Surface effects on phase transitions in ferroelectrics and dipolar magnets

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A phenomenological theory is developed to describe the change of the local spontaneous polarization in the vicinity of a free surface of a ferroelectric thin film which is kept between metallic electrodes. It is shown that depolarizing field effects reduce the deviation of this local polarization from its bulk value, as compared to surface effects on phase transitions in other systems. In particular, the critical exponents describing the behavior of the local polarization in the vicinity of the Curie temperature T_C are the same as the bulk exponents and only critical amplitudes are changed. This behavior contrasts to phase transitions in other systems (antiferroelectrics, ferro- and antiferromagnets, ordering alloys etc.) where different exponents are predicted. In order to improve upon this Landau-type theory by taking into account the effects of statistical fluctuations near T_C , recent results of renormalization-group theory are used to estimate logarithmic correction factors which should modify the critical behavior of the local polarization. Finally the experimental implications of our results are briefly discussed, and also a discussion of surface effects on the phase transition of dipolar magnets is given.

I. INTRODUCTION

It is well known that the polarization of small ferroelectric particles or thin films may behave differently as compared to bulk materials.¹⁻³ Many of these observations where the bulk polarization of the sample is observed can be accounted for by the fact that for minimizing the electrostatic energy of the sample a domain configuration is more favorable than a monodomain particle (or film).⁴ (This effect is also familiar from ferromagnets.^{5,6}) This explanation cannot explain strong size effects found with NMR measurements³ where the existence of a local polarization is probed on the atomic scale. Another source of size effects is due to depolarizing fields which are still present in the case where free-surface charges are available to partially compensate the polarization discontinuity at the surface of the ferroelectric.^{7,8}

A third source of surface effects with which we are concerned here is a change in the local polarization near the surface which is expected to occur over a distance comparable to the correlation length ξ of polarization fluctuations (Fig. 1). Far away from $T_C \xi$ is estimated for a ferroelectric to be of the order of a few lattice spacings only,⁹ and hence this effect is expected to become more pronounced close to T_C only since ξ diverges there.¹⁰ Related problems of orderparameter variations near surfaces have been considered for second-order phase transitions in superconductors,¹¹ suprafluids,¹² ordering or unmixing binary alloys,¹³ and magnetic systems.¹⁴⁻¹⁷ In the latter case at least a semiquantitative experimental verification of the theoretical concepts is due to lowenergy electron diffraction (LEED),¹⁸ magnetooptical studies,¹⁹ ferromagnetic resonance²⁰ and photoelectron spin polarization analysis,²¹ although some phenomena remain to be explained.²² It is expected that techniques to investigate similar surface effects in ferroelectrics will be available in the near future.

The generalization of the theories mentioned above¹¹⁻¹⁷ to ferroelectrics is nontrivial for two reasons: (i) The depolarizing field associated with the inhomogeneous distribution of the polarization close to the surface (Fig. 1) has to be included in the treatment. (ii) The temperature dependence of the critical singularities is more complicated than usually due to the presence of logarithmic correction factors²³⁻²⁹ to mean-field behavior. Although these correction factors have been obtained for a variety of static and



FIG. 1. Variation of the local polarization P(z) in the vicinity of a plane free surface situated at z = 0. The bulk polarization is denoted by P_{∞} while P_1 denotes the polarization at the surface. λ is the so-called extrapolation length, Ref. 11.

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dynamic quantities of ferroelectrics,²⁶⁻²⁹ so far only one experiment on triglycine sulfate (TGS)³⁰ has been suggested²⁶ to support these corrections. In fact, there is a considerable uncertainty in the theoretical predictions for the temperature interval around T_C where these corrections are appreciable.²⁷ Moreover it must be noted that the transitions of many ferroelectrics are (at least weakly) first order rather than second order.³¹ It may thus seem premature to extend the calculation of these corrections from bulk static and dynamic quantities in $pure^{26-28}$ and impure²⁹ ferroelectrics to surface quantities. But this problem is interesting because it also has an application to dipolar magnetic systems (like LiTbF₄, CrBr₃) where many of the predicted correction factors have clearly been revealed experimentally.^{32, 33} Our theoretical treatment of effect (ii) readily applies to these systems as well. With respect to depolarization effects the situation is of course different; compensating surface charges exist only in the case of the ferroelectric, while in the ferromagnet only domain formation is possible to reduce the magnetostatic energy.

II. PHENOMENOLOGICAL THEORY OF SURFACE EFFECTS IN FERROELECTRIC THIN FILMS

We consider a thin ferroelectric plate between two metallic electrodes in short-circuit conditions (Fig. 2). We assume that the easy axis of the uniaxial ferroelectric has the z direction perpendicular to the film, which we assume of infinite extent in x and y directions for simplicity. Without the electrodes a homogeneous spontaneous polarization $\vec{\mathbf{P}}_s = (0, 0, P_s)$ would give rise to a depolarizing field $\vec{\mathbf{E}}_D = -4\pi N \vec{\mathbf{P}}_s$, the depolarization factor N being N = 1 for our geometry. To remove the electrostatic energy associated with this field a strip-domain pattern is established, with domain width $l \propto 1/P_s$.⁴ In the case where a surface charge density σ is present a similar calculation (only the boundary conditions change)



FIG. 2. Sketch of geometric arrangement of a thin ferroelectric plate between metallic electrodes. For explanations cf. text.

yields $l \propto 1/(P_s + \sigma)$, i.e., a monodomain sample is obtained for complete compensation $(\sigma = -P_s)$.

However, in the case we are interested in the spontaneous polarization is not uniform in z direction. Since we assume the ferroelectric to be perfectly insulating, there are no space charges available which could compensate the depolarizing field locally everywhere. But a monodomain sample is still achieved for global compensation, i.e. (V is the volume of the sample),

$$\sigma = -\frac{1}{V} \int_{V} (\vec{\mathbf{P}} \cdot \vec{\mathbf{n}}) \, dV \,. \tag{1}$$

To derive Eq. (1) it is convenient to first consider the auxiliary problem (Fig. 2) of a charge q located a distance a from the upper metallic plate (A) and a distance b = L - a from the lower one (B).³⁴ The image charges induced on A, B are

$$q_A' = -q\left(\frac{L-a}{L}\right), \quad q_B' = -q\left(\frac{L-b}{L}\right)$$

A second charge -q located a distance $\vec{\delta} = (0, 0, \delta)$ from the first consequently yields

$$q_A^{\prime\prime} = q \left(\frac{L-a-\delta}{L} \right), \quad q_B^{\prime\prime} = q \left(\frac{L-b+\delta}{L} \right)$$

Hence the image charges associated with the dipole $\vec{p} = q \vec{\delta}$ are

$$q_A = q_A'' + q_A'' = -q \,\delta/L = -p/L$$
,
 $q_B = q_B' + q_B'' = q \,\delta/L = p/L$.

Introducing the unit vector \vec{n} directed perpendicular to the surface of the ferroelectric into the metal this result is hence rewritten $q_{A,B} = -\vec{p} \cdot \vec{n}/L$. The induced surface charge density then is $\sigma_{A,B} = -\vec{p} \cdot \vec{n}/V$. This result for a single dipole \vec{p} is readily generalized to a distribution of polarization $\vec{P}(\vec{r})$, and hence Eq. (1) results.

We next obtain the depolarizing field \vec{E}_d associated with the polarization distribution \vec{P} , using Maxwell's equations

$$\operatorname{curl} \vec{E} = 0, \quad \operatorname{div} \vec{D} \equiv \operatorname{div} (\vec{E} + 4\pi \vec{P}) = 0$$
(2)

[without external applied field we have $\vec{E} = \vec{E}_d = (0, 0, E_d)$, of course]. Since in the monodomain case $\vec{P}(\vec{r})$ depends on z only, Eq. (2) simplifies to

$$\frac{\partial E_d}{\partial z} = -4\pi \frac{\partial P}{\partial z}, \quad \vec{\mathbf{P}} = (0, 0, P) \quad , \tag{3}$$

which is readily integrated to yield

$$E_d(L^-) - E_d(z) = -4\pi [P(L^-) - P(z)] .$$
 (4)

The boundary condition between ferroelectric and metal is

$$\vec{D} \cdot \vec{n} \mid_{\text{metal}} - \vec{D} \cdot \vec{n} \mid_{\text{ferroelectric}} = 4 \pi \sigma .$$
 (5)

Since $\vec{D} = 0$ in the metal, Eq. (5) yields

$$E_d(L^-) + 4\pi P(L^-) = -4\pi\sigma$$

$$E_d(z) = -4\pi [P(z) + \sigma] \tag{6}$$

As noted above, in the uniform case $P(z) \equiv P_s$ the depolarizing field is compensated for $\sigma = -P_s$, while in the nonuniform case $E_d(z)$ is nonzero.

We now generalize the treatment of Refs. 14 and 15 to ferroelectrics, writing down a Landau expansion of the free energy in terms of P(z),

$$F = \int_{V} dV \left[\frac{1}{2} A P^{2}(\vec{r}) + \frac{1}{4} B P^{4}(\vec{r}) + \frac{1}{2} C (\nabla P)^{2} - \frac{1}{2} \vec{E}_{d}(\vec{r}) \vec{P}(\vec{r}) \right] + \int_{S} \frac{1}{2} C \lambda^{-1} \left[P^{2}(x, y, z = 0^{+}) + P^{2}(x, y, z = L^{-}) \right] dx dy$$

The constants A, B, and C in the volume term of Eq. (7) are

$$A = 2a'(T - T_C), \quad a' = \frac{2\pi}{C_+},$$
(8)

 C_+ being the Curie constant of the ferroelectric, and

$$B = 2a'T_C/P_0^2$$
, $P_0 = P(T=0)$,

and

$$C = 2a'T_C a_0^2 \quad , \tag{10}$$

where a_0 is a length of the order of the lattice spacing. Note that the last contribution to the volume term in Eq. (7) represents the self-energy of the depolarizing field (hence the factor $\frac{1}{2}$). The proportionality factor of the correction term proportional to the surface area S defines then the extrapolation length λ (see also Fig. 1).

Since in the monodomain sample P is a function of z only, the integrations over x and y in Eq. (7) are readily performed to yield (we now include an external field E_{ext})

$$F/S = \int_0^L dz \left[\frac{A}{2} P^2(z) + \frac{B}{4} P^4(z) + \frac{C}{2} \left(\frac{dP}{dz} \right)^2 - \frac{1}{2} E_d(z) P(z) - E_{\text{ext}} P(z) \right] + \frac{C}{2} \lambda^{-1} \left[P_1^2 + P_L^2 \right],$$

$$P_1 \equiv P(0^+), \quad P_L = P(L^-) . \tag{11}$$

Note that there is no factor $\frac{1}{2}$ in front of the term involving the external field, in contrast to the term involving the depolarizing field. We use now Eqs. (1) and (6) [in the form $\sigma = -\int_0^L P(z) dz /L$] to rewrite Eq. (11)

$$\frac{F}{S} = \int_0^L dz \left(\frac{A}{2} P^2(z) + \frac{B}{4} P^4(z) + \frac{C}{2} \left(\frac{dP}{dz} \right)^2 + 2\pi P^2(z) - E_{\text{ext}} P(z) \right) - \frac{2\pi}{L} \left(\int_0^L P(z) \, dz \right)^2 + \frac{C}{2} \lambda^{-1} \left(P_1^2 + P_L^2 \right).$$
(12)

The free energy is then minimized with respect to P, which yields

$$\frac{\delta F}{\delta P} = 0 = AP(z) + BP^{3}(z) - C \frac{d^{2}P}{dz^{2}} + 4\pi P(z) - E_{\text{ext}} - \frac{4\pi}{L} \int_{0}^{L} P(z) dz \quad .$$
(13)

The surface terms in Eq. (12) yield the boundary conditions

$$\frac{dP}{dz}\Big|_{z=0} - P_1 \lambda^{-1} = 0, \quad \frac{dP}{dz}\Big|_{z=L} + P_L \lambda^{-1} = 0.$$
(14)

In the homogeneous case $P(z) \equiv P_{\infty}$ (bulk ferroelectric) Eq. (13) reduces to the standard result

$$\frac{A}{C}P_{\infty} + \frac{B}{C}P_{\infty}^3 = \frac{E_{\text{ext}}}{C} , \qquad (15)$$

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(9)

(7)

of course. Note that Eqs. (13) and (14) differ from previous treatments of magnets where demagnetizing fields were neglected^{14, 15} by the appearance of the additional term

$$4\pi[P(z)-\int_0^L P(z) dz/L].$$

We now consider the solution of Eqs. (13) and (14) in the paraelectric regime where the cubic term $BP^{3}(z)$ can be neglected, and seek a symmetric solution

$$P_1 = P_L, \quad \frac{dP}{dz} \Big|_{z=0} = -\frac{dP}{dz} \Big|_{z=L}$$
 (16)

The solution of Eqs. (13) and (14) is then found by first introducing a constant R by

$$(4\pi/CL)\int_0^L P(z) dz = R ,$$

solving then the inhomogeneous differential equation and calculating R afterwards. The solution of Eq. (13) which satisfies Eq. (16) is $[\kappa^2 \equiv (A + 4\pi)/C]$

$$P(z) = 2C_2 e^{\kappa L/2} \cosh[(z - L/2)\kappa] + \frac{E_{\text{ext}}}{C\kappa^2} + \frac{R}{\kappa^2}, \quad (17)$$

which yields

$$R = \frac{4\pi/C}{1 - (4\pi/C)/\kappa^2} \left(\frac{4C_2 e^{\kappa L/2}}{\kappa L} \sinh \frac{\kappa L}{2} + \frac{E_{\text{ext}}}{C\kappa^2} \right).$$
(18)

The constant C_2 can be eliminated in favor of the

surface polarization P_1 , which yields

$$P(z) = \frac{(P_1 - E_{ext}/A) \cosh[(z - L/2)\kappa]}{\cosh(L\kappa/2 + 8\pi/L\kappa A) \sin(L\kappa/2)} + \frac{E_{ext}}{A} + \frac{P_1 - E_{ext}/A}{1 + (L\kappa A/8\pi) \cosh(L\kappa/2)} .$$
 (19)

Since

$$\frac{E_{\text{ext}}}{A} = \frac{E_{\text{ext}}}{[a'(T-T_C)]} = P_{\infty}$$

we obtain from Eq. (19) for $L \rightarrow \infty$ and in the case where

$$\kappa^2 \approx A/C = a'(T - T_C)/C \equiv \xi_b^{-2}$$

the standard result¹³⁻¹⁵

$$P(z) = \frac{(P_1 - P_{\infty})\cosh[(z - L/2)/\xi_b]}{\cosh(L/2\xi_b)} + P_{\infty} , \qquad (20)$$

i.e., the polarization differs appreciably from P_{∞} over the distance of the bulk correlation length ξ_b from the surface. Including the depolarization effect it is seen already from Eq. (19) that this effect is reduced: the length κ^{-1} stays finite at T_C , unlike ξ_b ; hence the length ξ in Fig. 1 has to be identified as κ^{-1} rather than the bulk correlation length. However, note that now the polarization does not reach P_{∞} in the center of the film, but the slightly different value ($\kappa L \gg 1$)

$$P\left(\frac{L}{2}\right) = P_{\infty} + \frac{P_1 - P_{\infty}}{\cosh(L\kappa/2)[1 + (8\pi/L\kappa A)\tanh(L\kappa/2)]} + \frac{P_1 - P_{\infty}}{1 + (L\kappa A/8\pi)\cosh(L\kappa/2)} \approx P_{\infty} - \frac{P_{\infty} - P_1}{1 + L\kappa A/8\pi} \quad , \quad (21)$$

i.e., the leading correction to P_{∞} is of order 1/L as $L \rightarrow \infty$. In the standard case without depolarizing fields it is readily seen from Eq. (20) that

$$P\left(\frac{L}{2}\right) = P + \frac{P_1 - P_{\infty}}{\cosh(L/2\xi_b)} \approx P_{\infty} - \frac{2(P_{\infty} - P_1)}{\exp(L/2\xi_b)} , \quad (22)$$

i.e., the leading correction to P_{∞} is exponentially small. This difference (Fig. 3) will have important consequences for the behavior of the surface susceptibility χ_s and surface polarization P_s , as will be discussed below.

We now obtain the surface layer polarization P_1 from using Eqs. (14) and (19) as

$$P_1 = \frac{\kappa \lambda P_{\infty} \tanh(\kappa L/2)}{1 + (\kappa \lambda + 8\pi/L \kappa A) \tanh(\kappa L/2)} .$$
 (23)

For $L \rightarrow \infty$ this result simplifies to

$$P_{1} = \frac{\kappa \lambda P_{\infty}}{1 + \kappa \lambda} = \frac{(4\pi/C + \xi_{b}^{-2})^{1/2} \lambda P_{\infty}}{1 + \lambda (4\pi/C + \xi_{b}^{-2})^{1/2}} .$$
(24)



FIG. 3. Distribution of polarization across a film of thickness L in the case where depolarizing fields are included (a) or not included (b).

Hence the surface layer susceptibility $\chi_1 \equiv \partial P_1 / \partial E_{ext}$ is $[t = (T - T_C)/T_C]$,

$$\chi_1 = \frac{(4\pi/C + \xi_b^{-2})^{1/2} \lambda \xi_b^2/C}{1 + \lambda (4\pi/C + \xi_b^{-2})^{1/2}} \propto \xi_b^2 \propto t^{-1} .$$
 (25)

Since the exponent γ_1 is defined by $\chi_1 \propto t^{-\gamma_1}$, we now have

$$\gamma_1 = \gamma_b = 1 , \qquad (26)$$

instead of the previous result¹⁵ $\gamma_1 = \frac{1}{2}$. Hence due to these depolarization effects there is no longer a difference between the critical exponents at the surface and in the bulk. But the critical amplitude $\hat{\chi}_1$

(defined via $\chi_1 = \hat{\chi}_1 t^{-\gamma_1}$) differs from the critical amplitude $\hat{\chi}_b$ [defined via $\chi_b = \hat{\chi}_b t^{-\gamma_b}$,

$$\chi_b \equiv \partial P_{\infty} / \partial E_{\text{ext}} = A^{-1} = (a'/T_C)^{-1} t^{-1}$$

i.e., $\hat{\chi}_b = T_C/a'$], since $\hat{\chi}_1 = \hat{\chi}_b / \{1 + [\lambda(4\pi/C)^{1/2}]^{-1}\}$. Next we consider the average polarization $\overline{P} = (1/L) \int_0^L P(z) dz$ of the film, which becomes

$$\overline{P} = \frac{\kappa^2 (P_1 - P_{\infty})}{4\pi/C + (\kappa L/2)(A/C)\cosh(\kappa L/2)} + P_{\infty}; \quad (27)$$

 \overline{P} differs from P_{∞} by a correction of order 1/L, as expected. Since the film has two equivalent surfaces, it makes sense to define the surface polarization $P_s^{(1)}$ via

$$P_{s}^{(1)} = \frac{L}{2} \left(P_{\infty} - \bar{P} \right) = \frac{\kappa^{2} \frac{1}{2} L \left(P_{\infty} - P_{1} \right)}{4\pi/C + (\kappa L/2) \left(A/C \right) \cosh(L\kappa/2)} \xrightarrow{\kappa} \frac{\kappa(P_{\infty} - P_{1})}{A/C} = \frac{P_{\infty} \kappa}{(A/C) \left(1 + \kappa \lambda \right)}$$
(28)

and hence the surface susceptibility¹⁵ $\chi_s^{(1)} \equiv \partial P_s^{(1)} / \partial E_{\text{ext}}$ becomes

$$\chi_{s}^{(1)} = \frac{C\kappa}{A^{2}} \frac{1}{1+\lambda\kappa}$$
$$= \frac{C(4\pi/C + \xi_{b}^{-2})^{1/2}}{(a'T_{C})^{2}[1+\lambda(4\pi/C + \xi_{b}^{-2})^{1/2}]} t^{-2} .$$
 (29)

Hence the exponent $\gamma_s^{(1)}$ defined by $\chi_s^{(1)} = \hat{\chi}_s^{(1)} t^{-\gamma_s^{(1)}}$ is $\gamma_s^{(1)} = 2$ in this case, in contrast to the usual situation

without depolarizing fields¹⁵ where $\kappa = \xi_b^{-1}$ and hence

$$\kappa \propto t^{1/2}, \quad \chi_s \propto t^{-3/2}, \quad \gamma_s = \frac{3}{2}$$

While in the case of the local susceptibility χ_1 at the surface we have found that the effect of the depolarizing field makes χ_1 closer to the bulk susceptibility χ_b , the contrary seems to occur for $\chi_s^{(1)}$. It must be noted, however, that the main contribution to $\chi_s^{(1)}$ as defined above [Eq. (28)] is not due to the variation of P(z) close to the surfaces of the film but rather due to the depression of polarization in the bulk of the film [Fig. 3(a)]. Hence it makes sense to consider the alternative definitions of surface polarization and surface susceptibility

$$P_{s}^{(2)} = \frac{1}{2} L \left[P(\frac{1}{2}L) - \overline{P} \right]$$

= $\frac{1}{2} L \left(P_{\infty} - P_{1} \right) \left(-\frac{1}{\cosh(L\kappa/2) \left[1 + (8\pi/L\kappa A) \tanh(L\kappa/2) \right]} + \frac{A/4\pi}{1 + (L\kappa/2) (A/4\pi) \coth(L\kappa/2)} \right)$ (30)

and hence the surface polarization of a ferroelectric "half-space" becomes

$$\lim_{L \to \infty} P_s^{(2)} \equiv \int_0^\infty dz \left[P_\infty - P(z) \right]$$
$$= \frac{P_\infty - P_1}{\kappa} = \frac{P_\infty}{1 + \lambda \kappa} . \tag{31}$$

In the case without depolarizing field $\kappa = (A/C)^{1/2} = \xi_b^{-1}$ and hence there would be no difference between $P_s^{(1)}$ and $P_s^{(2)}$. In our case, however, the limit $L \rightarrow \infty$ and the integration over z in Eqs. (27) and (31) are not interchangeable. From Eq. (31) we get the surface susceptibility $\chi_s^{(2)} \equiv \partial P_s^{(2)} / \partial E_{\text{ext}}$,

$$\chi_s^{(2)} = \frac{1}{A\kappa} \frac{1}{(1+\lambda\kappa)} = \frac{1}{a'T_C} \frac{1}{(4\pi/C + \xi_b^{-2})^{1/2}} \frac{1}{1+\lambda(4\pi/C + \xi_b^{-2})^{1/2}} t^{-1} .$$
(32)

Hence the exponent $\gamma_s^{(2)}$ defined by $\chi_s^{(2)} = \hat{\chi}_s^{(2)} t^{-\gamma_s^{(2)}}$ becomes $\gamma_s^{(2)} = 1 = \gamma_b$, as one would expect. Next we consider the case $T < T_C$. Since there exists for $E_{ext} = 0$ in the bulk a spontaneous polarization P_{∞}

$$P_{m} = (-A/B)^{1/2} = (a'T_{C}/B)^{1/2}t^{1/2}$$

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as given by Eq. (15), we may replace the term $(4\pi/L) \int_0^L P(z) dz$ in Eq. (13) simply by $4\pi P_{\infty}$ in the limit $L \rightarrow \infty$. Hence the equation to be solved becomes

$$\kappa^{2}P(z) + \frac{B}{C}P^{3}(z) - \frac{d^{2}P}{dz^{2}} = \frac{4\pi}{C}P_{\infty}.$$
 (33)

As usual¹⁵ we multiply this equation by dP/dz and integrate over z from z = 0 to $z = \frac{1}{2}L$ [in the center of the film we may put $P(z = \frac{1}{2}L) = P_{\infty}$ for $L \to \infty$] to find

$$\frac{\kappa^2}{2}(P_{\infty}^2 - P_1^2) + \frac{B}{4C}(P_{\infty}^4 - P_1^4) - \frac{1}{2}\left(\frac{dP}{dz}\right)^2 \Big|_{0}^{1/2}$$
$$= \frac{4\pi}{C}(P_{\infty}^2 - P_{\infty}P_1) . \quad (34)$$

In the bulk we have $(dP/dz)_{z-L/2} = 0$ while on the surface we may again use Eq. (14) to find

$$\kappa^{2}(P_{\infty}^{2} - P_{1}^{2}) + \frac{B}{2C}(P_{\infty}^{4} - P_{1}^{4}) + \lambda^{-2}P_{1}^{2} - \frac{8\pi}{C}(P_{\infty}^{2} - P_{1}P_{\infty}) = 0 , \quad (35)$$

which differs from previous results with no depolarizing fields by the additional term $-(8\pi/C)$ $\times (P_{\infty}^2 - P_1 P_{\infty})$. This term is a small correction only as long as the ratio $|C\kappa^2/8\pi| \gg 1$. Using our result for κ^2 this is seen to be equivalent to $|A/4\pi + \frac{1}{2}| \gg 1$ or $|A| \gg 4\pi$. Very close to T_C this inequality can never be fulfilled, of course, and then the depolarization corrections always are important. But this condition becomes valid not so close to T_C when T_C is very high (in units of the dipolar energy), which is possible if in addition to the dipolar interaction energy, strong short-range interactions are present. This is seen more clearly by invoking the molecular-field expression for the critical temperature (q is the local dipole moment at lattice site i)

$$k_B T_C = J(0) - q^2 \sum_{i(\neq j)} \frac{[1 - 3(z_i - z_j)^2 / (\vec{r}_i - \vec{r}_j)^2]}{|\vec{r}_i - \vec{r}_j|^3} , \quad (36)$$

where

$$J(\vec{\mathbf{k}}) = \sum_{i(\neq j)} J_{ij} \exp[i \vec{\mathbf{q}} (\vec{\mathbf{r}}_i - \vec{\mathbf{r}}_j)]$$

is the Fourier transform of the short-range interaction. The (dimensionless) constant A in Eq. (7) is expressed in mean-field theory as [a is the lattice spacing, and the sum in Eq. (36) is written as Γ/a^3 where Γ is a dimensionless constant of order unity depending on the lattice structure]

$$A = \frac{a^3}{q^2} k_B (T - T_C) = \frac{k_B T}{q^2/a^3} - \left[\frac{J(0)}{q^2/a^3} + \Gamma \right].$$
 (37)

Hence $|A| >> 4\pi$ can be fulfilled if the short-range energy J(0) strongly exceeds the dipolar energy (which is proportional to q^2/a^3). Although Eq. (36) is known to be accurate only within a numerical factor of order unity, this does not invalidate our orderof-magnitude argument.

We now solve Eq. (35) for P_1 in this limit where the depolarization effects are small, by first omitting the term $(8\pi/C)(P_{\infty}^2 - P_1P_{\infty})$ which yields the standard result¹⁵ $P_1^{(0)} = \lambda P_{\infty}/\xi_b^<$ with $\xi_b^< = [C/(-2A)]^{1/2}$. We now obtain corrections to $P_1^{(0)}$ linear in $4\pi/A$ by putting

$$P_1 = P_1^{(0)} + \frac{4\pi}{A} \delta P \tag{38}$$

and keep only terms up to order linear in δP in Eq. (35). The result, which is valid for $\pi/A \ll 1$, is

$$P_1 = \frac{\lambda}{\xi_b^{<}} P_{\infty} \left(1 - \frac{\pi}{A} - \frac{\lambda}{\xi_b^{<}} \right)^2 . \tag{39}$$

We now consider the inverse case where $|8\pi/C\kappa^2| >> 1$. From Eq. (35) it is clear that $P_1 \rightarrow P_{\infty}$ as $|8\pi/C\kappa^2| \rightarrow \infty$. Hence we now put

$$P_1 = P_{\infty} + \delta P \tag{40}$$

and keep only terms up to order $(\delta P)^2$ in Eq. (35). The result is

$$P_1 = P_{\infty} \left\{ 1 - \left[1 + \frac{\lambda}{(C/4\pi)^{1/2}} (1 - 2A/\pi)^{1/2} \right]^{-1} \right\}$$
(41)

Figure 4 shows the crossover between the two kinds



FIG. 4. Log-log plot of the relative polarization at the surface P_1/P_{∞} vs $\xi_b^{<}(4\pi/C)^{1/2} = (2\pi/a'T_C)^{1/2}(-t)^{1/2}$ for several choices of the ratio between extrapolation length λ and decay length $\kappa_{T_C}^{-1} = (C/4\pi)^{1/2}$. Full curves represent the exact solution of Eq. (35) while triangles and circles represent the simple approximations Eqs. (39) and (41), respectively.

of limiting behaviors given in Eqs. (39) and (41). As $T \rightarrow T_C$ Eq. (41) always becomes valid for small $[\lambda/(C/4\pi)^{1/2}]$, and we hence find that

$$P_1 = P_{\infty} \frac{1}{1 + (C/4\pi/\lambda)^{1/2}} , \qquad (42)$$

which implies $[P_1 \equiv \hat{P}_1(-t)^{\beta_1}, P_{\infty} = \hat{P}_{\infty}(-t)^{\beta_b}]$ that $\beta_1 = \beta_b = \frac{1}{2}$ but

$$\hat{P}_1 = \hat{P}_{\infty} [1 + (C/4\pi/\lambda)^{1/2}]^{-1} .$$
(43)

Hence we find that both above and below T_C the depolarization effects diminish the surface effects which otherwise would occur. The exponents of local quantities at the surface are the same as in the bulk, and only amplitude factors become changed. On the other hand, it can be shown from Eqs. (23) and (27) that the shift of the critical temperature $T_C^{(L)}$ of the film is given by

$$\frac{T_C^{(L)} - T_C}{T_C} = -\frac{a'}{T_C} \frac{8\pi}{\lambda L} \frac{1}{1/C + 1/4\lambda (\pi C)^{1/2}}, \quad L \to \infty$$
(44)

and hence the shift exponent ζ which is defined as $[T_C(L) - T_C]/T_C \propto L^{-\zeta}$ in our case is $\zeta = 1$ instead of $\zeta = 2$ as in the case without depolarizing fields.¹⁵ This stronger depression of $T_C^{(L)}$ of the film is related to the stronger depression of the polarization in the bulk of the film (Fig. 3).

Finally we consider the case of a general direction of the polarization of the uniaxial ferroelectric. In a monodomain both $\vec{E}_d(\vec{r})$ and $\vec{P}(\vec{r})$ can still depend on the z coordinate only, and from Eq. (2) it then follows that we still have $\vec{E}_d = (0, 0, E_d)$. Since in Eqs. (3)-(6) we have to replace the absolute value P(z)of the polarization by its component in z direction $P(z) \cos\theta$, where θ is the angle between $\vec{P}(z)$ and \vec{n} , in Eq. (11) we have to replace the term $-\frac{1}{2}E_d(z)P(z)$ by $-\frac{1}{2}(\cos\theta)E_d(z)P(z)$. As a consequence the constant $4\pi/C$ which appears in the length κ^{-1} and explicitly in Eqs. (17)-(42) has to be replaced by $(4\pi/C)\cos^2\theta$. In the case where the polarization direction is precisely parallel to the film $(\theta = \frac{1}{2}\pi)$ the depolarizing field vanishes and the results of the previous treatments¹⁵ are recovered.

III. LOGARITHMIC CORRECTION FACTORS

Very close to the critical point of a ferroelectric or dipolar magnetic system fluctuations become important and predictions for the critical behavior obtained on the basis of a Landau theory have to be modified.^{10,23} Renormalization-group theory³⁵ states that the "marginal dimensionality" d^* for dipolar systems is $d^* = 3$ rather than $d^* = 4$, as it is for systems with short-range interaction, and it predicts^{24, 25} that the leading critical singularities in both cases are the same. More specifically, it is concluded that the power-law singularities obtained for various quantities near the critical point from Landau theory have to be multiplied by correction factors of the form $|\ln|t||^p$, where the exponent p depends on the property considered, e.g.,

$$m_b \propto (-t)^{1/2} |\ln(-t)|^{1/3} ,$$

$$\xi_b \propto |t|^{-1/2} |\ln|t| |^{1/6} .$$
(45)

It is the aim of the present section to predict these factors $|\ln|t||^p$ which are expected to occur for the surface properties introduced above. Since renormalization-group calculations in restricted geometry are actually quite involved,^{16,17} we estimate these factors by a less rigorous method. For bulk quantities X it is well known that there is a relation between the exponent p_x at the marginal dimensionality and the first coefficient x_1 of the leading critical exponent x of the ϵ expansion ($\epsilon = d^* - d$),

$$X \propto |t|^{x}, \quad x = x_0 + x_1 \epsilon + O(\epsilon^2), \quad d < d^*, \quad (46)$$

$$K \propto |t|^{x_0} |\ln|t| |^{\rho_x}, \quad d = d^*.$$
(47)

This relation which we will justify below is

$$p_{\mathbf{x}} = 2x_1 \ . \tag{48}$$

We then suggest that Eq. (48) is valid for surface quantities as well, and use then the ϵ -expansion results of Lubensky and Rubin¹⁶ for x_1 in the case of P_1 together with scaling relations for surface quantities¹⁵ to obtain the logarithmic factors of interest.

In order to justify Eq. (48) we start from the renormalization-group expression for the singular part of the free energy $F_s(r,u)$ (where r and u are the coupling constants for terms quadratic and quartic in the *n*-component order-parameter field, respectively)³⁶

$$F_{s}(r,u) = -\frac{n}{16(4-n)} \frac{t^{2}}{u} \left[\left(1 + \frac{Bu(e^{\epsilon t^{*}} - 1)}{\epsilon} \right)^{(4-n)/(n+8)} - 1 \right],$$
(49)

and $l^* = \ln b^*$ where b^* is the factor of spatial rescaling. It is chosen such that $F_s(r(l^*), u(l^*))$ stays noncritical. One may choose $e^{l^*} \approx t^{-1/\lambda_t}$, where $\lambda_t = 2 - [(n+2)/(n+8)]\epsilon$ is the eigenvalue associated with a temperature perturbation at the nontrivial fixed point.³⁶ This yields $e^{\epsilon l^*} \approx t^{-\epsilon/2 + O(\epsilon^2)}$ and hence

$$F_{s}(r,u) = -\frac{n}{16(4-n)} \frac{t^{2}}{u} \left[\left(1 + \frac{Bu}{\epsilon} (t^{-\epsilon/2} - 1) \right)^{(4-n)/(n+8)} - 1 \right]$$
$$= -\frac{n}{16(4-n)} \left(\frac{t^{2-\alpha}}{u} \right) R^{(4-n)/(n+8)} + \frac{n}{16(4-n)} \frac{t^{2}}{u} ,$$

where

$$R = (1 - Bu/\epsilon)t^{\epsilon/2} + Bu/\epsilon$$

and we have identified the specific-heat critical exponent α in first-order ϵ expansion as $\alpha = \frac{1}{2} \epsilon (4-n)/(n+8)$. Equation (50) shows that F_s depends on the parameters r and u only in the scaled form $F_s \propto t^{2-\alpha} f(R)$. Moreover, the exponent of R in Eq. (50) is nothing but $\frac{1}{2}$ that of the linear term in the ϵ expansion of α . It is now easy to see that the term $R^{(4-n)/(n+8)}$ which for ϵ finite tends to a finite constant as $t \to 0$ yields a logarithmic correction factor as $\epsilon \to 0$. We then use

$$t^{\epsilon/2} = \exp[(\epsilon/2) \ln t] \approx 1 + (\epsilon/2) \ln t$$

to rewrite R

$$R = 1 - \frac{1}{2} B u \ln t + \frac{1}{2} \epsilon \ln t + O(\epsilon^2)$$
(51)

and hence it follows that for $\epsilon \rightarrow 0$ the most singular part of the free energy behaves as

$$F_{s} \propto t^{2} \ln|t| |^{(4-n)/(n+8)}$$

$$(=t^{2} \ln|t| |^{1/3} \text{ for } n=1) . (52)$$

and hence the specific heat $C_s \propto \partial^2 F_s / \partial t^2$ behaves as $C_s \propto |\ln|t| |^{1/3}$ for $d = d^*$ and uniaxial systems, as is well known. Since the specific-heat exponent $\alpha = 0$ in mean-field theory this result illustrates already the general relation Eq. (48). For the susceptibility χ_b Eq. (48) was demonstrated by Essam *et al.*³⁷ Since to order ϵ we have $\chi_b \propto \xi_b^2$ Eq. (48) also holds for the bulk correlation length. Finally we note the result for the order parameter P_{∞}^{36}

$$P_{\infty} \approx \frac{|t|^{\beta}}{2u^{1/2}} R^{1/3} ,$$

$$\beta = \frac{1}{2} - \frac{1}{6} \epsilon + O(\epsilon^2) (n = 1)$$
(53)

and Eq. (51) that

$$P_{m} \propto |t|^{1/2} \ln |t|^{1/3}$$

and thus again verify Eq. (48). From the result of Lubensky and Rubin 16

$$\beta_1 = 1 - \frac{3}{2(n+8)} \epsilon + O(\epsilon^2) , \qquad (54)$$

we hence conclude that at the marginal dimensionality

$$P_1 \propto |t|^1 |\ln|t| |^{3/(n+8)} (= |t|^1 |\ln|t| |^{1/3} \text{ for } n=1)$$
(55)

From the scaling law¹⁵

$$\gamma_1 = 2 - \alpha_b - \beta_b - \beta_1 \tag{56}$$

and the ϵ -expansion results for α_b , β_b , and β_1 one has the ϵ expansion for γ_1

$$\gamma_1 = \frac{1}{2} + \frac{2+n}{2(n+8)} \epsilon + O(\epsilon^2)$$
(57)

and we hence conclude that at the marginal dimensionality

$$\chi_{1} \propto |t|^{-1/2} |\ln|t| |^{(2+n)/(n+8)}$$

$$(= |t|^{1/2} |\ln|t| |^{1/3} \text{ for } n=1) . \quad (58)$$

Similarly, we use the scaling law¹⁵

$$\gamma_s = \gamma_b + \nu_b \tag{59}$$

to find the ϵ expansion

$$\gamma_{s} = \frac{3}{2} + \frac{3(n+2)}{4(n+8)}\epsilon + O(\epsilon^{2})$$
(60)

and hence estimate at the marginal dimensionality

$$\chi_{s} \propto |t|^{-3/2} |\ln|t| |^{3(n+2)/2(n+8)}$$

(= |t|^{-3/2} |\ln|t| |^{1/2} for n = 1). (61)

Equations (55), (58), and (61) should hold for short-range systems with d = 4 (which is accessible in Monte Carlo computer simulations, see, e.g., Ref. 38) and for dipolar systems for d = 3 in the case where the easy axis is parallel to the surface. To see how the treatment of Sec. II valid for general directions is modified by these logarithmic corrections, it is most convenient to use generalized Ginzburg-Landau theory.³⁹ Adjusting this method to our case the free-energy expression Eq. (7) is still used but the temperature dependence of the coefficients A, B, C, and λ has to be modified in order to reproduce the correct critical behavior of the dipolar system. Since^{24, 25}

$$X_b \propto |t|^{-1} |\ln|t| |^{(n+2)/(n+8)}$$

and $\chi_{b}^{-1} \propto A$, we must choose

$$A = 2a''(T - T_C) \left| \ln(T - T_C) \right|^{-1/3}$$

in our case. Since [Eq. (45)]

$$P_{m} \propto |t|^{1/2} \ln |t|^{1/2}$$

and [Eq. (15)] $P_{\infty} = -(A/B)^{1/2}$ we must have²⁶ $B = b' |\ln|t| |^{-1}$. Finally we conclude from Eq. (45)

(50)

and the relation $\xi_b = (C/A)^{1/2}$ that C is not modified by any logarithmic factor.

In order to obtain the logarithmic correction for λ it is most convenient to consider the case where the spontaneous polarization is parallel to the film, and hence we have the relation¹⁵

$$P_1 = (\lambda/\xi_b) P_{\infty} . \tag{62}$$

In the case with non-mean-field exponents this relation can hold within the framework of a generalized Landau theory only if λ also has a power-law singularity at T_C ,

$$\lambda = \hat{\lambda} t^{-x_{\lambda}} \tag{63}$$

and Eqs. (62) and (63) then imply a scaling law

$$x_{\lambda} = \nu_b + \beta_b - \beta_1 . \tag{64}$$

From Eq. (54) and the ϵ -expansion results for ν_b and β_b we find the exponent x_{λ} within first-order ϵ expansion as

$$x_{\lambda} = \frac{n+2}{4(n+8)}\epsilon + O(\epsilon^2) .$$
(65)

Invoking once more Eq. (48) we predict that at the marginal dimensionality

$$\lambda = \hat{\lambda}' |\ln|t| |1^{1/6} (n=1) .$$
(66)

Thus the extrapolation length diverges at the critical point. Since for a direction of the polarization not parallel to the film the length κ^{-1} stays finite, Eq. (24) implies that $P_1 \rightarrow P_{\infty}$ as $T \rightarrow T_C$ and hence $\hat{\chi}_1 = \hat{\chi}_b$. Equation (43) also implies $\hat{P}_1 = \hat{P}_{\infty}$. A different result, however, is obtained for the surface susceptibilities $\chi_s^{(1)}$ and $\chi_s^{(2)}$ since Eqs. (29) and (32) imply

$$\chi_s^{(1)} \propto t^{-2} |\ln|t| |^{1/2}, \quad \chi_s^{(2)} \propto t^{-1} |\ln|t| |^{1/6}.$$
 (67)

Concluding this section we emphasize that all results are based on the suggestion that Eq. (48) also holds for the critical behavior of surface quantities. Since Bray and Moore¹⁷ have argued that the critical exponents of surface quantities can be expressed in terms of bulk critical exponents only, this suggestion is quite plausible because Eq. (42) does hold for all bulk properties.

IV: ORDERED SURFACE LAYERS IN DIPOLAR SYSTEMS

In the mean-field treatment where depolarizing fields are omitted, it is well known¹⁵ that for negative extrapolation lengths λ (which may occur for sufficiently enhanced interaction strength at or close to the surface layer) the surface starts to order at a temperature T_c^s higher than the bulk T_c , which is given by

$$(T_C^s - T_C)/T_C = \frac{C}{a'T_C}(-\lambda)^{-2}$$
 (68)

This is no longer true in the case where the depolarizing fields are included if \vec{P} is not parallel to the surface. From Eqs. (25) and (35) it is seen that $T_C^s = T_C$ for negative extrapolation lengths λ not exceeding a critical value λ_c which is given by

$$\lambda_c = -\frac{(C/4\pi)^{1/2}}{\cos\theta}$$
 (69)

For $|\lambda| < |\lambda_c|$ the surface ordering starts at T_C^s given by

$$(T_C^s - T_C)/T_C$$

$$= \frac{C}{a'T_C} [(-\lambda)^{-2} - (-\lambda_c)^{-2}] \xrightarrow{\lambda - \lambda_c} \frac{2C}{a'T_C} \left| \frac{\lambda_c - \lambda}{\lambda_c^3} \right|.$$
(70)

If one relates λ to microscopic exchange parameters of a nearest-neighbor Ising model one finds that $\lambda^{-1} \propto \Delta - \Delta_c$, where Δ describes the enhancement of interaction within the surface layer (assuming that all other interaction constants are not modified by the existence of a free surface), and $\Delta_c = \frac{5}{4}$ for a simple cubic lattice.¹⁵ Defining, then, the crossover exponent ϕ at the multicritical point (T_C, Δ_c) in terms of $T_C^s - T_C \propto (\Delta - \Delta_c)^{1/\phi}$, Eq. (68) would imply $\phi = \frac{1}{2}$.¹³ In our case we find that a larger enhancement Δ_c is necessary for surface ordering and

$$T_C^{\mathbf{s}} - T_C \propto (\Delta - \Delta_{c'}) (\Delta_{c'} - \Delta_c) , \quad \Delta \to \Delta_c , \qquad (71)$$

which implies that now $\phi = 1$. This different behavior is illustrated in Fig. 5. Finally, Fig. 6 com-



FIG. 5. Schematic variation of the critical temperature T_c^s of the surface layer as function of the enhancement Δ of the interactions at the surface as compared to the bulk.

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FIG. 6. Schematic variation of polarization in the vicinity of the surface for negative extrapolation length λ in the case $\kappa^{-1} < |\lambda|$ (a) and in the case $\kappa^{-1} > |\lambda|$ (b), the latter case being possible only for $|\lambda| < |\lambda_c|$ and $T_C < T < T_C^s$. Note that κ^{-1} is not the bulk correlation length.

pares the variation of the polarization across the film in the regime $\Delta_c < \Delta < \Delta_c'$ (i.e., $\lambda < \lambda_c < 0$) to the behavior in the regime $\Delta > \Delta_c'$ (i.e., $\lambda_c < \lambda < 0$). In both cases the polarization at the surface is enhanced in comparison to that in the bulk. But in the first case the length κ^{-1} is smaller than $|\lambda|$ even at T_C and hence no spontaneous surface ordering is possible because it would have to carry a polarization induced over the whole bulk. Spontaneous surface ordering requires that κ^{-1} exceeds $|\lambda|$.

V. CONCLUSIONS

In this paper we have considered the critical behavior near the surface of a ferroelectric (or dipolar ferromagnet like LiTbF₄, CrBr₃ etc.). In our treatment we have assumed a monodomain sample, which for a direction of polarization not parallel to the surface of the thin film is possible only for a ferroelectric between metallic plates in short-circuit condition where a complete compensation of the depolarizing field by free-surface changes may occur. In magnets or ferroelectrics with other boundary conditions the depolarizing field is compensated by a strip domain structure. However, our treatment applies to this situation as well, as long as the domain size is by far larger than the thickness of the Bloch walls between the domains, a condition which usually is well satisfied. The results obtained here refer then to the variation of the order parameter within a domain a distance apart from the domain wall such that this distance distinctly exceeds the correlation length. Our results include:

(i) For temperatures not too close to T_C , i.e., where the bulk correlation length ξ_b satisfies the condition $\xi_b \ll (C/4\pi)^{1/2}\cos\theta$, where θ is the angle between the easy axis of the polarization and the direction normal to the surface, depolarization effects on the critical behavior are negligible. In this regime the critical behavior at the surface is correctly described by previous mean-field treatments, apart from temperatures very close to T_C , where logarithmic correction factors appear. These correction factors have been estimated using results of renormalization-group theory, and are listed in Eqs. (55), (58), and (61). Within a generalized Landau theory these results would require to use an extrapolation length diverging logarithmically as the critical temperature is approached. Clearly, these logarithmic corrections can become appreciable only when the dimensionless parameter A [Eqs. (7) and (37)] is small, which requires $k_B(T - T_C) \ll q^2/a^3$, where q is the dipole moment per ion and a is the lattice spacing. Since at the same time the above condition on the correlation length has to be satisfied, which can be rewritten

$$A = a^{3}k_{B}(T - T_{C})/q^{2} >> 4\pi\cos^{2}\theta$$
,

one can see these logarithmic effects only when the direction of polarization is (at least nearly) parallel to the film.

(ii) In the regime where $\xi_b \ge (C/4\pi)^{1/2} \cos\theta$ the order parameter decays from its value at the surface to the bulk value no longer over a distance ξ_b but already on a shorter distance

$$\kappa^{-1} = 1/(\xi_b^{-2} + 4\pi \cos^2\theta/C)^{1/2}$$

(Figs. 3 and 6). This fact has the consequence that the surface layer polarization P_1 and associated susceptibility χ_1 have the same critical exponents as the corresponding bulk quantities, only amplitude factors are different [Eqs. (25) and (43), and Fig. 4]. If one can come close enough to T_C that logarithmic corrections have to be taken into account we even predict that $P_1 = P_{\infty}$ and $\chi_1 = \chi_b$ asymptotically. Thus the depolarization effects strongly reduce the difference in critical behavior between surface and bulk. On the other hand, they produce a more pronounced depression of the polarization below P_{∞} in a thin film (Fig. 3); therefore now two different surface polarizations $P_s^{(1)}$ and $P_s^{(2)}$ and susceptibilities $\chi_s^{(1)}$ and $\chi_s^{(2)}$ have to be defined [Eqs. (28)-(32)] which have different critical behavior. We also find that now the shift of T_C varies like 1/L for film thicknesses $L \rightarrow \infty$ instead of $1/L^2$ as previously seen [Eq. (44)].

(iii) In the case where the interactions at the surface are enhanced it turns out that a larger enhancement Δ_c' is necessary to produce two-dimensional ordering at a temperature T_c^s higher than the bulk T_C (Fig. 5). However, this enhancement of T_c^s varies linearly with the interaction enhancement Δ rather than quadratic in $\Delta - \Delta_c$, as is found without depolarization effects.

(iv) With regard to the experimental verification of our results it must be remarked that we have considered the symmetric situation where the boundary condition on both surfaces of the film are precisely equivalent. Hence a rather complete compensation of depolarizing fields is possible, in contrast to the cases of asymmetric boundary conditions considered by Würfel and Batra, where a noncompensated depolarizing field may exist across the bulk of the film, and hence the free surfaces have a much more drastic effect on the film. Hence, in order to experimentally check the predictions of our theory, it is most important to provide the same boundary conditions at the surface of the film. Since we feel that none of the size and surface effects reported so far in the literature for ferroelectrics allow a meaningful comparison with our treatment, additional experiments are strongly suggested.⁴⁰ Most promising would also be the study of surface properties near T_C in dipolar magnets like LiTbF4, since there logarithmic corrections for bulk critical properties have been established much more clearly than for ferroelectrics.

(v) As a last comment we note that the present work adds nothing new to the critical properties at surfaces of antiferroelectrics or other systems undergoing structural phase transitions. For these systems the dipolar interaction should not make any relevant distinction as compared to systems with truly shortrange interaction, and hence (for uniaxial systems) the surface critical behavior should be the same as that of the Ising model studied extensively previously.

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