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Transient Space-Charge-Limited Currents in Photoconductor-Dielectric Structures

I. P. Batra, B. H. Schechtman, and H. Seki
IBM Research Laboratory, San Jose, California 95114

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The theory for transient space-charge-limited currents in photoconductor-dielectric structures is presented. The analysis gives the transient current density and voltage across the photoconductor as a function of time for various values of α. Here α is a parameter that characterizes the photoconductor-dielectric structure and depends upon the relative thicknesses and dielectric constants of the two regions. In two limiting cases, our results reduce to the direct-contact (no dielectric) and open-circuit (dielectric of infinite thickness) configurations already discussed in the literature. Our rigorous mathematical expressions, involving integrals of the exponential integral, differ somewhat from the original work on the direct-contact case, although numerical results are essentially the same. The general theory presented here will broaden the spectrum of techniques already available for determining drift mobilities in insulating solids, and the relative merits of various photoconductor-dielectric geometries are discussed.

I. INTRODUCTION

In 1962 the theory for transient space-charge-limited currents (TSCLC) in insulating materials was derived independently by Many and Rakavy[1] and by Helfrich and Mark.[2] Since then the theory has found wide application in the experimental determination of drift mobilities in a large number of materials, including anthracene,[3] iodine,[4] arsenic sulfide,[5] phthalocyanine,[6] and sulfur.[7] Most of these materials form molecular solids in which the carrier mobilities are low ($\lesssim 1\ cm^2/V\ sec$). For such low mobilities, Hall-effect measurements are impractical and the TSCLC technique has proved extremely valuable.

There are two major assumptions in the original TSCLC theory that impose limitations on experimental measurements. First, it is assumed that the generation of the carriers causes the electric field at the injecting electrode to drop to zero in a time short compared with the carrier transit time, and that the generation rate is sufficient to maintain zero field thereafter. The second constraint of the original theory is that the potential drop across the sample be held constant. One of the purposes of this paper is to generalize the TSCLC theory so that the constant-potential condition may be relaxed. As will be seen below, this should allow the interpretation of a wider variety of experimental situations and will relate the theory more closely to actual conditions existing in typical measurements.

One of the key results of the original theory is that the TSCLC exhibits a cusp (denoting the arrival at the collecting electrode of the first injected front of carriers) at a time equal to 0.787 times the Ohmic transit time. It is through the observation of this cusp that the drift mobility is often determined. Resolution of the cusp requires rapid commencement of carrier injection, and therefore most TSCLC experiments have made use of photoinjection through intense illumination with strongly absorbed light, which demands that at least one electrode be semitransparent. Typically, with electrodes of this type (e.g., very thin metal films on glass pressed against the sample or tin oxide layers), a barrier layer may exist between the contact and the photoconductor. This barrier would introduce uncertainty as to the actual voltage that appears across the photoconductor, and in severe cases the constant-voltage assumption of the original theory may not hold. In the treatment presented here, a dielectric is assumed to be in contact with the photoconductor. This analysis will also be applicable to cases in which a blocking layer is intentionally introduced, e.g., to eliminate dark injection. There may also be other undesirable effects due to direct contacts. For example, in experiments on phthalocyanine with transparent gold electrodes, Westgate and Warfield[6] observed strong effects due to surface trapping. Similarly, for evaporated gold on As_4S_4, Street and Gill[5] observed contact-dependent currents attributed to holes injected at the nonilluminated electrode. In such cases it would be desirable to remove the uncertainty of the contact effect by introducing a blocking layer, and this was in fact done in the As_4S_4 work. To interpret such measurements, one must either ignore any time dependence in the voltage drop across the photoconductor or take pains to ensure that the potential difference across the blocking layer is negligible. The theory developed in this work makes these approximations unnecessary and one can in fact treat these cases analytically.

A common alternative technique for measuring low mobilities is the time-of-flight or charge-counting method[8] with a weak pulse of injected carriers. This method has been used by Kepler[9] on anthracene, by Hartke[10] on amorphous Se, and by Szymanski and Labes[11] on tetracene, as well as by others. The usefulness of this procedure has been limited, however, since it is required that the injected charge be sufficiently small to keep the initial field distribution in the photoconductor and an adjacent dielectric unchanged during the measurement. This small-signal condition makes the direct observation of the drift current difficult, and instead a measure of the integrated current is obtained by monitoring the voltage induced on the dielectric. The transit time is observed as a turning point in the integrated current. This considerably limits the sensitivity of the experiment, especially in the presence of trapping or of non-negligible dark injection. The general theory given below allows one to combine the advantages of the charge-counting measurement (no metallic contacts) and the standard TSCLC measurement (high sensitivity) into one measurement, i.e., TSCLC with finite dielectrics. Furthermore, the direct-contact configuration treated in the original theory[1,2] follows as the special case of our analysis when we let the thickness of the dielectric shrink to zero. Our mathematical expressions, which we believe to be rigorous, differ somewhat from those of Many and Rakavy[1]; however, the numerical results are essentially unchanged.

Recently, Batra, Kanazawa, and Seki[12] treated the case of transient space-charge-limited conductivity in insulators for open-circuit conditions, i.e., one surface of the photoconductor is initially charged to a known voltage and left floating. In practice, this condition can be achieved by corona-charging the surface. This situation may be thought of as another special case of our generalized treatment such that the dielectric's thickness becomes very large. We refer to this configuration as the "corona" case in what follows. Here, since one surface is floating, the time derivative of the surface potential is the observable quantity; the external current is equal to zero. It appears that some of the problems mentioned earlier in connection with having direct contacts on the photoconductor may be alleviated by use of the corona configuration. Furthermore, the analytical details of the corona case turn out to be relatively simple, and this facilitates the application of the theory to the interpretation of experimental data. On the other hand, there are drawbacks which arise in the corona configuration. For one, the cusp (in the time derivative of the voltage) used to determine the transit time is not as pronounced as the cusp observed in other configurations, and this means that the effect of trapping or of non-negligible dark injection may impose more restrictive conditions on the observation of the transit time in corona measurements. Secondly, the use of corona charging makes it difficult to study the photoconductor under controlled ambient conditions, e.g., clean surfaces in vacuum. For these reasons and those discussed earlier, the treatment of a general dielectric layer in contact with the photoconductor as presented below should considerably broaden the spectrum of techniques already available for determining drift mobilities in insulating solids.

II. PHYSICAL MODEL

The configuration assumed for our photoconductor-dielectric structure is shown in Fig. 1. Light incident on the surface at $x = 0$ is totally transmitted through the dielectric, which is treated as a perfect insulator. The light is strongly absorbed at the interface between the dielectric and the photoconductor, generating electron-hole pairs. Only one type of carrier moves into the photoconductor under the applied field, leaving the other behind at the interface. The light is sufficiently intense to maintain zero electric field inside the photoconductor at $x = d$. One-dimensional plane-parallel geometry is assumed. The diffusion component of the current is neglected for mathematical simplicity. This is a good approximation[13] under typical experimental conditions. Furthermore, no specific treatment of trapping is included.

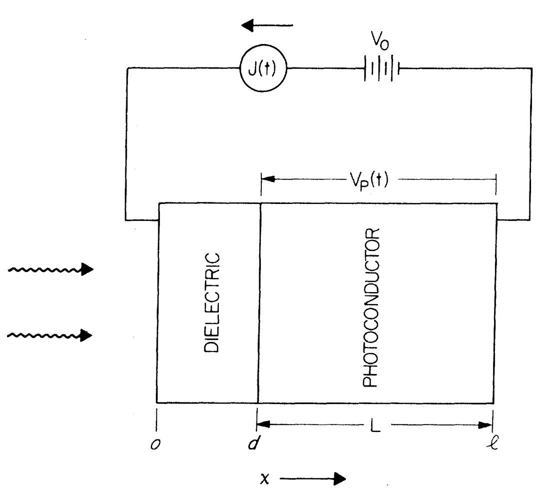

FIG. 1. Schematic representation of the dielectric-photoconductor structure. The voltage across the entire structure is maintained constant at V_0, whereas the voltage across the photoconductor $V_P(t)$ is time dependent.

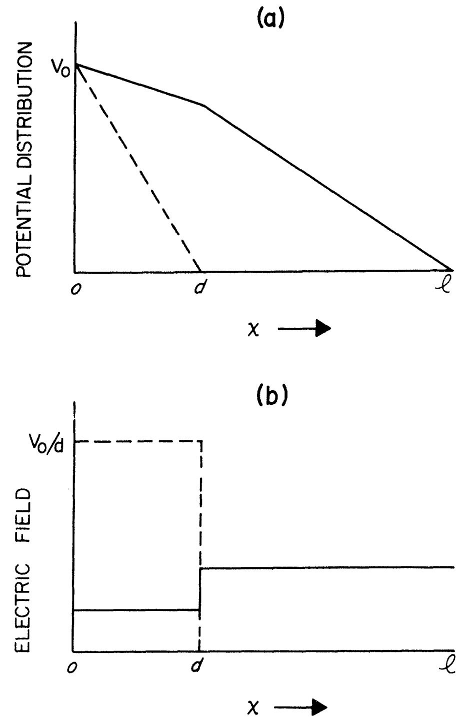

FIG. 2. (a) Potential distribution in the photoconductor-dielectric structure at $t = 0$ (solid) and $t \to \infty$ (dashed). (b) Electric field distribution in the photoconductor-dielectric structure at $t = 0$ (solid) and $t \to \infty$ (dashed).

As pointed out by Many and Rakavy, [1] in the case of trapping times very fast compared to the transit time, the results for no trapping apply if the free-carrier mobility is reduced by a factor $(1+\theta^{-1})^{-1}$, where θ is the equilibrium ratio of free carriers to trapped carriers. For slow trapping, Many and Rakavy[1] adopt a quasi-steady-state approximation which is not appropriate to our general model.

Note that in the presence of a dielectric of finite thickness, both the total current density $J(t)$ and the voltage $V_P(t)$ across the photoconductor are useful observables. This provides an advantage of flexibility and cross-checking capability when compared to the direct-contact case in which only $J(t)$ contains information or the corona case in which only $V_P(t)$ can be observed. In Fig. 2 we portray the potential and electric field distributions in the dielectric and the photoconductor. The solid lines depict the initial conditions and the dashed lines represent the conditions for $t \to \infty$. In the general case, unlike the direct-contact case, there is no steady-state field distribution in the photoconductor and hence no steady-state conduction current. The entire applied voltage finally appears across the dielectric.

III. CALCULATION

A. General Formulation of Problem

In this section the time-dependent flow of charge carriers in the photoconductor-dielectric structure is mathematically formulated. In the coordinate system chosen (Fig. 1), the dielectric lies between $x = 0$ and $x = d$ and the photoconductor extends from $x = d$ to $x = l$. The thickness of the photoconductor is $L \equiv l - d$. The interface ($x = d$) is illuminated continuously with intense light for all times $t \geq 0$, while the voltage across the *entire* structure is maintained constant.

As the photogenerated carriers move across the photoconductor, the portions of the voltage which appear across the photoconductor and the dielectric are time dependent, and in steady state the entire applied voltage appears across the dielectric. We are therefore interested in theoretically calculating the total current density $J(t)$, the voltage across the photoconductor $V_P(t)$, and the time rate of change of this voltage $\dot{V}_P(t)$ as a function of t. This is done for various values of the parameter α, where α is the ratio of the capacitance of the dielectric to the sum of the capacitances of the dielectric and the photoconductor.

In the absence of trapping and diffusion, the equations governing the flow of one type of carrier in the photoconductor are[1]

$$J_P^{(C)}(x, t) = q\mu n(x, t) E_P(x, t) \quad , \tag{3.1}$$

$$\frac{\partial J_P^{(C)}(x, t)}{\partial x} = -q \frac{\partial n(x, t)}{\partial t} \quad , \tag{3.2}$$

$$\frac{\partial E_P(x, t)}{\partial x} = \frac{4\pi q}{\kappa_P} n(x, t) \quad , \tag{3.3}$$

$$J_P(t) = q\mu n(x, t) E_P(x, t) + \frac{\kappa_P}{4\pi} \frac{\partial E_P(x, t)}{\partial t} \quad . \tag{3.4}$$

In the dielectric region, since the carrier density is always zero, the only equation we need to write is that for the total current density:

$$J_I(t) = \frac{\kappa_I}{4\pi} \frac{\partial E_I(x, t)}{\partial t} \quad . \tag{3.5}$$

Here the subscript P refers to the photoconductor and I refers to the dielectric. $J_P^{(C)}(x, t)$ is the conduction-current density, $E(x, t)$ is the electric field, $n(x, t)$ is the free-carrier density, q is the carrier charge ($q < 0$ for electrons), μ is the carrier mobility, $\kappa_P(\kappa_I)$ is the dielectric constant of the photoconductor (dielectric), and $J(t)$ is the total current density. Continuity of current demands that

$$J_P(t) = J_I(t) = J(t) \quad . \tag{3.6}$$

We now discuss the boundary and initial conditions relevant to our problem. As stated earlier, the interface $x = d$ is illuminated with continuous intense light such that the field $E(d^+, t)$ can be set equal to zero for all $t \geq 0$. Next, since the voltage across the entire structure is maintained constant for all times, the integral $\int_0^l E(x, t)\, dx = V_0$, where V_0 is the externally applied voltage. We define

$$V_P(t) = \int_0^l E_P(x, t)\, dx$$

and

$$V_I(t) = \int_0^d E_I(x, t)\, dx \quad .$$

With these definitions, the constant-voltage condition becomes $V_P(t) + V_I(t) = V_0$. Before the onset of injection, i.e., for $t \leq 0^-$, the voltage distributes itself capacitively between the dielectric and the photoconductor, since initially there is no charge present anywhere in the structure. If we define $\alpha \equiv C_I/(C_P + C_I)$, then $V_P(0) = \alpha V_0$ and $V_I(0) = (1-\alpha) \times V_0$. Here $C_I \equiv \kappa_I/4\pi d$ is the geometrical capacitance per unit area of the dielectric and $C_P \equiv \kappa_P/4\pi L$ is that of the photoconductor. At $t = 0$, the fields in the two regions are uniform (since no charge redistribution has yet taken place) and are given by $V_I(0)/d$ and $V_P(0)/L$, respectively.

From the above considerations, the boundary conditions can be expressed mathematically as

$$V_P(t) + V_I(t) = V_0, \qquad t \geq 0 \tag{3.7}$$

$$V_P(0) = \alpha V_0 \; , \tag{3.8}$$

$$V_I(0) = (1-\alpha) V_0 \; , \tag{3.9}$$

$$E_P(x, 0) = V_P(0)/L. \quad x > d \tag{3.10}$$

$$E_I(x, 0) = V_I(0)/d, \quad x < d \tag{3.11}$$

$$E(d^+, t) = 0, \qquad t \geq 0 \tag{3.12}$$

$$n(x, 0) = 0, \qquad x \neq d$$
$$n(x, t) = 0, \qquad x < d \quad . \tag{3.13}$$

B. Method of Solution

We are primarily interested in calculating $J(t)$ and $V_P(t)$ for various values of α. The method used here is similar to the one employed by Many and Rakavy.[1] The essential difference is that in their calculation $V_P(t)$ is constant, whereas in the present treatment the time dependence of $V_P(t)$ is itself one of the quantities of interest. Integrating (3.4) with respect to x from d to l and making use of Eq. (3.3) and the boundary conditions (3.7) and (3.12), we obtain

$$J_P(t) = C_P \tfrac{1}{2} \mu E_P^2(l, t) - C_P \dot{V}_I(t) \quad . \tag{3.14}$$

Since there is no space charge in the dielectric, we can set $J_I(t) = C_I \dot{V}_I(t)$. Equating this to $J_P(t)$, we get

$$\dot{V}_I(t) = -\dot{V}_P(t) = (C_P/C_I)\, \alpha \tfrac{1}{2} \mu E_P^2(l, t)$$

$$= (1-\alpha)\tfrac{1}{2}\mu E_P^2(l,t)\ , \tag{3.15}$$

where

$$\alpha \equiv C_I/(C_P+C_I)\quad . \tag{3.16}$$

Then

$$J(t) = J_I(t) = (C_P\alpha)\tfrac{1}{2}\mu E_P^2(l,t)\ , \tag{3.17}$$

and the voltage at any time t_β can be expressed in terms of its value at t_γ through the relation

$$V_P(t_\beta) = V_P(t_\gamma) - (1-\alpha)\tfrac{1}{2}\mu \int_{t_\gamma}^{t_\beta} E_P^2(l,t)\,dt\ . \tag{3.18}$$

To compute the total current density and the voltage, it is essential to calculate the electric field at the collecting surface ($x=l$) for all times. A total differential equation for $E_P(l,t)$ can be obtained by substituting (3.17) into (3.4) and particularizing to $x=l$. This leads to

$$\frac{dE_P(l,t)}{dt} + \frac{4\pi q\mu}{\kappa_P}\, n(l,t)E_P(l,t) = \frac{\alpha\mu}{2L}E_P^2(l,t)\ , \tag{3.19}$$

where $n(l,t)$ is the carrier density at the collecting surface. To arrive at the explicit t dependence of $n(l,t)$, the concept of the carrier-flow lines[1] is introduced through the relation

$$\frac{dx(t)}{dt} = \mu E(x(t),t)\quad , \tag{3.20}$$

where, since the mobile carriers exist only in the photoconductor, the flow lines have meaning only in that region. Equations (3.1)–(3.4) may be combined into one equation for E_P and another for n, defined along the flow lines $x(t)$:

$$\frac{dE_P(x(t),t)}{dt} = \frac{4\pi}{\kappa_P}J(t)\quad , \tag{3.21}$$

$$\frac{dn(x(t),t)}{dt} = -\frac{4\pi q\mu}{\kappa_P}n^2(x(t),t)\quad . \tag{3.22}$$

Equation (3.22) may be easily integrated from some time t_d to t to give the dispersion of carriers along a flow line. The result is

$$n(x(t),t) = \frac{n(x(t_d),t_d)}{1+n(x(t_d),t_d)(4\pi q\mu/\kappa_P)(t-t_d)}\ . \tag{3.23}$$

Equation (3.23) enables one to solve (3.19) for $E_P(l,t)$, from which $J(t)$ and $V_P(t)$ can be computed. This is accomplished in Sec. III C.

C. Expressions for Transient Current and Voltage

It is clear from Eqs. (3.17) and (3.18) that $J(t)$ and $V_P(t)$ are readily determined, once we know $E_P(l,t)$. This requires knowledge of $n(l,t)$. For the determination of $n(l,t)$, it is convenient to divide the problem into various time zones which we proceed to discuss now.

Zone I. This corresponds to that time interval for which $t \geq 0^+$, but no carriers have yet arrived at $x=l$. Suppose it takes time t_1 for the first front of injected carriers to arrive at $x=l$. Then for $t<t_1$, $n(l,t)=0$; and the solution of Eq. (3.19), subject to the boundary conditions (3.8) and 3.10), is

$$E_P(l,t) = (\alpha V_0/L)(1-\alpha t/2t_A)^{-1},\quad 0\leq t\leq t_1 \tag{3.24}$$

where

$$t_A \equiv L^2/\mu(\alpha V_0) = L^2/\mu V_P(0)\quad , \tag{3.25}$$

which represents the Ohmic transit time corresponding to an initially applied voltage $\alpha V_0 = V_P(0)$.

Substituting (3.24) in (3.15), (3.17), and (3.18) gives, for $0\leq t\leq t_1$,

$$\begin{aligned} \dot V_P(t) &= -(1-\alpha)(\alpha V_0/2t_A)(1-\alpha t/2t_A)^{-2}\ , \\ J(t) &= (\alpha C_P)(\alpha V_0/2t_A)(1-\alpha t/2t_A)^{-2}\ , \\ V_P(t) &= (\alpha V_0)\,\frac{1-t/2t_A}{1-\alpha t/2t_A}\ , \end{aligned} \tag{3.26}$$

where we have set $t_\gamma=0$ and $t_\beta=t$ in Eq. (3.18). The flow line of the leading front is obtained by introducing the value of $E_P(l,t)$ from Eq. (3.24) into Eq. (3.20). This gives

$$x(t)-d = \int_0^t \mu E_P(l,t')\,dt' = -(2L/\alpha)\ln(1-\alpha t/2t_A),\quad 0\leq t\leq t_1\ . \tag{3.27}$$

The transit time t_1 is obtained from (3.27) by setting $x(t_1)=l$ and using (3.25):

$$t_1 = (2t_A/\alpha)(1-e^{-\alpha/2})\ . \tag{3.28}$$

Zone II. This zone treats the transit of those carriers which originate from $x=d$ after the leading front and up to the particular front which originates when $E_P(d^+,t)$ has just become zero. We assume that the time it takes $E_P(d^+,t)$ to drop to zero is vanishingly small. Hence all carriers treated in zone II originate at $t=0^+$. The flow lines in this region are characterized by the field $E_P(d^+,0^+)$ which the corresponding front feels just as it emerges from the interface. The last carriers in zone II see the field $E_P(d^+,0^+)=0$, and their arrival at the collecting surface $x=l$ at $t=t_2$ marks the end of this zone. To integrate Eq. (3.19) in this region again requires an expression for $n(l,t)$. Since the carrier density at $x=d^+$ is assumed to be very large, Eq. (3.23) may be rewritten as

$$n(x(t),t) = [(4\pi q\mu/\kappa_P)(t-t_d)]^{-1}\ , \tag{3.29}$$

where t_d now represents the departure time of the flow line under consideration. As the various fronts arrive at $x=l$, the carrier density there may be expressed from Eq. (3.29) as

$$n(l,t) = [(4\pi q\mu/\kappa_P)t]^{-1},\ t_1\leq t\leq t_2 \tag{3.30}$$

since $t_d=0^+$ for all fronts in zone II.

Equation (3.19) now becomes

$$\frac{dE_P(l,t)}{dt}+\frac{E_P(l,t)}{t}=\frac{\alpha\mu}{2L}E_P^2(l,t) \quad . \tag{3.31}$$

This equation can be linearized through the substitution $y=1/E_P(l,t)$ and the resulting differential equation can be readily integrated. The result is

$$E_P(l,t)=\{t[a-(\alpha\mu/2L)\ln t]\}^{-1}, \quad t_1\le t\le t_2 \tag{3.32}$$

where a is a constant of integration. This constant can be determined by equating $E_P(l,t_1)$ as determined from (3.32) with $E_P(l,t_1)$ as determined from (3.24). Calculating a from this requirement and substituting back into (3.32), we get

$$E_P(l,t)=\frac{\alpha V_0}{L}\frac{e^{\alpha/2}t_1}{t}\left(1-(e^{\alpha/2}-1)\ln\frac{t}{t_1}\right)^{-1}, \quad t_1\le t\le t_2 \quad . \tag{3.33}$$

Now substituting (3.33) into (3.15) and (3.17), we obtain, for $t_1\le t\le t_2$,

$$\dot{V}_P(t)=-(1-\alpha)\frac{\alpha V_0}{2t_A}e^{\alpha}\left(\frac{t_1}{t}\right)^2\left(1-(e^{\alpha/2}-1)\ln\frac{t}{t_1}\right)^{-2},$$

$$J(t)=\alpha C_P\frac{\alpha V_0}{2t_A}e^{\alpha}\left(\frac{t_1}{t}\right)^2\left(1-(e^{\alpha/2}-1)\ln\frac{t}{t_1}\right)^{-2} . \tag{3.34}$$

To obtain an explicit expression for the voltage across the photoconductor at any time t in the interval $t_1\le t\le t_2$, Eq. (3.18) is rewritten in the form

$$V_P(t)=V_P(t_1)-M(t,t_1)/C_I \quad , \tag{3.35}$$

with

$$M(t,t_1)\equiv\int_{t_1}^{t}J(t')\,dt' \quad , \tag{3.36}$$

where $J(t')$ is given by (3.34) and $V_P(t_1)$ can be obtained from (3.26). The integral $M(t,t_1)$ has been evaluated in the Appendix. It is clear from the way $M(t,t_1)$ is evaluated that it is not permissible to set $\alpha=0$. To obtain $V_P(t)$ for $\alpha=0$ it is straightforward to integrate $\dot{V}_P(t)$ in (3.34) after setting $\alpha=0$, and this is done in Sec. IV A. The remainder of this section is restricted to $\alpha>0$. Substituting the value of $M(t,t_1)$ from the Appendix in (3.35), we get

$$\frac{V_P(t)}{\alpha V_0}=\frac{1}{\alpha}-\frac{(1+A)(1-\alpha)}{\alpha}\left\{\left[\frac{t_1}{t}\left(A-\ln\frac{t}{t_1}\right)^{-1}\right]+e^{-A}\left[E_1\left(\ln\frac{t}{t_1}-A\right)-E_1(-A)\right]\right\}, \tag{3.37}$$

where

$$A\equiv(e^{\alpha/2}-1)^{-1} \quad ,$$

$$E_1(u)\equiv\int_u^{\infty}(e^{-z}/z)\,dz \quad , \quad u\neq 0 \quad . \tag{3.38}$$

The integral $E_1(u)$ is called the exponential integral.[14]

To compute t_2, we must calculate the flow lines in zone II. Integrating (3.21), we obtain

$$E_P(x(t),t)=E_P(d^+,0^+)+(4\pi/\kappa_P)\int_0^t J(t')\,dt' \quad . \tag{3.39}$$

Now integrating (3.20) and using (3.39), we get

$$x(t)=d+\mu E_P(d^+,0^+)t+(4\pi\mu/\kappa_P)S(t) \quad , \tag{3.40}$$

where

$$S(t)\equiv\int_0^t dt'\int_0^{t'}J(t'')\,dt'' \quad . \tag{3.41}$$

In calculating $S(t)$ caution must be exercised to use the appropriate expressions for the current in different time regions; namely, for $0\le t''\le t_1$ one must use (3.26) and for $t_1\le t''\le t_2$ one must use (3.34). To avoid any confusion, we will use subscripts I and II to distinguish between the zone-I and zone-II expressions for the current.

We can now write

$$S(t)=\int_0^t dt'\int_0^{t'}J_{\rm I}(t'')\,dt'' \quad , \quad 0\le t\le t_1 \tag{3.42}$$

$$S(t)=\int_0^{t_1}dt'\int_0^{t'}J_{\rm I}(t'')\,dt''+\int_{t_1}^{t}dt'\int_0^{t_1}J_{\rm I}(t'')\,dt''+\int_{t_1}^{t}dt'\int_{t_1}^{t'}J_{\rm II}(t'')\,dt'' \quad , \quad t_1\le t\le t_2 \quad . \tag{3.43}$$

Most of the integrals involved in the above expressions are quite straightforward and require only tedious algebra. The last term in Eq. (3.43) is more involved. It can be written in terms of $M(t',t_1)$ and simply becomes $\int_{t_1}^{t}M(t',t_1)\,dt'$. An explicit expression for M has been obtained in the Appendix and can be used here. With $S(t)$ thus determined, substitution in (3.40) gives

$$x(t)=d+\mu\left(E_P(d^+,0^+)-\frac{\alpha V_0}{L}\right)t-\frac{2L}{\alpha}\ln\left(1-\frac{\alpha t}{2t_A}\right), \quad 0\le t\le t_1 \tag{3.44}$$

$$x(t)=l+\mu\left(E_P(d^+,0^+)-\frac{\alpha V_0}{L}\right)t+\frac{2L}{\alpha}\left[\ln\left(\frac{A}{A-\ln(t/t_1)}\right)+\frac{e^{-A}}{t_1}\int_{t_1}^{t}dt'\,\{E_1[\ln(t'/t_1)-A]-E_1(-A)\}\right], \quad t_1\le t\le t_2 \tag{3.45}$$

where A and $E_1(u)$ have been defined previously.

It is clear that for a given value of α, various flow lines are being characterized by $E_P(d^+,0^+)$. For $E_P(d^+,0^+)=\alpha V_0/L$, both (3.44) and (3.45) give $x(t_1)=l$. This represents the leading front. Successive flow lines are characterized by decreasing values of $E_P(d^+,0^+)$ [e.g., $E_P(d^+,0^+)=0.9(\alpha V_0/L)$, $0.8(\alpha V_0/L)$, ...]; and the last flow line in zone II, which arrives at $x=l$ at $t=t_2$, has $E_P(d^+,0^+)=0$. To obtain t_2 we set $t=t_2$, $x(t)=l$, and $E_P(d^+,0^+)=0$ in (3.45). The result is

$$\ln\left(\frac{A}{A-\ln(t_2/t_1)}\right)+\frac{e^{-A}}{t_1}\int_{t_1}^{t_2}dt'\,E_1\left[\ln\left(\frac{t'}{t_1}\right)-A\right]$$

$$-\frac{(t_2-t_1)e^{-A}E_1(-A)}{t_1}-\tfrac{1}{2}\alpha\frac{t_2}{t_A}=0\ . \qquad (3.46)$$

This is a transcendental equation containing an integral of the exponential integral and is to be solved for t_2. We have solved this equation numerically and the results will be discussed later.

Zone III. The flow lines in this zone originate from $x=d^+$ at various times t_d following the onset of injection ($t=0^+$). It has been pointed out by Many and Rakavy[1] that

$$n(l,t)=\{(4\pi q\mu/\kappa_P)[t-t_d(t)]\}^{-1}\ ,$$

but the explicit t dependence of $t_d(t)$ is unknown. Analytic solutions in this zone have not been obtained, and it was deemed unnecessary to proceed with any approximation methods since the information to be gained would be of limited practical value.

In all cases except $\alpha=1$ (no dielectric present), the photoconductor continues to discharge until the entire applied voltage appears across the dielectric and the total current drops to zero. For $\alpha=0$ (open circuit) the total current is zero at all times. For $\alpha=1$ there is a finite steady-state current.[15] This particular region has been called zone IV in Ref. 1.

IV. SPECIAL CASES

It is valuable to discuss separately two important limiting cases: $\alpha=0$ and $\alpha=1$. The case $\alpha=0$ implies a dielectric of zero capacitance in contact with a photoconductor. This corresponds to the configuration where one surface of the photoconductor is left floating and the other is grounded. The floating surface is initially charged to a voltage $V_P(0)$ and its subsequent decay upon exposure to strongly absorbed intense light is investigated. The photoinduced discharge characteristics in this geometry have been discussed both experimentally and theoretically in Ref. 12, and for $\alpha=0$ our treatment reproduces the expressions obtained there. The other extreme ($\alpha=1$) implies a dielectric of infinite capacitance in contact with a photoconductor. This is accomplished by shrinking the dielectric thickness to zero, and is the case discussed by Many and Rakavy.[1] Our expressions essentially reduce to theirs, with the exception of our Eqs. (3.44)–(3.46) for the flow lines in zone II. We believe that the present expressions are rigorous and we will elaborate upon the differences from the earlier work[1] below.

A. Corona-Discharge Case ($\alpha=0$)

To derive the expressions valid for the corona-discharge case we let $\alpha\to0$ and $V_0\to\infty$ such that the product $\alpha V_0=V_P(0)$ is finite. This is the initial voltage to which the floating surface is charged at $t=0$. We readily see from (3.26) and (3.34) that

$$\dot V_P(t)=-V_P(0)/2t_T\ ,\quad 0\le t\le t_T \qquad (4.1a)$$

$$\dot V_P(t)=-(L^2/2\mu)\,1/t^2,\quad t\ge t_T \qquad (4.1b)$$

$$J(t)=0\ ,\quad t\gtrless 0\ . \qquad (4.2)$$

In zone I, $V_P(t)$ is obtained from (3.26); however, to obtain $V_P(t)$ in zone II we must integrate (4.1), since (3.37) is not valid for $\alpha=0$. The results are

$$V_P(t)=V_P(0)(1-t/2t_T)\ ,\quad 0\le t\le t_T \qquad (4.3a)$$

$$V_P(t)=(L^2/2\mu)\,1/t,\quad t\ge t_T\ . \qquad (4.3b)$$

In Eqs. (4.1)–(4.3) we have defined

$$t_T\equiv L^2/\mu V_P(0)\ . \qquad (4.4)$$

From (3.25) and (3.28) we see that for the present case $t_A=t_1=t_T$.

Furthermore, since $J(t)=0$, we notice from (3.20) and (3.21) that the flow lines are given by

$$x(t)=d+\mu E_P(d^+,0^+)\,t\ .$$

The last flow line in zone II is characterized by $E_P(d^+,0^+)=0$ and takes infinite time to reach $x=l$. Therefore, $t_2\to\infty$ and in the corona case there is no zone III. These results have been obtained earlier and good agreement with experiment has already been reported.[12]

B. No Dielectric ($\alpha=1$)

This corresponds to the situation where the entire applied voltage is directly across the photoconductor. Clearly, then,

$$\dot V_P(t)=0\ ,\quad t\ge0$$
$$V_P(t)=V_0\ ,\quad t\ge0\ . \qquad (4.5)$$

The above equations result when one sets $\alpha=1$ at appropriate places in Eqs. (3.26), (3.34), and (3.37). The quantity of interest in the present case is the total current density $J(t)$, and for $t\le t_1$ it is obtained from (3.26) by setting $\alpha=1$. The result is

$$J(t)=(\kappa_P\mu V_0^2/8\pi L^3)(1-t/2t_0)^{-2}\ ,\quad 0\le t\le t_1 \qquad (4.6)$$

where according to (3.28)

$$t_1=2t_0(1-e^{-1/2})\ . \qquad (4.7)$$

Here

$$t_0\equiv t_A=L^2/\mu V_0\ . \qquad (4.8)$$

These results are in agreement with results already reported in the literature.[1,2]

In zone II we have from (3.34)

$$J(t)=\frac{\kappa_P\mu V_0^2}{8\pi L^3}\left(\frac{t_1}{t}\right)^2\left[1-(e^{1/2}-1)\ln\left(\frac{t}{t_1}\right)\right]^{-2}\ ,\qquad t_1\le t\le t_2 \quad . \tag{4.9}$$

The equation from which t_2 may be determined is the particular case of (3.46) with $\alpha=1$. In this case we have $A=1.541$ and $E_1(-1.541)=-3.42$. The transcendental equation for t_2 then becomes

$$\ln\left(\frac{1.541}{1.541-\ln(t_2/t_1)}\right)+\frac{0.214}{t_1}\int_{t_1}^{t_2}dt'\times E_1\left[\ln\left(\frac{t'}{t_1}\right)-1.541\right]+\frac{0.732(t_2-t_1)}{t_1}-\frac{1}{2}\frac{t_2}{t_0}=0. \tag{4.10}$$

This equation is somewhat different from the equation one obtains from Many and Rakavy's work[1] and we have not been able to show the mathematical equivalence of the two. The solution to Eq. (4.10) was obtained numerically. The result is

$$t_2=1.514\,t_0=1.924\,t_1 \quad . \tag{4.11}$$

The numerical values quoted by Many and Rakavy[1] are practically identical to these.

The flow lines in zone II can be obtained from (3.44) and (3.45) by making appropriate substitutions. The results are

$$x(t)=d+\mu\left(E_P(d^+,0^+)-\frac{V_0}{L}\right)t-(2L)\ln\left(1-\frac{t}{2t_0}\right)\ ,\qquad 0\le t\le t_1 \tag{4.12}$$

$$x(t)=l+\mu\left(E_P(d^+,0^+)-\frac{V_0}{L}\right)t+(2L)\left\{\ln\left(\frac{1}{e^{1/2}-1}\right)-\ln\left[\frac{1}{e^{1/2}-1}-\ln\left(\frac{t}{t_1}\right)\right]\right\}+\frac{2L}{t_1}\exp\left(-\frac{1}{e^{1/2}-1}\right)\times\int_{t_1}^{t}dt'\left\{E_1\left[\ln\left(\frac{t'}{t_1}\right)-\frac{1}{e^{1/2}-1}\right]-E_1\left(-\frac{1}{e^{1/2}-1}\right)\right\},\qquad t_1\le t\le t_2 \quad . \tag{4.13}$$

Our flow-line expressions differ from the earlier work in two respects. First, for reasons not obvious to us, Many and Rakavy do not give any expression for flow lines in zone II for $0\le t\le t_1$, even after stating that flow lines in zone II all originate at $t=0$. In other words, the expression corresponding to our Eq. (4.12) is missing in their paper. Second, the mathematical form of (4.13) is quite different from Eq. (39) of the earlier work and we have not been able to show that the two expressions are identical. We believe that our results are rigorous, even though numerical evaluation of the expressions does not show any significant differences.

For the case $\alpha=1$, since a constant voltage is maintained across the photoconductor at all times, steady state here consists of a constant current flowing through the external circuit. In the steady state, Eq. (3.4) becomes

$$J_s=q\mu n_s(x)E_s(x)\ , \tag{4.14}$$

where the subscript s refers to the steady-state values. With the help of (3.3) and the boundary condition (3.12) we get

$$E_s(x)=(8\pi J_s/\kappa_P\mu)^{1/2}(x-d)^{1/2} \quad . \tag{4.15}$$

Integrating both sides with respect to x from d to l and making use of the condition that $\int_d^l E_s(x)\,dx=V_0$, we obtain

$$J_s=9\kappa_P\mu V_0^2/32\pi L^3 \quad . \tag{4.16}$$

This is the well-known expression for the steady-state current.[15]

To calculate the flow lines in the steady state, we recall that each front arriving at $x=l$ after $t=t_2$ was subject to $E_P(d^+,t_d)=0$, where t_d is the front's departure time. Furthermore, J is independent of t and therefore (3.40) and (3.41) lead to

$$x(t)=d+(4\pi\mu/\kappa_P)J_s\int_{t_d}^{t}dt'\int_{t_d}^{t'}dt'' \quad .$$

Here the flow lines are characterized by their respective departure times t_d. Substituting the value of J_s from (4.16), we get

$$x(t)=d+\tfrac{9}{16}[(\mu V_0)^2/L^3](t-t_d)^2 \quad . \tag{4.17}$$

If t_a denotes the arrival time, then the transit time $t_s=t_a-t_d$ in steady state is obtained by setting $x(t_a)=l$. Since $l-d=L$, Eq. (4.17) gives

$$t_s=\tfrac{4}{3}t_0 \quad . \tag{4.18}$$

Thus in steady state all flow lines take $1.334t_0$ to arrive at the collecting surface. It is interesting to point out that the carrier transit times exhibit a behavior somewhat analogous to that of the transient current itself. The transit time of the leading front is $t_1=0.787t_0$. For subsequent fronts, the transit time increases until for the last front in zone II it reaches $t_2=1.514t_0$. It decreases thereafter, and the steady-state transit time takes the value $1.334t_0$.

V. NUMERICAL RESULTS AND DISCUSSION

The treatment presented in this paper is valid for all values of α from 0 to 1, where α has been defined as the ratio of the capacitance of the dielectric to the sum of the capacitances of the dielectric and the photoconductor. The case $\alpha=0$ corresponds to the open-circuit condition and $\alpha=1$ signifies the absence of the dielectric altogether. It is instructive at this stage to investigate how the physical properties like the transient current, voltage, etc., vary with time for various values of α.

In Fig. 3 we have plotted t_1/t_A and t_2/t_A as a function of α. We are normalizing t to t_A because this is the true Ohmic transit time corresponding to the voltage that appears across the photoconductor at $t=0$. For obtaining t_1/t_A values we made use of Eq. (3.28). One notices from Fig. 3 that these values start out from unity for $\alpha=0$ and drop to 0.787 for $\alpha=1$. In other words, the leading front of carriers has the shortest transit time in the absence of the dielectric. This is easy to understand physically because, as the leading front moves away from the photogeneration region, the battery must supply more charge to keep the voltage constant. This increases the field ahead of the leading front and lowers the transit time. In the presence of the dielectric the battery still supplies some additional charge, but not quite as much, since a somewhat lower field existing over a larger region of space can still satisfy the requirement $\int_0^l E(x,t)dx = V_0$. Thus, in the presence of the dielectric the field ahead of the leading front in the photoconductor region stays lower than in the absence of the dielectric.

The task of obtaining t_2/t_A for different values of α is much more involved because in this case one has to solve the rather complicated transcendental equation (3.46). Extra complications arise because the integral of the exponential integral is involved, and even to solve (3.46) numerically one therefore needs to know the value of the exponential integral at very closely spaced intervals. The problem then goes beyond the scope of standard tables of the exponential integral. The computations were carried out numerically, and the results are shown in the dashed curve of Fig. 3. For $\alpha=1$, the value of t_2/t_A is 1.514 and it tends towards infinity as $\alpha \to 0$. Physically speaking, the last front under open-circuit conditions ($\alpha=0$) never arrives at the collecting electrode, whereas in the absence of the dielectric ($\alpha=1$), the end of zone II occurs earliest, for the same reason which made t_1/t_A shortest in this case.

Figure 4 is a plot of the normalized transient current density as a function of t/t_A for various values of α, and Fig. 5 displays the behavior of the normalized negative time derivative of the voltage across the photoconductor. We define normalized current and voltage decay rate through the relations

$$j(t) = \frac{J(t)}{\kappa_P \mu (\alpha V_0)^2 / 4\pi L^3} , \tag{5.1}$$

$$\dot{v}_P(t) = \frac{\dot{V}_P(t)}{\mu (\alpha V_0)^2 / 2L^2} . \tag{5.2}$$

In terms of these normalizing constants we can immediately write from (3.26) and (3.34)

$$j(t) = \tfrac{1}{2}\alpha\Theta, \quad -\dot{v}_P(t) = (1-\alpha)\Theta,$$

where

$$\Theta \equiv H(t_1 - t) f_1(t,\alpha) + H(t - t_1) f_2(t,\alpha) , \quad 0 \le t \le t_2 \tag{5.3}$$

and where

$$f_1(t,\alpha) = (1 - \alpha t / 2t_A)^{-2} , \tag{5.4}$$

$$f_2(t,\alpha) = e^{\alpha}(t_1/t)^2 [1 - (e^{\alpha/2} - 1)\ln(t/t_1)]^{-2} , \tag{5.5}$$

and $H(\tau)$ is the Heaviside step function

$$H(\tau) = 0, \quad \tau \le 0$$
$$= 1, \quad \tau > 0 . \tag{5.6}$$

It is clear from Eq. (5.3) that for $\alpha=0$, $j(t)=0$, and for $\alpha=1$, $\dot{v}_P(t)=0$, as expected from physical considerations.

Equation (5.3) has been used to plot $j(t)$ in Fig. 4. The steady-state value for the current for the

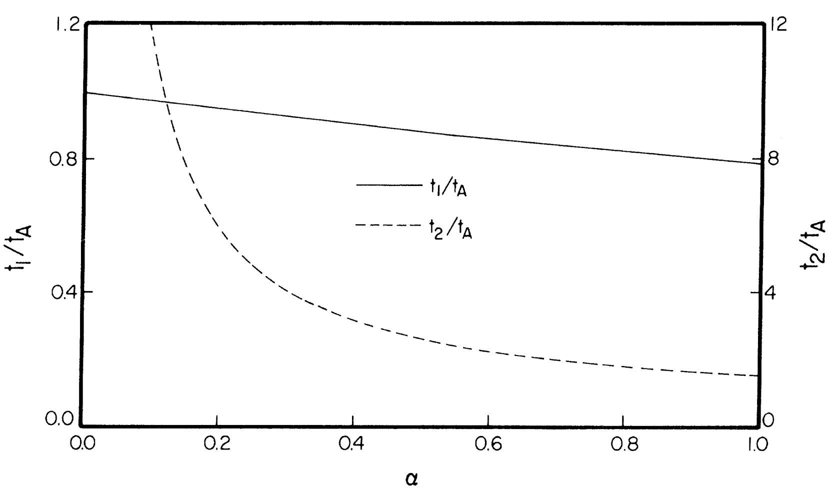

FIG. 3. α dependence of t_1/t_A (solid curve) and t_2/t_A (dashed curve). See text for the explanation of symbols.

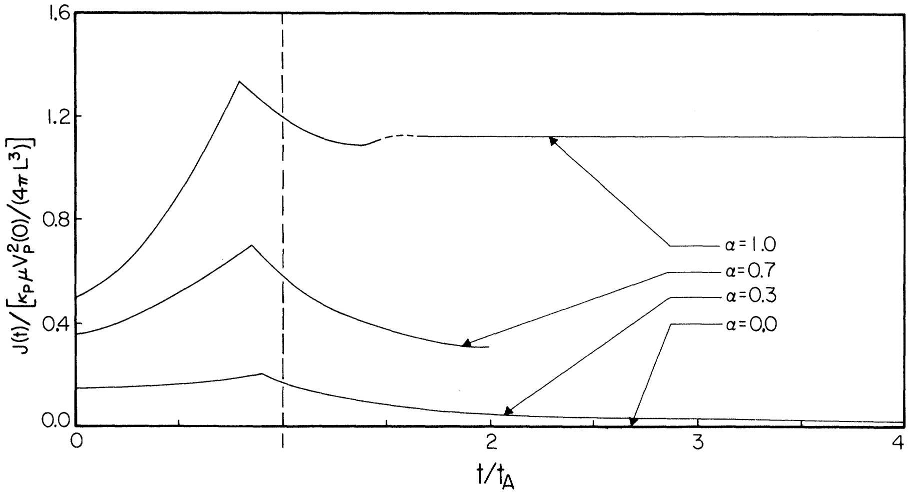

FIG. 4. Time dependence of the normalized current density for various values of α. The vertical dashed line indicates the position of the Ohmic transit time. See text for the explanation of symbols.

case $\alpha = 1$ is obtained from Eq. (4.16) and is found to be equal to $\frac{9}{8}$ in normalized units. Several general features of the curves should be noticed. The initial values at $t = 0$ are proportional to α and reach the value 0.5 for $\alpha = 1$. In the presence of a dielectric ($\alpha < 1$), the electric field in the photoconductor changes more slowly than for no dielectric ($\alpha = 1$); hence, the greater the relative thickness of the dielectric (i.e., the smaller the value of α), the smaller the current, even after normalizing out the effect of smaller initial photoconductor voltage. In the extreme case of an infinitely thick dielectric ($\alpha = 0$), the current vanishes. For all values of α, $J(0) \propto V_P^2(0)/L^3$ and the experimental verification of this relationship would ensure that one is in the space-charge-limited regime. For a given α, the current increases with increasing t until it reaches a maximum at $t = t_1$. The reasons for the existence of the maximum have been explained earlier.[1,2] It can be demonstrated analytically that for all $\alpha > 0$, the current maximum at $t = t_1$ takes the form of a cusp. This is done by computing the derivative of $J(t)$ at $t = t_1$ from the left and from the right and showing that the two derivatives are unequal. In units of $J(t_1)/t_A$, we can write

$$D^- - D^+ = \frac{\alpha}{1 - e^{-\alpha/2}} = \frac{2}{t_1/t_A} \quad , \tag{5.7}$$

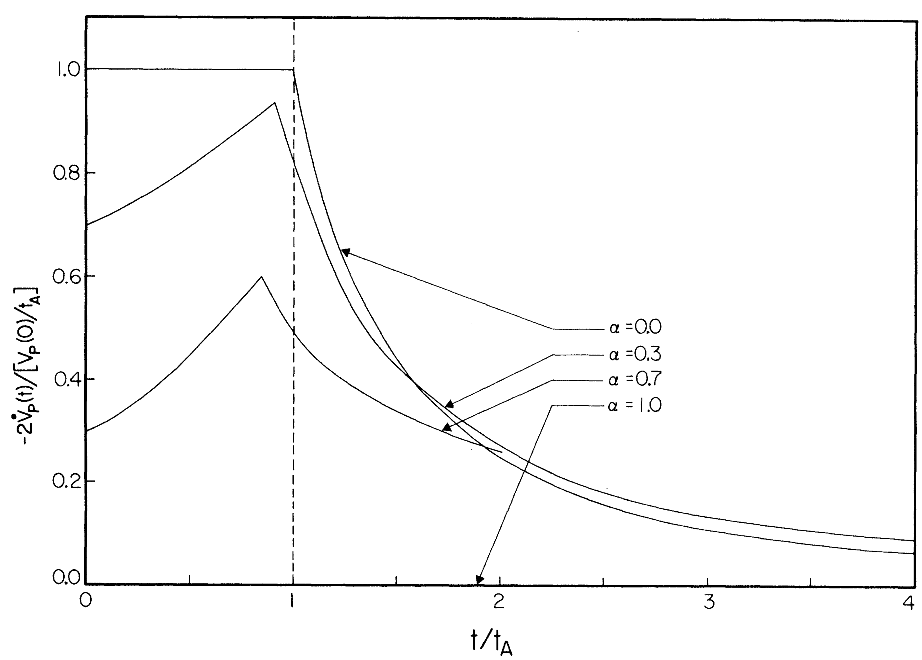

FIG. 5. Time dependence of the normalized negative time derivative of the photoconductor voltage for various values of α. The vertical dashed line indicates the position of the Ohmic transit time. See text for the explanation of symbols.

where D^- is the time derivative of $J(t)$ computed at $t=t_1^-$ using Eq. (3.26) and D^+ is computed at $t=t_1^+$ using (3.34). One finds that $D^- \neq D^+$ and hence there is a cusp at $t=t_1$. In fact, for $\alpha=1$, $D^- - D^+ = 2.54$, and for $\alpha \to 0$, $D^- - D^+ \to 2.0$. The cusp is therefore sharpest for $\alpha=1$ and our value for $D^- - D^+$ agrees with the value quoted in the literature.[2] Therefore, if one is interested in using the position of the current cusp at $t=t_1$ as a bench mark for determining drift mobilities, it would be sensible to work with a relatively high value of α, especially if carrier trapping and/or finite injection rise time limit the experimental resolution. We also see from Fig. 4 that the position of the cusp moves closer and closer to the Ohmic transit time $(t=t_A)$ as α decreases, for reasons explained earlier. The ratio $J(t_1)/J(0)$ varies from 1.0 to 2.7 as α goes from 0.0 to 1.0.

Note that the value of $J(0)$ may be used as an independent method of determining the drift mobility. Relatively small values of α afford the best opportunity for determining $J(0)$, since the initial portions of the $J(t)$ curves are most flat for these cases. This method for measuring mobility requires knowledge of κ_P, in addition to $V_P(0)$ and L, both of which were also needed in the transit-time mobility determination. On the other hand, if the mobility is known from other considerations, the value of $J(0)$ may be used to determine κ_P. Finally, an additional check on the value of the mobility may be obtained by fitting experimental data for $J(t)$ in zone II to the theoretical curves, since the current here also depends upon mobility.

In Fig. 5 we have plotted $-\dot{v}_P(t)$ from Eq. (5.3) as a function of t/t_A for various values of α. These curves differ from those for $J(t)$ by the factor $2(1-\alpha)/\alpha$, which leads to somewhat different properties. For $\alpha=1$, $\dot{v}_P(t)=0$ for all times. As α decreases, $|\dot{v}_P(t)|$ in zone I increases and reaches a maximum value for $\alpha=0$.

Under open-circuit conditions $(\alpha=0)$ there is no carrier injection after $t=0^+$. Each injected front moves with a uniform velocity. Since there are no carriers leaving the system, $\dot{v}_P(t)$ is constant for $t \leq t_1$. For the reasons discussed earlier, a cusp occurs in $\dot{v}_P(t)$ at $t=t_1$, and the position of this cusp moves closer to the Ohmic transit time t_A as α tends towards zero. The carrier mobility could be determined from the value of t_1 if L and $V_P(0)$ are known. As was discussed for $J(t)$, the cusp at $t=t_1$ is more sharply defined for larger values of α. This would be an important consideration if trapping and/or finite-rise-time effects limit experimental resolution.

For all cases except $\alpha=0$, $\dot{v}_P(t)$ has a finite slope at $t=0$. As $\alpha \to 0$ the slope of $\dot{v}_P(t)$ in zone I becomes vanishingly small, thereby facilitating the accurate determination of $\dot{v}_P(0)$. Thus it is advantageous to choose a small value of α if the drift-mobility evaluation is to be carried out using the initial value of $\dot{v}_P(t)$. The time dependence of $\dot{v}_P(t)$ for $t>t_1$ can be used to check the value of mobility obtained by either of the two aforementioned methods. For $\alpha=0$, the $1/t^2$ dependence of $\dot{v}_P(t)$ in this region provides an especially nice check, which agrees well with experimental data on amorphous selenium.[12]

Figure 6 shows the time variation of the normalized voltage $v_P(t) \equiv V_P(t)/\alpha V_0$ for various values of α. In calculating these curves use has been made of Eqs. (3.26) and (3.37). For $\alpha=1$, there is no decay at all, and the voltage decays more rapidly for smaller values of α. In the open-circuit case $(\alpha=0)$, the voltage has already dropped to 50% of the initial applied voltage by $t=t_1$. Because $\dot{V}_P(t)$ is in this case a constant, $V_P(t)$ shows a linear decrease.

Finally, it should be remarked that our general treatment is applicable to the experimentally important configuration in which the photoconductor is sandwiched between two dielectrics. If for C_I one now takes the combined effective geometrical capacitance of the two dielectrics in series, the mathematical analysis remains unchanged. The field distribution $E_P(x,t)$ *inside the photoconductor* is the same as in the corresponding single-dielectric geometry. In the steady state, the electric field inside the photoconductor is zero, and the entire applied voltage appears across the two dielectrics.

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APPENDIX: EVALUATION OF $M(t,t_1)$

We wish to evaluate the definite integral

$$M(t,t_1)=\int_{t_1}^{t} J(t')\,dt' \quad , \tag{A1}$$

where $J(t')$ is given by (3.34). Substituting that expression in the above equation, we get

$$M(t,t_1)=\lambda t_1 \int_{t_1}^{t} \frac{dt'}{(t')^2[A-\ln(t'/t_1)]^2}\,, \tag{A2}$$

with

$$\lambda \equiv C_P \alpha V_0 (1+A)\ , \quad A \equiv (e^{\alpha/2}-1)^{-1}, \tag{A3}$$

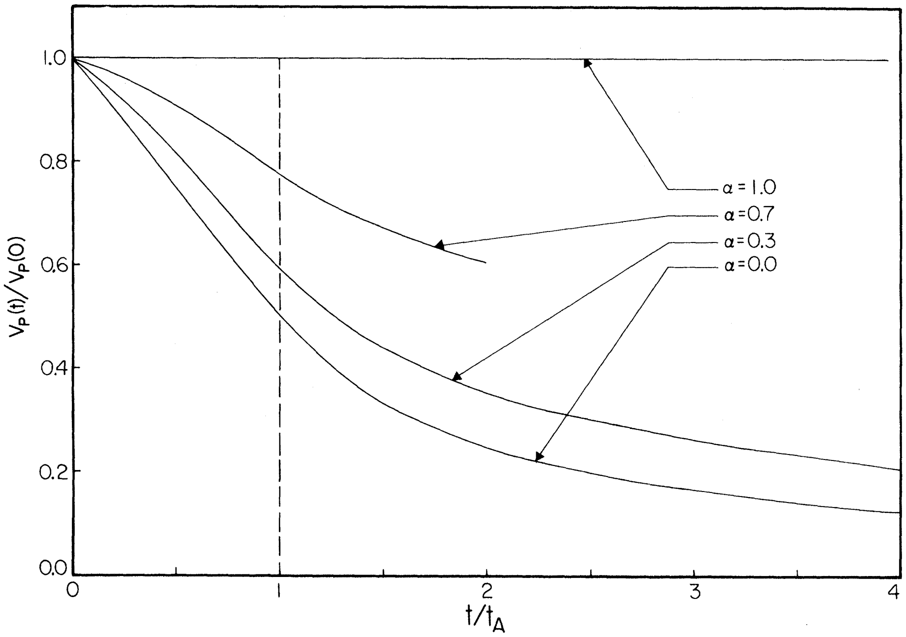

FIG. 6. Time dependence of the normalized voltage across the photoconductor for various values of α. The vertical dashed line indicates the position of the Ohmic transit time. See text for the explanation of symbols.

where, in arriving at this form for λ, we used the relation $t_1 = (2t_A/\alpha)(1 - e^{-\alpha/2})$. Also, since we divided the integrand by $(e^{\alpha/2} - 1)^2$, the results are not valid for $\alpha = 0$. Now substituting $y = A - \ln(t'/t_1)$, we get

$$M(t, t_1) = \lambda\, e^{-A} \int_{A-\ln(t/t_1)}^{A} e^y dy/y^2 \quad . \tag{A4}$$

A partial integration at this stage gives

$$M(t, t_1) = \lambda \{ (t_1/t)[A - \ln(t/t_1)]^{-1} - A^{-1} + e^{-A} \int_{A-\ln(t/t_1)}^{A} (e^y/y)\, dy \} \, . \tag{A5}$$

The last integral on the right-hand side of (A5) can be written in terms of the exponential integral[14] $E_1(u) \equiv \int_u^\infty (e^{-z}/z)\, dz$ through the substitution $y = -z$. Proceeding in this manner, we obtain the following expression for the desired integral:

$$M(t, t_1) = C_P \alpha V_0 (1 + A) \{ (t_1/t)\, [A - \ln(t/t_1)]^{-1} - A^{-1} + e^{-A} [E_1(\ln(t/t_1) - A) - E_1(-A)] \} \, , \tag{A6}$$

which is valid for all values of α except $\alpha = 0$.

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