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Ultrasonic-Attenuation Studies in Superconducting Lead*

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Measurement of the attenuation of 10-, 30-, 50-, and 70-MHz longitudinal sound waves, propagated in the [100], [110], and [111] directions, have been conducted on the normal and superconducting states of lead. Analysis of the data has been presented allowing for the possibility of multiple energy gaps. The data have also been compared to theoretical calculations for a strong coupling model.

INTRODUCTION

Recent studies[1-3] involving the attenuation of ultrasound in superconducting lead have based their data analysis on the existence of a single energy gap. In view of other recent superconductivity studies[4,5] it seems quite possible that the superconducting state of lead might involve more than one energy gap. Our intent has been to analyze the experimental data in such a manner as to allow for the observation of multiple energy gaps. Furthermore there is the question of whether superconducting lead is associated with weak[6] or strong coupled[7] interactions. We have made a comparison of our data with Woo's calculations for a strong coupling model.

Our basic experimental study of the ultrasonic attenuation was conducted on unstrained lead for 10-, 30-, 50-, and 70-MHz longitudinal sound waves propagated in the [110] and [111] directions. The data for waves propagated in the [100] direction were taken on a strained crystal previously studied, in the unstrained state, by Deaton.[1] This crystal was strained by the application of a small impulsive shock. Comparison of these results with Deaton's results should provide some additional qualitative information regarding the strain effects on the ultrasonic attentuation.

SAMPLE PREPARATION

The samples were prepared from a single crystal obtained from Research Crystals, Inc., with a specified purity of 99.9999%. The samples were spark cut, x-ray oriented, and spark polished to the desired crystallographic direction. The prepared samples were in the form of cubes of approximately 5 mm on a side. Before application of a binder, the sample faces were alternately subjected to applications of dilute nitric acid and dilute sodium hydroxide, interspaced with washings of distilled water.

The quartz transducers were cut for a fundamental frequency of 10 MHz and polished for overtone operation. The bonding material between sample and transducer was either Nonaq stopcock grease or Dow Corning 250 000 centistokes silicone fluid.

APPARATUS

The cryostat assembly was of the standard configuration described in earlier publications.[1]

The single-ended pulse-echo technique was used in these studies. Figure 1 shows a block diagram of the electronic equipment which is primarily an Arenberg impedance-matched system. The Arenberg Model W. A. 600-D wide-band amplifier was modified to give better stability. The display scanner allows one to monitor the voltage of any part of the trace diplayed on the oscilloscope. The display scanner was then set to monitor the voltage of the peak of the first echo. This voltage was fed to the Y axis of a Moseley Model 2D-2A X-Y recorder. The X axis was driven by the output of a germanium resistance thermometer. With this configuration the height of the first echo (in volts) was plotted as a function of temperature over a range of 2 to 12 °K. As a secondary check, the voltage of the first echo was measured at different temperatures by means of a Tektronix 547 Oscilloscope and Tektronix Type W Comparator unit.

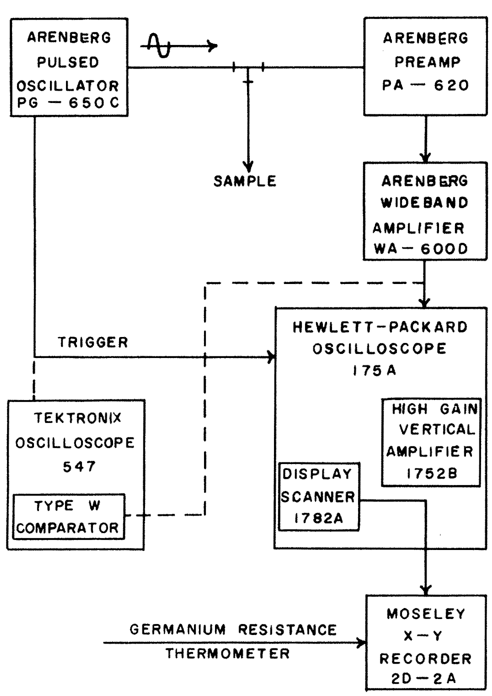

FIG. 1. Block diagram of experimental electronic equipment.

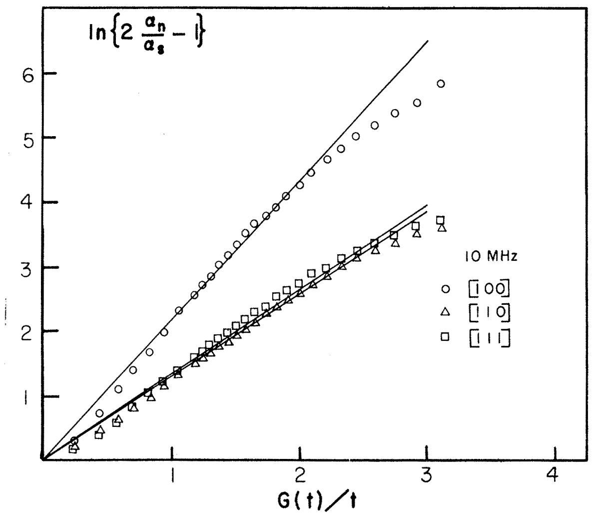

FIG. 2. Plot of $\ln(2\alpha_n/\alpha_s - 1)$ versus $G(t)/t$ for various directions in Pb at 10 MHz. The attenuations in the normal and superconducting states are α_n and α_s, respectively. The ratio of superconducting energy gaps is $G(t)$ and the ratio of temperature to the transition temperature is t.

Much care was taken to determine if the attenuation showed an amplitude dependence; none was present.

The superconducting data were obtained in the absence of a magnetic field. The normal-state data were obtained by quenching the superconductive properties in a magnetic field of 1000 G. In order to avoid flux trapping the samples were always warmed to temperatures above the transition temperature T_c, and then lowered to the lower temperature to begin an experiment.

DATA ANALYSIS

The plots of echo voltage versus temperature were converted to attenuation (dB/cm) versus temperature plots by using the transition temperature as a reference level. The data were then adjusted to account for a temperature-independent background attenuation.[2,8] The values for the superconducting attenuation α_s and normal-state attenuation α_n were obtained and the basic analysis began with the standard BCS equation[6]

$$\alpha_s/\alpha_n = 2(1 + e^{\Delta(T)/kT})^{-1} , \qquad (1)$$

where $2\Delta(T)$ represents the temperature-dependent superconducting energy gap.

In order to determine the superconducting energy gap, Eq. (1) was rearranged[5] to give

$$2[\Delta(0)/kT_c]G(t)/t = 2\ln(2\alpha_n/\alpha_s - 1) , \qquad (2)$$

where $G(t) = \Delta(T)/\Delta(0)$ and $t = T/T_c$.

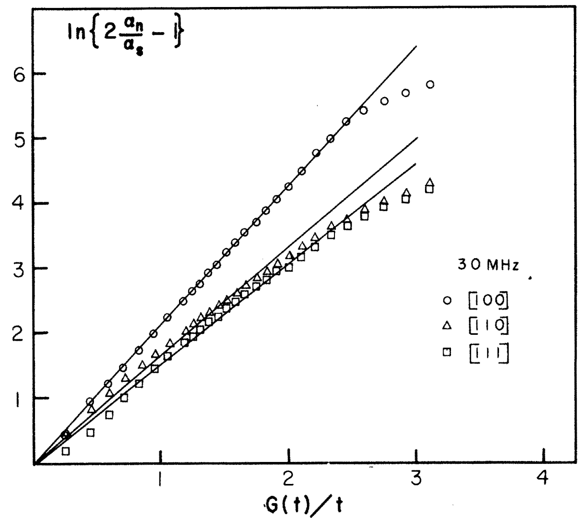

FIG. 3. Plot of $\ln(2\alpha_n/\alpha_s - 1)$ versus $G(t)/t$ for various directions in Pb at 30 MHz.

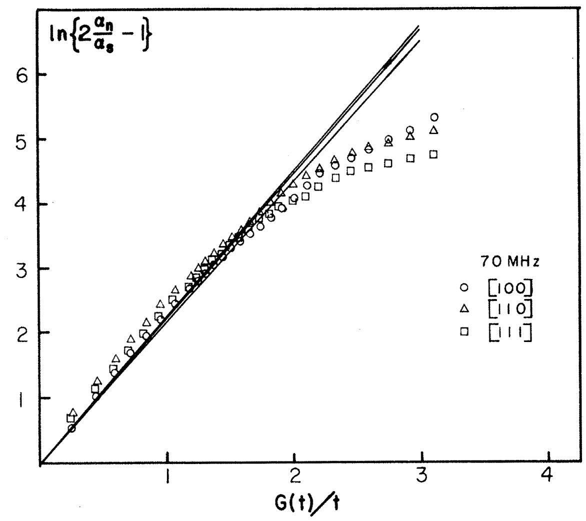

FIG. 5. Plot of $\ln(2\alpha_n/\alpha_s - 1)$ versus $G(t)/t$ for various directions in Pb at 70 MHz.

The values for $G(t)/t$ were calculated for an ideal superconductor in accordance with the work of Mühlschlegel.[9] A plot of $G(t)/t$ versus $\ln(2\alpha_n/\alpha_s - 1)$ will yield a slope which should equal $\Delta(0)/kT_c$. Any major deviation from a straight-line plot should indicate the presence of more than one energy gap. Figures 2–5 readily show that, for our data, there is a strong deviation from a single straight-line plot. A complete set of zero-temperature primary energy gaps are given in Table I. The secondary gaps cannot be precisely determined from our data. However, our determinations of the secondary gaps, $2\Delta(0)/kT_c$, from these plots give values of 2.14 for the strained [100] crystal and 1.50 for the unstrained [110] and [111] crystals. The strained crystal in both the initial and secondary energy gaps exhibits higher values than do the unstrained crystals.

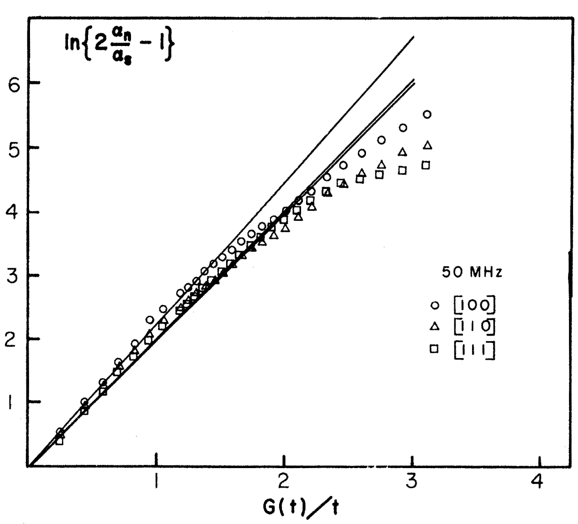

FIG. 4. Plot of $\ln(2\alpha_n/\alpha_s - 1)$ versus $G(t)/t$ for various directions in Pb at 50 MHz.

Figure 6 gives a plot of $2\Delta(0)/kT_c$ versus frequency. At these frequencies, the strained crystal maintained a fairly constant energy gap with an average value of 4.38. The unstrained crystals displayed a definite frequency dependence.

Figures 7–9 give plots of α_s/α_n versus T/T_c for our data along with plots of the BCS calculations utilizing a zero-temperature energy gap of $2\Delta(0) = 3.52kT_c$. The difference between the strained crystal results and the results given by Deaton[1] for the same crystal, in the unstrained state, is immediately apparent. Deaton's results revealed a marked variation with frequency, whereas our results gave no frequency dependence.

A comparison of our data to a strong coupling

TABLE I. Values of $2\Delta(0)/kT_c$ for various directions and frequencies in Pb.

Direction of wave propagation	Frequencies (MHz)	$2\Delta(0)/kT_c$
[100] (strained)	10	4.33
	30	4.25
	50	4.47
	70	4.45
[110] (unstrained)	10	2.58
	30	3.33
	50	4.03
	70	4.33
[111] (unstrained)	10	2.63
	30	3.05
	50	3.98
	70	4.47

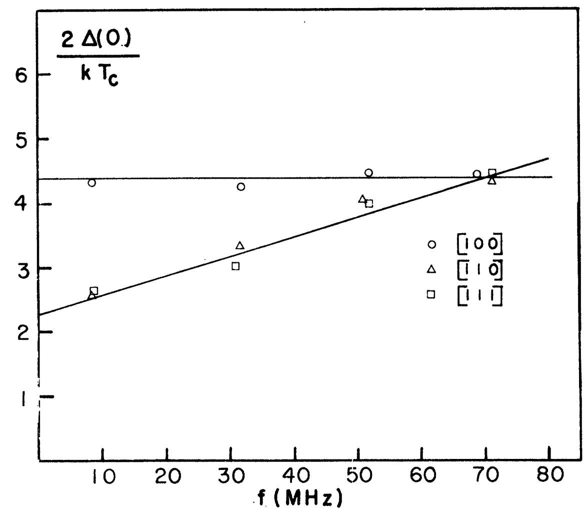

FIG. 6. Plot of $2\Delta(0)/kT_c$ versus the frequency f in MHz.

model was accomplished with the aid of calculations by Woo.[7,10] Of immediate interest is the behavior of the attenuation ratio in the near vicinity of the transition temperature. Figures 10–12 display our data along with the strong coupling model and the BCS-theory curve.

CONCLUSION

The first observation to be noted is the absence of anisotropy of the energy gap for the unstrained crystals. However, the difference between strained and unstrained states was quite apparent. Our results for the [100] direction appear to be in mild contradiction to a previous study by Fate *et al.*[3] Their results indicated that the primary energy gap for a strained lead crystal exhibited a small frequency dependence. In another earlier study by Deaton[1] on unstrained lead, the primary

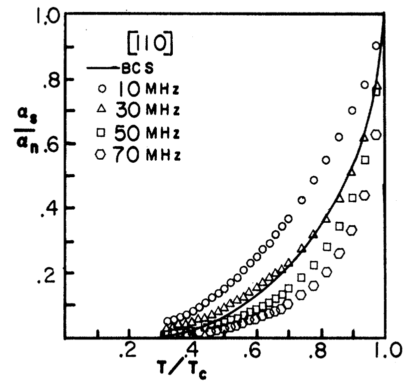

FIG. 8. Plot of the attenuation ratio α_s/α_n versus the temperature ratio T/T_c in the [110] direction in Pb for various frequencies.

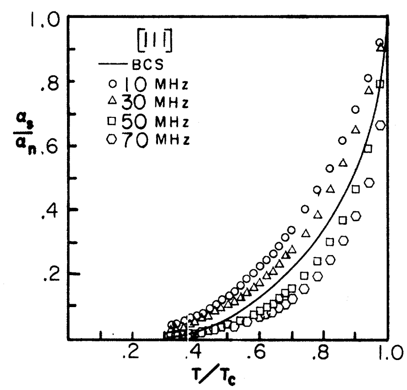

FIG. 9. Plot of the attenuation ratio α_s/α_n versus the temperature ratio T/T_c in the [111] direction in Pb for various frequencies.

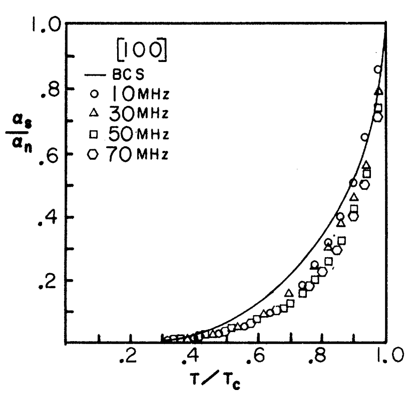

FIG. 7. Plot of the attenuation ratio α_s/α_n versus the temperature ratio T/T_c in the [100] direction in Pb for various frequencies.

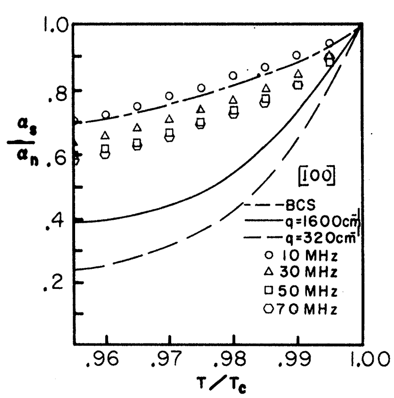

FIG. 10. Comparison of data to the calculations of Woo and the BCS theory about the transition temperature for the [100] direction in Pb.

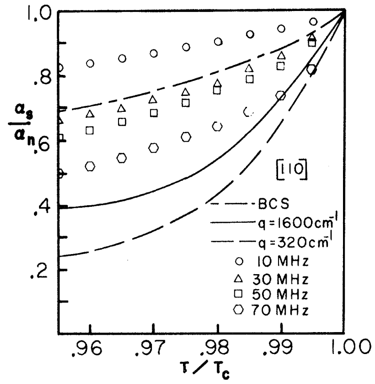

FIG. 11. Comparison of data to the calculations of Woo and the BCS theory about the transition temperature for the [110] direction in Pb.

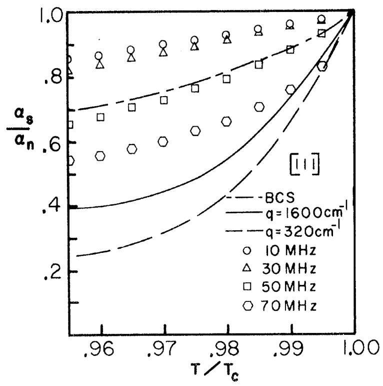

FIG. 12. Comparison of data to the calculations of Woo and the BCS theory about the transition temperature for the [111] direction in Pb.

energy gap was found to exhibit a frequency dependence in accordance with our results for unstrained crystals. Furthermore, this frequency dependence for the unstrained lead was in the reverse order to that found by Fate in the strained crystal. These results would indicate that pure unstrained lead crystals exhibit a definite frequency dependence of the primary energy gap. As the crystal is strained this frequency dependence becomes diminishingly small and eventually it reappears in the reverse order under more severe strains. It is common belief that this frequency dependence, or its lack, is due to the density of dislocations within the specimen. Since we presently have no measure of the dislocation density of any of these samples it would be superfluous to draw any definite conclusions.

The second observation is concerned with what appears to be a second energy gap with the magnitude being roughly half that of the initial gap. Based on our observations and those of Newcomb and Shaw[5] it would appear that the secondary energy gap could be more precisely studied at higher frequencies (above 0.50 GHz). Higher frequencies appear to reveal the secondary energy gap at temperatures much closer to the transition temperature. This would allow more accurate determinations of attenuation, based on this gap, with respect to temperature. This would allow the secondary energy gap to be seen at lower values of $G(t)/t$ thereby allowing precise values of the slope to be obtained.

The last observation is concerned with the comparison of our data to the strong coupling model. It appears that lead exhibits an intermediate type of behavior falling between the weak and strong coupling models.

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