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Plasma Effects on Transition Radiation from Metal Foils*

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The transition radiation emitted from a thin metal foil bombarded by electrons is calculated, taking into account the excitation of plasma waves in the foil. The correction the plasma waves introduce varies greatly in different cases and can be of the order of 10% or greater at the frequency at which the radiation has its peak intensity.

I. INTRODUCTION

Transition radiation is the radiation emitted due to the passage of a charged particle through an inhomogeneous medium, for example, as in crossing the boundary between two media of different dielectric constants. This radiation, first discussed by Frank and Ginzburg[1] in 1945, has been the subject of increasing interest in recent years.

The transition radiation is a consequence of the boundary conditions appropriate to the given situation. To calculate the transition radiation, one applies these conditions to the fields at the boundary crossed by the particle, while the media on either side of the boundary are described in some suitable way.

Problems in electromagnetic theory involving boundaries, such as transition radiation or the derivation of Fresnel's formulas, are usually worked under the assumption that the media in question are described by their dielectric constants, permeabilities, or refractive indices. Recently, Sauter[2] pointed out that this approach failed to take into account the possibility of excitation of plasma waves in the case of metals, so that, for instance, there should be corrections to Fresnel's formula for the reflection coefficient of a metal surface in certain circumstances.

It is of interest to consider what effect these plasma waves would have on transition radiation, since the targets usually used for experimental investigations are thin foils of aluminum, silver, or similar metals. Forstmann[3] has done such a calculation for electrons incident (obliquely) from vacuum on a semi-infinite metal. This paper reports the calculations for a thin metallic film.

II. FUNDAMENTAL EQUATIONS

The equation of motion of a degenerate electron gas in the metal is given by

$$\dot{\vec{v}}(\vec{r},t)+f\vec{v}(\vec{r},t)=-\frac{e}{m}\vec{E}(\vec{r},t)-\frac{W^2}{n_0}\nabla n(\vec{r},t) \quad , \qquad (1)$$

where the constant $W^2=\frac{3}{5}v_F^2$ for a completely degenerate electron gas with a Fermi velocity v_F. Further, $\vec{v}$ is the velocity of an element of the electron gas, n_0 is the equilibrium number density of the electrons (and constant density of the positive ions), n is the deviation of the electron number density from n_0, $-e/m$ is the charge to mass ratio of an electron, f is the inverse of the relaxation time of the electrons, and $\vec{E}$ is the total electric field inside the metal. Assume that $n \ll n_0$.

The charge and current density of the plasma are, respectively, $\rho = -en$ and $\vec{J}_p = -e(n_0+n)\vec{v}$; since $\vec{v}$ is small, the second term will be neglected in the current density. The equation of continuity reads

$$n_0 \nabla \cdot \vec{v} = \dot{n} \ . \qquad (2)$$

The electromagnetic field, which governs the motion of metallic electrons through (1), is in turn determined through Maxwell's equations by the total charge density $\rho = -en + \rho_c$ and the total current density $\vec{J} = -en_0\vec{v} + \vec{J}_c$, where ρ_c and $\vec{J}_c$ are densities of sources not belonging to the electronic plasma; in this problem, they will be the densities appropriate to the incident charged particle. Maxwell's equations are

$$\nabla \times \vec{E} = -(1/c)\dot{\vec{H}} , \qquad (3)$$

$$\nabla \times \vec{H} = (1/c)\dot{\vec{E}} + (4\pi/c)(-en_0\vec{v} + \vec{J}_c) \ , \qquad (4)$$

$$\nabla \cdot \vec{E} = 4\pi(-en + \rho_c) \ , \qquad (5)$$

$$\nabla \cdot \vec{H} = 0 \ . \qquad (6)$$

Now everything is Fourier-analyzed in time according to

$$f(t) = (2\pi)^{-1/2} \int_{-\infty}^{\infty} f(\omega) e^{-i\omega t} d\omega \ .$$

The same symbol is used for both the original function and its Fourier transform.

Applying $\nabla \cdot$ to (1) yields

$$-i\tilde{\omega} \nabla \cdot \vec{v} = -(e/m) \nabla \cdot \vec{E} - (W^2/n_0)\nabla^2 n \ , \qquad (7)$$

where $\tilde{\omega} = \omega + if$. Substituting (2) and (5) into (7) gives

$$\nabla^2 n + (\omega^2/c^2)\epsilon' n = -(\omega_p^2/eW^2)\rho_c \ , \qquad (8)$$

where $\epsilon' = (c^2/W^2)(\tilde{\omega}/\omega)\epsilon$, $\epsilon = 1 - \omega_p^2/\omega\tilde{\omega}$, and $\omega_p^2 = 4\pi e^2 n_0/m$. ω_p is then the plasma frequency. From (1), (3), and (4), we obtain

$$\nabla^2 \vec{H} + (\omega^2/c^2)\epsilon \vec{H} = -(4\pi/c)\nabla \times \vec{J}_c \ . \qquad (9)$$

Other necessary quantities can be calculated once n and $\vec{H}$ are found. Manipulation using (1) and (4) gives

$$\vec{E} = i\frac{c}{\omega}\frac{1}{\epsilon}\nabla \times \vec{H} - i\frac{c}{\omega}\eta\nabla n - i\frac{4\pi}{\omega}\frac{1}{\epsilon}\vec{J}_c \ , \qquad (10)$$

$$\vec{J}_p = \frac{c}{4\pi}\frac{\epsilon - 1}{\epsilon}\nabla \times \vec{H} + \frac{c}{4\pi}\eta\nabla n - \frac{\epsilon - 1}{\epsilon}\vec{J}_c \ , \qquad (11)$$

where the abbreviation $\eta = 4\pi i e W^2/\epsilon c\tilde{\omega}$ is used.

The problem thus takes the form of a set of coupled differential equations, which are to be solved given the source terms ρ_c and $\vec{J}_c$ and the boundary conditions. One boundary condition is that the normal component of $\vec{J}_p$ must vanish at the surfaces of the foil, since the plasma electrons cannot leave the metal. Other boundary conditions follow in the well-known way from Maxwell's equations; these are that $\vec{E}$ and $\vec{H}$ be continuous at both boundaries. Of course, not all these conditions are independent. For instance, the continuity of the normal component of $\vec{E}$ follows from the tangential continuity of $\vec{H}$ and the vanishing of the normal component of $\vec{J}_p$.

III. RESULTS

Let an electron be normally incident upon a metal foil of thickness d bounded by vacuum on both sides. The parallel planes of the foil are located at $z=0$ and $z=d$. The direction of the electron's flight is taken as the z axis. Let $\vec{u} = (0, 0, u)$ be the electron's velocity.

The Fourier-transform method is used to solve the equations. The transform taken is

$$f(x, y, z, \omega) = (2\pi)^{-1}\int_{-\infty}^{\infty} dk_x \int_{-\infty}^{\infty} dk_y \times f(k_x, k_y, z, \omega)\exp[i(k_x x + k_y y)] \ .$$

The external sources are $\rho_c = -e\delta(\vec{r} - \vec{u}t)$ and $\vec{J}_c = -e\vec{u}\delta(\vec{r} - \vec{u}t)$, whose transforms are

$$\rho_c = -(2\pi)^{-3/2}(e/u)e^{iKz} ,$$

$$\vec{J}_c = -e(2\pi)^{-3/2}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} e^{iKz} \ ,$$

where $K = \omega/u$.

Equations (8) and (9) are now ordinary differential equations. The solution of (8) is

$$n = N_+ e^{iUz} + N_- e^{-iUz} + N_0 e^{iKz} \ , \qquad (12)$$

where $U^2 = (\omega^2/c^2)\epsilon' - \kappa^2$, $\kappa^2 = k_x^2 + k_y^2$, $\mathrm{Im}U > 0$,

$$N_0 = (2\pi)^{-3/2}\omega_p^2/uW^2(K^2 - U^2) \ .$$

N_+ and N_- are to be determined from the boundary conditions. In the same way, the magnetic field inside the metal is

$$\vec{H} = \vec{h}_+ e^{iVz} + \vec{h}_- e^{-iVz} + \vec{h}_0 e^{iKz} \quad (0 < z < d) \ , \qquad (13)$$

where $V^2 = (\omega^2/c^2)\epsilon - \kappa^2$, $\mathrm{Im}V > 0$,

$$\vec{h}_0 = \frac{C}{K^2 - V^2}\begin{pmatrix} -k_y \\ k_x \\ 0 \end{pmatrix} \ , \quad C = \left(\frac{2}{\pi}\right)^{1/2}\frac{e}{ic} \ .$$

In writing the solutions to (9) in free space, one must keep in mind that only waves outgoing from the metal are acceptable, that is, solutions corresponding to waves traveling in the positive (negative) z direction in the region $z < 0$ ($z > d$). The solutions are then

$$\vec{H} = \vec{h}' e^{-iV'z} + \vec{h}_0' e^{iKz} \ , \quad (z < 0) \qquad (14)$$

$$\vec{H} = \vec{h}'' e^{iV'z} + \vec{h}_0' e^{iKz} \ , \quad (z > d) \qquad (15)$$

where $V'^2 = \omega^2/c^2 - \kappa^2$ and the real parts of V and V' have the same sign. It is easy to see that this sign is the same as that of ω, so that the solutions indeed represent waves traveling in the correct directions. It also can be easily shown that Fourier components for which $\omega^2/c^2-\kappa^2<0$ contribute nothing to the energy flux, using the requirement that the fields must be real. Thus, V' can always be taken as purely real. Furthermore, we find that

$$\vec{h}_0'=\frac{C}{K^2-V'^2}\begin{pmatrix}-k_y\\ k_x\\ 0\end{pmatrix} .$$

The boundary conditions now allow the determination of the unknown coefficients in Eqs. (12)–(15). This determination is to be carried out for given values of the Fourier variables k_x and k_y. It will be found convenient to choose a new set of coordinate axes such that $k_y=0$, that is, to choose the new x axis (to be called the l axis) to have the same direction as the tangential component of the wave vector. The new y axis will be called the m axis. It is understood that the point at which the particle enters the metal is the origin of coordinates. It will be useful later on if the original set of coordinate axes are defined by running the x axis through the point of observation. Let the angle between the l and x axes be denoted by φ.

From the continuity of $\vec{H}$ across the boundaries and the vanishing of its divergence, it is now easy to show that the l and z components of $\vec{h}'$ and $\vec{h}''$ vanish. This will turn out to lead to an important result – the complete polarization of the transition radiation for normal incidence of the charged particle. The vanishing of the z component of the plasma current and the continuity of the l component of the electric field at both boundaries are then sufficient to evaluate the m components of $\vec{h}'$ and $\vec{h}''$.

We obtain the following results for the Fourier components of the radiation fields:

$$h_m'=C\frac{A_2A_7-A_1A_6}{A_2^2-A_1^2},\quad h_m''=C\frac{A_2A_6-A_1A_7}{A_2^2-A_1^2}\,e^{-iV'd} ,$$

where $A_7=A_5-A_3b+A_4b,\quad A_6=A_3+A_4+A_5b$,

and $b=e^{i\omega d/u}$,

$$A_1=V'+\frac{i}{\epsilon}V\cot Vd-i\frac{\kappa^2}{U}\frac{\epsilon-1}{\epsilon}\cot Ud ,$$

$$A_2=-(i/\epsilon)\,[V\csc Vd-(\kappa^2/U)(\epsilon-1)\csc Ud] ,$$

$$A_3=\kappa\left(\frac{1}{D_1}-\frac{1}{\epsilon}\frac{1}{D_2}-\frac{\epsilon-1}{\epsilon}\frac{1}{D_2'}\right) ,$$

$$A_4=-i\kappa\,(F\cot Vd-G\cot Ud) ,$$

$$A_5=i\kappa\,(F\csc Vd-G\csc Ud) ,$$

$$G=\frac{\epsilon-1}{\epsilon}\left(\frac{U}{D_2'}+\frac{\kappa^2}{U}\frac{1}{D_1}\right),\quad F=\frac{1}{\epsilon}V\left(\frac{1}{D_1}-\frac{1}{D_2}\right) ,$$

$$D_1=\frac{1}{K^2-V'^2},\quad D_2=\frac{1}{K^2-V^2},\quad D_2'=\frac{1}{K^2-U^2} .$$

The right-hand term in each A_i gives the correction to the result caused by taking into account the longitudinal oscillation in the metal.

To find the polarization of the radiation, we invert the Fourier transform to find the x and y components of the magnetic vector. Recalling how we defined our coordinates, we have

$$\begin{aligned}H_x&=(2\pi)^{-3/2}\int\int\int h_m'\sin\varphi\\&\quad\times\exp[i(k_xx+k_yy-V'z-\omega t)]\,dk_x dk_y d\omega\\&=(2\pi)^{-3/2}\int\int\int h_m'\sin\varphi\\&\quad\times\exp[i(\kappa x\cos\varphi-V'z-\omega t)]\,\kappa\, d\kappa\, d\varphi\, d\omega\\&=0\end{aligned}$$

on carrying out the φ integration. However, H_y does not vanish. These results are true for the fields on both sides of the plate. Thus we have obtained the result mentioned earlier – the complete polarization of the transition radiation in the plane containing the line of incidence and the observation ray (i.e., with the $\vec{E}$ vector in that plane – $E_x\neq 0,\ E_y=0$).

The remaining integration is carried out by the saddle-point method. Let θ be the angle between the electron's path and the observation ray. Then the result is that the energy per unit solid angle per unit frequency interval is

$$\frac{d^2I}{d\Omega d\omega}=\frac{e^2\cos^2\theta}{\pi^2c}\left|A\right|^2 , \tag{16}$$

where $A=(a_2a_7-a_1a_6)/(a_2^2-a_1^2)$ for $z<0$,

$$A=(a_2a_6-a_1a_7)/(a_2^2-a_1^2)\ \text{ for } z>d ,$$

$$a_7=a_5-a_3b+a_4b,\quad a_6=a_3+a_4+a_5b ,$$

$$a_1=r+i\frac{t}{\epsilon}\cot td-i\frac{\epsilon-1}{\epsilon}\frac{s^2}{t'}\cot t'd ,$$

$$a_2=i\left(-\frac{t}{\epsilon}\csc td+\frac{\epsilon-1}{\epsilon}\frac{s^2}{t'}\csc t'd\right) ,$$

$$a_3=s\beta^{-1}\left(\frac{1}{d_1}-\frac{1}{\epsilon d_2}-\frac{\epsilon-1}{\epsilon}\frac{1}{d_2'}\right) ,$$

$$a_4=-is(f\cot td-g\cot t'd) ,$$

$$a_5=is\,(f\csc td-g\csc t'd) ,$$

$$\epsilon'=\epsilon(\tilde{\omega}/\omega)(c^2/W^2),\quad r=(\omega/c)\cos\theta ,$$

$$s=(\omega/c)\sin\theta,\quad \beta=(u/c) ,$$

$$t\,(\text{or } t')=(\omega/c)[\epsilon\ (\text{or } \epsilon')-\sin^2\theta]^{1/2} ,$$

$$d_1=(\omega/c)(\beta^{-2}-\cos^2\theta) ,$$

$$d_2\ (\text{or } d_2')=(\omega/c)[\beta^{-2}-\epsilon\ (\text{or } \epsilon')+\sin^2\theta] ,$$

$$f=\frac{t}{\epsilon}\left(\frac{1}{d_1}-\frac{1}{d_2}\right),\quad g=\frac{\epsilon-1}{\epsilon}\left(\frac{t'}{d_2'}+\frac{s^2}{t'd_1}\right) .$$

Again, the right-hand term in each a_1-a_5 gives the correction caused by taking into account the longitudinal plasma waves. It is easy to see that if W^2 goes to zero, which means no longitudinal waves, all these terms vanish. In this case, our result reduces to that given by Garibyan and Chalikyan[4] and by Ritchie and Eldridge[5] for the transition radiation from a thin foil due to normally incident electrons. In these papers the foil is described by a dielectric constant $\epsilon(\omega)$. Our ϵ becomes the dielectric constant, whose meaning is given by the constitutive relation $\vec{D}=\epsilon\vec{E}$, in the limit where the plasma effect is ignored. Actually, the two papers cited do not specify the frequency dependence of $\epsilon(\omega)$.

In certain cases there will be Čherenkov radiation. In deforming the path of integration from the real κ axis to the line of steepest descent through the saddle point [which turns out to occur at $\kappa_0=(\omega/c)\sin\theta$], we will cross a pole of the integrand under certain circumstances. The residue at this pole will then contribute to the integral; it will turn out that this term gives the Čherenkov radiation. The pole will occur for κ such that $A_2=\pm A_1$. If we are interested in thin foils for which the electron travels only a very small distance in the metal, the Čherenkov radiation may be ignored.

Let us now compare the result (16) to the results of the old theory[4,5] which did not take into account the plasma effect. First, it should be noted that Silin and Fetisov[6] have already given a formula for the transition radiation yield from a metal slab taking into account the possibility of propagation of longitudinal waves in the medium. From this formula, they estimate that consideration of the longitudinal waves will not lead to very great changes in the predicted transition radiation yield as compared to formulas, assuming that only transverse waves can propagate in the medium; the corrections might be about 1%.

Their method involves taking spatial dispersion into account. To compare this method to that of the present paper, write

$$\vec{D}=\vec{E}+i(4\pi/\omega)\vec{J}_p \quad .$$

Now if there are no external sources, it is very easy to show (by taking Fourier transforms in an infinite medium) that

$$D_i=[\epsilon\delta_{ij}-(W^2/\omega\tilde{\omega})k_ik_j]E_j \quad .$$

The term in brackets can be broken up into transverse and longitudinal components in the usual way:

$$D_i=[\epsilon^T(\delta_{ij}-k_ik_j/k^2)+\epsilon^L k_ik_j/k^2]E_j \quad ,$$

from which

$$\epsilon^T=\epsilon, \quad \epsilon^L=\epsilon-W^2k_ik_j/\omega\tilde{\omega} \ ,$$

which corresponds to the spatial dispersion of the dielectric permeability assumed by Silin and Fetisov, where W^2 is equal to their α.

Now if we assume that the incident particle is nonrelativistic and the dielectric permeability is not too large (so that, for example, terms in $\beta^2\epsilon$ can still be neglected compared to terms in β), we can examine (16) in this approximation, which is the same as that made in Ref. 6. With this approximation Eq. (16) gives the same result as Eq. (5) of Ref. 6, with the exception of the coefficients of terms involving $1/t'$. This is to be expected since these coefficients are modified by changes in the boundary conditions; in Ref. 6 the normal component of the field of radiation going inside the metal vanishes on the surface, whereas in the present paper it is the normal component of the plasma current which must vanish on the surface. Nevertheless, the order of magnitude does not change.

The approximation of (16) to the formula from which Silin and Fetisov estimated the longitudinal-wave corrections to be insignificant would seem to be predictable if one considers the magnitudes of the various terms in (16). If ϵ (and therefore t) are of order 1, then ϵ' is of order 10^5 and t' is of order 10^2. Thus, the right-hand terms in a_1–a_5, which give the corrections, seem to be of order 10^{-2} compared to the other terms and so can be neglected.

Yet these approximations are misleading. For example, consider a very thick foil. In the limit in which d becomes infinite, it is easy to see that dI is proportional to

$$d\omega\, d\theta \left| (1-\beta^2+\beta t)/(1+\beta t)(1-\beta^2\cos^2\theta) + (1/\beta t')[1/(1+\beta t')-(1-\beta^2)/(1-\beta^2\cos^2\theta)]\right|^2 .$$

The left-hand term is of order 1; the right-hand term is the correction term. If we assume $|t'|=100$ and $\beta=\frac{1}{4}$ (corresponding to 16-keV electrons), then we have approximately $dI\sim|1+0.04|^2$ or a correction of about 8%. This correction can, of course, be considerably greater; Forstmann,[3] using values for sodium (giving $|t'|\approx 23$) and $\beta=\frac{1}{3}$, obtained a 28% correction. (The thick-foil limit is not essential in showing a relatively large correction; it is merely an aid in making an estimate.)

To compare formula (16) to previous theories without longitudinal-wave corrections,[4,5] it is best simply to compute the numerical values given by the two formulas for particular values of the parameters. To give an example, consider a hypothetical metal whose Fermi velocity is 0.01 of that of light and for which $\lambda_p=2000$ Å and

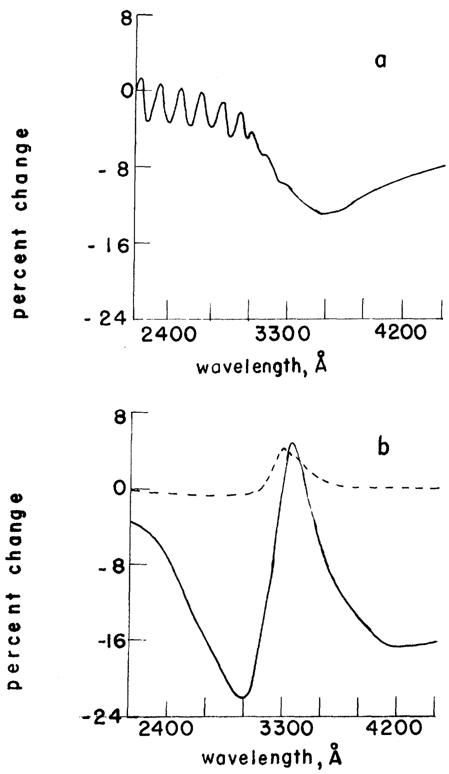

FIG. 1. Percent change in predicted intensity of transition radiation per unit solid angle per unit wavelength interval due to longitudinal waves in target. Curves are drawn for silver foils bombarded by 30-keV electrons at normal incidence; direction of observation makes an angle of 45° with direction of incidence. (a) Foil thickness $d = 100$ Å; note the fine structure at high frequencies. (b) Solid line is for $d = 300$ Å; broken line is for $d = 500$ Å.

$\lambda_f = 50\,000$ Å, where $\lambda_p = 2\pi c \omega_p^{-1}$, $\lambda_f = 2\pi c f^{-1}$. Suppose a 30-keV electron ($\beta = 0.33$) impinges on a foil of this material and the resulting radiation is observed at an angle $\theta = 30°$ in the region $z > d$.

As is well known,[4,5] the intensity of the transition radiation has a sharp peak at the plasma frequency. If the foil is 100 Å thick, there is a decrease in specific intensity at the peak of 18% (expressed as a percentage of the intensity predicted by the theory without plasma corrections) due to the corrections. At 200-Å thickness, the correction at the peak is very small; whereas at 300 Å the intensity should increase by 7% and at 500 Å by 30%. These values vary greatly with changes in the parameters. For example, if the velocity of the electron is $\beta = 0.25$ (energy about 16 keV), the computation for the 500-Å case predicts a very large gain at the peak of about 280%. In this connection it is instructive to consider the cotangent and cosecant terms in (16).

The figure shows plots of the percent change in specific intensity versus wavelength for a material for which $\lambda_p = 3280$ Å, $\lambda_f = 49\,560$ Å, and the Fermi velocity is 0.0047 of that of light, bombarded by 30-keV electrons. These constants have been chosen since they are roughly similar to those of real metals used for experimental work in this field, and should therefore give a good idea of the effect of the correction terms. The results are for observations in the region $z > d$.

It is interesting that the percent change for the 100-Å foil shows a fine structure at frequencies higher than ω_p. Similar fine structure is always predicted in a calculation of the longitudinal-wave corrections whenever the foil is very thin. It is interesting that Shieh and Ritchie,[7] applying the hydrodynamical approach to foils bombarded not by particles but by electromagnetic waves, found the same sort of fine structure due to the plasma corrections in Fresnel's formulas. However, in the transition-radiation case where the intensity falls off rapidly away from the plasma-frequency peak, it seems unlikely that the fine structure can be observed.

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