Comparison of Keldysh and perturbation formulas for one-photon absorption

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(Received 12 April 1979)

It is shown that the one-photon absorption edge in direct-gap crystals predicted by the Keldysh formula agrees with the results of conventional perturbation theory to within a factor of order unity. This is in contrast to the frequently stated opinion that the Keldysh formula is incorrect or invalid when the photon multiplicity is small.

I. INTRODUCTION

Keldysh[1] derived an expression for the electronic transition rate between the valence and conduction bands of a direct-gap crystalline solid, when the electron simultaneously absorbs an arbitrary number of photons from an intense electromagnetic radiation incident on the crystal. He assumed that the transitions took place not between the unperturbed Bloch states as in the conventional perturbation theory, but between the Stark-shifted initial and final states, and that the electronic wave functions were described by the Houston functions.[2]

In obtaining his final expression for the multiphoton transition probability Keldysh assumed that the ratio of the photon energy to the band-gap energy was "sufficiently small." This assumption has given rise to considerable misunderstanding about the Keldysh formula. Many workers[3–7] have stated, without amplification, that the Keldysh formula is inaccurate or even invalid, when the photon multiplicity is less than four. However, if one follows the Keldysh derivation, it is not clear how small the photon energy should be compared to the band-gap energy, in order to obtain Kelysh's final result. The smallness of the photon energy compared to the gap energy is used only in the evaluation of certain integrals [Eqs. (15) and (29) of Ref. 1] by the saddle-point method. While the accuracy of the saddle-point method may increase with decreasing photon energy,[8] it is not clear how much error is introduced if the above energy is comparable to the gap energy. In the following, we show that the one-photon absorption edge resulting from Keldysh's formula agrees with that predicted by conventional first-order perturbation theory to within a factor of the order of unity.

II. RESULTS

Under Keldysh's stated conditions of applicability of his formula, the "effective band gap" in the presence of the electromagnetic radiation can be replaced by the field-free band gap, and the term proportional to the intensity of the radiation that occurs in the exponential function can be omitted. Close to the one-photon absorption edge, $\hbar\omega \simeq E_g$, only the first term in the series expansion of the Dawson integral[9] encountered in the Keldysh formula need be retained. With these approximations, close to the one-photon absorption edge, the Keldysh transition rate per unit volume [Eq. (41) of Ref. 1] becomes

$$W \simeq \left(\frac{2\omega}{9\pi}\right)\left(\frac{m^*\omega}{\hbar}\right)^{3/2}\left(\frac{e^2E_0^2}{16m^*\omega^2E_g}\right) \times \left(2-\frac{2E_g}{\hbar\omega}\right)^{1/2} \exp 2 \ . \tag{1}$$

The resulting one-photon absorption coefficient is thus

$$\alpha_K(\hbar\omega \simeq E_g) \simeq \frac{2^{3/2}e^2\sqrt{m^*}\ 7.389}{c\,\hbar^2\sqrt{\epsilon_\infty}9}(\hbar\omega - E_g)^{1/2} \ , \tag{2}$$

while the corresponding one-photon absorption edge obtained from conventional first-order perturbation theory is[10]

$$\alpha_p(\hbar\omega \simeq E_g) \simeq \frac{2^{3/2}e^2\sqrt{m^*}}{c\,\hbar^2\sqrt{\epsilon_\infty}}(\hbar\omega - E_g)^{1/2} \ . \tag{3}$$

From Eqs. (2) and (3) we obtain

$$\frac{\alpha_K}{\alpha_p}(\hbar\omega \simeq E_g) \simeq 0.82 \ . \tag{4}$$

III. CONCLUSION

The Keldysh one-photon absorption edge agrees very well with that predicted by conventional perturbation theory. It is quite possible that Keldysh's use of Houston functions instead of the ordinary Bloch functions for the electronic wave functions increased the apparent "accuracy" of his derivation and this increased accuracy compensated for the error introduced by his use of the saddle-point method. In fact, Narducci *et al.*[11] had earlier noted that the Keldysh formula accurately predicted the frequency dependence and numerical values of the one-photon absorption coefficients in GaAs and InSb both at the absorption edges and away from them. The accuracy of the Keldysh formula could probably be further improved by replacing the saddle-point integrations in his derivation by exact integrations.

ACKNOWLEDGMENT

One of the authors (A. V.) expresses his sincere gratitude to Dr. Ed. Fenton of NRC, Canada for making available his notes on the Keldysh formula, and for valuable discussions. One of the authors (A. V.) would like to thank NRC, USA for a postdoctoral resident Research Associateship.

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