# Lattice Mobility of Holes in III-V Compounds

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(Received 6 January 1970)

We review the circumstances under which polar mode scattering is important in the III-V compounds and conclude that it is the dominant intrinsic scattering mechanism only in the $n$-type direct-gap materials. For $n$-type indirect-gap materials, the most important scattering mechanisms are acoustic and nonpolar optical (NPO) mode intra- and intervalley scattering. For $p$-type III-V materials, we find that the dominant hole-scattering mechanism is a combination of acoustic and NPO mode scattering. This is in disagreement with previous treatments of hole mobility which emphasized polar mode scattering. It should be noted, however, that the usual expression for polar mobility was derived for nondegenerate $s$-like bands and is therefore not applicable to the complex $p$-like valence bands of the III-V compounds. Using acoustic $(E_{AC})$ and NPO mode $(E_{NPO})$ deformation potentials which are essentially the same as those obtained for $p$-Ge and $p$-Si [$3 < E_{AC} < 8$ eV and $(E_{NPO}/E_{AC})^2 \approx 4$], we obtain excellent fits for the magnitude and temperature dependence of published Hall-mobility data for $p$-type GaP, GaAs, InP, and AlSb. A simple expression is given which shows that the variation in Hall mobility for $p$-type group-IV and group-III-V materials is attributable to small variations in material parameters rather than different scattering mechanisms.

## I. INTRODUCTION

For purposes of discussing scattering mechanisms in III-V compounds, it is convenient to distinguish three classes of materials: (i) $n$-type direct gap, (ii) $n$-type indirect gap, and (iii) $p$ type. Of these, the high-mobility $n$-type direct-gap materials (e.g., GaAs and InSb) have received by far the greatest amount of attention. In the direct-gap semiconductors, the $s$-like symmetry of the low-effective-mass $\Gamma$ conduction-band minimum is responsible for the high mobilities principally because the $s$-like wave functions do not couple to nonpolar optical (NPO) modes.[1] Under these circumstances, the mobility is an admixture of acoustic and polar optical mode scattering. Because of the low effective mass, polar modes provide the dominant contribution.[1-4] In $n$-type indirect-gap and $p$-type materials, one is no longer justified in assuming, as is often done,[5,6] that polar modes continue to dominate. Recent work on electron transport[7] and free-carrier absorption[8] in $n$-GaP has emphasized the importance of acoustic and NPO mode intra- and intervalley scattering. Polar mode scattering appears to play a very minor role in this material. In the case of $p$-type materials, it is well known that the agreement between the magnitude of the calculated polar mobilities and experimental room-temperature data is fairly good.[4-6] Nevertheless, we believe, based on the following

experimental and theoretical observations, that this agreement is fortuitous:

(a) The expression used to calculate the polar mobility,[2,3,9] was derived for a simple nondegenerate $s$-like band and is not applicable[10] to degenerate $p$-like valence bands. Any calculation of scattering in the valence bands of III-V compounds must take account of the multiple-sheeted energy surfaces and the consequent possibility of interband scattering.

(b) The *temperature dependence* of the calculated polar mobility is in very poor agreement with the data for $p$-type materials. In fact, the most striking feature of these data is that they follow a power-law temperature dependence ($T^{-\beta}$ with $\beta \approx 2.3-2.4$) which can be explained by combining acoustic and NPO mode scattering in precisely the same way as for Ge and Si.

(c) Recent free-carrier absorption measurements in $p$-GaP[11] at 300 °K indicate that the hole scattering is dominated by acoustic and NPO mode mode scattering and that polar mode scattering is much weaker.

In Sec. II, we will discuss hole scattering in $p$-type III-V compounds and present an expression for the Hall mobility which is in excellent agreement with published data for GaP,[12] GaAs,[13] InP,[14] and AlSb.[15] Electron scattering in direct-[1-6] and indirect-[4,7,8] gap $n$-type III-V compounds has been adequately discussed in the literature and will be summarized in Sec. III.

## II. SCATTERING OF HOLES

### A. Theory

The valence bands of III-V semiconductors are, aside from small effects caused by the lack of inversion symmetry of the zinc-blende lattice, quite similar to those of the group-IV semiconductors. They consist of three atomic $p$-like bands, of which two are degenerate at $k = 0$ as shown in Fig. 1. The third band is split off from the other two by the spin-orbit interaction and for most low-field transport work can be ignored, though it can be important in both high-field transport and optical (free-carrier) absorption. In order to treat the two-band transport problem correctly, one must, in principle, solve a set of coupled Boltzmann equations, taking into account intra- and interband scattering together with the detailed shapes of the energy surfaces. This has been done for acoustic, [16-19] NPO mode, [16,17,19] and ionized impurity[18] scattering and the results have been successfully applied to the valence bands of Ge. Unfortunately, the methods used[16-19] in solving the coupled Boltzmann equations involve the relaxation-time approximation and hence are not applicable to polar mode scattering.[20] There have, to date, been no proper calculations of the effects of polar mode scattering on transport in the valence bands of III-V compounds.[10] It is clear, however, that whatever polar mode scattering there is will be *in addition* to the acoustic and NPO mode scattering. Furthermore, there is a remarkable similarity between the temperature dependence of the Hall mobility for $p$-type III-V semiconductors and for $p$-Ge (Ref. 21) and $p$-Si (Ref. 22). All have lattice mobilities of the form $\mu \propto T^{-\beta}$ with $\beta \approx 2.3$. This suggests that the
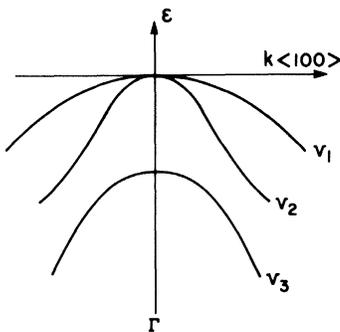
polar mode contribution is negligible. We now investigate the contribution is negligible. We now investigate the extent to which the Hall-mobility data for $p$-type III-V and group-IV materials can be explained in a unified way by invoking acoustic and NPO mode scattering alone.

The detailed theories[16-19] of scattering in degenerate valence bands of the form shown in Fig. 1 are difficult to apply. In interpreting their Hall data for $p$-Ge, Brown and Bray[21] used a somewhat simpler approach[23] which, in the lattice scattering regime, may be described as follows. It is assumed that both heavy $(m_1^*)$ and light $(m_2^*)$ holes have the same average lattice relaxation time characterized phenomenologically by $\langle \tau \rangle$. This assumption is based on a simple density-of-states argument which suggests that for both heavy holes and light holes the scattering rate into the heavy-hole band will dominate.[24] Thus, $\langle \tau \rangle$ is calculated using the effective mass $m_1^*$. In this approximation, we have for the heavy- and light-hole drift mobilities

$$\mu_1 = e\langle \tau \rangle / m_1^*,$$
$$\mu_2 = e\langle \tau \rangle / m_2^* = (m_1^*/m_2^*)\mu_1. \tag{1}$$

The total lattice mobility is then given by

$$\mu = (p_1\mu_1 + p_2\mu_2)/(p_1 + p_2), \tag{2}$$

where $p_1$ and $p_2$ are the average densities of holes in valence bands $v_1$ and $v_2$ (see Fig. 1). Assuming simple parabolic bands, $p_1$ and $p_2$ are related by

$$p_1/p_2 = (m_1^*/m_2^*)^{3/2}, \tag{3}$$

which gives $\quad \mu = \left(\dfrac{r + r^{3/2}}{1 + r^{3/2}}\right) \mu_1, \tag{4}$

where $\quad r = m_1^*/m_2^* > 1. \tag{5}$

The multiplicative $r$ factor in Eq. (4) is plotted in Fig. 2 and is seen to be a relatively weak function of $r$, taking values between 1.20 and 1.33 for the physically reasonable range $2 < r < 20$.

To calculate $\mu_1$, we average the energy-dependent relaxation time $\tau$ over a Boltzmann distribution of carrier energies in the usual way:

$$\langle \tau \rangle = \langle v^2 \tau \rangle / \langle v^2 \rangle, \tag{6}$$

where $v$ is the velocity $(v^2 = 2\mathcal{E}/m_1^*)$ and $\mathcal{E}$ is the hole energy. The relaxation time is taken to be

$$1/\tau = 1/\tau_{AC} + 1/\tau_{NPO}, \tag{7}$$

where $\tau_{AC}$ and $\tau_{NPO}$ are the relaxation times for acoustic and NPO mode scattering, respectively. Conwell[25] has shown that $1/\tau_{NPO}$ is given by

$$1/\tau_{NPO} = (1/\tau_{AC})\{C[(1 + \Theta/xT)^{1/2}$$



FIG. 1. Schematic representation of the valence bands in III-V compounds ignoring the small terms linear in $k$.
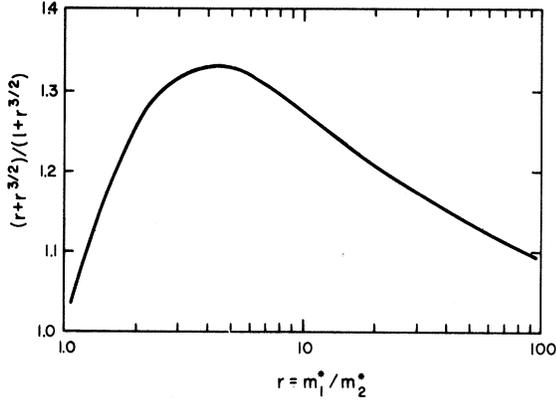
FIG. 2. Plot of the $r$ factor which appears in Eq. (4). The maximum occurs at $r=4$ and has a value of $\frac{4}{3}$. Values of $r=m_1^*/m_2^*$ for Si, Ge, and the III-V compounds are in the range $3 < r < 8$.

$$+ e^{\Theta/T}(1 - \Theta/xT)^{1/2}]\}, \tag{8}$$

in which $x = \mathcal{E}/kT$,

$$C = \tfrac{1}{2}\eta\,(\Theta/T)(e^{\Theta/T} - 1)^{-1},$$

$$\eta = (E_{\mathrm{NPO}}/E_{\mathrm{AC}})^2, \tag{9}$$

and

$$\frac{1}{\tau_{\mathrm{AC}}} = \frac{\sqrt{2}\,(m_1^*)^{3/2}(kT)^{3/2}x^{1/2}}{\pi\hbar^4\rho\bar{u}^2}\,E_{\mathrm{AC}}^2.$$

In these equations, $\Theta$ is the characteristic temperature of the zone-center optical phonons, [16] $E_{\mathrm{AC}}$ and $E_{\mathrm{NPO}}$ are, respectively, deformation potentials for acoustic and NPO modes, [26] $\rho$ is the density, and $\bar{u}$ is an average sound velocity. Substituting Eq. (7) into Eq. (6) gives

$$\langle \tau \rangle = \frac{4}{3\sqrt{\pi}}\int_0^\infty \frac{x^{3/2}\,e^{-x}}{1/\tau_{\mathrm{AC}} + 1/\tau_{\mathrm{NPO}}}\,dx. \tag{10}$$

Combining Eqs. (1) and (4) with Eqs. (8)–(10), we derive the following expression for the total lattice mobility:

$$\mu = \left(\frac{r + r^{3/2}}{1 + r^{3/2}}\right) S(\Theta, \eta, T)\mu_{\mathrm{AC}}. \tag{11}$$

Here $\mu_{\mathrm{AC}}$ is the acoustic mobility given by the familiar equation

$$\mu_{\mathrm{AC}} = \frac{2^{3/2}\pi^{1/2}e\hbar^4\rho\bar{u}^2}{3(m_1^*)^{5/2}(kT)^{3/2}E_{\mathrm{AC}}^2}, \tag{12}$$

and $S$ is given by

$$S = \int_0^\infty \frac{xe^{-x}\,dx}{1 + C[(1 + \Theta/xT)^{1/2} + e^{\Theta/T}(1 - \Theta/xT)^{1/2}]}. \tag{13}$$

The integral $S$ is shown in Fig. 3 as a function of $T/\Theta$ for various values of the parameter $\eta$. In computing this integral, the term $(1 - \Theta/xT)^{1/2}$

(which arises from phonon emission) is set equal to zero when $\Theta/xT > 1$. In general, the integral must be performed numerically, but in the Appendix we give an approximate expression for $S$ which allows hand calculation of $\mu$ to an accuracy of a few percent. In Fig. 4, we show $\mu$ as a function of $T/\Theta$ for reasonable values of $\eta$. The qualitative behavior of $\mu$ can be described as follows. At low temperatures ($T/\Theta < 0.1$), $S \approx 1$, and $\mu$ has the $T^{-3/2}$ temperature-dependence characteristic of acoustic mode scattering. As $T$ increases, $S$ decreases sharply and eventually (when $T/\Theta > 1$) approaches a constant value so that $\mu$ again has a $T^{-3/2}$ temperature dependence. The transition region ($0.1 < T/\Theta < 1$) has a rather steep temperature dependence which can be approximated[17] by a power law $T^{-\beta}$ with $\beta$ determined by $\eta$. For $\eta = 1, 2, 3, 4$, and $5$, one obtains $\beta \approx 1.8$, $2.0$, $2.2$, $2.3$, and $2.4$, respectively.

For purposes of calculation, it is convenient to write Eq. (11) in the form

$$\mu = 3.2\times10^{-5}\left(\frac{r + r^{3/2}}{1 + r^{3/2}}\right)\left(\frac{\rho\bar{u}^2}{(m_1^*/m_0)^{5/2}E_{\mathrm{AC}}^2}\right)$$

$$\times S(\Theta, \eta, T)T^{-3/2}, \tag{14}$$

where $\mu$ is in $\mathrm{cm}^2/\mathrm{V}$ sec, $E_{\mathrm{AC}}$ is in eV, $\rho$ is in $\mathrm{g/cm^3}$, $\bar{u}$ is in cm/sec, and $m_0$ is the free-elec-
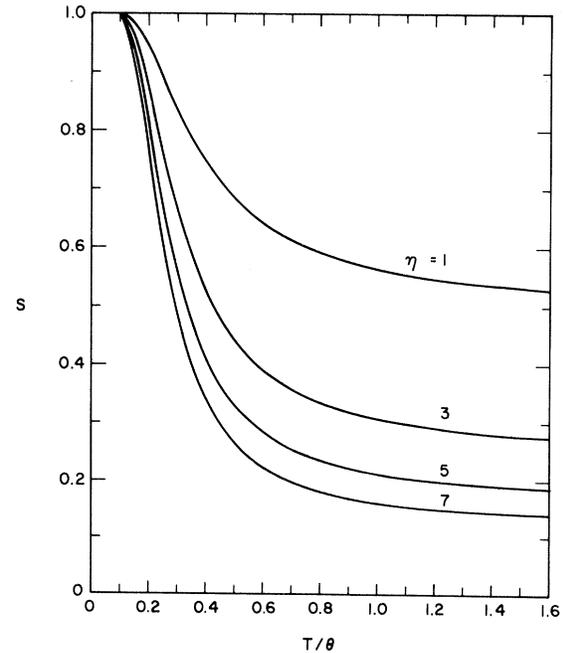


FIG. 3. Temperature dependence of the integral $S$ for various values of $\eta = (E_{\mathrm{NPO}}/E_{\mathrm{AC}})^2$. These curves were obtained by numerical integration of Eq. (13).
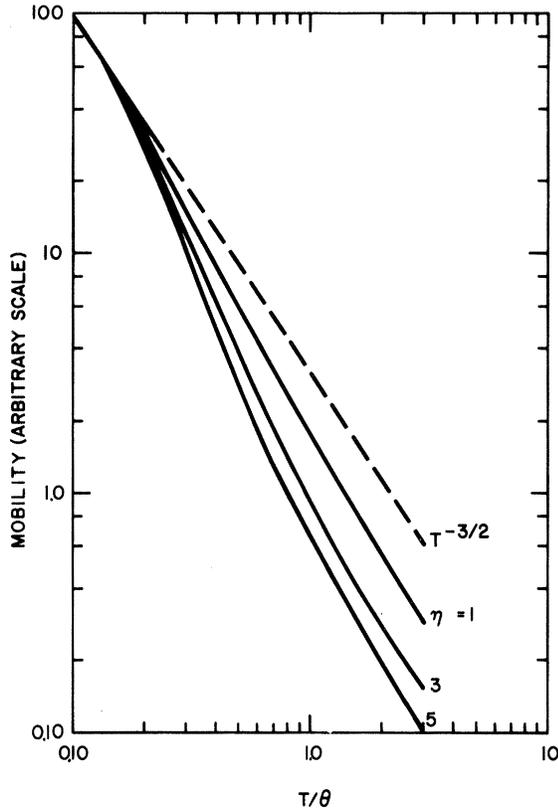
FIG. 4. Temperature dependence of the hole mobility [Eq. (14)] assuming combined acoustic and NPO mode scattering for various values of $\eta = (E_{NPO}/E_{AC})^2$.

tron mass. From Eq. (14), it is clear that there are seven physical parameters to be specified: $m_1^*$, $m_2^*$, $\rho$, $\bar{u}$, $\Theta$, $E_{AC}$, and $\eta$. The values of $\rho$ and

$\Theta$ are well known for most materials and present little difficulty. Since both longitudinal and transverse phonons are involved in the scattering,[16] the appropriate sound velocity is an average

$$\bar{u}^2 = \tfrac{1}{3} u_L^2 + \tfrac{2}{3} u_T^2, \qquad (15)$$

where $u_L$ and $u_T$ are, respectively, the velocities of longitudinal and transverse sound waves averaged over all directions.[16] Since these velocities are known for most materials, $\bar{u}$ is not an adjustable parameter. The light-hole mass $m_2^*$ enters only through the insensitive $r$ factor shown in Fig. 2 and hence the choice of this parameter is not critical. This leaves only $m_1^*$, $E_{AC}$, and $\eta$ as important adjustable parameters. As noted above, the temperature dependence of $\mu$ is determined by $\eta$ while the absolute magnitude is determined by the product $(m_1^*)^{-5/2} E_{AC}^{-2}$.

### B. Comparison with Experiment

In Table I, we have listed values for the parameters needed in Eq. (14) for Ge, Si, GaP, GaAs, InP, and AlSb.[27] The first six columns contain all the material parameters except $E_{AC}$. The values of $E_{AC}$ listed in the seventh column were chosen to give agreement between Eq. (14) and the room-temperature experimental Hall mobilities $\mu$(expt) listed in column 8. We have assumed that the Hall and drift mobilities are directly comparable $(\mu_H/\mu_D \approx 1)$. For pure acoustic mode scattering in a simple band, the correct ratio would be $\mu_H/\mu_D = 3\pi/8$. In the absence of detailed calculations of $\mu_H$ for the valence bands, the assumption $\mu_H/\mu_D \approx 1$ would appear to be reasonable.

In Fig. 5, we show complete fits of the lattice scattering portions of the data for Ge, GaAs, and

TABLE I. Parameters used in calculating hole mobilities [Eq. (14)] in group-IV and group-III-V materials.

| Material | $\rho$ (g/cm$^3$) | $\Theta$[a,b] (°K) | $m_1^*/m_0$[a] | $m_2^*/m_0$[a] | $\bar{u}$ (10$^5$ cm/sec)[a] | $S$ ($\eta = 4$, $T = 300$ °K) | $E_{AC}$[c] (eV) | $\mu$(expt)[d] (300 °K) | $\beta$[d] $\mu \propto T^{-\beta}$ |
|---|---|---|---|---|---|---|---|---|---|
| Ge | 5.32 | 430 | 0.35 | 0.04 | 3.63 | 0.30 | 3.7 | 2050 | 2.33 |
| Si | 2.33 | 730 | 0.50 | 0.16 | 6.82 | 0.45 | 7.9 | 350 | 2.3 |
| GaP | 4.13 | 580 | 0.88[e] | 0.16[e] | 4.76 | 0.37 | 5.7 | 120 | 2.2–2.3 |
| GaAs | 5.31 | 410 | 0.50[f] | 0.068[f] | 3.90 | 0.30 | 5.3 | 400 | 2.3–2.4 |
| InP | 4.79 | 480 | $\begin{Bmatrix} 0.50^g \\ 0.60 \\ 0.70 \end{Bmatrix}$ | 0.10[g] | 3.81[h] | 0.32 | $\begin{Bmatrix} 7.9 \\ 6.3 \\ 5.2 \end{Bmatrix}$ | 160 | 2.3–2.4 |
| AlSb | 4.22 | 480 | $\begin{Bmatrix} 0.40^i \\ 0.50 \\ 0.60 \end{Bmatrix}$ | 0.07[i] | 3.72 | 0.32 | $\begin{Bmatrix} 6.1 \\ 4.6 \\ 3.7 \end{Bmatrix}$ | 400 | >2 |

[a] P. Aigrain and M. Balkanski, *Selected Constants Relative to Semiconductors* (Pergamon, Oxford, England, 1961).

[b] Reference 5, p. 118.

[c] $E_{AC}$ was adjusted to make Eq. (14) agree with $\mu$ (expt) at 300 °K.

[d] References for experimental data are cited in text.

[e] R. A. Faulkner (private communication).

[f] A. K. Walton and U. K. Mishra, J. Phys. C **1**, 533 (1968).

[g] Assumed to be intermediate between InSb and GaP.

[h] Estimated using empirical relation. See R. W. Keyes, J. Appl. Phys. **33**, 3371 (1962).

[i] Assumed to be similar to GaAs.

less, if we assume that the $m_1^*$ values listed in Table I are accurate and we adjust $E_{AC}$ as in the case of Ge and Si, we obtain the values shown in Table I. These values for $E_{AC}$ were used to compute the curves shown in Fig. 5 and all fall in the range $3 < E_{AC} < 8$ eV. The range of acoustic mode deformation potentials for the III-V compounds is comparable with those for the elemental semiconductors. We believe that this is a consequence of the similarities in their valence bands and crystal structures. The differences in hole mobility values for the group-IV and group-III-V $p$-type materials are attributable to (small) differences in material parameters rather than different scattering mechanisms.

It is appropriate at this point to discuss the extent to which polar modes may be ruled out as an effective scattering mechanism in the valence bands of III-V compounds. The expression usually quoted for the polar mode mobility has the temperature dependence[9]

$$\mu_{PO} \propto (\Theta/T^{1/2}(e^{\Theta/T} - 1)G\,1)\,e^{-\ell}, \qquad (16)$$

where $G(1)e^{-\ell}$ is given graphically in Ref. 9 and is a slowly varying function of $T$ when the plasma frequency $\omega_p$ is small (i. e., when screening is negligible). At high temperatures $(T \gtrsim \Theta)$, $\mu_{PO}$ approaches a $T^{-1/2}$ temperature dependence which is in strong disagreement with the experimental data. In Fig. 6, we show a comparison between the temperature dependence of Eq. (14) (with $\eta = 4$) and

FIG. 5. Comparison between theoretical (solid lines) and experimental (open circles) mobilities for Ge (Ref. 21), GaAs (Ref. 13), and GaP (Ref. 12). The dashed lines have $T^{-3/2}$ slopes and correspond to pure acoustic mobilities. Parameters used in the calculation are given in Table I and discussed in the text.

GaP to illustrate the agreement between experimental and theoretical temperature dependences.[28] In calculating the mobilities given in Fig. 5, we have used $\eta = (E_{NPO}/E_{AC})^2 = 4$ in all cases. This is in agreement with previous results for $p$-Ge[16,17,21] and $p$-Si,[17] and as was mentioned above, is a consequence of the approximately $T^{-2.3}$ temperature dependence. In the cases of Ge and Si, all other parameters entering in Eq. (14) are well known except $E_{AC}$. By fitting the magnitude of $\mu$ at 300 °K, we obtain $E_{AC}(Ge) \approx 3.7$ eV and $E_{AC}(Si) \approx 7.9$ eV. In the III-V compounds, the hole effective masses are not well known. Therefore, one should not place great confidence in the values of $E_{AC}$ obtained by fitting the magnitude of $\mu$ at 300 °K. Neverthe-



FIG. 6. Comparison between the temperature dependence of the polar mobility [Eq. (16)] and the mobility resulting from combined acoustic and NPO mode scattering [Eq. (14)] with $\eta = 4$.

Eq. (16) (with $\omega_p = 0$) for $T/\Theta > 0.3$. If Eq. (16) were in agreement with the data at $T/\Theta = 0.5$, it would be in disagreement by a factor of 2 at $T/\Theta = 1$. It is possible that a correct calculation of the polar mobility in $p$-type materials would give a different temperature dependence than predicted by Eq. (16). It is also possible that one can obtain better agreement between Eq. (16) and the experimental data by using temperature-dependent dielectric constants.[29] We simply note that Eq. (16) does not agree with experiment whereas Eq. (14) is in excellent agreement.

It is also worthwhile to point out that polar mode scattering occurs preferentially in the forward direction[2] (the matrix element for scattering is proportional to $q^{-2}$ where $q$ is the phonon wave vector) and hence involves small momentum changes. This would reduce its effectiveness for interband scattering and would tend to decouple the bands $v_1$ and $v_2$. Under these circumstances, the total polar mobility would be *higher* than what would be predicted by using a density-of-states effective mass $m_d^* \approx m_1^*$ in Eq. (16), as has been done in previous calculations of the polar mobility in $p$-type III-V semiconductors.[4-6,12,14]

### III. SUMMARY AND CONCLUSIONS

The scattering mechanisms which appear to be dominant in the III-V compounds can be summarized as follows.

#### A. *n*-Type Direct-Gap Materials

These materials (InSb, InAs, InP, GaSb, and GaAs) have $s$-like conduction-band minima with relatively small effective masses. The $s$-like symmetry of the wave functions prohibits NPO mode scattering and the low $m^*$ makes acoustic mode scattering ineffective. Of the remaining intrinsic scattering mechanisms, polar mode scattering is the most important.[4,30,31] Energetic carriers such as those involved in high-field transport or free-carrier absorption can occupy states which no longer have pure $s$-like symmetry. This necessitates a careful analysis and reconsideration of the scattering processes for each case. In extreme cases, the subsidiary minima of the conduction band become energetically accessible and one has the additional possibility of intervalley scattering.

#### B. *n*-Type Indirect-Gap Materials

The conduction-band minima of AlP, AlSb, and GaP are at or near the $X$ point where effective masses tend to be larger and the symmetry is no longer purely $s$-like. Thus, both acoustic and NPO mode scattering are more important than for

direct-gap materials. In addition, one has the possibility of intervalley scattering. It has been shown[7,8] that acoustic and NPO mode intra- and intervalley scattering are dominant over polar mode scattering in $n$-GaP. The larger effective masses and the extra scattering mechanisms made possible by the many-valleyed conduction bands undoubtedly explain the fact that mobilities in $n$-type indirect-gap III-V materials are much lower than those in the direct-gap materials.[6]

#### C. *p*-Type Materials

In principle, one should expect acoustic, NPO mode, and polar mode scattering to be present in the valence bands of III-V compounds. We have pointed out that there is no adequate theoretical expression for polar mode scattering in $p$-type materials and that the usual expression [Eq. (16)] fails to explain the temperature dependence of the Hall mobility. In view of the great similarities between group-IV and group-III-V compounds, it seems reasonable to expect that the *nonpolar* scattering mechanisms which have been successfully treated for $p$-Ge and $p$-Si are also important in the III-V materials. Pursuing this analogy, we have shown that the magnitude and temperature dependence of the Hall mobility for $p$-type GaP, GaAs, InP, and AlSb can be fitted following a treatment first developed by Brown and Bray[21] for $p$-Ge. The two adjustable parameters $E_{AC}$ and $\eta = (E_{NPO}/E_{AC})^2$ are quite similar for all the materials considered and are physically reasonable.

In conclusion, we feel that previous claims regarding the universal importance of polar mode scattering in the III-V semiconductors are unjustified. Polar modes are clearly dominant at 300 °K in the $n$-type direct-gap materials but not in the indirect gap or $p$-type compounds. A complete discussion of hole mobility must await a proper treatment of polar mode scattering but it appears that acoustic and NPO mode scattering are sufficient to explain the data.

### ACKNOWLEDGMENTS

### APPENDIX

It can be shown that Eq. (13) approaches a high-temperature limit of

$$S_\infty = (1 + \eta)^{-1} \qquad \text{(A1)}$$

as $T/\Theta \to \infty$. This gives the ultimate contribution of NPO mode scattering (e.g., if $\eta = 4$, the high-temperature mobility will be only one-fifth of what it would have been with acoustic mode scattering alone). For purposes of hand calculation, $S$ can be approximated by the expression

$$S \approx (1 + A\eta)^{-1}, \qquad \text{(A2)}$$

where $A = (\Theta/T)H/(e^{\Theta/T} - D) \qquad \text{(A3)}$

with $H$ and $D$ given as functions of $\eta$ in Table II. The values of $H$ and $D$ given in Table II for a particular value of $\eta$ will assure an accuracy of better than 1% for $T/\Theta \lesssim 1.5$ when Eq. (A2) is used to approximate $S$. When the values of $H$ and $D$ for

$\eta = 4$ are used to determine $A$, the error in $S$ is $< 3\%$ for $1 \le \eta \le 6$ and $T/\Theta \lesssim 1.5$.

TABLE II. Values of $H$ and $D$ which optimize the approximation given by Eq. (A2) for various values of $\eta = (E_{NPO}/E_{AC})^2$.

| $\eta$ | $H$ | $D$ |
|---|---|---|
| 1 | 1.51 | 0.795 |
| 2 | 1.42 | 0.857 |
| 3 | 1.37 | 0.892 |
| 4 | 1.34 | 0.914 |
| 5 | 1.32 | 0.929 |
| 6 | 1.30 | 0.941 |
| 7 | 1.29 | 0.950 |
| 8 | 1.28 | 0.957 |

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[20]For a discussion of the relaxation-time approximation and its inapplicability to the case of polar mode scattering see A. C. Beer, *Galvanomagnetic Effects in Semiconductors* (Academic, New York, 1963), Sec. 25, p. 278.

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[22]F. J. Morin and J. P. Maita, Phys. Rev. <u>96</u>, 28 (1954).

[23]H. Brooks, *Advances in Electronics and Electron Physics* (Academic, New York, 1955), Vol. 7.

[24]If $\tau_{ij}$ represents the relaxation time for scattering from band $i$ to band $j$, the assumption of equal relaxation times for light and heavy holes can be expressed as $\tau_{11} \approx \tau_{21} \ll \tau_{12} \approx \tau_{22}$. It is relatively easy to show that, at equilibrium, $\tau_{12}/\tau_{21} = (m_1^*/m_2^*)^{3/2} \equiv r^{3/2}$ and that the fraction of time that any given hole spends in $v_1$ is $f_1 = r^{3/2}/(1 + r^{3/2})$. As pointed out by Brown and Bray (Ref. 21), however, the assumptions $\tau_{11} \approx \tau_{21}$ and $\tau_{12} \approx \tau_{22}$ are empirical.

[25]E. M. Conwell, J. Phys. Chem. Solids <u>8</u>, 236 (1959).

[26]The deformation potentials $E_{AC}$ and $E_{NPO}$ should be regarded as phenomenological parameters, and are not related in a simple way to the deformation-potential parameters of more detailed scattering theories. This is because the effect of pressure on the valence bands is much more complicated than the simple shift of the band edge envisaged in the deformation-potential approximation. It should be noted that in particular $E_{AC}$ and $E_{NPO}$ are not derivable from the rate of change of the energy gap with pressure since this involves both conduction- and valence-band shifts.

[27]Data for InSb and InAs are available but these materials have very narrow gaps and one runs into complications of intrinsic conduction and electron-hole scattering. We were unable to find any data for high-quality AlP, AlAs, or GaSb.

[28]The data for Si, InP, and AlSb can be fitted equally well and are excluded from Fig. 5 solely because they are so similar to the GaP and GaAs results.

[29]M. Aven and B. Segall, Phys. Rev. <u>130</u>, 81 (1963).

[30]H. Ehrenreich, J. Appl. Phys. <u>32</u>, 2155 (1961).

[31]Recent work by D. L. Rode [Phys. Rev. B (to be published)] shows that piezoelectric scattering can be extremely important at low temperatures. Rode also finds that the temperature dependence of the dielectric constants will not result in the reduction of the high-temperature mobility suggested by Aven and Segall (Ref. 29).