[19] The internal energy of a magnetic system may be written in terms of the static spin-spin correlation function $\Gamma(R_s, T)$ as $U(T)=-\sum_s \nu_s \mathcal{J}(R_s)\Gamma(R_s, T)$, where ν_s is the number of spins in the shell of radius R_s surrounding a local-moment site, and $\mathcal{J}(R)$ is the Heisenberg exchange energy [see, e.g., Ref. 5]. In the case of nearest-neighbor interactions the sum reduces to $U(T)=-z\mathcal{J}\Gamma(T)$, with z the number of neighboring magnetic moments, so that the heat capacity is $c_v=-z\mathcal{J}d\Gamma/dT$. Thus a sharp heat-capacity peak implies that Γ is falling steeply toward zero at the ordering temperature. While this argument is correct only for nearest-neighbor coupling of constant local moments, the qualitative conclusion that short-range order is suppressed above a sharp specific-heat peak is more generally applicable.

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Domain Nucleation in Uniaxial Ferromagnets*

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We use linearized micromagnetic theory to calculate the nucleation field and the form of the nucleation mode for finite-thickness platelets of uniaxial ferromagnets in arbitrarily oriented uniform external magnetic fields. To carry out this calculation, we use the results of the theory in the limit of very thick platelets, a generalization of the switching theory of Stoner and Wohlfarth. The nucleation field and mode for smaller specimens are then found as first-order deviations from the corresponding quantities of thick platelets. The nucleation mode has the form of incipient strip domains parallel to the field component in the platelet plane.

I. INTRODUCTION

The domain structures formed in thin platelets of hexagonal ferrites and rare-earth orthoferrites have been observed in a variety of external-field geometries and demagnetization cycles.[1–9] Under many conditions, the patterns exhibit a striking regularity. Attempts to interpret the observations on the basis of micromagnetics have been confined to an analysis of domain nucleation in a field lying in the plane of the platelet[10,11] or in a field lying along the easy axis of magnetization,[12,13] and to a qualitative discussion of domain nucleation in a field inclined at a small angle to this plane.[14]

In the present work, we extend the theory to demagnetization in an arbitrarily oriented field. The theory is based on the linearized micromagnetic equations which treat deviations from uniform magnetization as small quantities. This micromagnetic problem is a self-consistent-field problem in the sense that the linearized equations contain the unknown orientations of the uniform magnetization at nucleation, and thus, in effect, require their own solution in order to be formulated.[15] The calculation uses, as a starting point, the recently obtained solution for the nucleation field in the limit of thick specimens.[16] Using these results as the zeroth-order approximation, the nucleation field and mode for smaller samples may be found as first-order deviations by treating the free-energy minimization as an eigenvalue problem.

In Sec. II, we summarize the procedure (given in detail in Ref. 16) for obtaining the nucleation field function or switching threshold curves in the limit of thick specimens. This is done both for the convenience of reference and to provide a basis for showing, in a later section, the equivalence of this approach to the linearized micromagnetic equations under the appropriate limiting assumptions. Section III contains the solution of the micromagnetic problem of the uniaxial plate in an arbitrarily oriented uniform external magnetic field. It is shown that the nucleation mode has the form of incipient strip domains parallel to the component of field in the platelet plane. This invalidates the conjecture[14] that a checkerboard pattern may be nucleated. The micromagnetic solution also provides the switching threshold curve for thinner specimens.

II. SWITCHING THRESHOLD CURVE FOR THICK UNIAXIAL FERROMAGNETS

We use a coordinate system whose axes coincide with the principal axes of an ellipsoidal specimen.[16] The easy direction of magnetization is assumed to coincide with the x axis, the applied field lies in the x-z plane at an angle ϕ from the z axis, and the uniform magnetization lies in the x-z plane at an angle θ from the z axis (Fig. 1).[17]

The domain nucleation curve is obtained by equating to zero both the first and second variations of the free energy with respect to the magnetization vector **M**. The calculation differs from the usual switching-

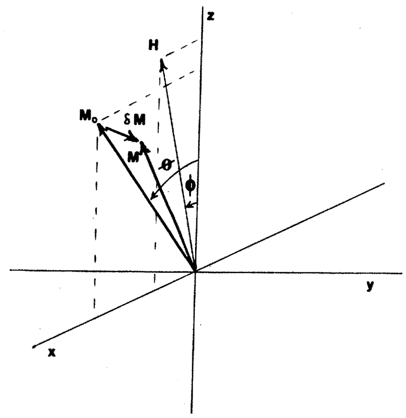

FIG. 1. Field and magnetization geometry.

curve calculation[18] in that $\delta\mathbf{M}$ is an arbitrary function of position rather than a constant. It can be shown that the exchange energy density and part of the dipolar energy density decrease with specimen size and may be neglected for large enough specimens. In this limit, the nucleation threshold is given by

$$\tfrac{1}{2}[1-(D_x-D_z)/k]\sin 2\theta = h\sin(\theta-\phi), \qquad (2.1)$$

$$[1-(D_x-D_z)/2k]\cos 2\theta+(D_x+D_z)/2k = h\cos(\theta-\phi), \qquad (2.2)$$

where

$$h = MH/2K, \qquad (2.3)$$

$$k = K/2\pi M^2. \qquad (2.4)$$

H is the applied field, $K>0$ is the magnetocrystalline anistropy constant, and D_x and D_z are the demagnetizing factors $(D_x+D_y+D_z=1)$. It is assumed that the anisotropy energy F_K is given by

$$F_K = -K\sin^2\theta \qquad (2.5)$$

and that the magnetostrictive energy is negligible or else can be incorporated in the magnetocrystalline anisotropy. Equations (2.1) and (2.2) furnish a relation between h and ϕ which constitutes the domain nucleation or switching curve for thick samples.

III. NUCLEATION FIELD AND MODE OF UNIAXIAL PLATE IN ARBITRARILY ORIENTED MAGNETIC FIELD

We consider a ferromagnetic plate lying between $x=\pm\frac{1}{2}T$ in the coordinate system of Fig. 1 (such a plate is, of course, a degenerate form of an ellipsoid). Let the uniform magnetization before nucleation have direction cosines $(\alpha_0, 0, \gamma_0)$; superposed on this will be the nucleation mode with direction cosines (α, β, γ).

The free energy of a ferromagnet in equilibrium is a minimum; when this condition is met, the torque acting on the magnetization vanishes everywhere:

$$\mathbf{M}\times\mathbf{H}_{\text{eff}}=0, \qquad (3.1)$$

where

$$\mathbf{M}=(M_x,M_y,M_z), \quad \mathbf{H}=H(\sin\phi,\ 0, \cos\phi),$$

$$M_x=M(\alpha_0+\alpha), \qquad M_y=M\beta, \qquad M_z=M(\gamma_0+\gamma),$$

where the effective field is given by

$$M\mathbf{H}_{\text{eff}}=C(\nabla^2M_x\hat{x}+\nabla^2M_y\hat{y}+\nabla^2M_z\hat{z})+(2KM_x/M)\hat{x} - M\left(\frac{\partial U}{\partial x}\hat{x}+\frac{\partial U}{\partial y}\hat{y}+\frac{\partial U}{\partial z}\hat{z}\right) + M(H\sin\phi\hat{x}+H\cos\phi\hat{z}) \qquad (3.2)$$

and C is the exchange constant and U is that portion of the magnetostatic potential arising from the magnetization. One may identify each term of H_{eff} with the exchange, anisotropy, dipolar, and applied field, respectively. Note specifically that the exchange energy and the size-dependent part of the dipolar energy are not excluded in this calculation, in contrast to the calculation in Sec. II. Here the anisotropy constant K is positive.

The x and y components of Eq. (3.1) are then

$$\frac{C}{M^2}(M_y\nabla^2M_z-M_z\nabla^2M_y)-\left(M_y\frac{\partial U}{\partial z}-M_z\frac{\partial U}{\partial y}\right) + M_yH\cos\phi=0, \qquad (3.3)$$

$$\frac{C}{M^2}(M_z\nabla^2M_x-M_x\nabla^2M_z)+\frac{2}{M^2}KM_xM_z - \left(M_z\frac{\partial U}{\partial x}-M_x\frac{\partial U}{\partial z}\right) + H(M_z\sin\phi-M_x\cos\phi)=0, \qquad (3.4)$$

and U satisfies Poisson's equation,

$$\nabla^2U=4\pi\nabla\cdot\mathbf{M}. \qquad (3.5)$$

We linearize Eqs. (3.3) and (3.4) by

$$M_z=M[1-(M_x/M)^2-(M_y/M)^2]^{1/2}\cong\gamma_0-(\alpha_0/\gamma_0)\alpha,$$

so that

$$\nabla^2M_z=-M(\alpha_0/\gamma_0)\nabla^2\alpha,$$

and write Eqs. (3.3)–(3.5) in dimensionless notation by using

$$h=MH/2K, \qquad k=K/2\pi M^2, \qquad S=T/2T_0,$$

$$T_0=(C/2M^2)^{1/2}, \qquad \xi=2\pi x/T,$$

$$\eta=2\pi y/T, \qquad \zeta=2\pi z/T, \qquad u=(2/C)^{1/2}\Phi.$$

Φ is the first-order part of U and does not include the zeroth-order term $4\pi M_x \sin\theta$. This gives

$$\left[-\nabla'^2+\frac{2S^2kh\cos\phi}{\pi\cos\theta}\right]\beta+\frac{S}{2\pi}\frac{\partial\mu}{\partial\eta}=0, \quad (3.6)$$

$$\left[-\nabla'^2+\frac{S^2\kappa}{\pi}\right]\alpha+\frac{S\cos\theta}{2\pi}\left[\cos\theta\frac{\partial\mu}{\partial\xi}-\sin\theta\frac{\partial\mu}{\partial\eta}\right]=0, \quad (3.7)$$

$$\nabla'^2U=4S\left[\frac{\partial\alpha}{\partial\xi}-\tan\theta\frac{\partial\alpha}{\partial\zeta}+\frac{\partial\beta}{\partial\eta}\right], \quad (3.8)$$

where

$$\kappa=2kh\cos(\theta-\phi)-2k\cos2\theta-2\sin^2\theta. \quad (3.9)$$

The boundary conditions are

$$\partial\alpha/\partial\xi=0,\qquad \partial\beta/\partial\xi=0\qquad \text{at } \xi=\pm\pi,$$
$$u_{\text{in}}=u_{\text{out}}\qquad \text{at } \xi=\pm\pi,$$
$$(\partial u/\partial\xi)_{\text{out}}=-4S\alpha+(\partial u/\partial\xi)_{\text{in}}\qquad \text{at } \xi=\pm\pi.$$

The general solution of the system of equations (3.6)–(3.8) is

$$\begin{aligned}\alpha&=A\sin m\eta\cos(p\xi-n\zeta),\\ \beta&=B\cos m\eta\sin(p\xi-n\zeta),\\ u&=V\sin m\eta\sin(p\xi-n\zeta),\end{aligned} \quad (3.10)$$

where p, m, and n are related by the vanishing of the secular determinant

$$\begin{vmatrix} p^2+e^2+(S^2\kappa/\pi) & 0 & (S/2\pi)\cos^2\theta(p+n\tan\theta) \\ 0 & p^2+e^2+(2khS^2\cos\phi/\pi\cos\theta) & Sm/2\pi \\ 4S(p+n\tan\theta) & 4Sm & -(p^2+e^2) \end{vmatrix}$$

and $e^2=m^2+n^2$.

For most platelets of experimental interest, the quantity S (which is of the order of the platelet thickness divided by a Bloch wall thickness) is much greater than unity. We now make the assumption (which will be shown to be self-consistent later) that in the nucleation mode e^2 is of order S and κ is of order S^{-1}; then the secular equation becomes approximately

$$p^6+\frac{S^2p^2}{\pi}\left[p^2\left(\frac{2kh\cos\phi}{\cos\theta}+\kappa\right)+2\cos^2\theta(p+n\tan\theta)^2\right]+\frac{2khS^4\cos\phi}{\pi^2\cos\theta}\left[p^2\kappa+2\cos^2\theta(p+n\tan\theta)^2\right]$$
$$+\frac{2S^2}{\pi}\left(\frac{e^2kh\cos\phi}{\cos\theta}+m^2\right)\left(e^2+\frac{S^2\kappa}{\pi}\right)=0 \quad (3.11)$$

which, to the same approximation, has the roots

$$p_{1,2}=\pm i(2S^2kh\cos\phi/\pi\cos\theta)^{1/2},$$
$$p_{3,4}=\pm i(2S^2\cos^2\theta/\pi)^{1/2},$$
$$p_{5,6}=n\tan\theta\pm p_0,$$

where

$$p_0^2=-(\kappa+2\cos\theta)^{-2}\left[2\kappa n^2\sin^2\theta+(\kappa+2\cos^2\theta)\times\left(\frac{\pi e^2}{S^2}+\kappa\right)\left(e^2+\frac{m^2\cos\theta}{kh\cos\phi}\right)\right]. \quad (3.12)$$

Thus, the eigenfunction will be of the form

$$\alpha=\sin m\eta\sum_{i=1}^{6}A_i\cos(p_i\xi-n\zeta), \quad (3.13)$$

with similar forms for β and u. Equations (3.6)–(3.8) furnish relations between the A_i, B_i, and V_i:

$$V_i=\frac{-2\pi[p_i^2+e^2+S^2\kappa/\pi]}{S\cos^2\theta(p_i+n\tan\theta)}A_i,$$

$$B_i=\frac{m}{\cos^2\theta(p_i+n\tan\theta)}\frac{[p_i^2+e^2+S^2\kappa/\pi]}{[p_i^2+e^2+S^2h_1/\pi]}A_i, \quad (3.14)$$

where $h_1=2kh\cos\phi/\cos\theta$.

The reduced potential outside the magnet is of the form

$$u=\exp[-(m^2+n^2)^{1/2}(\xi-\pi)]\sin m\eta\times\sum_i V_i\sin(p_i\pi-n\zeta),\qquad \xi>\pi \quad (3.15)$$

$$u=-\exp[-(m^2+n^2)^{1/2}(\xi-\pi)]\sin m\eta\times\sum_i V_i\sin(p_i\pi-n\zeta),\qquad \xi<-\pi$$

which vanishes properly at infinity.

The boundary conditions, Eqs. (3.10), furnish the needed relations to determine the values of the constants A_i, B_i, and U_i. The resulting system of six linear homogeneous equations can be expressed in terms of the A_i alone by means of Eqs. (3.14):

$$\sum_{i=1}^{6}\{p_i\sin p_i\pi\}A_i=0, \quad (3.16)$$

$$\sum_{i=1}^{6}\{p_i\cos p_i\pi\}A_i=0, \quad (3.17)$$

$$\sum_{i=1}^{6}\left\{\frac{p_i}{\cos^2\theta\,(p_i+n\tan\theta)}\times\frac{[p_i^2+e^2+S^2\kappa/\pi]}{[p_i^2+e^2+S^2h_1/\pi]}\cos p_i\pi\right\}A_1=0, \quad (3.18)$$

$$\sum_{i=1}^{6}\left\{\frac{p_i}{\cos^2\theta\ (p_i+n\tan\theta)}\times\frac{[p_i^2+e^2+S^2\kappa/\pi]}{[p_i^2+e^2+S^2h_1/\pi]}\sin p_i\pi\right\}A_i=0, \quad (3.19)$$

$$\sum_{i=1}^{6}\left\{(e\sin p_i\pi+p_i\cos p_i\pi)\frac{2\pi}{S}\times\frac{[p_i^2+e^2+S^2\kappa/\pi]}{[\cos^2\theta\ (p_i+n\tan\theta)]}+4S\cos p_i\pi\right\}A_i=0, \quad (3.20)$$

$$\sum_{i=1}^{6}\left\{(e\cos p_i\pi-p_i\sin p_i\pi)\frac{2\pi}{S}\times\frac{[p_i^2+e^2+S^2\kappa/\pi]}{[\cos^2\theta\ (p_i+n\tan\theta)]}-4S\sin p_i\pi\right\}A_i=0, \quad (3.21)$$

which has solutions only if the determinant of the coefficients vanishes.

It can be shown[19] that to the approximation in which we are working, the determinant will vanish if

$$\frac{\pi(e^2+S^2\kappa/\pi)\cos 2p_0\pi}{p_0\cos^2\theta}-S^2\sin^2 p_0\pi=0.$$

Since the coefficient of $\cos 2p_0\pi$ is of order S^{-1}, this implies $p_0\simeq\frac{1}{2}$ and hence from Eq. (3.12)

$$\kappa=-\frac{h_1\cos^2\theta}{2[n^2h_1(1+\tan^2\theta)+m^2(2+h_1)]}-\frac{\pi(m^2+n^2)[m^2(2+h_1)+n^2h_1]}{S^2[n^2h_1(1+\tan^2\theta)+m^2(2+h_1)]}. \quad (3.22)$$

Maximization of Eq. (3.22) with respect to m and n yields

$$m=\left[\frac{S^2}{2\pi}\frac{(kh\cos\phi\cos^2\theta)}{(kh\cos\phi+\cos\theta)}\right]^{1/4},$$

$$n=0,$$

$$\kappa=-\left[\frac{2\pi kh\cos\phi\cos^2\theta}{S^2(kh\cos\phi+\cos\theta)}\right]^{1/2}. \quad (3.23)$$

There is no stationary point $m\neq 0$, $n\neq 0$, and Eqs. (3.23) give the absolute maximum. It is the mode belonging to this eigenvalue that may be expected to nucleate. Note here that we have shown the self-consistency of the assumption made earlier that, for the nucleation mode, (m^2+n^2) is of the order S and κ is of order S^{-1}.

We see from Eqs. (3.23) that the nucleation mode has the form of incipient strip domains parallel to the component of field in the plane of the platelet. This refutes the previous conjecture[14] that the incipient domain structure is a checkerboard pattern. The micromagnetic solution also permits calculation of the switching threshold curve for thinner samples. The switching threshold conditions are now

$$\tfrac{1}{2}[1-(D_x-D_z)/k]\sin 2\theta=h_n\sin(\theta-\phi_n), \quad (3.24)$$

$$[1-(D_x-D_z)/2k]\cos 2\theta+(1/2k)(D_x+D_z)(1+\kappa)=h_n\cos(\theta-\phi_n), \quad (3.25)$$

where the subscript n refers to thin samples, and where from Eq. (3.23)

$$\kappa=\frac{-4\pi\delta k}{T}\left(\frac{h_\infty\cos\phi\cos^2\theta}{kh_\infty\cos\phi+\cos\theta}\right)^{1/2}. \quad (3.26)$$

[The subscript infinity refers to thick samples and $\delta=(C/2K)^{1/2}$ is the nominal Bloch wall thickness.] Equation (3.24) is just Eq. (2.1) rewritten at nucleation; Eq. (3.25) is the generalization for nondegenerate ellipsoidal specimens of Eq. (3.9) which was written for $D_x=1$.

Equations (3.24) and (3.25) may be solved for h_n and ϕ_n for given θ using the values of h_∞ and ϕ_∞ obtained from the threshold-switching curve expressed by Eqs. (2.1) and (2.2). The corresponding threshold-switching curves are shown in Fig. 2 for the case $D_x=1$, $k=2$ and 4, and $2\pi\delta/T=0.2$. The switching curve of the thinner specimen always lies outside the curve corresponding to the thicker sample. The interpretation of the threshold curve is the same as that of Ref. 16; namely, instability occurs when ϕ and θ are in different quadrants and h crosses the curve from interior to exterior. Also, instability always occurs when h crosses into the loop of the threshold curve below hard axis saturation. Thus, for any given demagnetization cycle which starts with a saturated specimen, domain nucleation (or switching) will occur sooner for thick specimens than for thinner ones.

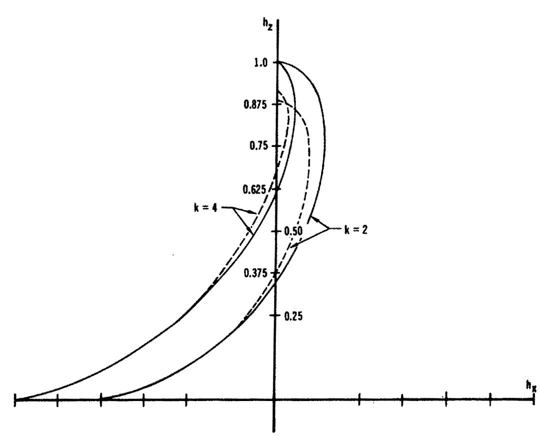

FIG. 2. One quadrant of the switching threshold curves for $D_x=1$, $k=2$, and 4 for very thick sample and one sample where $2\pi\delta/T=0.2$. Solid lines: thick plate; dashed lines: thin plate.

The threshold curves, of course, reduce properly to their thick-specimen values in the limit $T/\delta \to \infty$. It was shown in Ref. 16 that in the thick-specimen limit nucleation of domains always precedes switching by the uniform rotation mechanism of Stoner and Wohlfarth[18] because the uniform rotation switching astroid always lies outside the full curves of Fig. 2. This is no longer true of the dashed curves of the figure which refer to thinner platelets; indeed, the present theory, in essence, predicts single-domain behavior for conditions under which the domain-nucleation threshold curve lies outside the astroid. This criterion is, of course, a function of the orientation of the uniform magnetization. Comparing the switching equation of the Stoner-Wolhfarth theory with Eq. (3.25) of the present section shows that it will be met if

$$(D_x+D_z)(1+\kappa)+(D_x-D_z)\cos 2\theta > 0. \quad (3.27)$$

Thus, for the platelet geometry ($D_x=1$), using κ as given by Eq. (3.26) we find

$$\frac{T}{\delta} \leq \frac{4\pi k}{(1+\cos 2\theta)} \left(\frac{h_\infty \cos\phi_\infty \cos^2\theta}{k h_\infty \cos\phi_\infty + \cos\theta}\right)^{1/2}.$$

For most of the range of θ, this inequality will require a specimen so thin that our approximation no longer holds. Thus the inequality is only useful in the vicinity of $\theta=\frac{1}{2}\pi$, and of course it is always satisfied at $\theta=\frac{1}{2}\pi$. We must conclude that our approximation does not permit us to establish an improved criterion for single-domain behavior. We do confirm the result of Forlani and Minnaja[13] that domain nucleation cannot occur in an easy axis demagnetization cycle, but the present calculation makes it clear that the result arises from a singularity. The theory predicts domain nucleation for angles quite close to $\frac{1}{2}\pi$ if the plate is thick enough.[20]

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