film 1 which diffuse into film 2. The fact that the energy gap, Δ_3, of film 3 was also influenced by the extraction current (Figs. 4 and 5) is a direct proof that the recombination phonons from film 1 play an important role. Because the nonequilibrium phonons are not taken into account in the simple calculation, the value of τ_0 obtained by fitting the experimental data at voltages less than $(\Delta_1-\Delta_2)/e$ is somewhat uncertain. Nevertheless, it is encouraging to note that the required value of τ_0, 80 ns, is in good agreement with that obtained in our charge relaxation measurements on Al films with similar transition temperatures. To assess the importance of the nonequilibrium phonons, a more detailed calculation that takes this effect into account is clearly required. We hope that such a calculation will become available in the near future.

It is interesting to note that the extraction process effectively cools[20] the quasiparticles in film 2, in contrast to the Eliashberg mechanism in which the quasiparticle energies are increased. As quasiparticles are removed by tunneling, pairs flow into the extracted volume, where they are broken to form quasiparticles with the absorption of phonons of energy $\geqslant 2\Delta_2$. Thus, the net intrinsic result is a reduction in both the quasiparticle and phonon populations in film 2. As a further consequence, gap enhancement by extraction would not be necessarily limited to low T_c materials if the diffusion of the $2\Delta_1$ phonons into film 2 could be prevented, or at least reduced. On the other hand, gap-enhancement experiments involving the Eliashberg mechanism are likely to be confined to low T_c materials, in which τ_0 is long, because phonon production is an intrinsic part of the process.

ACKNOWLEDGMENTS

We thank Dr. Brooke Gregory and Dr. Thomas Lemberger for helpful comments on the manuscript. One of us (J. C.) is grateful to the members of the H. C. Ørsted Institute, Copenhagen, for their kind hospitality while this manuscript was written. This work was supported by the Division of Materials Sciences, Office of Basic Energy Sciences, U.S. Department of Energy. One of us (J. C.) would like to thank the Guggenheim foundation for financial support.

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[17]Δ_2 and Δ_3 were measured with $I_e=0$. With $V_e=(\Delta_1-\Delta_2)/e$, we deduced the value of Δ_1 using the (enhanced) value of Δ_2 determined from the detector junction. The depression of Δ_1 by quasiparticle injection should be negligible because the reduced temperature of Al(1) is relatively low, <0.91 for A, and <0.97 for B, at temperatures close to T_{c2}.

[18]From Eqs. (3.11) and (3.12), we have

$$\begin{aligned}\delta f_\epsilon = \Bigg\{ & G_\epsilon - \tau_0^{-1}(k_B T_{c2})^{-3} \\ & \times \int_{-\infty}^{\infty} d\epsilon' \left[\frac{1}{2}\left(1-\frac{\epsilon\epsilon'}{EE'}+\frac{\Delta_{2T}^2}{EE'}\right)(E+E')^2[f_T(E)+n_T(E+E')] \right. \\ & \left. \quad + \frac{1}{2}\left(1-\frac{\epsilon\epsilon'}{EE'}-\frac{\Delta_{2T}^2}{EE'}\right)(E-E')^2\{\Theta(E-E')[-f_T(E)-n_T(E-E')]+\Theta(E-E')[-1+f_T(E)-n_T(E'-E)]\}\right]\delta f_{\epsilon'}\Bigg\} \\ & \times \Bigg\{ \tau_0^{-1}(k_B T_{c2})^{-3}\int_{-\infty}^{\infty} d\epsilon' \left[\frac{1}{2}\left(1-\frac{\epsilon\epsilon'}{EE'}+\frac{\Delta_{2T}^2}{EE'}\right)(E+E')^2[f_T(E')+n_T(E+E')+\delta f_{\epsilon'}] + \frac{1}{2}\left(1-\frac{\epsilon\epsilon'}{EE'}-\frac{\Delta_{2T}^2}{EE'}\right)(E-E')^2 \right. \\ & \left. \quad \times \left\{\Theta(E-E')[1-f_T(E')+n_T(E-E')-\delta f_{\epsilon'}]+\Theta(E'-E)[f_T(E')+n_T(E'-E)+\delta f_{\epsilon'}]\right\}\right]\Bigg\}^{-1} .\end{aligned}$$

In our simple iteration scheme, we start with $\delta f_\epsilon=0$ for all ϵ in the right-hand side of this equation to generate a new form of δf_ϵ. We then iterate as many times as necessary to obtain a convergent solution.

[19]The BCS gap equation can be written

$$\frac{\Delta}{\Delta_0}=\exp\left(-\int_{-\infty}^{\infty}\frac{d\epsilon}{E}[f_T(E)+\delta f_\epsilon]\right) ,$$

where $E=(\epsilon^2+\Delta^2)^{1/2}$ and Δ_0 is the zero-temperature gap. By expanding this equation to first order in $\delta\Delta\equiv\Delta-\Delta_T$ for small δf_ϵ, we obtain Eq. (3.13).

[20]J. L. Paterson (unpublished).