# Critical behavior of order-parameter fluctuations in liquid $^4$He near $T_\lambda$

B. Lambert, R. Perzynski, and D. Salin
*Laboratoire d'Ultrasons, Université Pierre et Marie Curie, Tour 13, 4 place Jussieu, 75230 Paris Cedex 05, France*
(Received 27 October 1978)

We have measured the critical attenuation of first sound in liquid $^4$He from 550 to 1700 MHz. We can describe the attenuation due to the fluctuations of the order parameter up to 1.7 GHz, using experimental data at lower frequencies from various authors. Above $T_\lambda$, the fluctuations are the only phenomena and are analyzed with a scaling function of $\omega\tau_2$ with a characteristic time $\tau_2 = 2 \times 10^{-12} \times t^{-1.062}$ sec, $t = |(T - T_\lambda)/T_\lambda|$. Below $T_\lambda$, several phenomena are present. In the range of 1 GHz, the main contribution is due to the fluctuations. We demonstrate experimentally that the fluctuation attenuation is not symmetric about $T_\lambda$. We fit our data with a scaling function of $\omega\tau_2$.

## I. INTRODUCTION

During the last ten years, many investigations have provided important data on critical phenomena and especially on the $\lambda$ transition of liquid helium. The critical attenuation and dispersion of first sound at low frequencies ($\lesssim 1$ MHz) have been first described in terms of order-parameter relaxations (Landau and Khalatnikov[1]). Experiments (for instance from Williams and Rudnick[2]) have confirmed the existence of these relaxations, but the data exhibit an additional attenuation due to the fluctuations of the order parameter which are not so well known.

Recently Buchal and Pobell[3] have performed first-sound measurements over a wide range of low frequencies (2 – 600 kHz). Above $T_\lambda$ they described the fluctuation attenuation with a scaling function of $\omega\tau_2$,[4] $\tau_2$ being a characteristic time of the interaction between first- and second-sound waves. Below $T_\lambda$ they analyzed the relaxations as previously but more precisely, assuming the symmetry of the fluctuation attenuation around $T_\lambda$.

In a higher-frequency range, Imai and Rudnick[5] performed an experiment at 1 GHz. It is not possible to fit these data as was done for the low-frequency case, especially for the relaxation attenuation which becomes small compared with the fluctuation attenuation. A complete description in 1 GHz range requires data over a wide-frequency range.

We have measured the attenuation and dispersion of first sound from 550 to 1700 MHz in liquid helium. This experiment is a good opportunity to study the fluctuations of the order parameter. We compare these data to the low-frequency ones. As a result we analyze these fluctuations in the same way from 2 kHz to 1.7 GHz, above and below the transition.

## II. EXPERIMENTAL AND THEORETICAL PRESENTATION

Several phenomena may contribute to the critical attenuation and dispersion of the first sound in $^4$He around $T_\lambda$. In this part we present two of these: the first involves the relaxations of the order parameter and the second involves its fluctuations.

### A. Relaxation

Historically, the mechanism considered first was the relaxation of the order parameter to its equilibrium value after an initial disturbance. The first-sound waves are scattered by these order-parameter variations. The problem has been treated by Landau and Khalatnikov.[1] They found a nonequilibrium order parameter decaying exponentially in time, with a relaxation time $\tau_{LK}$, a resulting attenuation, and a dispersion of first-sound velocity. This process contributes only below the transition ($T < T_\lambda$) where the time average of the order parameter is nonzero.

It is known that Landau theory in its original form is not a good description for critical $^4$He properties. However it is possible[6] to describe the attenuation due to order-parameter relaxations in $^4$He by a linear coupling between first and second sound. One obtains for $\alpha_R$ and $D_R$

$$\alpha_R = \frac{\Delta u_1}{u_1^2} \frac{\omega^2 \tau_2}{1 + \omega^2 \tau_2^2} \ , \tag{1}$$

$$D_R = u_1(\omega) - u_2(0) = \Delta u_1 \frac{\omega^2 \tau_2^2}{1 + \omega^2 \tau_2^2} \ , \tag{2}$$

where $\tau_2$ is the order-parameter relaxation time $\tau_2 = \tau_2^0 t^{-x}$, and $t = |(T - T_\lambda)/T_\lambda|$ is the reduced temperature. $\Delta u_1$ is known from thermodynamic relations and is the difference between the first-sound velocity at infinite frequency, $u_1(\infty)$, and the one at zero frequency, $u_1(0)$ $[u_1 = (\partial P/\partial \rho)_s^{1/2}]$.

Equation (1) states that the critical attenuation $\alpha_R$ has a maximum for $\omega\tau_2 = 1$ and vanishes at the transition. It is well-known experimentally that the critical attenuation is nonzero at $T_\lambda$, and for $T > T_\lambda$ there is also a critical attenuation. Thus we have to invoke another process.

### B. Fluctuations

The first-sound waves are also scattered by the order-parameter variations that are the fluctuations around the equilibrium value. These processes contribute on both sides of the transition. Several theoreticians[7,8] have discussed this problem. Using a mode-mode coupling method (one first-sound wave breaking up into two order-parameter fluctuations) and from general scaling arguments, Kawasaki[7] obtains the following scaling form for the fluctuations contribution to the critical first-sound attenuation[7]:

$$\alpha_F = C_F \omega^{1+s/x'} f_F(\omega\tau_F) \quad , \tag{3}$$

where $C_F$ is a constant, $s$ is a small positive number, and $\tau_F = \tau_F^0 t^{-x'}$ is a characteristic time of the order-parameter critical dynamics ($x'$ being its critical exponent).

We must note that this expression is valid only when the first-sound wavelength is much greater than the critical fluctuation correlation length ($\lambda >> \xi$).

### C. Characteristic times

In the transition vicinity, several elementary mechanisms can exist: Swift and Khadanoff[9] have made a detailed study of this problem; they have considered the breaking up, near $T_\lambda$, of a transport mode into several other modes. They have found two kinds of important phenomena. The first one involves multiple production of first-sound waves with a characteristic time $\tau_1 = \xi/u_1 \propto t^{-2/3}$ with $\xi$ the coherence length ($\xi = \xi_0 t^{-\nu}$, $\nu \sim \frac{2}{3}$) and $u_1$ the first-sound velocity. The second important phenomenon is an interaction between first- and second-sound waves with a characteristic time $\tau_2 = \xi/u_2 \propto t^{-1}$ with $u_2$ the second-sound velocity ($u_2 \sim u_{20} t^w$, $w \sim \frac{1}{3}$).

These two times differ by their critical exponents and by their order of magnitude. As $u_1 >> u_2$, for every temperature we have $\tau_1 << \tau_2$. So $\tau_2$ is the leading characteristic time for low frequencies ($\omega\tau_2 << 1$). For higher frequencies, $\tau_1$ becomes more important.

### D. Summary of experimental results at low frequency ($\omega/2\pi < 1$ MHz)

At low frequencies, critical attenuation has been analyzed for $T > T_\lambda$ due to the fluctuations of the order parameter and for $T < T_\lambda$ as the sum of both fluctuations and relaxations.[2]

$$\alpha^+ = \alpha_F^+ \text{ for } T > T_\lambda \ ,$$

$$\alpha^- = \alpha_F^- + \alpha_R \text{ for } T < T_\lambda \ .$$

The fluctuation part of the attenuation is assumed to be symmetric versus $T_\lambda$.[2]

For the same $t$ we have $\alpha_F^+ = \alpha_F^- = \alpha^+ \equiv \alpha_F$. So, with this assumption, the relaxation contribution to attenuation corresponds to the remaining part, for $T < T_\lambda$, $\alpha_R = \alpha^- - \alpha^+$. Then the experimental results of $\alpha_R$ and also of the dispersion $D_R$ are well fitted by expressions (1) and (2). The involved characteristic time is the time related to the second-sound velocity $\tau_{LK} \equiv \tau_2 = \tau_2^0 t^{-x}$ with[3] $x = \nu + w = 1.062$. The value of $\tau_2^0$ is adjusted from experimental results,[3] $\tau_2^0 = 2.01 \times 10^{-12}$ sec.

The fluctuations ($\alpha_F = \alpha^+$) contribution to the attenuation has been analyzed with a scaling function of $\omega\tau$ for at least four decades of $\omega\tau$.[3]

$$f_F(\omega\tau) = \omega\tau/(c + \omega\tau) \tag{4}$$

with $c = 0.506$ and $\tau = \tau_2$.

Such an empirical scaling function with $c = 1$ has been first suggested by Ahlers.[4] Recently, Kroll[10] has attempted to calculate $f_F(\omega\tau)$ using mode-mode coupling and renormalization-group theory: the results [see for example Fig. 16 of Ref. 3(a)] disagree with experiments at low frequencies.

From 2 – 600 kHz, the characteristic time involved for relaxations and fluctuations is the same. We must note that both of these have been determined separately.

Buchal and Pobell[3] have also found a scaling function for dispersion of first sound. But the function explains only experimental results on two decades of $\omega\tau$ and diverges near $T_\lambda$.

### E. First-sound attenuation at higher frequencies

Earlier experiments[2] from 600 kHz to 3.17 MHz have been analyzed in the same way; because of the restricted frequency range the conclusions were less refined.

From 10 MHz to 1 GHz, three series of experiments have been performed: from 10 – 270 MHz,[11,12] at 650 MHz,[13] and at 1 GHz.[5] Tozaki and Ikushima[11] have presented their own results, and reviewed the experimental data from 600 kHz to 1 GHz. As a result, they observed that for $T > T_\lambda$, the attenuation follows a scaling function of $\omega\tau_2$ (with $\tau_2 \propto t^{-x'}$, $x' = 1$); below $T_\lambda$ the conclusions were not so clear (Fig. 3 of Ref. 11), but we can remark that for frequencies larger than 160 MHz we can not assume that the fluctuations part of the attenuation is symmetric: for the same value of $t$, the branches $\alpha^-$ and $\alpha^+$ can intersect and sometimes $\alpha^+ > \alpha^-$. The last experiment performed at high frequency is at 9 GHz,[14] the attenuation is measured in the films for $T < T_\lambda$ and its interpretation involved the characteristic time $\tau_1 \propto t^{-2/3}$.

Thus it seemed interesting to perform an experiment in the GHz frequency range in order to clarify these points. The purpose of this paper is to describe the results of such an experiment.

## III. EXPERIMENT

We have performed a measurement of first-sound attenuation and velocity in $^4$He around the λ point (1.4 – 2.4 K) at high frequencies (550 – 1700 MHz).

### A. Principle

We use a classical ultrasonic transmission method through a sample of liquid $^4$He. An elastic and periodic deformation, generated by a transducer, propagates through helium. We measure the intensity and the phase of this deformation after crossing the sample.

The cell is made of two *X*-cut quartz rods separated by helium. Because of the strong attenuation of helium in our range of frequencies and temperatures, we need a very thin sample, a few micrometers thick.

The transmitted signal has to cross two quartz-helium boundaries. Because of the impedance mismatch between quartz and helium the transmission loss is ~27 dB at each interface. This *ab initio* loss implies that the incident signal must be rather large. The transducer is made of ZnO layers deposited on platinum with a fundamental resonance at 1200 MHz ($\frac{1}{2}\lambda$). The frequency response is essentially constant from 500 to 2400 MHz; with an incident power of 100 mW, the first echo reflected by quartz at 1 GHz and 4 K is about 80 dB above the noise. A classical superheterodyne system is used for detection.

Good experimental conditions require very parallel faces. The quartz rods are cut in the laboratory with a face parallelism precision of 10 sec of arc. A mylar ring defines a space for helium between the two quartz. Various rings ranging in thickness 3 – 8 $\mu$m were used. The system is then aligned with a goniometer and the parallelism of the two interfaces is better than one minute of arc.

### B. Experiment

In order to obtain, for different frequencies, the attenuation and velocity of first sound versus the temperature, we measure the relative variations of the transmitted signal (in dB) and the variations of phase. The phase $\phi$ is measured by interference and the reference is the first reflected echo in the emitting quartz.

The cell is immersed in an helium bath and the temperature is measured through the helium-saturated vapor pressure of the bath. The accuracy of our temperature measurement is better than 1 mK.

### C. Direct experimental results

The first point is that there is no critical dispersion ($D = u_\omega - u_0$) of first-sound velocity in our range of frequencies (550 – 1700 MHz) and temperature (1.5 – 2.3 K) within the precision of our data (Fig. 1). The accuracy of our phase measurement is

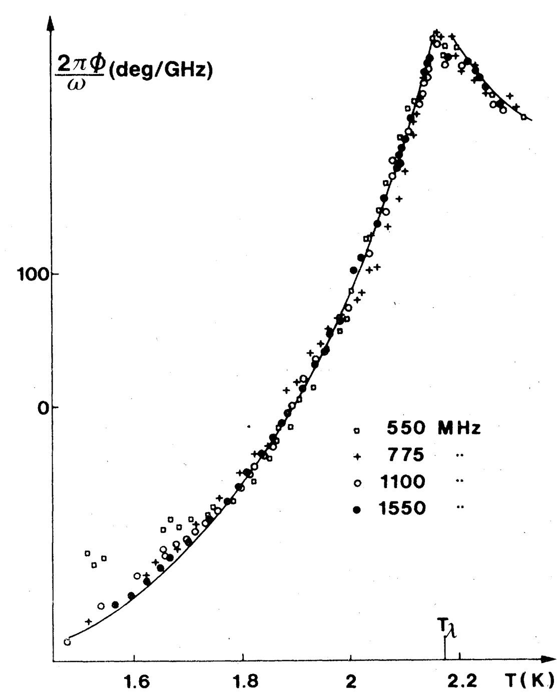


FIG. 1. Variation of the ratio $2\pi\phi/\omega$ vs $T$ for different frequencies. The full line corresponds to $2\pi\phi/\omega = 2\pi d\Delta u_1/u_1^2$ obtained from $d = 4.1\mu \pm 0.2$ and from the variations $\Delta u_1$ of the velocity vs temperature (Refs. 15 and 16).

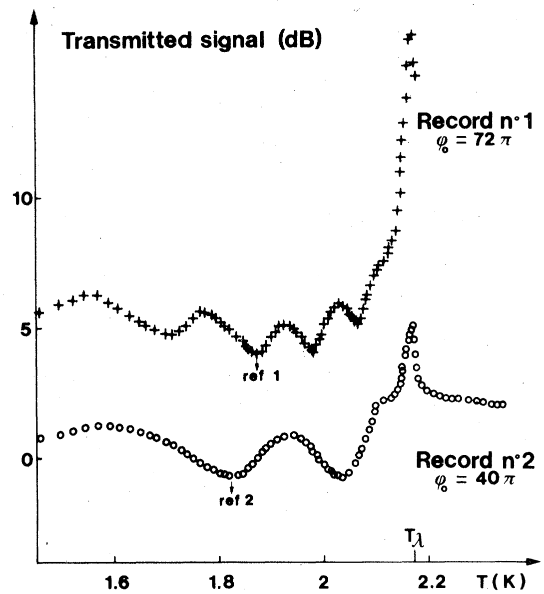


FIG. 2. Two records of the transmitted signal vs $T$ obtained at 1100 MHz with two sample thicknesses ($d_1 \sim 7.5\mu$m and $d_2 \sim 4\mu$m).

$\Delta\phi/\phi \sim 10^{-3}$, and the precision on the velocity ($\Delta\phi/\phi = \Delta u/u$) is then $\Delta u \sim 0.2$ m/sec. Very close to $T_\lambda$, this accuracy is even worse because the signal is very small. This rather bad precision is due to our small sample thickness $d \sim 5$ $\mu$m.

From an extension of low-frequency measurements, we expect at 1 GHz,

$$u_\omega - u_0 \sim 0.05 \text{ m/sec} \quad \text{for } |T - T_\lambda| \sim 40 \text{ mK}$$

and

$$u_\omega - u_0 \sim 0.2 \text{ m/sec} \quad \text{for } |T - T_\lambda| \sim 4 \text{ mK} .$$

These values are of the order of magnitude of our precision. This means that we can not measure this dispersion.

The intensity of the transmitted signal exhibits, at all frequencies, a critical divergence at the $\lambda$ point (Fig. 2), and also exhibits some extra oscillations. As shown above, the first-sound velocity $u_1$ in helium varies with temperature. These velocity variations induce phase variations of the transmitted signal ($\phi = \omega d/u_1$). Because the transmitted signal is the sum of different reflections in the sample, the phase variations will cause oscillations in the transmitted intensity through wave interference.

We have now to relate these experimental records to the attenuation in the sample.

## IV. TRANSMISSION COEFFICIENT THROUGH THE SAMPLE

We first make this calculation in the case of a perfect parallelism between the two interfaces. We consider a system of three media with two parallel boundaries. If we designate $k_{\rm I}$, $\rho_{\rm I}$, $v_{\rm I}$ the wave vector, the density, and the acoustic velocity in the quartz, and respectively $k_{\rm II}$, $\rho_{\rm II}$, $v_{\rm II}$ those in helium, we define the complex impedance ratio of the two media

$$z_0 e^{-i\theta} = \frac{k_{\rm II} v_{\rm II} \rho_{\rm II}}{k_{\rm I} v_{\rm I} \rho_{\rm I}} . \tag{5}$$

All these quantities are real except

$$k_{\rm II} = k' + ik'' ,$$

$z_0 \sim 2.2 \times 10^{-3}$ and $\theta$ characterizes the attenuation of the medium ($\tan\theta = k''/k' = \alpha/k'$), $\theta$ being small here, i.e., $\theta \sim \alpha/k' \sim \alpha d/\phi$.

A short calculation gives the transmission-coefficient intensity (with $z_0 << 1$, which is the case in our experiment)

$$T_{\rm int} \simeq \frac{8 z_0^2}{\cosh 2k''d - \cos 2k'd} . \tag{6}$$

We must note that this formula is not valid when $k''d << 1$, but this expression suits our experimental conditions where the transmitted signal is always small.

To the first order, the phase $\phi$ of the transmitted signal is equal to $k'd$. Equation (6) gives, other parameters being constant, the oscillations of $T_{\rm int}$ with $\phi$. This expression (6) of the transmission allows us to explain some records where the parallelism is particularly good, but usually we need a more refined expression of the transmission coefficient, taking into account a slight defect of parallelism ($<1'$). We have made this calculation assuming several things:

(a) Under oblique incidence both transverse and longitudinal waves are created in the quartz; a longitudinal wave brings much more energy (about $10^4$ more, for an incidence of $1'$ between He and quartz) than the transverse ones; then we neglect the transverse waves.

(b) The reflection and transmission coefficients at a single boundary depend on the incidence angle to the second order only; when the incidence angle is small we are allowed to consider at the first order these coefficients as constant.

The total transmission coefficient in amplitude can be rewritten from calculations very similar to the optical "air corner"; we obtain

$$T_{\text{ampl}} = \frac{1}{2\epsilon}\int_{-\epsilon}^{+\epsilon} T(\beta)\, d\beta \ ,$$

with (7)

$$T(\beta) = 4z_0 e^{-i\theta} \sum_{n=0}^{\infty} \left(e^{ik_{\text{II}}(d+\beta)}\right)^{2n+1} \ ,$$

$d+\beta$ being the thickness of the sample, and $\beta$ varying between two extreme values $\pm\epsilon$. ($2\epsilon$ is the difference of thickness due to the defect of parallelism.)

After some straightforward calculations,[17] one can obtain, assuming $\cosh k''\epsilon \sim 1$ and $\sinh k''\epsilon \sim k''\epsilon$,

$$T_{\text{int}} = \frac{z_0^2}{\epsilon^2 |k_{\text{II}}|^2} \left[ \text{arctanh}^2 \left( \frac{2\gamma \sin\phi \cosh\theta\phi}{\cosh^2\theta\phi - \cos^2\phi + \gamma^2} \right) + \arctan^2 \left( \frac{2\gamma \cos\phi \sinh\theta\phi}{\cosh^2\theta\phi - \cos^2\phi - \gamma^2} \right) \right] \ , \tag{8}$$

if

$$\cosh^2(\theta\phi) - \cos^2\phi - \gamma^2 > 0 \ ,$$

with $\gamma = \sin k'\epsilon$.

If $\cosh^2\theta\phi - \cos^2\phi - \gamma^2 < 0$ ($\gamma$ and $\cos\phi$ being positive) $\arctan^2(\cdots)$ must be transformed into $(\pi - \arctan^2|\cdots|)$ for continuity.

This expression is only valid for sample thicknesses and attenuations not too small ($k''d \gtrsim 1$) and for a small value of $\epsilon$ ($k''\epsilon << 1$).

We analyze our results with such a transmission coefficient. The value of $\gamma$ can not be determined experimentally, even through frequency variations. We consider $\gamma$ as an adjustable parameter. From the experimental records and with the calculated expression of $T_{\text{int}}$ we can obtain the ultrasonic attenuation versus temperature for each of our frequencies. One can see, for instance, on Figs. 2 and 3, that from two different records at the same frequency with different sample thicknesses, our analysis gives the same curve for the attenuation. This demonstrates that Eq. (8) connects very well our records with the attenuation, even if this formula is a little bit complicated.

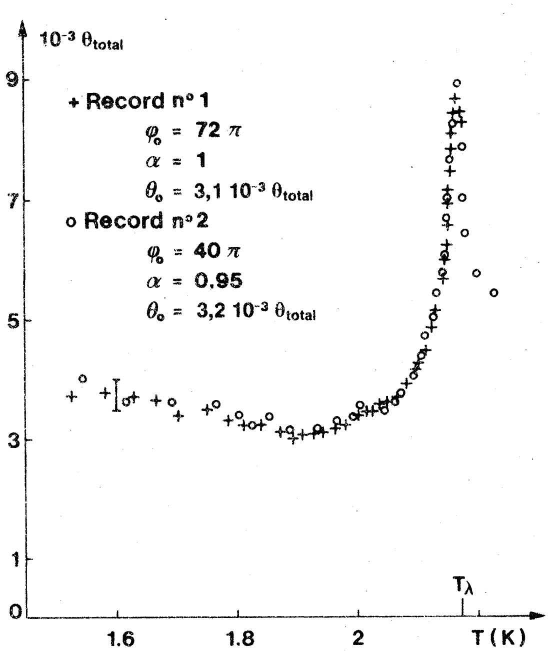


FIG. 3. Values of $\theta_{\text{total}}$ at 1100 MHz obtained from the records No. 1 and No. 2 of Fig. 2.

In our analysis we must assume that $\gamma$ is a constant during a whole record. This assumption is only valid for certain values of $\gamma$.

Our nonparallel analysis needs a calibration point: we choose it in the temperature range of 1.8 – 1.9 K. We know from previous experiments at 1 GHz [5] (and from our own results for strictly parallel samples) that the first-sound attenuation is almost constant in this range of temperature. We assume, and we verify this assumption in good parallelism cases, that in this range the total attenuation is proportional to $\omega^2$. Our analysis is done with $\theta = 2.9 \times 10^{-3}\omega/2\pi$ ($\omega/2\pi$ in GHz) for $T \sim 1.8 - 1.9$ K (which is in agreement with Ref. 5). We obtain by this method a total accuracy $\Delta\theta = 5 \times 10^{-4}$.

## V. CRITICAL VARIATIONS OF ULTRASONIC ATTENUATION

We want to isolate the part of the attenuation purely due to the phase transition. From our experiments we obtain

$$\theta(T) = k''/k' = [v(T)/\omega]\alpha(T) \ .$$

This value of $\theta$ is the sum of a critical part $\theta_C$ and a noncritical one $\theta_B$, which is our background.

### A. Noncritical $\theta$: $\theta_B$

The choice of $\theta_B$ is a delicate problem. At low frequencies other researchers[3] covering a small range of critical temperature chose a temperature-independent background up to 271 MHz; Tozaki and Ikushima[11] assumed a background attenuation linear in temperature. At 1 GHz, an estimate[5] has been made of the background attenuation due to classical losses: shear viscosity and thermal conductivity for $T > T_\lambda$ and only shear viscosity for $T < T_\lambda$. The result is that $\theta_B$ is proportional to $\omega$. For $T > T_\lambda$ we take into account the contribution of shear viscosity and thermal conductivity (see Fig. 4); for $T < T_\lambda$ there is an additional term to $\theta_B$. The elementary excitations (phonons and rotons) give, at low temperatures, a contribution which peaks at 1.5 K and 1 GHz.[5] This additional part of $\theta_B$ proportional to $\omega$ is very difficult to calculate precisely[18]; we estimate the total attenuation $\theta_B$ for $T < T_\lambda$ by a smoothly linear temperature-dependent contribution. The variations of $\theta_B$ are shown on Fig. 4. The contribution due to shear viscosity for $T < T_\lambda$ is shown for comparison. The value of $\theta_B$ is about 20% of the maximum total attenuation; the frequency dependence of $\theta_B$ is taken linear. We have verified that a weak variation of the slope of $\theta_B$ has a small incidence on the scaling functions described later in the text; we take into account this variation in order to estimate the accuracy of the scaling functions.

### B. Critical attenuation $\theta_C$

Some of our results on the critical variations are plotted in Fig. 5. As we can see in this figure, and as we mentioned before, it is no longer allowed to assume that the fluctuations are symmetric versus $T_\lambda$. (For certain values of $t$, we have $\alpha^+ > \alpha^-$). We shall then separately analyze our results above and below $T_\lambda$; we begin by the critical attenuation

$$\alpha_\lambda = \alpha(T_\lambda) = (\omega/v)\theta_C(T_\lambda) = (\omega/v)\theta_\lambda$$

at the $\lambda$ point.

#### *1. Variations with frequency of the critical attenuation at the $\lambda$ point*

The value of the attenuation $\alpha$ at the transition gives the coefficient $c_F\omega^{1+s/x'}$ of Eq. (3) $[f_F(\omega\tau(T_\lambda)) = 1]$. The study of $\alpha_\lambda$ over a broad range of frequency would give a measurement of $1+s/x$; from 2 to 600 kHz, $1+s/x = 1.15$; but if the results up to 1 GHz are added to the previous results, a fit requires $1+s/x = 1.33$.[11] We have to note that values of $\alpha_\lambda$ depend on the estimation of $\theta_B$ which varies sensibly from one work to another. Our own values of $\alpha_\lambda$ are compatible with $1+s/x = 1.33$ up to 1200 MHz; for higher frequencies we need a higher value of $1+s/x$.

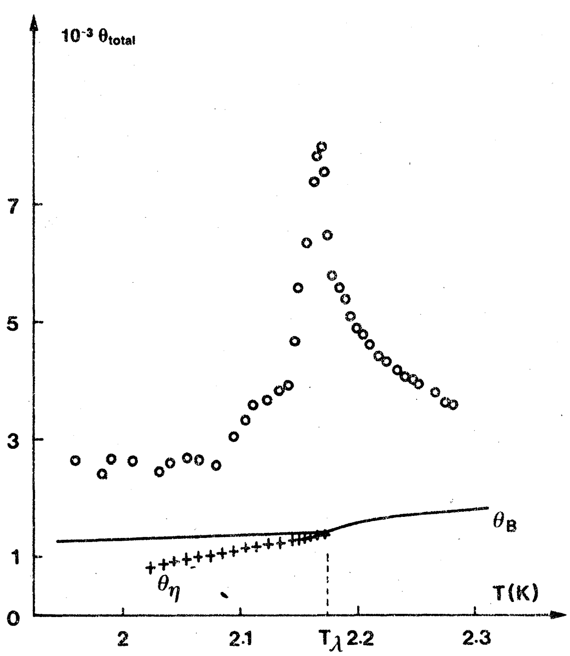


FIG. 4. Circles show the variations of $\theta_{total}$ at 775 MHz; the full line is the background $\theta_B$ (see the text); the crosses correspond to the shear viscosity contribution $\theta_\eta$ for $T < T_\lambda$.

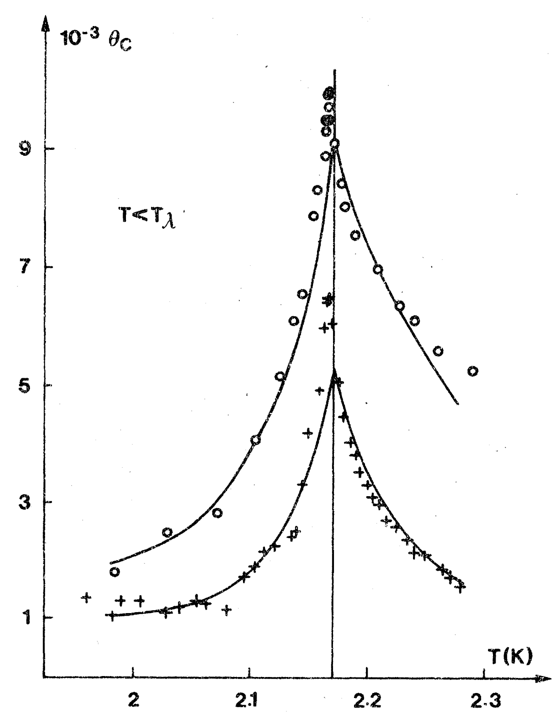


FIG. 5. Variations of $\theta(T)$ at 775 MHz (crosses) and 1690 MHz (circles); the full lines correspond to $\theta/\theta_\lambda = \omega\tau_2/(c+\omega\tau_2)$ with $\tau_2 = 2\times10^{-12}t^{-1.062}$ sec (Ref. 3); $c = 0.5$ for $T > T_\lambda$ and $c = 1$ for $T < T_\lambda$.

### 2. *Critical attenuation above $T_\lambda$*

Figure 6 presents the values of $\alpha/\alpha_\lambda = \theta/\theta_\lambda$ vs $(2\pi/\omega)(T - T_\lambda)$ for $T > T_\lambda$. It shows that from 10 MHz to 1.7 GHz, $\alpha/\alpha_\lambda$ is function of $\omega\tau$ with $\tau \sim t^{-1}$. In order to connect these results with the low frequency ones we have to use a more precise value of the critical exponent of $\tau$. We analyze our results with the value obtained by Buchal and Pobell[3] from 2 to 600 kHz ($\tau_2 = 2 \times 10^{-12} t^{1.062}$ sec) and also Eq. (4) with $c$ as a free parameter.

Our conclusion is that one can describe the critical attenuation from 2 kHz to 1.7 GHz for $T > T_\lambda$ with the same scaling function (see for instance, Fig. 7)

$$\frac{\alpha_c^+}{\alpha_\lambda} = \frac{\theta_c^+}{\theta_\lambda} = \frac{\omega\tau_2}{c + \omega\tau_2} \ , \tag{9}$$

with $c = 0.50 \pm 0.05$. This result is in good agreement with Buchal and Pobell's work.[3]

For 1.5 and 1.7 GHz, there is a small systematic deviation from this law of about 10%. We do not have any explanation about this point.

### 3. *Critical attenuation below $T_\lambda$*

From the experimental data (see Fig. 5), the fluctuations contribution below $T_\lambda$ is of smaller amplitude than above. Moreover a fluctuation term alone can not explain the attenuation maximum that we observe a few millikelvin below $T_\lambda$. Between 550 and 1690 MHz this maximum has in fact a small amplitude (5 – 10% of the total attenuation) and occurs in temperatures ranging from the transition to 10 or 20 mK below $T_\lambda$. In a first analysis we will ignore this small maximum and try a study of the critical attenuation in terms of fluctuations for $20 < |T - T_\lambda| < 200$ mK.[19]

*a. Fluctuations of order parameter.* For $T < T_\lambda$, expression (9) is no longer useful but we may take as trial function the one proposed by Ahlers[4]

$$\frac{\alpha_{\mathrm{Fl}}^-}{\alpha_\lambda} = \frac{\theta_{\mathrm{Fl}}^-}{\theta_\lambda} = \frac{\omega\tau_2}{1 + \omega\tau_2} \ . \tag{10}$$

Using this function we fit the critical attenuation for $T < T_\lambda$ (except in the vicinity of the small maximum close to $T_\lambda$) from 550 – 1690 MHz (see Fig. 8). Our experimental accuracy gives the constant 1 in the trial function with a precision of 10%. It is important to note that the same characteristic time $\tau_2$ is used in both expressions (9) and (10) to describe the fluctuations above and below $T_\lambda$.

In our frequency range, and for $T < T_\lambda$, we describe the critical attenuation as the sum of a fluctuation part [Eq. (10)] plus a small contribution which gives a small maximum close to $T_\lambda$. Before discussing more about the fluctuations we shall study this small maximum.

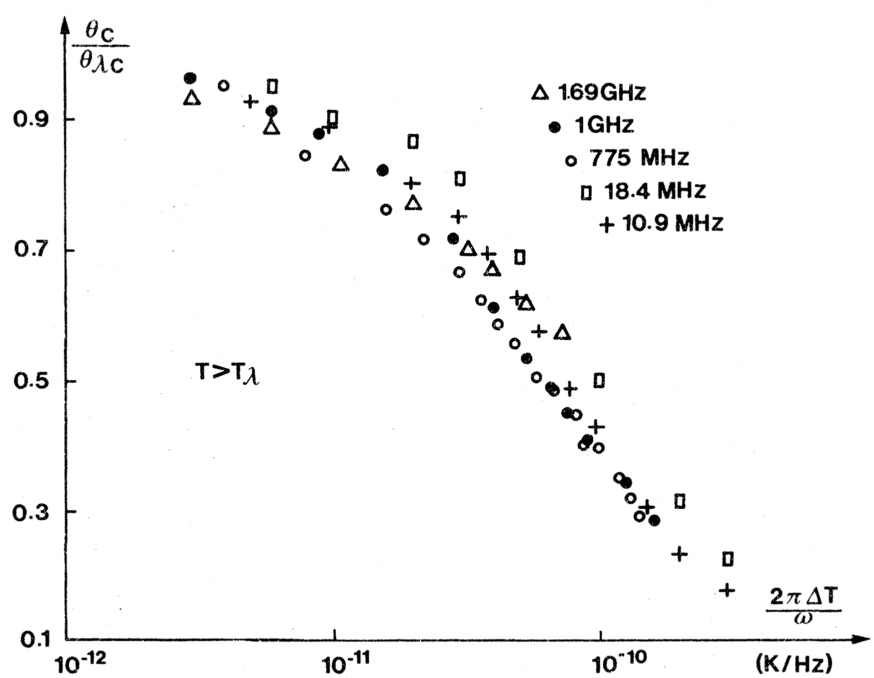


FIG. 6. Scaling plot $\theta_C/\theta_{\lambda C}$ vs $2\pi(T - T_\lambda)/\omega$ for $T > T_\lambda$ from 10.9 MHz to 1.69 GHz. The data at 10.9 MHz and 18.4 MHz are from Ref. 11 and at 1 GHz from Ref. 5.

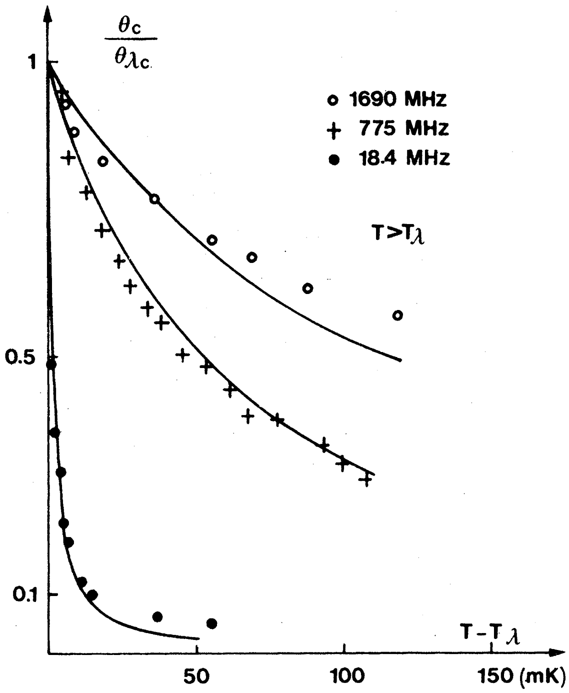


FIG. 7. $\theta_C/\theta_{\lambda C}$ vs $T-T_\lambda$ for $T>T_\lambda$ at different frequencies; the full lines correspond to $\theta/\theta_\lambda=\omega\tau_2/(0.5+\omega\tau_2)$ with $\tau_2=2\times10^{-12}t^{-1.062}$ sec (Ref. 3). The data at 18.4 MHz are from Ref. 11.

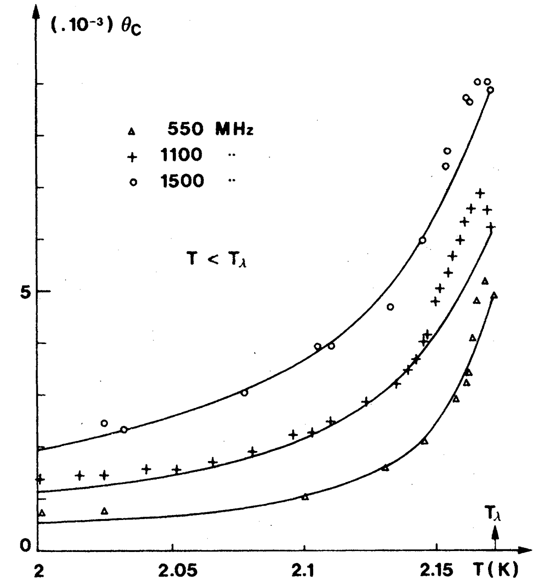


FIG. 8. Variation of $\theta_C$ vs $T$ at different frequencies for $T<T_\lambda$. The full lines correspond to $\theta/\theta_\lambda=\omega\tau_2/(1+\omega\tau_2)$ with $\tau_2=2\times10^{-12}t^{-1.062}$ sec.[3]

*b. Residual attenuation for $T<T_\lambda$.* This residual attenuation is obtained after two successive subtractions ($\theta_B$ and then $\theta_{\rm Fl}$). Its amplitude $\theta_r$ (residual) is about $10^{-3}$ and is essentially constant from 550 – 1700 MHz; the accuracy of this value is quite bad, 50 to 100%; hence the residual attenuation $\theta_r$ is approximatively proportional to the frequency.

The *apparent* maximum of the *total* critical attenuation moves off the transition as the frequency increases: from 2 mK at 550 MHz, it comes to 5 mK at 1690 MHz; and so does the residual attenuation maximum: for 550 MHz this maximum occurs at $|T-T_\lambda|=5\pm3$ mK and $14\pm3$ mK for 1500 MHz. In our frequency range we have roughly $\Delta T\propto\omega$. This maximum of attenuation which exists at any frequency can be interpreted at low frequencies as a relaxation contribution [Eq. (1)] of characteristic time $\tau_2$; in this low-frequency range its amplitude is bigger than the fluctuation attenuation. At higher frequencies we can no longer describe it in terms of relaxation. The order of magnitude of this maximum and its variation with frequency (proportional to $\omega$) are in agreement with the low-frequency relaxation, but the temperature where the maximum occurs does not agree with the low-frequency results: at 1 GHz, $\omega\tau_2=1$ corresponds to $\Delta T=40$ mK and not 10 mK as we found experimentally. For this maximum other phenomena may be invoked. One can think of order-parameter relaxations of characteristic time $\tau_1=\xi/u_1$, corresponding to first-sound waves interactions.[14] But at 1 GHz, with $\xi_0\sim1$ Å and $u_1\sim230$ m/sec, $\omega\tau_2=1$ occurs when $\Delta T\sim0.4$ mK, which is not at all where the maximum peaks experimentally.

To explain this small maximum, Lyuksyutov and Pokrovskii[20] have proposed a process of first-sound diffusion by quasistatic order-parameter fluctuations. This process also does not fit our data: the temperature maximum occurs also for $k\xi=\omega\tau_1\sim1$, so that at 1 GHz, $\Delta T\sim0.4$ mK. Moreover this process would give an attenuation proportional to $\omega^3$ which we do not observe experimentally.

In conclusion none of these phenomena can, by itself, explain the residual critical attenuation for $T<T_\lambda$. Anyway we have no argument to say that the observed phenomenon is a pure one; it may result from a mixing of several different phenomena.

## VI. DISCUSSION ABOUT FLUCTUATIONS

We shall now return to a larger discussion of the fluctuations. We have established above that the fluctuations for $T>T_\lambda$ are described from 2 kHz to 1.7 GHz by the same scaling function (9) with the same characteristic time $\tau_2$. Below $T_\lambda$ our high-frequency experiments lead to another scaling function (10) with the same characteristic time $\tau_2$. This result seems to be in contradiction with the low-frequency results, where the fluctuations below $T_\lambda$

are described by Eq. (9). Two interpretations are possible:

(a) The fluctuations for $T < T_\lambda$ may evolve from low frequencies to high frequencies; that would mean that we have two types of fluctuations (low and high frequencies). But for $T > T_\lambda$, the same trial function (9) fits the results on a really wide range of frequencies from 2 kHz to 1.7 GHz.

(b) The other possibility is that fluctuations had not been settled precisely at low frequencies for $T < T_\lambda$ by the symmetrization method. The fact that at low frequencies the relaxation maximum is much bigger than the fluctuation contribution is a handicap for a good evaluation of fluctuations below the transition. As this problem vanishes around 1 GHz, we can study, in a good way, fluctuations from 550 to 1690 MHz. We can think that expression (10) is also valid for low frequencies. To test this idea, it would be necessary to reanalyze low-frequency results for $T < T_\lambda$ with a fluctuation attenuation described by a scaling function (10).

The study of relaxation for $T < T_\lambda$, done by Buchal and Pobell[3] from 2 to 600 kHz, is not affected by our procedure. Subtracting Eq. (9) or the other Eq. (10) fluctuations function of the total attenuation, at 600 kHz, changes only the amplitude of the relaxation peak without moving its position in temperature (the amplitude was a free parameter in the analysis). In this range of frequencies, fluctuations and relaxations have a contribution to attenuation of the same order; an inaccurate fluctuation contribution (9) instead of Eq. (10) has a weak influence on the apparent behavior of the relaxation peak, anyway up to 600 kHz; for higher frequencies (for example, 163 MHz[11,12]), it becomes necessary to use Eq. (10) to analyze the data.

From 2 kHz to 1.7 GHz, we can describe the critical attenuation due to fluctuations of the order parameter by the scaling function (10) above $T_\lambda$ and Eq. (9) below $T_\lambda$, with the same characteristic time

$$\tau_2 = 2\times 10^{-12} t^{-1.062}\ \text{sec} .$$

We have to note that nothing justifies the scaling functions (9) or (10). Scaling arguments predict only the same critical exponent on both sides of the transition for the characteristic time $\tau_2$. Swift and Kadanoff[9] using mode-mode coupling theory associated with scaling arguments predict that the critical times $\tau_1$ and $\tau_2$ are symmetric versus $T_\lambda$.

At this point, our description of the fluctuations around $T_\lambda$ uses one characteristic time $\tau_2$ and two different scaling functions of Eqs. (9) and (10). These two scaling functions differ only by the value of the constant $c$ in Eq. (4); $c = 1 \pm 0.1$ below $T_\lambda$, and $c = 0.5 \pm 0.05$ above the transition. An alternative way to fit the data is to use one scaling function and two different characteristic times,

$$\frac{\alpha_{\mathrm{Fl}}}{\alpha_\lambda} = \frac{\theta_{\mathrm{Fl}}}{\theta_\lambda} = \frac{\omega\tau}{1+\omega\tau} , \tag{11}$$

with

$$\tau \equiv \tau^- = \tau_2 \quad \text{for } T < T_\lambda ,$$

$$\tau \equiv \tau^+ = 2\tau^- = 2\tau_2 \quad \text{for } T > T_\lambda .$$

$\tau^-$ and $\tau^+$ depend on temperature in the same way. $\tau_2$ is still the relaxation time which describes the attenuation due to the relaxation of the order parameter (interaction between first- and second-sound waves). This description is valid from 2 kHz to 1.7 GHz, remembering our last discussion of Buchal and Pobell's results for $T < T_\lambda$.

If we formulate our results in such a way, we have a ratio $\tau^+/\tau^- = 2$ for the same values of $t$. We can suggest a comparison for the value of this ratio with recent results of renormalization-group calculations.[21-23] To do this, we assume that

$$\frac{\tau^+}{\tau^-} = \frac{\xi^+}{\xi^-} = 2 . \tag{12}$$

Now we try to estimate the ratio $\xi^+/\xi^-$. The renormalization-group theory allows us to define and evaluate some universal ratios. For instance, the thermal conductivity $\lambda_m$ for $T > T_\lambda$ can be written[21]

$$\lambda_m = R_\lambda g_0 C_p^{1/2} \xi_+^{1/2} , \tag{13}$$

where $R_\lambda$ is an universal number, $g_0$ is a dynamic coupling constant, $C_p$ is the specific heat, and $\xi^+$ is the correlation length for $T > T_\lambda$. The experimental value $R_\lambda^{\exp} = 0.3$ is in reasonable agreement with the calculated one $R_\lambda^{\mathrm{th}} = 0.36$.[22] In the same way, another universal amplitude ratio $R_m$ may be defined from the characteristic frequency $\omega_m(k)$ at $T_\lambda$. The value of $R_m$ is difficult to reach experimentally[21]; an estimate gives $R_m^{\mathrm{th}} = 0.54$.[22]

In the case of helium, we obtain a relation between $R_m$ and $R_\lambda$,[21]

$$\frac{\xi^+}{\xi^-} = \left(\frac{R_m}{R_\lambda}\right)^2 . \tag{14}$$

The theoretical values of $R_m$ and $R_\lambda$ give a ratio $(\xi^+/\xi^-)_{\mathrm{th}} = 2.25$. Due to the rather large inaccuracies inherent in methods of calculations,[22] such a good agreement is probably fortuitous. It would be interesting to compare the ratio $\xi_0^+/\xi_0^-$ with the other experimental values of the correlation length. For $T < T_\lambda$ values of $\xi_0^-$ are deduced from the measurement of second-sound velocity[23] or from its damping.[24] Unfortunately for $T > T_\lambda$, the correlation length can not be directly measured. So it is not possible to compare the ratio of the experimental values of $\xi_0$ with the theoretical one or with our own results.

## VII. CONCLUSION

As low-frequency elastic waves provide a good opportunity to study the relaxations of the order parameter below $T_\lambda$, high-frequency elastic waves provide a good way to study the fluctuations of the order parameter. As a result we have shown that the fluctuations are not symmetric about $T_\lambda$.

Using only one characteristic time $\tau_2$ which also is the relaxation time of the order parameter at low frequencies, we find two scaling functions to describe the fluctuations above and below $T_\lambda$. In an alternative way, it is also possible to explain the experimental results with only one scaling function and two different characteristic times. The theoretical data do not allow us to choose between these two possibilities.

---

[*] Associated with the Centre National de la Recherche Scientifique.

[1] L. D. Landau and I. M. Khalatnikov, in *Collected papers of L. D. Landau*, edited by D. ter Haar (Gordon and Breach, New York, 1965), p. 626.

[2] R. D. Williams and I. Rudnick, Phys. Rev. Lett. 25, 276 (1970).

[3] (a) C. Buchal and F. Pobell, Phys. Rev. B 14, 1103 (1976); (b) R. Carey, C. Buchal, and F. Pobell, Phys. Rev. B 16, 3133 (1977).

[4] G. Ahlers, J. Low Temp. Phys. 1, 609 (1969).

[5] J. S. Imai and I. Rudnick, Phys. Rev. Lett. 22, 694 (1969); D. E. Commins and I. Rudnick, in *Proceedings of the Thirteenth International Conference of Low Temperature Physics, Boulder, 1972,* edited by W. J. O'Sullivan, K. D. Timmerhaus, and E. F. Hammel (Plenum, New York, 1973), Vol. 1, p. 356.

[6] V. L. Pokroskii and I. M. Khalatnikov, JETP Lett. 9, 149 (1969); I. M. Khalatnikov, Sov. Phys. JETP 30, 268 (1970).

[7] K. Kawasaki, Phys. Lett. A 31, 165 (1970); K. Kawasaki, Int. S. Magn. 1, 171 (1971).

[8] P. C. Hohenberg, in *Proceedings of the International School "Enrico Fermi,"* edited by M. S. Green (Academic, New York, 1971).

[9] J. Swift and L. P. Kadanoff, Ann. Phys. (N.Y.) 50, 312 (1968).

[10] Results of the Kroll's calculations are unpublished. They are partly presented in Ref. 3(b).

[11] K. I. Tozaki and A. Ikushima, Phys. Lett. A 59, 458 (1977).

[12] K. I. Tozaki and A. Ikushima, J. Low Temp. Phys. (to be published).

[13] W. Heenicke, G. Winterling, and K. Dransfeld, Phys. Rev. Lett. 22, 170 (1969).

[14] J. Joffrin, B. Lambert, D. Salin, J. Phys. Lett. 37, L255 (1976).

[15] W. M. Whitley and C. E. Chase, Phys. Rev. 158, 200 (1967).

[16] M. Barmatz and I. Rudnick, Phys. Rev. 170, 224 (1968).

[17] Detailed calculations are presented in: R. Perzynski, thèse de 3ème cycle de Physique (Université P. et M. Curie, 1978) (unpublished).

[18] M. A. Woolf, P. M. Platzman, and M. G. Cohen, Phys. Rev. Lett. 17, 294 (1966).

[19] These results have been partly presented in J. Phys. Coll. 38, C6 - 148 (1978).

[20] I. F. Lyukoyutov and V. L. Pokrovskii, JETP Lett. 25, 392 (1977).

[21] B. I. Halperin, P. C. Hohenberg, and E. D. Siggia, Phys. Rev. B 13, 1299 (1976).

[22] P. C. Hohenberg and B. I. Halperin, Rev. Mod. Phys. 49, 435 (1977).

[23] P. C. Hohenberg, A. Aharony, B. I. Halperin, and E. D. Siggia, Phys. Rev. B 13, 2986 (1976).

[24] G. Ahlers, in *The Physics of Liquid and Solid Helium,* edited by K. H. Bennemann and J. B. Ketterson (Wiley, New York, 1976), Chap. II.