Sine-Gordon chain as a model for a two-dimensional interface

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The sine-Gordon chain (SGC) is used as a model of the solid-on-solid (SOS) model of a two-dimensional interface. In the SGC model the energy scale for creation of edges (solitons) differs from the energy scale for the creation of disorder. The roughness of a surface and the melting of a surface occur at controllably different temperatures. The SOS and SGC models are compared.

I. INTRODUCTION

Recently there has been a good deal of interest in the properties of the interfacial layer which separates the phases in a two-phase system.[1,2] In this context the Ising models, among others, have been extensively studied.[3] Although the study of these models has led to substantial agreement as to the nature of the interface in two-dimensional systems,[2] there are ambiguities as to the properties of the interface in three-dimensional systems. The reason is that the interface of a two-dimensional system is a one-dimensional system, while the interface of a three-dimensional system is a two-dimensional system. The purpose of this paper is to describe the properties of the interface of a two-dimensional system using the sine-Gordon chain.[4] Both the static and dynamic properties of the sine-Gordon chain have been extensively studied.[4–7] By using the sine-Gordon chain as model of the two-dimensional (2-D) interface we confirm much of what is known and gain some insight into the behavior of the interface that is of interest and that may be helpful in providing some understanding of the three-dimensional interface.

In Sec. II the column model (the solid-on-solid or SOS model[2]) of the interface is described. In Sec. III we describe the sine-Gordon model of the interface and compare its properties to those of the column model.

II. COLUMN MODEL OF A TWO-DIMENSIONAL INTERFACE

The column model is considered in the context of a two-dimensional near-neighbor Ising system.[2] In this model the interface of up and down spins has no overhangs (see Fig. 1) so that the energy of the system is given in terms of the relative height of neighboring columns of spin

$$\Delta E(n_1,\ldots,n_{N+1}) = \sum_{i=1}^{N} J_2 |n_{i+1} - n_i| \ , \tag{1}$$

where n_i is the height of column i above the fiducial height 0. This expression measures the departure of the interfacial energy from $(N+1)J_0$, its value when the interface is flat (see Fig. 1), due to the relative motion of neighboring columns. The surface free energy is the energy required to create the flat surface (fiducial surface) plus the free energy associated with the thermal fluctuations permitted by $\Delta E(n_1,\ldots,n_{N+1})$, i.e.,

$$f(T) = (N+1)J_0 - k_B T \ln \Delta Z \ , \tag{2}$$

where

$$\Delta Z = \sum_{n_1,\ldots,n_{N+1}} \delta_{n_1,0}\delta_{n_{N+1},M} e^{-\beta \Delta E(n_1,\ldots,n_{N+1})} \ . \tag{3}$$

In writing ΔZ as in Eq. (3) we have chosen to fix the ends of the interface at 0 and M. From Eq. (1) it is clear that the energy of a configuration depends only upon the perimeter of the configuration, see Fig. 1(b).

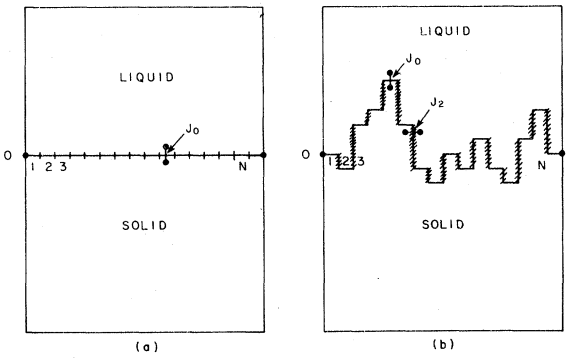

FIG. 1. Fiducial interface, from which energies are measured, is horizontal at column height 0 for all columns (a). The interaction across the top of a column is J_0. When the surface is disordered, perimeter is created (b). Each unit of perimeter costs energy J_2. At $T \neq 0$ it is energetically favorable to create some perimeter in order to recover the free energy of "mixing". From Eqs. (17) and (18), $f/N = J_0 + J_2 p + k_B T \ln[(1+p)^{1/2} - p]$, $p = P/N$.

We define the perimeter

$$P(n_1,\ldots,n_{N+1}) = \sum_{i=1}^{N} |n_{i+1} - n_i| \ ; \tag{4}$$

it is a measure of the length across which pieces of up-spin and down-spin material face one another. The average value of P is

$$P = -\frac{\partial}{\partial x} \ln \Delta Z \ , \tag{5}$$

where $x = \beta J_2$. A measure of the average size of the surface at site $l+1$ is given by

$$\langle (n_{l+1} - \langle n_{l+1} \rangle)^2 \rangle \equiv \langle \Delta n_{l+1}^2 \rangle \ . \tag{6}$$

The partition function ΔZ is found in terms of the solution to a transfer matrix problem,

$$\Delta Z = \sum_{n_1,\ldots,n_{N+1}} \delta_{n_1,0} \delta_{n_{N+1},M} T(n_1 n_2) \times T(n_2 n_3) \cdots T(n_N n_{N+1}) \ , \tag{7}$$

where

$$T(n_2 n_1) = \exp(-x|n_2 - n_1|) \ . \tag{8}$$

Upon introducing the complete set of states generated by $T(mn)$,

$$\begin{aligned} &\sum_m T(nm)\psi_\nu(m) = \lambda_\nu \psi_\nu(n) \ , \\ &\sum_n \psi_\nu^*(n)\psi_\nu(n) = \delta_{\mu,\nu} \ , \\ &\sum_\nu \psi_\nu^*(n)\psi_\nu(m) = \delta_{n,m} \ , \end{aligned} \tag{9}$$

we obtain

$$\Delta Z(N;x) = \sum_\nu \psi_\nu^*(M)\psi_\nu(0)\lambda_\nu^N \tag{10}$$

and

$$\langle F(n_{l+1}) \rangle = \frac{1}{\Delta Z} \sum_{\mu\nu} \psi_\nu^*(M)\psi_\mu(0) F_{\nu\mu} \lambda_\nu^{M-l} \lambda_\mu^l \ , \tag{11}$$

where

$$F_{\nu\mu} = \sum_n \psi_\nu^*(n) F(n) \psi_\mu(n) \ . \tag{12}$$

The solutions to Eq. (9) are the wave functions

$$\psi_\nu(n) = e^{i\nu n} \ , \tag{13}$$

$-\pi \leq \nu \leq \pi$, with eigenvalue

$$\psi_\nu = \frac{\sinh x}{\cosh x - \cos \nu} \ . \tag{14}$$

For the $M = 0$ case, ΔZ is dominated by the $\nu = 0$ eigenvalue and the free energy, energy, and entropy are

$$\frac{f(T)}{N} = J_0 - k_B T \ln(\tanh \tfrac{1}{2} x) \ , \tag{15}$$

$$\frac{u(T)}{N} = \frac{J_2}{\sinh x} + J_0 \ , \tag{16}$$

$$\frac{Ts(T)}{N} = \frac{J_2}{\sinh x} - k_B T \ln(\tanh \tfrac{1}{2} x) \ . \tag{17}$$

For the perimeter (energy/J_2) we have

$$\frac{P}{N} = \frac{1}{\sinh x} \tag{18}$$

and since $\langle n_{l+1} \rangle = 0$

$$\langle \Delta n_{+1}^2 \rangle = \frac{l(N-l)}{N} \frac{1}{\sinh^2 \frac{1}{2} x} \ . \tag{19}$$

These results are the same as those of Leamy, Gilmer, and Jackson[2] who discuss them at length. In addition, there are other notable features:

(i) As pointed out by Zittartz and Mueller-Hartman[8] the condition $f(T)/N = 0$ leads to $\sinh x_c = 1$ or $k_B T_c$ equal to the Ising-model transition temperature, for $J_0 = J_2$. The 2-D surface free energy vanishes at $T = T_c$, the transition temperature for the 2-D Ising model. More generally

$$\sinh x_c = 1 + \tfrac{1}{2}(e^{x_c \epsilon} - 1) \ , \tag{20}$$

where $\epsilon = 1 - J_0/J_2$.

(ii) From comparison of Eqs. (16) and (18) we have

$$u(T)/N = J_2 P(T) + J_0 \ , \tag{21}$$

i.e., the internal energy is that associated with the production of *new* surface or perimeter.

(iii) The phase transition at which $f(T_c)/N = 0$ (at T_c it costs no free energy to mix regions of up and down spin) is effected because the positive energy expended to produce perimeter, $J_2 P(T)$, in turn produces a region of disorder in which a free energy of "mixing" works to lower the total free energy. At fixed $T < T_c$ the interface achieves a size determined by the balance between these two competing contributions to the free energy. Finally at T_c the free energy of "mixing" drives the total free energy below zero and the interface becomes unstable.

(iv) In Fig. 2 we show the profile of the surface (with the ends fixed at 0) given by Eq. (19). At $l \simeq \frac{1}{2} N$ the surface fluctuates over a distance of order $\sqrt{N}$;

$$\langle \Delta n^2 \rangle \simeq \tfrac{1}{4} N \frac{1}{\sinh^2 \frac{1}{2} x} \ . \tag{22}$$

In terms of the perimeter the surface profile is given

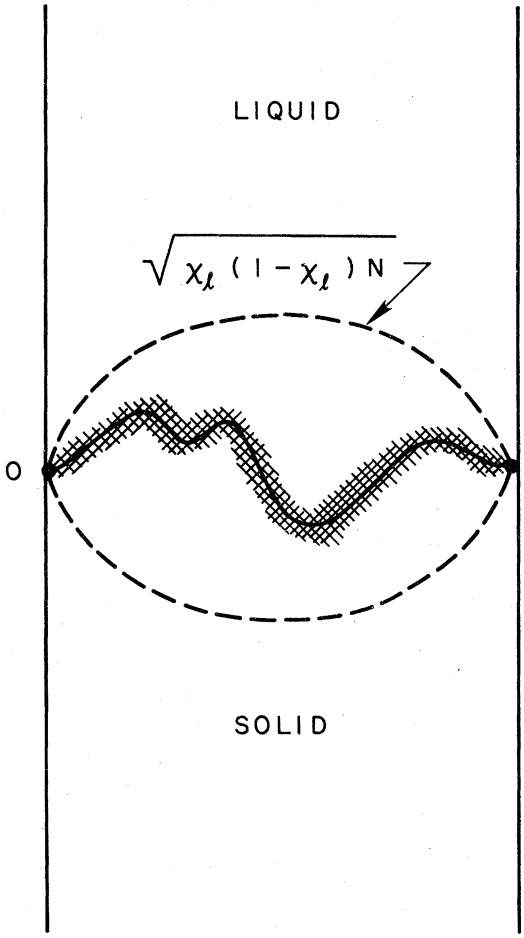

FIG. 2. Location of interface as a function of position. The interface is pinned at $n=0$ at $i=0$, $N+1$. Thermal fluctuations permit it to reside in a region of space of size $N\times\sqrt{N}$ (within the dashed "football") at T_c. However a disordered region, intensive in size, is found in the vicinity of the average position of the interface (cross hatched region).

by $(x_l = l/N)$

$$\langle \Delta n_{l+1}^2 \rangle = N x_l (1-x_l)\,[P/(P^2+N^2)^{1/2} - P] \quad . \quad (23)$$

Upon averaging $\langle \Delta n_{l+1}^2 \rangle$ over the length of the surface, we find

$$\langle \Delta n^2 \rangle_{\rm av} \equiv \sum_l \frac{1}{N} \langle \Delta n_l^2 \rangle = \tfrac{1}{6} N \frac{P}{(P^2+N^2)^{1/2} - P} \quad . \quad (24)$$

At T_c, $P(T_c) = N$, $\langle \Delta n^2 \rangle_{\rm av} = 0.4024N$; at $T \ll T_c$ the perimeter is small compared to N, $P \simeq \exp -\beta J_2$, and $\Delta n^2 \simeq P$; at $T \gg T_c$ the perimeter is large compared to N, $P \sim NT/T_c$, and $\langle \Delta n^2 \rangle_{\rm av} \sim P^2/M$. At T_c there is approximately one unit of perimeter per unit length along the surface [see Fig. 3(b)]. At $T \ll T_c$ there

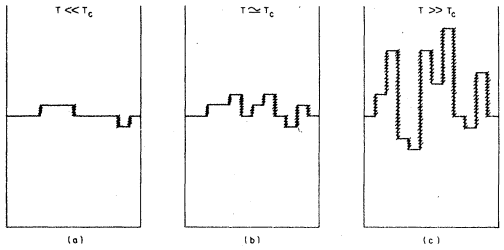

FIG. 3. Qualitative appearance of the interface as a function of temperature. At $T \ll T_c$ there are very few edges per unit of surface, $P \ll N$. At $T = T_c$ there is essentially one edge for each unit of surface, $P \simeq N$. At $T \gg T_c$ there is lots of edge per unit of surface. Of course at $T \gg T_c$ the configuration of the system (melting) differs markedly from those admitted by the SOS model.

are large regions of smooth surfaces between edges [see Fig. 3(a)]. At $T \gg T_c$ the surface is rough [see Fig. 3(c)], i.e., the surface behaves as if it random walked from 0 to N in N steps each of length P/N.

(v) Although the description above is of an interface which occupies a region of size $(\langle \Delta n^2 \rangle_{\rm av})^{1/2} \times N \simeq N^{3/2}$, this is not the region over which disorder occurs in the crystal [otherwise $s(T) \propto N^{3/2}$]. A locally disordered region accompanies the chain as it moves back and forth between the extremes measured by $\langle \Delta n_{l+1}^2 \rangle$ (illustrated in Fig. 2); a properly extensive entropy is associated with the locally disordered region.

(vi) The notion of "rough" and "smooth" are well defined for this two-dimensional model and have meaning with respect to the relative motion of columns separated by distance Δ. A measure of the correlation in the height of nearby columns is given by

$$C_l(\Delta) \equiv \langle n_{l+1} n_{l+\Delta+l} \rangle - \langle n_{l+1}^2 \rangle = x_l \Delta (\sinh^2 \tfrac{1}{2} x)^{-1} \quad .$$

In the middle of the surface, $x_l = \frac{1}{2}$, at $T \ll T_c$ the columns are correlated over $\Delta \simeq \exp +\beta J_2$; at $T = T_c$ the columns are correlated at most over a distance of order 1. The surface has properties at $T < T_c$ and $T > T_c$ that are reasonably described by "smooth" and "rough." There is no "roughening" transition but rather a rapid change in the length scale over which column height correlations occur for T near T_c.

The results we described here are for a two-dimensional system with finite but large N. For a 1-cm long interface $N \simeq 10^8$, $(\langle \Delta n^2 \rangle) \simeq 10^4$. (Corrections to the results are of order 10^{-4}.[9])

One might imagine measuring some of the properties described here in the ^{3}He on Grafoil system using NMR techniques.

III. SINE-GORDON MODEL OF THE TWO-DIMENSIONAL INTERFACE

For the sine-Gordon model of the two-dimensional interface we take the energy to be given by

$$E(\theta_1,\ldots,\theta_{N+1}) = \sum_{i=1}^{N} [\tfrac{1}{2} E_2(\theta_{i+1}-\theta_i)^2 - E_1\cos\theta_i] \ , \quad (25)$$

where θ_i, the phase of pendulum i, is a continuous variable identified with the column height, $n_i \leftrightarrow \theta_i/2\pi$, and the $\cos\theta_i$ potential causes θ_i to prefer value $2\pi n$. The torsion spring, of strength E_2, attempts to keep θ_{i+1} equal to θ_i and plays a role qualitatively similar to J_2 in Eq. (1). The sine-Gordon model of the interface permits an alternative picture of the statics and dynamics of the interface,[5-7] e.g., the dynamics of the advance of the interface maps onto the phase evolution of the sine-Gordon chain.[6] Equation (25) corresponds to a discrete Gaussian model,[2,3] in which the discreteness is enforced preferentially but not exactly. In analogy with Eq. (2) the surface free energy is

$$f(T) = (N+1)J_0 - k_B T \ln Z_{SG} \ , \quad (26)$$

where

$$Z_{SG} = \frac{1}{(2\pi)^{N+1}} \int d\theta_1 \cdots d\theta_{N+1} \delta(\theta_1)\delta(\theta_{N+1}-\phi) \times \exp[-\beta E(\theta_1,\ldots,\theta_{N+1})] \ , \quad (27)$$

where ϕ is the total phase evolution along the chain. In place of the transfer-matrix techniques used above, transfer-integral techniques are employed. Instead of Eqs. (8)–(12) we have

$$T(\theta_2\theta_1) = e^{\lambda_1\cos\theta_1} e^{-(\lambda_2/2)(\theta_2-\theta_1)^2} \ , \quad (28)$$

where $\lambda_1 = \beta E_1$ and $\lambda_2 = \beta E_2$. This transfer-integral problem involves a Fredholm equation of the second kind with polar kernel and generates the complete set of states

$$\begin{aligned} &\int d\theta_2 T(\theta_2\theta_1)\psi_\nu(\theta_2) = e^{-\beta\epsilon_\nu}\psi_\nu(\theta_1) \ , \\ &\int d\theta \phi_\nu(\theta)\psi_\mu(\theta) = \delta_{\nu,\mu} \ , \\ &\sum_\nu \phi_\nu(\theta)\psi_\nu(\theta') = \delta(\theta-\theta') \ , \end{aligned} \quad (29)$$

where $\phi_\nu(\theta_1)$ is the left-hand eigenfunction of $T(\theta_2\theta_1)$. We obtain

$$Z_{SG}(N;\lambda_1\lambda_2) = \sum_\nu \phi_\nu(\Phi)\psi_\nu(0)e^{-N\beta\epsilon_\nu} \quad (30)$$

and

$$\langle F(\theta_{l+1})\rangle = Z_{SG}^{-1} \sum_{\mu\nu} \phi_\nu(\Phi)\psi_\mu(0)F_{\mu\nu}e^{-N\beta\epsilon_\mu} \times e^{-l\beta(\epsilon_\nu-\epsilon_\mu)} \ , \quad (31)$$

where

$$F_{\mu\nu} = \int d\theta \, \phi_\mu(\theta)F(\theta)\psi_\nu(\theta) \ . \quad (32)$$

Equations (28)–(32) have been studied extensively both analytically[5-7] and numerically.[10] For purposes of describing the essential elements of the physics the sine-Gordon model can be treated in the continuum approximation in which analytic methods are more readily applicable.[11] In the continuum approximation we replace the discrete chain by a continuous line of uniform density. Then, the transfer integral reduces to the differential equation

$$\left(-\frac{1}{2\beta^2 E_2}\frac{d^2}{d\theta^2} - E_1\cos\theta\right)\psi_\nu(\theta) = \epsilon_\nu\psi_\nu(\theta) \ , \quad (33)$$

where $\epsilon_\nu = e_\nu - \frac{1}{2}\ln(2\pi\lambda_2)$, $\phi_\nu = \psi_\nu^*$, and the partition function is

$$Z_{SG} = \sum_\nu \psi_\nu(\Phi)\psi_\nu(0)e^{-N\beta\epsilon_\nu} \ . \quad (34)$$

By describing the interface by the sine-Gordon equation in the continuum approximation we have done away with discreteness in the second dimension. (The use of $E_1\cos\theta$ removes discreteness in the motion of each column; the continuum approximation replaces the column to column discreteness by a continuous interface.) We have gone to the continuum approximation to be able to use the analytic description of the transfer integral provided by the Mathieu equation, Eq. (33). It is known that the basic features found in the description of the physics in this approximation are retained as one goes away from it.[5,10]

The results of a sine-Gordon chain description of the interface are [although it is not necessary we again work at $\phi = 0$ (Ref. 9)]:

(i) The Helmholtz free energy vanishes at T_c given by

$$f(T) = J_0 + \epsilon_0 = 0 \ , \quad (35)$$

where $\epsilon_0 = e_0 - \frac{1}{2}[k_B T\ln(2\pi\lambda_2)]$ and e_0 is the ground-state eigenvalue of Eq. (33). In Fig. 4 we plot ϵ_0 as a function of β^*. For the choice $J_0 = \pi^2 E_2$ we have $f(T) = 0$ for $k_B T_c = 1.88 J_0$ which is in reasonable agreement with the Ising model ($2.27J_0$) and discrete Gaussian model ($1.74J_0$).[12,13]

(ii) At low temperatures where the tight-binding approximation is valid for ψ_ν in Eq. (33) we have

$$\langle\theta_{l+1}^2\rangle - \langle\theta_{l+1}\rangle^2 = \frac{l(l-N)}{N^2}N(T) \ , \quad (36)$$

where

$$N(T) = N\beta|t_0|\exp(-\beta E_\phi) \ ,$$

[$|t_0| = 4(E_1/E_2)^{1/2}$ and $E_\phi = 8(E_1E_2)^{1/2}$]; the number

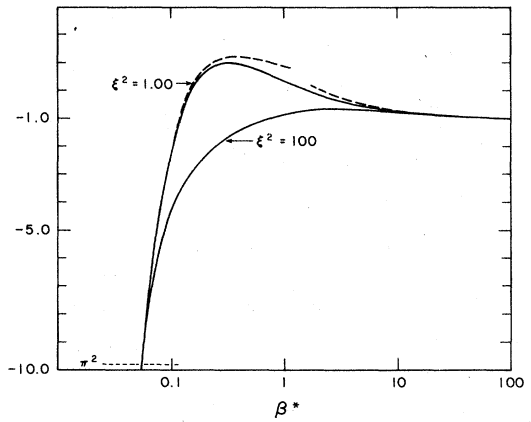

FIG. 4. Ground-state energy eigenvalue as a function of temperature. The ground-state energy eigenvalue of Eq. (29), in units of E_1 is plotted as a function of $\beta^* = \beta E_1$. For $E_2 = J_0/\pi^2$ the free energy in Eq. (35) vanishes at $\epsilon_0/E_1 = -\pi^2$. This occurs at $\beta^* \simeq 0.05$. The heavy line labeled $\xi^2 = 1$ is from numerical solution to the transfer-integral equation. The dashed lines are analytic asymptotes from solution to the corresponding Mathieu equation, Eq. (33). At $\xi^2 > 1$ the Mathieu equation becomes a better and better approximation to the exact transfer integral (in the relevant β^* range the Mathieu equation is already very good at $\xi^2 = 1.0$). For $1 \leq \xi \leq 1.00 (\xi^2 = E_2/E_1)$ the behavior of ϵ_0/E_1 is relatively insensitive to ξ^2.

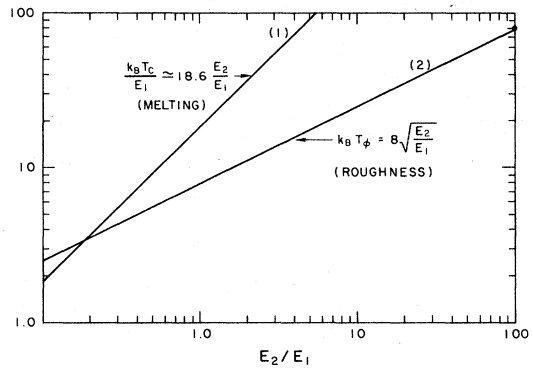

FIG. 5. Melting temperature [from Eq. (35)] and the temperature that separates smoothness from roughness, $k_B T = 8(E_1 E_2)^{1/2}$, as a function of E_2/E_1.

$N(T)$ is the number of thermally activated kinks or solitons on the chain.[6] In the column model the interface fluctuates and advances in a gradient by the creation and motion of edge; in the sine-Gordon chain model of the interface fluctuations correspond to the creation of pairs of kinks and the interface advances through the motion of these kinks (or solitons). The solitons are the sine-Gordon chain equivalent of an edge. The total perimeter on the chain is

$$P \simeq N(T)\xi \ , \tag{37}$$

where $\xi = (E_2/E_1)^{1/2}$ is a measure of the size of a "kink." The entire chain is covered with perimeter at $N(T)$ such that

$$N(T)\xi \simeq N \tag{38}$$

or at $k_B T \simeq E_\phi = 8(E_2 E_1)^{1/2}$. The energy E_ϕ, the energy of kink formation, is a measure of the energy required to form an edge (or one unit of perimeter). At $k_B T >> E_\phi$ the number of kinks on the chain is large $[N(T)\xi > N]$ so that the kink concept, useful in a description of low-temperature behavior, becomes useless. At $k_B T >> E_\phi$ the surface has no edgelike features because the energy to create these is much less than $k_B T$. In this limit the interface should be termed "rough." At $k_B T << E_\phi$ the interface is smooth; there are few kinks on the chain. The growth and fluctuations of the interface are governed by the creation of kink pairs (soliton-antisoliton pairs) and their motion. We identify E_ϕ with the temperature at which smooth to rough behavior occurs. We can compare this to the melting temperature [the temperature at which $f(T)/N = 0$, and at which the notion of a surface fails]. This is done in Fig. 5 as a function of E_2/E_1. We note from Fig. 5 that melting and roughness are characterized by two different temperatures. In the latter, roughness occurs before melting for $E_2/E_1 \gtrsim 0.20$.

Certainly this result is a consequence of the physics we have used. The discrete Gaussian model corresponds to $E_2/E_1 \to 0$. In the limit $E_2/E_1 \to 0$ the kink in our model becomes meaningless and the model should be replaced by the DG model. By keeping E_2/E_1 finite we maintain that there are two energy scales: (i) E_ϕ, the energy to create an edge and thus the energy that determines the temperature at which enough edges are present to cause roughness and (ii) E_2, the energy required to move neighboring columns with respect to one another and thus the energy that determines the melting temperature.

We may gain further insight into the nature of the behavior of the surface in a growth process by examining the response of the interface to an external field that drives it forward. In the column model such an external field is a chemical potential gradient[2]

$$-\Delta\mu \sum_{i=1}^{N+1} n_i \ . \tag{39}$$

In the sine-Gordon chain model such an external field is a constant torque on each pendulum[6,7]

$$-E_0 \sum_{i=1}^{N+1} \theta_i \ . \tag{40}$$

The response of the sine-Gordon chain to an external field given by Eq. (30) has been described in the heavy damping limit using the Smolouchowski equation.[7] The behavior of the "current" ("current" is the phase evolution in the sine-Gordon chain model; "current" is the rate of column motion in the column

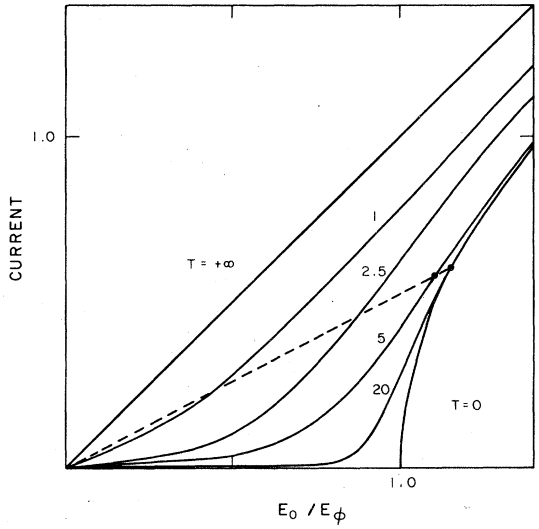

FIG. 6. Current as a function of temperature and external field. The current (as a ratio of its value at T to its value at very high temperature) is plotted as a function of field, E_0/E_ϕ, for several values of temperature (measured in units of $\beta^* = \beta E_\phi$). This plot follows from the demonstration by Guyer and Miller (Ref. 7) (that the current as a function of field and temperature on the SGC is essentially the same as that which flows in a single-particle system with a field-dependent barrier) and use of the results of V. Amegaokar and B. I. Halperin, Phys. Rev. Lett. 22, 1364 (1969).

model) as a function of temperature is shown in Fig. 6. The notable features of the results are: (a) At $T=0$ no current flows (the interface does not move) until $E_0 = E_1$. (b) At $T \neq 0$, $E_0 << E_1$, $k_B T << E_\phi$ the current is due to spontaneous kink pair creation and subsequent kink motion *along* the chain (the interface advances by edge creation and edge motion). See Fig. 7. (c) At $k_B T > E_\phi$ the current is due to large regions of the chain of no particular size or structure moving forward as a unit.

It is the qualitative difference between this case and (b) above that is described by Burton and Cabrera[14] and to which the notion of smoothness and roughness are applied.

The roughening temperature, the temperature at which the current is one-half of its high-temperature value for a given E_0, is a measure of the temperature at which the interface motion evolves from domination by edge motion at lower T to domination by gross motions at higher T (see Fig. 6). As $E_0/E_1 \to 0$ this temperature approaches E_ϕ. This is in agreement with previous discussion of the sine-Gordon chain; the qualitative properties of current evolution are characterized by the energy required to create a kink. Thus the surface responds *smoothly* or *roughly* to an external field at $k_B T \lesssim E$ and $k_B T \gtrsim E_\phi$.

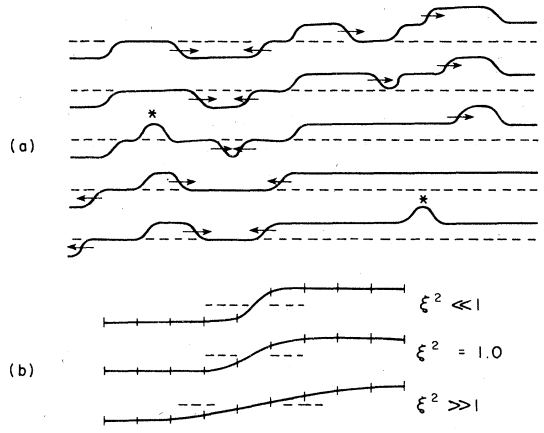

FIG. 7. Interface evolution as a function of time. Surface evolution is due to kink creation, motion, destruction, etc. (a). The size of the edges (kinks) is set by the ratio E_2/E_1 (b).

IV. CONCLUSIONS

We have used the sine-Gordon chain (SGC) as a model for the interface between a liquid and solid (as a model of the SOS model). In the SGC model the column height discreteness of the DG model is replaced by continuous (but preferentially integral) height and the column to column discreteness is replaced by a continuous interface. The role of edges in the SOS model is played by the solitons of the SGC. In the SGC model the energy scale for edge creation (soliton creation) differs from the energy scale for column to column disorder. It is the concentration of edges on the interface that determine the qualitative properties of the interface, e.g., its smoothness or roughness, its response to a chemical potential gradient, etc. It is the column to column motion that leads to the disorder which finally makes it energetically preferable for the system to create more surface, i.e., for the interface to breakup and for the solid to "melt."

Use of the SGC model of the two-dimensional interface permits one to think about edge and interface dynamics in the way in which one thinks about the dynamics of the SGC. For example, is it possible that the $t^{4/3}$ time evolution observed by Schneider and Stoll[4] will occur in a physical system?

ACKNOWLEDGMENTS

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