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Direct and recoil-induced electron emission from ion-bombarded solids

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The kinetic emission of secondary electrons from ion-bombarded solid surfaces is split into two contributions, a direct one caused by ionizing collisions between the bombarding ion and target atoms, and an indirect one originating from ionizing collisions undergone by recoil atoms with other target atoms. The direct contribution, which has been treated by several authors in previous studies, shows a behavior that is determined primarily by the electronic stopping power of the bombarding ion, while the indirect contribution is nonproportionally related to the nuclear stopping power. This latter contribution is known to be quite important' for heavy-ion bombardment at keV energies, and is shown to be of crucial importance for the understanding of the energy dependence of the electron yield in such cases. The model is shown to give consistent results for copper bombarded with electrons, protons, and noble-gas ions within the accuracy of the treatment.

I. INTRODUCTION

It is well known that the interaction of ionizing radiation with solid surfaces can lead to emission of electrons. In the case of charged-particle bombardment, this phenomenon is most often called secondary-electron emission. The present study deals with the particular case of energetic-ion bombardment.

Conventionally, one distinguishes between potential and kinetic secondary-electron emission for this type of bombardment. Potential emission is associated with the neutralization of an impinging ion before it reaches the surface, and is therefore rather independent of the ion energy as long as the ion velocity is lower than the relevant atomic electron velocities. ' Kinetic emission is associated with the ionizing collisions taking place in the target; the dependence on ion energy is determined by the frequency of ionizing collisions.² In particular, there is a more or less well-defined threshold toward low energies and a gradual drop-off at high velocities.

While the distinction between surface and bulk processes in electron emission appears to be a reasonable approach in the qualitative analysis, it is not obvious whether a separation into *potential* and *kinetic* emission can always be made. Indeed, ionizing collisions undergone by heavy ions at moderate velocities may be determined by electron promotion processes³ not unlike those proposed for ion neutralization at

the surface.¹ The following treatment concentrates on bulk phenomena and, where comparison is made with experimental results, those cases are considered where potential emission is a minor effect.

It is evident that the yield of emitted electrons is closely connected with the deposition of electron excitation energy near the target surface. This in turn, α according to Sternglass,² is determined primarily by the electronic stopping power of the bombarding ion, with some possible modification due to the transport of energy by energetic electrons.

The Sternglass theory refers to a range of projectile velocities with negligible nuclear stopping and ignores nuclear scattering. However, it is well known that at moderate and low projectile velocities, energy deposition profiles may be distorted drastically by the effect of nuclear scattering. ⁴ Even more important, in the region where nuclear stopping dominates, electronic excitation may be substantially due to recoil atoms, in particular for ions of high atomic number.^{5,6} Therefore, it is necessary to consider the effect of recoilinduced secondary-electron emission, whenever bombardment is done under conditions where a substantial fraction of the energy loss of the bombarding particle is due to nuclear stopping. The theory of Parilis and Kishinevskii,⁷ a common reference standard for low-velocity ion-electron emission, does not take account of recoil-induced processes in its original form, and consequently predicts a strict proportionality of the electron yield with the electronic stopping power

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(at velocities well above threshold). As pointed out by Dorozhkin et al. 8 for bombardment with mercury. ions, this neglect causes drastic differences between the predicted and the measured variation of the electron yield with the atomic number of the target. A theoretical treatment on the basis of a simple stopping model was subsequently given by Vinokurov $et \ al.⁹$

In the following, an attempt is made to formulate an expression for the electron yield in general terms familiar from stopping theory which takes account of both direct and indirect processes as well as scattering of the bombarding particle and energy transport by recoil atoms. While the theory is too crude in its present stage to properly account for energy transport by energetic secondary electrons, it does reveal the influence of recoil processes on the dependence of the electron yield on the mass and energy of the bombarding ion.

II. BASIC RELATIONSHIP

The starting point is the following relation:

 (1) $\gamma = \Lambda D$.

where γ is the secondary-electron yield (average number of emitted electrons per bombarding ion), A is a constant depending on the properties of the material, in particular the surface, and D is the mean energy per unit depth, deposited in electronic excitation at the surface by a bombarding ion. Excitation energy is considered "deposited" if the level of excitation is sufficiently low so that further relocation can be ignored. Thus, energy transport by energetic secondary electrons as well as recoiling atoms is included in the definition of D . In general, D depends on the atomic numbers and masses of ion and target atoms, as well as the energy and angle of incidence of the bombarding ion. (For crystalline targets, D would also depend on the orientation of the surface relative to the ion beam.)

^A relationship analogous to Eq. (I) was previously derived explicitly for the sputtering yield, 10 in which case nuclear quantities replace the corresponding electronic ones. A transport theoretical calculation along similar lines has been performed by one of the present authors to justify Eq. (1) for the secondaryelectron vield.^{11} Details of this calculation will be published separately. The main assumptions underlying the calculation are presumably more restrictive than those necessary for Eq. (I) to be valid, since simple binary-collision dynamics was assumed in the transport calculation for collisions between moving particles (ions, recoil atoms, or electrons) and electrons. This assumption, which is reasonably well justified in the analogous case of cascades of elastic nuclear collisions, breaks down for heavy-ion bombardment except at particle velocities well above the

fission-fragment region. Therefore, it may be more appropriate in the present context to find a physical argument for the feasibility of Eq. (I). The essence of this argument is the same as that in Refs. 2 and 7, although the conclusion is formulated in more general terms.

In order for Eq. (1) to be approximately valid, it is of crucial importance tnat the vast majority of emitted electrons have very low energies, of the order of the work function of the material, i.e., a few eV. Quite different' relationships would have to be expected if the high-energy tail of the spectrum were considered.

It is well known from the theory of multiple ionization phenomena, as well as the theory of radiation damage, that the amount of some cumulative radiation effect is roughly proportional to the available energy' at primary energies far above threshold, i.e., when cascade multiplication is important. In this region the respective cross section is unimportant, while near threshold the ionization or damage production rate hinges on the respective cross section. Thus, if a primary ion excites electrons to a sufficiently high level so that secondary excitation by primary electrons contributes substantially to the total flux of low-energy electrons in the bombarded target, then this flux hinges on the available energy, i.e., the energy deposited in a surface layer with a thickness of the order of the range of low-energy electrons.

Conversely, if the primary radiation creates only low-energy excitation, so that secondary excitations by primary electrons become unimportant for electron emission, then the excitation cross section rather than the deposited energy determines the electron yield, and Eq. (I) is inapplicable.

In case of substantial electron excitation by recoil atoms, the above requirement needs to be strengthened in the sense that also electrons excited by recoil atoms should have sufficiently high energy to cause an appreciable amount of secondary excitation for Eq. (I) to be valid. m for Eq. (1) to be valid.
Quantitative estimates in radiation damage¹² and

ionization¹³ theory indicate that the gap between the two limiting regimes, near threshold and far above threshold, respectively, is rather small.

The question of whether or not the above assumptions are fulfilled is tested most directly by inspection of the energy spectra of emitted electrons. Indeed, in order that Eq. (1) be valid, a spectrum should exhibit the usual tail, known from electron-electron emission,¹⁴ extending up to at least a few times the worl function of the material. The functional shape of this tail is determined mainly by the electron stopping power at low energies.¹¹

Mechanisms of excitation to energy 1evels far above the binary collision limit are weil established in above the binary collision limit are well established
the case of ion-atom collisions.¹⁵ Indeed, wheneve an ion-atom or atom-atom system produces x rays,

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Auger electrons will be produced at a rate depending on the fluorescence yield. In the present context, such Auger electrons contribute to the yield mainly by exciting secondary electrons inside the target. This does not preclude a substantial fraction of lowenergy electrons to arise from Auger processes.⁷

In the present investigation, the constant Λ will be treated as a material parameter, the value of which is to be determined experimentally. In a subsequent pato be determined experimentally. In a subsequent
per,¹¹ it will be shown that the absolute number of emitted electrons agrees well with the prediction based on transport theory, so long as those parts of the energy spectrum of the electrons are concerned for which reasonably certain values for the electron stopping power are available,

The deposited-energy density can be written in the form

$$
D = D(x, E, \cos \phi) \tag{2}
$$

where x is the depth inside the target, E is the initial ion energy, and ϕ is the angle of incidence; the quantity entering into Eq. (1) is $D(x=0,E,\cos\phi)$. If D is integrated over the depth variable, we obtain the total amount of energy per ion that ends up in electronic excitation within the target

$$
\eta^*(E, \cos \phi) = \int_0^\infty dx \ D(x, E, \cos \phi) \ . \tag{3}
$$

Apart from the loss of energy through the target surface, this is identical with the quantity $n(E)$ introduced by Lindhard et al .⁵

If, as a first approximation, the complications brought about by the presence of the target surface are ignored and the target is considered as imbedded into an infinite medium with a reference plane at $x = 0$, then the function $D(x, E, \cos \phi)$ is determined by (i) the trajectory of the impinging ion and its cross section for ionizing collisions; (ii) the number and energy distribution of recoil atoms, their trajectories, and their cross section for ionizing collisions; (iii) the trajectories of excited electrons.

Upon integration, cf. Eq. (3), the spatial coordinate drops out, and $\eta(E)$ becomes dependent essentially on the relative significance of the cross sections for ionizing collisions and elastic nuclear collisions.⁵

Accurate estimates of $\eta(E)$ are available.^{5,16} With regard to $D(x,E,\cos\phi)$, computations have been performed by Brice¹⁷ and Winterbon.¹⁶ In both sets of computations, energy transport by electrons is ignored. Energy transport by recoil atoms is included explicitly in Winterbon's work where it is important; the effect is treated with much less rigor in Brice's work, but it is also less important there since Brice considers higher ion velocities.

Neither author tried to explicitly separate the contributions due to electronic and nuclear stopping of the ion, according to

$$
D = D_{(p)} + D_{(r)} , \t\t(4)
$$

where $D_{(p)}$ is the excitation profile resulting from ionizing collisions undergone by the bombarding ion (including secondary collisions undergone by the excited electrons), and $D_{(r)}$ is the excitation profile resulting from ionizing collisions undergone by recoil atoms set in motion by either the bombarding ion or recoil atoms (also including secondary collisions undergone by the excited electrons).

Since $D_{(p)}$ is determined essentially by the electronic stopping power of the ion, while $D(r)$ hinges on the nuclear stopping power and the stopping parameters of recoiling atoms, the two quantities must exhibit very different dependencies on ion energy. It is this feature that is the subject of exploration in the present investigation. From this point of view, it is not important whether the calculations are done accurately enough to explicitly include energy transport by excited electrons, since this factor is common to both $D_{(p)}$ and $D_{(r)}$. This effect is ignored but mention is made that in the transport calculation applicable to high-ion velocities, $¹¹$ it is shown explicitly to be a</sup> minor effect with regard to the determination of the profile.

For similar reasons, the effect of a real surface on the energy deposition profile is ignored, as it is noted that a major part of this effect is common to both contributions to Eq. (4).

The splitting of the deposited-energy function according to Eq. (4) is also called for by a more technical consideration, i.e., construction of the depth profile $D(x)$. Indeed the two *profiles* $D_{(p)}(x)$ and $D_{(r)}(x)$ can be expected to behave quite differently in their variation with depth; while $D_{(p)}$ is most con-
veniently determined by "direct" methods.¹⁷ reliable veniently determined by "direct" methods, 17 reliable values for $D_{(r)}$ are more likely to result from mo-
ment methods.^{4, 16} Thus, construction of the *sum* of the two profiles by *either* method may lead to addithe two profiles by *either* method may lead to addi-
tional uncertainties^{16,17} that can readily be avoided by constructing the two contributions separately.

III. DETAILS

Within the range of validity of linear transport theory, i.e., for not too high density of energy deposition, the usual transport equation⁴ is obeyed by $D(x,E,\cos\phi)$,

$$
-\cos\phi \frac{\partial D}{\partial x} = N \int d\sigma \left(D - D' - D'' - \sum D''_e \right), \quad (5)
$$

where N is the number of atoms per unit volume, $d\sigma$ is the differential cross section, D_t is the deposited energy density for a moving target atom, D_e is the corresponding quantity for a moving electron,

and

$$
D' = D\Big(x, E - T_n - \sum T_{ei}, \cos\phi'\Big) , \qquad (6a)
$$

$$
D_t'' = D_t(x, T_n, \cos \phi'') \tag{6b}
$$

$$
\sum D_e'' = \sum_i D_e(x, T_{ei}, \cos \phi'')
$$
, (6c)

 T_n and T_{ei} being the energy transferred to the nucleus and an individual electron, respectively, in the collision with an atom. The notation is similar to that of Lindhard et al.⁵ Analogous (simpler) equation of Lindhard *et al.*⁵ Analogous (simpler) equation
are satisfied by D_t and D_e .^{5,11} ϕ' and ϕ'' are angles made by the direction of motion of the respective particle and the x axis.

If the usual separation⁵ is made between electronic and nuclear collisions, Eq. (5) reduces to the form

$$
-\cos\phi \frac{\partial D}{\partial x} = N \int d\sigma_n (D - D' - D''_t)
$$

$$
+ N \int d\sigma_e \Big[D - D' - \sum D'_e \Big] , \qquad (7)
$$

which is an essential simplification in that the separate cross sections $d\sigma_n$ and $d\sigma_e$ for nuclear and electronic collisions enter the picture.

The conventional method of moments is utilized in the solution of Eq. (7). The explicit equations are of the well-known type⁴ and will not be repeated here. It is important, however, to note an explicit difference between the treatment of zero- and higher-order moments.

Integrating over the spatial coordinate we obtain from Eq. (7)

$$
0 = N \int d\sigma_n (\eta - \eta' - \eta''_t) + N S_e \left(\frac{d\eta}{dE} - 1 \right), \qquad (8)
$$

where

$$
\eta = \int_{-\infty}^{\infty} dx \ D \tag{9a}
$$

$$
\eta_t = \int_{-\infty}^{\infty} dx \ D_t \ , \tag{9b}
$$

$$
\sum T_{ei} = \int_{-\infty}^{\infty} dx \sum D_e^{''};
$$
 (9c)

the last relationship means that practically all elec-

tronic excitation energy ends up in electronic motion. Furthermore, in Eq. (8)

$$
S_e = \int d\sigma_e \sum T_{ei} \tag{10}
$$

Finally, the following approximation has been made:

$$
\mathbf{y}, \text{ the following approximation has been made:}
$$
\n
$$
\eta' = \eta \Big(E - \sum T_{ei} \Big) \cong \eta(E) - \sum T_{ei} \frac{d\eta}{dE} \,. \tag{11}
$$

Equation (8) is identical with the one applied by Lindhard et al.⁵

With regard to higher moments, an additional simplification arises from the neglect of energy transport by electrons. Then, with the exception of the nor-

malization, Eq. (8), the term $\sum D_e^{\prime\prime}$ in Eq. (7) may be disregarded.

Consider next the function $D_{(p)}$ introduced in Eq. (4). According to its definition, it has to obey the following transport equation:

$$
-\cos\phi \frac{\partial D_{(p)}}{\partial x} = N \int d\sigma_n (D_{(p)} - D'_{(p)})
$$

$$
+ N \int d\sigma_e \left[D_{(p)} - D'_{(p)} - \sum D''_e \right],
$$
(12a)

which differs from Eq. (7) only by the absence of the recoil term D_t'' . On the other hand, the density $D_{(r)}$ in Eq. (4) obeys the complementary equation

$$
-\cos\phi \frac{\partial D_{(r)}}{\partial x} = N \int d\sigma_n (D_{(r)} - D'_{(r)} - D''_1)
$$

(7)
$$
+ N \int d\sigma_e (D_{(r)} - D'_{(r)}) . \quad (12b)
$$

Adding Eqs. (12a) to (12b) one readily obtains Eq. (7), observing Eq. (4).

The corresponding relationships for the zero-order moments read

$$
0 = N \int d\sigma_n \left(\eta_{(p)} - \eta'_{(p)} \right) + N S_e \left(\frac{d\eta_{(p)}}{dE} - 1 \right), \qquad (13a)
$$

$$
0 = N \int d\sigma_n \left(\eta_{(r)} - \eta'_{(r)} - \eta''_{(l)} \right) + N S_e \frac{d\eta_{(r)}}{dE} \quad . \tag{13b}
$$

If the following rough approximation⁵ is applied,

$$
\eta' = \eta (E - T_n) \cong \eta(E) - T_n \frac{d\eta}{dE} \tag{14}
$$

to both $\eta_{(p)}$ and $\eta_{(r)}$, it is found that

$$
\eta(\rho)(E) = \int_0^E dE' \frac{S_e(E')}{S_e(E') + S_n(E')}
$$
 (15a)

and

$$
\eta_{(p)}(E) = \int_0^E dE \frac{\zeta_{\epsilon}(E') + S_n(E')}{S_{\epsilon}(E') + S_n(E')}
$$
\n
$$
\eta_{(r)}(E) = \int_0^E dE' \frac{\int_0^{YE'} d\sigma_n(E', T_n) \eta_t(T_n)}{S_{\epsilon}(E') + S_n(E')}
$$
\n(15b)

with $\gamma = 4M_1M_2(M_1 + M_2)^{-2}$, M_1 and M_2 being masses of ion and target atom, respectively, and $S_n = \int d\sigma_n T_n$ is the nuclear stopping cross section. Since $N(S_e + S_n)$ is the total stopping power, Eq. (15a) can be rewritten in the form

$$
\eta_{(p)}(E) = \int_0^R dx' \, NS_e(E(x')) \quad , \tag{15c}
$$

where R is the total path length. Thus, except for corrections due to straggling and scattering, the term $D_{(p)}$ in Eq. (4) is proportional to the electronic stopping power of the ion. It is obvious that this form dominates in the case of dominating electronic stop-

ping, $S_e \gg S_n$.

Let us consider the case of dominating nuclear stopping, and approximate the cross section by the usual power form^{4,5}

$$
d\sigma_n = CE^{-m}T_n^{-1-m}dT_n , \quad 0 \le T_n \le \gamma E , \qquad (16a)
$$

where C is a well-defined constant, and $0 < m < 1$. Assume some relation

$$
\eta_t(T_n) = k_\omega(T_n)^\omega \tag{16b}
$$

for the moment, with unspecified parameters ω and k_{ω} . Then, Eq. (15b) reads

$$
\eta_{(r)}(E) = \frac{1 - m}{\omega(\omega - m)} \gamma^{\omega - 1} k_{\omega} E^{\omega}
$$

$$
= \frac{1 - m}{\omega(\omega - m)} \eta_{r}(\gamma E) . \tag{17}
$$

Thus, the amount of energy ending up in electronic excitation via recoiling atoms is, apart from a numerical factor, proportional to the corresponding quantity due to one single recoiling atom at maximum recoil energy. In order to determine the energy deposited at the target surface, a construction of the spatial distribution $D_{(r)}(x, E, \cos \phi)$ belonging to $\eta_{(r)}(E)$ is needed. In the case of dominating nuclear stopping, the term containing $d\sigma_e$ in Eq. (12b) can be ignored. Then, this equation differs from those treated previously⁴ only by the normalization, i.e., the zeroth moment, cf. Eq. (17). In particular, the following scaling relation holds 4 :

$$
D_{(r)}(x, E, \cos \phi) = \eta_{(r)}(E) \frac{NC}{E^{2m}}
$$

$$
\times \Delta \left(\frac{x}{E^{2m}/NC}, \cos \phi\right), \qquad (18)
$$

where Δ is some function that can be constructed from the higher moments over $D_{(r)}$, the latter following by a standard procedure from moment equations that are identical with those discussed in Ref. 4.

$$
D_{(r)}(0,E,\cos\phi) = \eta_{(r)}(E) \frac{NC}{E^{2m}} \Delta(0,\cos\phi) \ . \tag{19}
$$

FIG. 1. Factor β as a function of the mass ratio M_2/M_1 .

 $cidence¹¹$:

$$
D_{(r)}(0, E, 1) = \frac{(1 - m)^2}{\omega(\omega - m)} \gamma^{m-1} \Delta(0, 1)
$$

$$
\times NS_n(E) \left[\frac{\eta_t(\gamma E)}{\gamma E} \right]
$$

$$
= \beta_r NS_n(E) \left[\frac{\eta_t(\gamma E)}{\gamma E} \right].
$$

In order to facilitate calculations the factor β , is depicted in Fig. ¹ as a function of the mass ratio M_2/M_1 for $\omega = 1.2^{11}$ Table I gives the function

$$
\tilde{\eta}(\epsilon) = \eta_t(E) a M_2/Z_1 Z_2 e^2 (M_1 + M_2)
$$

as a function of the reduced energy $\epsilon = E a M_2 / Z_1 Z_2 e^2 (M_1 + M_2)$,⁵ on the basis of Winterbon's tables.¹⁶ Three representative values of the reduced electronic stopping power $k_L \epsilon^{1/2}$ have been chosen.²³

TABLE I. Reduced function $\tilde{\eta}(\epsilon)$ as a function of the reduced energy ϵ .

uial are identical with those discussed in Ref. 4.				
The quantity entering into Eq. (4) is then of the general form	ϵ	$k_I = 0.1$	$k_1 = 0.15$	$k_I = 0.2$
$D_{(r)}(0,E,\cos\phi) = \eta_{(r)}(E) \frac{NC}{F^{2m}} \Delta(0,\cos\phi)$. (19)	0.005 0.01	6.12×10^{-4} 1.36×10^{-3}	8.81×10^{-4} 1.95×10^{-3}	1.13×10^{-3} 2.49×10^{-3}
This determines the dependence on ion energy of the second contribution to the secondary-electron yield.	0.02 0.05 0.1	3.04×10^{-3} 8.80×10^{-3} 0.0195	4.33×10^{-3} 0.0124 0.0275	5.49×10^{-3} 0.0156
For qualitative orientation, we can construct $D(r)$ from the first few terms in the Edgeworth expan- sion. ^{4,10} More refined methods utilizing a large number of moments have been discussed in the	0.2 0.5	0.0453 0.134	0.0619 0.183	0.0344 0.0767 0.224
	1.0 2.0	0.319 0.779	0.429 1.01	0.511 1.18
literature. ^{4, 18, 19} By combination of Eqs. (17) and (19) the following	5.0 10.0	2.60 6.37	3.16 7.33	3.55 7.97
relation can be obtained for perpendicular ion in-				

The function $D_{(p)}$ has a discontinuity at the target surface. Indeed, if scattering of the ion and energy transport by excited electrons are ignored, $D_{(p)}$ has the form of a step function in the region around $x=0$, transport by excited electrons are ignored, *D*,
the form of a step function in the region aro
 $x = 0$,
 $D_{(p)}(x, E, \cos \phi) = \Theta(x) \frac{NS_e(E)}{\cos \phi}$, for $x \sim 0$

$$
D_{(p)}(x, E, \cos \phi) = \Theta(x) \frac{N S_e(E)}{\cos \phi} , \text{ for } x \sim 0 , \quad (20)
$$

where

$$
\Theta(x) = \begin{cases} 1, & x \ge 0 \\ 0, & x < 0 \end{cases}
$$

If ion scattering is included, mainly an additional (continuous) contribution to $D_{(p)}$ arises from backscattered ions. Whether or not this effect has to be taken into account may be judged from an inspection taken into account may be judged from an inspection
of the corresponding reflection coefficients. $20-22$ Energy transport by excited electrons broadens the profile and causes the surface value to be smaller. Ion scattering, of course, is important only for ions lighter than the target atoms, but the degree of broadening of the discontinuity depends substantially on the angular and energy spectrum of primary excited electrons, and therefore a more complex dependence on atomic number and energy of the ion is expected. The total effect of these two corrections on Eq. (20) is hard to assess on the basis of the present calculations. Therefore, as a first approximation these effects have been ignored. The influence of recoiling electrons has been included to some extent in Ref. 11.

IV. COMPARISON WITH EXPERIMENT

The basic relation for the secondary-electron yield γ , Eq. (1), can now be written

$$
\gamma = \Lambda [D_{(p)}(x = 0, E, \cos \phi) + D_{(r)}(x = 0, E, \cos \phi)].
$$
\n(21)

In the evaluation of $D_{(p)}$ and $D_{(r)}$ for perpendicul $\lim_{x \to a} \frac{\partial}{\partial x} \frac{\partial}{$ powers according to Lindhard et al.^{23,24} have beer used. The recoil-induced distribution $D_{(r)}$ was constructed by the Edgeworth expansion including moments up to the third order.

In Figs. 2 and 3 the prediction of Eq. (21) is com-In Figs. 2 and 3 the prediction of Eq. (21) is conserved with yields measured by Holmen *et al.*²⁵ for krypton and xenon ions on polycrystalline copper. The secondary-electron yield varies significantly with ion energy, but the extracted material constant

$$
\Lambda_{\exp} = \frac{\gamma_{\exp}}{D_{(p)}(0, E, 1) + D_{(r)}(0, E, 1)}
$$
(22)

varies only about 15% for krypton ions in the energy range ⁴⁰—⁴⁰⁰ keV. For xenon ions the variation is even less in the region ¹⁰⁰—⁴⁰⁰ keV.

FIG. 2. Extracted quantity Λ_{exp} from Eq. (22), the experimental secondary-electron yield, and the surface values of the primary and recoil-induced distributions as a function of ion energy for krypton on copper.

In Table II the average values of Λ_{exp} obtained for krypton and xenon ions are listed together with the average values of Λ_{exp} for electrons, protons, and ions of helium, neon, argon, and copper. For the case of electronic bombardment the calculations by Spencer²⁶ were applied to determine the surface value of $D¹¹$ For protons the stopping power²⁷ has been used in accordance with Eq. (20), while for helium and neon ions $D(0,E, 1)$ was found from the tables and neon ions $D(0, E, 1)$ was found from the tables
of Brice.¹⁷ For copper and argon ions the values for $D(0,E, 1)$ are taken from a subsequent paper²⁵ where also the electronic stopping has been taken into account in Eq. (12b).

Corrections for potential emission have been applied according to Kishinevskii.²⁸ The extracted
values of Λ_{exp} are found to lie within 45% of the
value $\Lambda = 52 \lambda/\text{keV}$ for widely different bom value $\Lambda_{\rm exp}$ = 52 Å/keV for widely different bombard ment conditions. This small variation is found encouraging.

FIG. 3. Extracted quantity Λ_{exp} from Eq. (22), the perimental secondary-electron yield, and the surface values of the primary and recoil-induced distributions as a function of ion energy for xenon on copper.

٠ Beam particle and reference		$\Lambda_{\rm exp}$ (Å/keV)
e^-	29	40.2
H^+	25	75.8
$He+$	25	53.3
$Ne+$	25	49.8
Ar^+	25	42.8
$Cu+$	25	51.1
Kr^+	25	39.5
Xe^+	25	30.0

TABLE II. Average value of Λ_{exp} for different bombarding particles. (Target: Cu).

V. DISCUSSION

Standard references on the theory of ion-induced electron emission are the papers by Sternglass' and Parilis and Kishinevskii⁷ for high- and low-velocity bombardment, respectively. The present theoretical model contains these limiting cases, since in both papers, the number of electrons excited is assumed proportional to the available energy, and hence, the electron yield related to the electronic stopping power. The description by the present authors is more comprehensive in the sense that it allows for substantial multiple or back scattering of the impinging ion and, even more important, indirect (recoil-induced) excitation processes are included, and demonstrated to be important for heavy-ion bombardment. Such processes can be neglected in the high-velocity limit to which the Sternglass theory refers, but can hardly be ignored whenever the energy loss is predominantly nuclear.

The present theory is less explicit than the two mentioned^{2,7} with respect to the electronic stopping power of the impinging ion. Indeed, rather than using a particular theoretical model, the present authors would prefer the best available experimental stopping power data to be applied in accurate calculations. On the other hand, the present description leaves the factor Λ unspecified except for the assumption that it may be a material constant independent of the type of bombarding particle. As mentioned previously, an explicit evaluation of this factor has been made by one of the authors¹¹ for the limiting case of highvelocity bombardment where binary scattering events dominate. Inserting feasible values for the lowvelocity electron stopping power and a conventional model for the surface barrier, one obtains a value of A that is consistent with experimental values. However, this value is not completely independent of the excitation spectrum for primary electrons, and the latter is well known to vary substantially over the range of ion types and energies aimed at in this work, Therefore, the variation of Λ_{exp} as apparent from Table I need not only express the inaccuracy of the evaluation of the deposited energy function as well as experimental error, but may also indicate a real dependence of Λ on the spectrum of primary electrons.

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