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Influence of Self-Induced Magnetic Field on the Stability of an Electron-Hole Plasma in Parallel Electric and Magnetic Fields*

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The Kadomtsev-Nedospasov theory of helical instability of a gaseous plasma has been applied to semiconductors by several authors. We have extended this theory to include the force arising from a self-induced magnetic field. Such a force pulls the plasma to the center. Helical instability is due to the flux of charged particles to or from the surface, and thus depends on the surface conditions. We have treated the effect of the self-induced field as a small perturbation on this surface effect. It is predicted that the stability increases or decreases if the flux due to the self-induced field adds or substracts from the flux due to boundary conditions.

INTRODUCTION

The onset of current oscillations[1–4] in semiconductors placed in parallel electric and magnetic fields is in some cases due to helical instabilities set up in the material. The theory of helical instability was first presented by Kadomtsev and Nedospasov.[5] The original Kadomtsev-Nedospasov paper is on gaseous plasma. Their ideas have been applied to plasma in semiconductors by several authors.[6–8] One solves the problem of two fluids moving through each other. The hydrodynamic equations describing the positive and the negative fluids connect the forces acting on the fluids with the fluxes they produce.

The present paper differs from the work of other authors by including the forces arising from the self-induced magnetic field in the hydrodynamic equations. Furthermore, we assume that the recombination properties of the material in the volume differ from those of the surface; and we use different parameters to describe their rates of recombination. It is shown that if these parameters are equal, no instabilities can occur.

CALCULATIONS

We assume an intrinsic semiconductor, i.e., the total number of free electrons and holes are equal. The free electrons (holes) behave as a nondegenerate gas. In the presence of external field, the densities of electrons (holes) may be functions of position and time. We assume the density of free electrons to be equal to that of the free holes everywhere in the sample. An internal electric field may exist in the material, although we neglect the existence of space charge. This assumption of quasineutrality is reasonable, for only small differences in the densities can produce very strong internal electric fields.

Denoting by ± quantities associated with the holes and electrons, we have the two fluid equations

$$\vec{\Gamma}_\pm + D_\pm \nabla n \mp \mu_\pm n \vec{E} \mp \mu_\pm \vec{\Gamma}_\pm \times \vec{B} = 0\,, \tag{1}$$

and the equations of continuity

$$\frac{\partial(n-\bar{n})}{\partial t} + \nabla\cdot\vec{\Gamma}_\pm = -(\eta-\bar{\eta})\xi\,, \tag{2}$$

where n is the number density of free electrons or holes, and $\vec{\Gamma}_\pm$ are the particle fluxes (i.e., $\vec{\Gamma}_\pm = n\vec{v}_\pm$), $n\vec{v}_\pm$ are the drift velocities, $D_\pm = \mu_\pm k T_\pm/e$ are the Einstein diffusion constants, $\mu_\pm$ are the mobilities, and $T_\pm$ are the temperatures of holes and electrons. $\vec{E}$ and $\vec{B}$ are the actual (external and internal) electric and magnetic fields. $1/\xi$ is the mean bulk lifetime for excess carriers, and $\bar{n}$ is the density at which no net recombinations or generations take place. $\bar{n}$ may be a function of external electric fields and is roughly of the order of the average density of charged particles in the sample. The continuity equation can only assume the form of Eq. (2) in the special case when the recombinations and regenerations are monomolecular. The validity of such a form may depend on the material. In Eq. (1) we assume that the fluxes are in equilibrium with the instantaneous values of the forces. This is a good approximation as long as we investigate processes that are slower than $1/\tau_\pm$, where $\tau_\pm = \mu_\pm m_\pm/e$ are the mean times between successive collisions with the lattice of electrons and holes.

Collisions of free electrons and holes with the lattice vibrations alone are considered, and collisions between electrons and holes are neglected. We solve for $\vec{\Gamma}_\pm$ from Eq. (1) and substitute it in (2); we thus have

$$\begin{aligned}\nabla\cdot\big[\mu_\pm^2\cdot D'_\pm\vec{B}(\nabla n\cdot\vec{B}) \mp \mu_\pm^2\mu'_\pm n\vec{B}(\vec{B}\cdot\vec{E}) + D'_\pm\nabla n\\ \mp D'_\pm\mu_\pm(\vec{B}\times\nabla n) \mp \mu'_\pm n\vec{E} + \mu_\pm\mu'_\pm n(\vec{B}\times\vec{E})\big]\\ = \frac{\partial n}{\partial t} + (n-\bar{n})\xi\,. \end{aligned}\tag{3}$$

Here we have $D'_\pm = D_\pm/(1+\mu'_\pm B_{0z}^2)$ and $\mu'_\pm = \mu_\pm/(1+\mu_\pm^2 B_{0z}^2)$. For a sample in the shape of a cylinder, we use cylindrical coordinates with the z axis along the direction of applied electric field E_{0z} and magnetic field B_{0z}. In addition to external magnetic field, an internal magnetic field $B_\phi(r)$ will be induced,

$$B_\phi(r) = \frac{e}{c^2}\frac{1}{r}\int_0^r n(r')v_z r'\,dr'\,, \tag{4}$$

where $v_z = v_z^+ - v_z^-$ are the components of the drift velocities. In the literature, $B_\phi(r)$ has usually been neglected. We assume a simple form for B_ϕ to be

$$B_\phi = e\,v_z n_a r/c^2\,, \tag{5}$$

where n_a is the average density of charges. The above form of B_ϕ seems reasonable if v_z is assumed independent of position.

STEADY STATE SOLUTION

In this case, we assume $\partial n/\partial t = 0$ and $\partial n/\partial\phi = \partial n/\partial t = 0$, $\partial E_z/\partial z = 0$, and $E_\phi = 0$. Eliminating E_{0r} between Eqs. (3), we have

$$\frac{\partial^2 n}{\partial r^2} + \left(\frac{1}{r} + Ar\right)\frac{\partial n}{\partial r} - (\beta_0^2 - 2A)\,n + \beta_0^2\bar{n} = 0, \tag{6}$$

where
$$AR^2 = \frac{\mu'_-\mu'_+(\mu_- + \mu_+)}{\mu'_+ D'_- + \mu'_- D'_+}\;\Phi E_{0z}R^2\,,$$
$$\beta_0^2 R^2 = \frac{\mu'_- + \mu'_+}{\mu'_+ D'_- + \mu'_- D'_+}\;\xi R\,, \tag{7}$$

and
$$\Phi = B_\theta/R\,. \tag{8}$$

B_θ is the induced magnetic field at the boundary of the cylinder $r = R$. The boundary conditions are

$$\vec{\Gamma}_\pm(R) = S[n(R) - n_s]\,. \tag{9}$$

Equation (9) relates the flux at the boundary with the surface recombination velocity S. n_s is the density at the boundary at which surface recombination does not take place. n_s depends on the nature of the surface of the material.

Neglecting the induced magnetic field, the solution of (6) is

$$n = N_0 I_0(\beta_0 r) + \bar{n}\,, \tag{10}$$

where

$$N_0 = -R\frac{S}{\xi}(\bar{n} - n_s)\Big/ \left(\frac{R}{\beta_0^2}\frac{dI_0(\beta_0 r)}{dr}\bigg|_{r=R} + R\frac{S}{\xi}I_0(\beta_0 R)\right). \tag{11}$$

Including the induced magnetic field, we transform Eq. (6) to the form

$$\frac{\partial^2 N}{\partial r^2} + \left(\frac{1}{r} + Ar\right)\frac{\partial N}{\partial r} - (\beta_0^2 - 2A)N = 0\,, \tag{12}$$

$$\text{where}\quad N = n - [\beta_2^0/(\beta_0^2 - 2A)]\bar{n}\,. \tag{13}$$

Treating the induced magnetic field as a perturbation, we try a solution of Eq. (12) (up to first order in Φ)

$$N = N_0[I_0(\beta_0 r) + a_1 I_0(\beta_1 r)]\,, \tag{14}$$

where a_1 is assumed linear in Φ. From the methods of perturbation, it follows that

$$a_1 = \left(A\int_0^R r^2 \frac{dI_0(\beta_0 r)}{dr} I_0(\beta_1 r)\,dr\right) \times \left[(\beta_1^2 - \beta_0^2)\int_0^R I_0^2(\beta_1 r) r\,dr\right]^{-1} \equiv AR^2 Q\,. \tag{15}$$

Using the boundary conditions (9), we have

$$R\Gamma(R) = RS(n(R) - n_s) = \int_0^R (n(r) - \bar{n})\xi r\,dr\,. \tag{16}$$

The right-hand side of Eq. (16) merely states that the total rate of generation over the cross section of the sample is equal to the total rate of generation on the surface. Thus, we have

$$N_0 R^2 Q\left[\int_0^R I_0(\beta_1 r) r\,dr - \frac{RS}{\xi} I_0(\beta, R)\right] = \frac{2\bar{n}R^2}{\beta_0}\left(\frac{S}{\xi R} - \frac{1}{2}\right) \tag{17}$$

$$\text{and}\quad n \simeq N_0[I_0(\beta_0 r) + a_1 I_0(\beta_1 r)] + [1 + (2A/\beta_0^2)]\bar{n}\,, \tag{18}$$

where a_1 and β_1 are determined from Eq. (15) and (17), respectively.

The radial component of the internal electric field E_{0r} in the steady state can be determined from the condition that the radial flux of the electrons and of the holes must be equal,

$$E_{0r} = \frac{\alpha}{n}\frac{\partial n}{\partial r} - a\,r\,\Phi\,E_{0z}\,, \tag{19}$$

$$\text{where}\quad \alpha = (D_+' - D_-')/(\mu_+' + \mu_-')$$
$$\text{and}\quad a = (\mu_-\mu_-' - \mu_+\mu_+')/(\mu'+\mu_+')\,. \tag{20}$$

TIME-DEPENDENT SOLUTION

Here we introduce

$$n = n_0 + n_1(t) \quad\text{and}\quad \vec{E} = \vec{E}_0 + \vec{E}_1(t)\,, \tag{21}$$

where n_0 and $\vec{E}_0$ are the steady state values of n and $\vec{E}$, while n_1 and $\vec{E}_1$ are their time-dependent parts.

We assume a helical form for the density n_1 and the electrostatic potential V_1 from which $\vec{E}_1$ is derived. Thus,

$$n_1 = f(r)\,e^{i(\omega t + m\theta + kz)} \quad\text{and}\quad V_1 = g(r)\,e^{i(\omega t + m\theta + kz)}\,, \tag{22}$$

where ω is the angular frequency, m (set = +1) and k are the wave numbers in the azimuthal and z directions. We assume

$$f(r) = f_0 J_1(\kappa_0 r) \quad\text{and}\quad g(r) = l_0 J_1(\kappa_0 r)/n_0\,, \tag{23}$$

where f_0 and l_0 are independent of r. κ_0 is determined from (9). Thus,

$$\int_0^R J_1(\kappa_0 r) r\,dr = (RS/\xi)J_1(\kappa_0 R)\,. \tag{24}$$

The expressions for n and V are substituted into Eqs. (3) and the resulting equations are multiplied by $J_1(\kappa_0 r)$ and integrated from 0 to R. The resulting expressions are similar in form to those[5–8] obtained neglecting the self-magnetic field and can be written in the form

$$A_+ f_0 + B_+ l_0 = 0 \quad\text{and}\quad A_- f_0 + B_- l_0 = 0\,, \tag{25}$$

where it is assumed that $\omega = \omega_1 + i\omega_2$. The condition that Eqs. (25) have a nontrivial solution is

$$A_+ B_- - A_- B_+ = 0\,. \tag{26}$$

We separate Eq. (26) into its real and imaginary parts, and obtain two equations involving ω_1 and ω_2,

$$A_1\omega_1 + B_1\omega_2 + C_1 = 0 \quad\text{and}\quad A_2\omega_1 + B_2\omega_2 + C_2 = 0\,. \tag{27}$$

Solve for ω_2. Instability sets in if $\omega_2 < 0$. Therefore, at the threshold, we have

$$A_1 C_2 - A_2 C_1 = 0\,. \tag{28}$$

It is known that the electron's (hole's) temperature and mobility are functions of the applied electric field E_{0z}. Dependence of the electron and hole temperature on the electric field is a well investigated subject.[9] If one assumes that the scattering is due to interaction with acoustic modes, one obtains simple expressions[10] for field-dependent mobility and electron or hole temperatures. In sufficiently strong fields, we have

$$\mu_\pm(E_{0z}) = \mu_\pm^0 (C_s/\mu_\pm^0 E_{0z})^{1/2} (32/3\pi)^{1/4} \tag{29}$$

and $$T_\pm(E_{0z}) = T_\pm^0 (\mu_\pm^0 E_{0z}/C_s)(3\pi/32)^{1/2}, \tag{30}$$

where $T_\pm^0$, $\mu_\pm^0$ are the temperatures and mobilities of electrons or holes at vanishing electric fields and C_s is the velocity of sound in the medium.

If we neglect $\mu_\pm^2 B_{0z}^2$ in comparison with unity, Eq. (28) has the form for $Z_1 < 1$ (keeping terms up to Z_1^2):

$$b(\mu_- + \mu_+) B_{0z}[H_1(\Phi R/B_{0z}) + H_2 Z_1^{1/2} + H_3(\Phi R/B_{0z}) Z_1 + H_4 Z_1^{3/2} + H_5(\Phi R/B_{0z}) Z_1^2] + H_6 + H_7 Z_1 + H_8 Z_1^2 \geqslant 0, \tag{31}$$

where $b = E_{0z}R/(N_1)^{1/2}\dfrac{k}{e}(T_+ + T_-)$, $Z_1 = k^2R^2/N_1$,

$$\frac{1}{n_0}\frac{dn_0}{dr} = P \equiv P_1 + P_2 AR^2,$$

and $$P_1 = -\frac{1}{n_0} R \frac{S}{\xi}(\bar{n} - n_s)\frac{dI_0(\beta_0 r)}{dr}\frac{1}{D}, \tag{32}$$

$$P_2 = -\frac{1}{n_0}\left[\bar{n}\frac{2}{\beta_0^2 R^2}\left(\frac{R^2}{2} + R\frac{S}{\xi}\right)\frac{dI_0(\beta_0 r)}{dr} + R\frac{S}{\xi}(\bar{n} - n_s) Q \frac{dI_0(\beta_0 r)}{dr}\right]\frac{1}{D}, \quad Q = \frac{a_1}{AR^2}$$

$$D \simeq R(S/\xi)(1 + \tfrac{1}{4}\beta_0^2 R^2) + \tfrac{1}{2}R^2(1 + \tfrac{1}{8}\beta_0^2 R^2).$$

We use the notation

$$\langle F(r) \rangle \equiv \int_0^R F(r) J_1(\kappa_0 r) dr/(1/R^2) \times \int_0^R J_1^2(\kappa_0 r) r\, dr$$

in defining

$$N_1 = \left\langle \frac{d}{dr}\left[r\frac{d}{dr}J_1(\kappa_0 r)\right]\right\rangle - \left\langle \frac{1}{r}J_1(\kappa_0 r)\right\rangle,$$

$$N_2 = \left\langle \frac{d}{dr}[r P_1 J_1(\kappa_0 r)]\right\rangle, \quad N_3 = \langle P_1 J_1(\kappa_0 r)\rangle, \tag{33}$$

$$N_4 = (1/R^2)\left\langle r^2 \frac{dJ_1(\kappa_0 r)}{dr}\right\rangle, \quad N_5 = \frac{1}{R^2}\langle r^2 P_1 J_1(\kappa_0 r)\rangle,$$

$$N_6 = \left\langle \frac{d}{dr}[r P_2 J_1(\kappa_0 r)]\right\rangle, \quad N_7 = \langle P_2 J_1(\kappa_0 r)\rangle;$$

$$H_1 = G_1^2 G_4/(N_1)^{1/2}, \quad H_2 = -G_3 G_1, \quad H_3 = -(2G_4 - G_5)G_1/(N_1)^{1/2}, \quad H_4 = G_3, \quad H_5 = (G_4 - G_5)/(N_1)^{1/2}, \quad H_6 = G_1^2 G, \quad H_7 = -G_1(2G - G_1), \quad H_8 = G + 2G_1; \tag{34}$$

and $G = 1 - \beta_0^2 R^2/N_1$, $G_1 = (N_1 - N_2)/N_1$, $G_2 = N_2/N_1$, $G_3 = N_3/N_1$, $G_4 = 2 + N_4$, $G_5 = 2 + N_5$.

The equation obtained by differentiating Eq. (31) with respect to k is

$$-b(\mu_- + \mu_+)B_{0z}[H_2 + 2H_3(\Phi R/B_{0z})Z_1^{1/2} + 3H_4 Z_1 + 4H_5(\Phi R/B_{0z})Z_1^{3/2}] = 2H_7 Z_1^{1/2} + 4H_8 Z_1^{3/2}. \tag{35}$$

Equation (35) together with Eq. (31) give the value of $k = k_c$.

Eliminating E_{0z} between (31) and (35), we have

$$(3H_2H_8 - H_4H_7)Z_1^2 + 4(H_1H_8 - H_5H_6) \times (\Phi R/B_{0z})Z_1^{3/2} + (H_2H_7 - 3H_4H_6)Z_1 + 2(H_1H_7 - H_3H_6)(\Phi R/B_{0z})Z_1^{1/2} - H_2H_6 = 0. \tag{36}$$

We can calculate the angular frequency ω_1 in the same approximation, noting from Eq. (31) that $b\mu_\pm^2 B_{0z}^2$ is of the order $\mu_\pm B_{0z}$:

$$b_1(G_1 - Z_1)^2 = -b(\mu_-^2 - \mu_+^2)B_{0z}^2(G_1^2 + G_3^2 + G_1 Z_1)Z_1^{1/2}, \tag{37}$$

where $$b_1 = \frac{\mu_- + \mu_+}{\mu_-\mu_+}\frac{\omega_1 R^2}{N_1(k/e)(T_+ + T_-)}.$$

Thus, the frequency can be calculated when k and E_{0z} are determined.

DISCUSSION

Consider first the steady state solution in the limit $\Phi = 0$, i.e., neglect the influence of the azimuthal magnetic field. In this case, the solution is given by Eq. (10). This solution is of the standard form[6–8] containing Bessel functions. However, the coefficient N_0 of I_0 vanishes if $\bar{n} = n_s$. An examination of our solution (10) shows that the density at $r = 0$ increases in the steady state if $\bar{n} > n_s$. It seems possible theoretically that one could have $\bar{n} < n_s$, in which case there would be decrease of density at the origin in the steady state. Previous authors[6,7] have imposed the boundary condition that the density of the plasma is zero at the boundary. There is reasonable agreement of their theories with some of the recent experiments.[11,12] In the frame work of the present theory, a zero density at the boundary is achieved by requiring $(n_s/\bar{n}) \to 0$ and $(B/R\xi) \to \infty$.

We believe that the density at the boundary is best approximated by $\bar{n} > n_s$ and $(S/R\xi) > 1$. $\bar{n}$ and n_s are the parameters that characterize the difference in the recombination properties of the bulk and the surface. $\bar{n} > n_s$ corresponds to a state of lower density at the boundary than in the bulk. It should be noted, that if the boundary properties are not different from those of the bulk, i.e., $\bar{n} = n_s$, our solution (10) gives a constant density $\bar{n}$.

If $\Phi \neq 0$, our solution (18) has additional terms. These terms are due to the self-induced magnetic field. Throughout this paper, it is assumed that the effect of the self-induced field is small. The density profile is now directly dependent on the magnitude of the applied electric field. In Eq. (10), the electric field appears indirectly through $\mu_\pm$ and $T_\pm$. We observe that the self-induced magnetic field always pulls plasma inward, but the effect of the boundary produces a flux out if $n_s < \bar{n}$.

Equations (31), (36), and (37) are derived on the assumption that E_{0z} is sufficiently large so that the use of the relations (29) and (30), and the assumption $\mu_\pm^2 B_{0z}^2 \ll 1$ is valid. It is possible to compute numerically the critical values of E_{0z}, k, and ω, from (31), (36), and (37).

We observe from Eq. (31) that if the steady state solution is a constant density (i.e., $\bar{n} = n_s$ and $\phi = 0$) instability is not possible in any field; and from Eq. (36), we note that no unique Z_1 exists if the steady state is a constant.

The general forms of Eqs. (31), (36), and (37) are similar to those derived by Glicksman.[6] Except that our equations contain the influence of the azimuthal magnetic field, the present calculation reveals the need for properties at the boundary to differ from those of the bulk.

In order to get approximate values of the above quantities, we expand the Bessel functions in terms of their arguments and keep terms up to third power of the arguments. The approximation has been made by Hurwitz.[8] We then obtain from Eq. (36) the threshold value of $Z_1^{1/2}(k)$,

$$Z_1^{1/2} = \left(\frac{1}{2}\right)^{1/2} \left(1 - \frac{\beta_0^2 R^2}{\kappa_0^2 R^2}\right)^{1/2} - \frac{1}{2}\,\frac{1}{\kappa_0 R}\left(\frac{S}{R\xi}\,\frac{\beta_0^2 R^2}{\kappa_0^2 R^2}\,\frac{\bar{n} - n_s}{\bar{n}}\right)^{-1} \frac{\Phi R}{B_{0z}}, \tag{38}$$

and from Eq. (31) the threshold value of $E_{0z}\, B_{0z}$,

$$b\,(\mu_- + \mu_+) B_{0z} = (2)^{1/2}\left[\left(\frac{S}{R\xi}\,\frac{\beta_0^2 R^2}{\kappa_0^2 R^2}\,\frac{\bar{n} - n_s}{\bar{n}}\right)^{-1} + 9\right] \times \left[1 + \frac{(2)^{1/2}}{\kappa_0 R}\left(\frac{S}{R\xi}\,\frac{\beta_0^2 R^2}{\kappa_0^2 R^2}\,\frac{\bar{n} - n_s}{\bar{n}}\right)^{-1} \frac{\Phi R}{B_{0z}}\right]. \tag{39}$$

The critical frequency ω, obtained from Eq. (37), is

$$b_1 = 6\,(\mu_- - \mu_+) B_{0z}\left[\left(\frac{S}{R\xi}\,\frac{\beta_0^2 R^2}{\kappa_0^2 R^2}\,\frac{\bar{n} - n_s}{\bar{n}}\right)^{-1} + 2\right] \times \left[1 + \frac{(2)^{1/2}}{2}\,\frac{1}{\kappa_0 R}\left(\frac{S}{R\xi}\,\frac{\beta_0^2 R^2}{\kappa_0^2 R^2}\,\frac{\bar{n} - n_s}{\bar{n}}\right)^{-1} \frac{\Phi R}{B_{0z}}\right]. \tag{40}$$

If we neglect the Φ terms, the threshold values for $Z_1^{1/2}$, $E_{0z}B_{0z}$, and ω_1 are similar to those of Hurwitz, except that our solutions are in terms of $\bar{n} - n_s$. The terms involving $(\bar{n} - n_s)/\bar{n}$ and $S/R\xi$ become small when $(n_s/\bar{n}) \ll 1$ and $(S/R\xi) \gg 1$. In order to determine the magnitude of Φ-dependent terms in Eqs. (38), (39), and (40), we estimate $\kappa_0 R$ and $\beta_0 R$. From Eq. (24) if $(S/\xi R) > 1$ then $J_1(\kappa_0 R) \simeq 0$, i.e., $(\kappa_0 R)^2 \sim 8$. From Eq. (7) if $\mu_- > \mu_+$, $(\beta_0 R)^2 \sim (es/\mu_- kT_-)$. We thus have the Φ-dependent term as

$$(3\,\mu_- kT_-/es)\,(\Phi R/B_{0z}).$$

If B_{0z} is small and R large, this term can be about 0.1. Thus, the Φ-dependent term may affect the results by 10%. In its absolute magnitude, this effect cannot be detected because we can never switch off the self-induced magnetic field. We note that the Φ-dependent term increases the critical field necessary for the onset of instability. This is in accord with Glicksman's[6] theory, since the self-induced magnetic field increases the density gradient, thus, increasing the forces which oppose the instability. Glicksman has shown that a decreased density gradient did reduce the threshold for instability.

We observe, that the Φ-dependent terms are size dependent. This is understandable; for a larger R leads to a stronger self-induced field: Thus, the influence of the self-induced field can be measured from the size dependence of Z_1 and b and b_1.

Another possibility for the measurement of the influence of the Φ-dependent term exists.

The number density of electrons and holes in the material can be modified by illuminating the sample with appropriate radiation. If the attenuation length of the radiation is large compared with the dimensions of the sample, the plasma density may be increased uniformly in the sample. A higher density of charged carriers affects only the Φ-dependent term. Thus, if the carrier density is increased by a factor of 2, the Φ-dependent term alone will increase by a factor of 2. One may thus be able to measure the influence of the self-induced magnetic field on the stability of the plasma.

Thus, it is possible to check the predictions of the present theory with experiments suggested above. Recent results[11,12] cannot be used to separate the effects of the self-induced field.

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Exact Solution of a Boltzmann-Equation Model for Oscillatory Photoconductivity*

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The photoconductive response of some semiconductors to monoenergetic excitation shows structure which is associated with the emission of optical phonons. A model for oscillatory photoconductivity based on the Boltzmann equation has been suggested and partly investigated by Stocker and Kaplan (SK). An exact solution of the SK Boltzmann equation is presented in this paper. A model valid for small electric fields is derived from the exact solution which retains the main features of the SK model in a form particularly amenable to calculation. The properties of the small-field model are illustrated by two series of calculations. In the first series of calculations, the composition of a dip in the photoresponse caused by the proximity of an optical phonon emission threshold to the electron injection energy is analyzed. The shape and "intensity" of a dip depend on the relative values of the recombination lifetime, the strength of acoustical and optical phonon scattering, and the electric field. A second series of calculations was designed to show how the periodic repetition of dips in the photoresponse as a function of electron injection energy can be inhibited or destroyed by competition from other optical phonons.

I. INTRODUCTION

The photoconductive response of certain semiconductors exhibits periodic dips as a function of the monoenergetic exciting radiation. These "oscillations" were first observed in the extrinsic photoconductivity of the InSb : Cu system[1] and have since been seen in many systems.[2] More recent observations of the extrinsic photoconductivity of Si have revealed dips which are repeated no more than once, if at all.[3] In all cases, the dips are thought to arise from the onset of optical phonon emission.

The phenomenon has been treated theoretically in terms of a model based on the Boltzmann equation.[4] The main innovation of the Stocker-Kaplan (SK) model as compared with similar models for electron transport[5] is the explicit inclusion of an electron generation term. It is this property of the SK model which suggested a scheme leading to an exact solution.

The theoretical development of the SK model as described in the original paper[4] is inadequate in several respects. First, the calculations are based on an expansion in spherical harmonics of the Boltzmann distribution function in which only the first two terms are retained. This truncation approximation is arbitrary and the resulting equations are complicated. A solution was found only for a limited region and special circumstances. Second, the calculations do not consider more than one type of optical phonon process. The experimental results for Si, which has three distinct optical phonon processes operative,[6] suggest that there is a competition between the various optical phonon emission processes that tends to prevent the periodic repetition of the dips. Even accepting the truncation approximation, it is not clear that the original treatment can be extended to include more than one type of phonon process.

It is the purpose of this paper to present an exact