Critical-current hysteresis of superconducting proximity-effect bridges under phonon injection

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A dc electric current is injected through the transverse normal-metal strip of Ag-Sn proximity-effect bridges to generate phonons which propagate through and further weaken the superconductivity in the proximity-effect-induced weak region by creating quasiparticles (by pair-breaking mechanisms). Consequently a decrease of the critical-current hysteresis is observed in the current-voltage characteristics of these bridges. These results cannot be adequately accounted for by simple heating. Further, we find that Parker's modified heating theory of nonequilibrium superconductors has to be invoked for a qualitative understanding of the observed phenomena.

I. INTRODUCTION

The study of the properties of superconductors driven into nonequilibrium states by external mechanisms such as quasiparticle injection, phonon injection, and photon injection has been a subject of current interest.[1-6] Most of the recent experimental results on the study of nonequilibrium superconductors have been interpreted in terms of Parker's T^* model[7] (also known as the "modified heating theory of nonequilibrium superconductors"), where the quasiparticles are assumed to remain in both thermal and chemical equilibrium at an effective temperature T^* greater than the helium-bath temperature, which is in contrast to a model proposed by Owen and Scalapino,[8] where the quasiparticles are considered to be in thermal equilibrium with the lattice at the bath temperature T but not in chemical equilibrium with the pair state. The Owen-Scalapino model predicts a first-order phase transition to the normal state at a large density of excess quasiparticles which presently is supported by inadequate experimental observation.[3,6] It has been speculated[9,10] that possibly a dynamic intermediate state or a simple thermal inhomogeneity[11] might be responsible for the absence of this first-order phase transition in these earlier experiments.

In this paper, we report on our experimental results that demonstrate the effect of phonon injection on the critical-current hysteresis observed in the current-voltage (I-V) characteristics of long Ag-Sn proximity-effect bridges. We observe that these superconducting proximity-effect bridges undergo a second-order phase transition to the normal state at excess phonon injections in the entire range of temperatures studied, which appears to be consistent with Parker's T^* model. Our results are not adequately accountable by simple heating, and the observed deviations from a simple heating model are in a direction apparently consistent with predictions of Parker's T^* model.

II. EXPERIMENTAL DETAILS

The structures used in our experiments are thin-film crossed strips of silver and tin (see inset Fig. 1) prepared by sequential vacuum evaporation of these materials onto clean glass substrates. (The details of the sample preparation have been described elsewhere.[12]) The thicknesses of the silver and tin strips vary between 0.1 and 0.4 μm. The overlay sections are of lengths varying between 100 and 500 μm and of width $\simeq 200 \mu$m. (These dimensions are comparable with those of Notarys and Mercereau[13] for structures made of soft superconductors that have been observed to show Josephson-like effects for a reasonable range of parameters, viz., lengths 0.3–150 μm, widths $1-10^3$ μm, Sn thickness 0.03–0.3 μm.) The normal-state resistance has been typically of the order of 1 Ω. In the overlay region, superconductivity is depressed by the proximity effect of the normal metal on the superconductor.[13] (If the overlay region is too long, the weak coupling between the two strongly su-

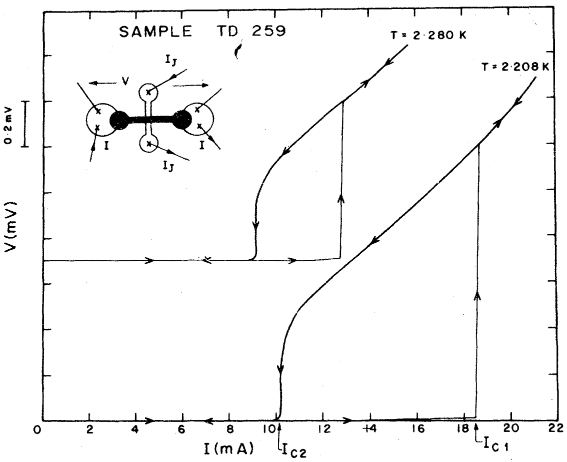

FIG. 1. *I-V* characteristics of a typical proximity-effect bridge (TD 259) at two different temperatures. The inset shows the sample geometry: dark areas—tin; light areas—silver; × indicate solder contacts of electrical leads.

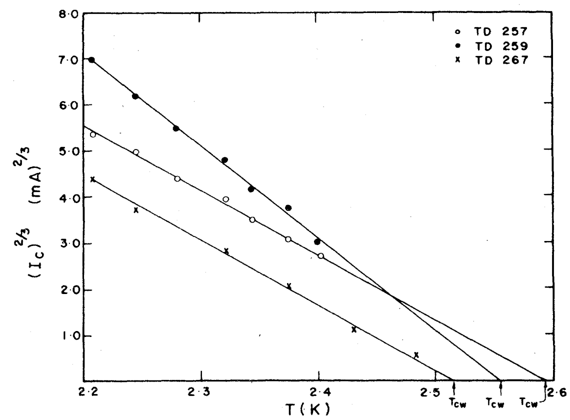

FIG. 2. Critical-current dependence on bath temperature for a few proximity-effect bridges.

perconducting adjoining films is destroyed by fluctuations, and the film acts as three separate but electrically connected superconductors.) The sample is mounted vertically in a liquid-helium bath inside a glass cryostat, which can be pumped down to lower the temperature below 1.6 K. A manostat is used to control the temperature, and the temperature is measured by monitoring the vapor pressure over the helium bath. A superconducting niobium shield is used to minimize stray fields.

The *I-V* traces of the proximity-effect bridges are experimentally obtained by sweeping the voltage across a 25-Ω shunt resistor in parallel with the series combination of the proximity-effect bridges and a 10-Ω current detecting resistor. The voltage across the two strongly superconducting regions and the current through the proximity-effect bridge are simultaneously recorded on an $x-y$ plotter. A dc electric current I_J can also be injected through the transverse normal-metal strip using a separate battery.

III. RESULTS

Figure 1 shows the observed *I-V* characteristics of a typical proximity-effect bridge at two different bath temperatures T, which exhibit the general features observed in previous experiments,[13,14] viz.,a maximum zero-voltage current I_c and an excess supercurrent I_S (the current through the bridge I being given by $I = I_S + V/R$), which becomes equal to $\frac{1}{2}I_c$ for $V >> I_cR$, where R is the normal-state resistance of the bridge. Hysteresis of the critical current is also observed exhibiting two metastable critical currents I_{c1} and I_{c2} as seen in Fig. 1. The *I-V* traces obtained by successive sweeps (at a fixed bath temperature) show a well-defined metastable critical current (I_{c1} or I_{c2}) whose spread is less than 2% of its magnitude. It is also observed that the first critical current $I_{c1}^{2/3}$ varies linearly with T (see Fig. 2) as observed by previous workers,[13–15] and which can be expressed in the form

$$I_{c1}(t) = I_{c1}(0)(1-t)^{3/2} \ , \qquad (1)$$

where $I_{c1}(0)$ is the extrapolated zero-temperature first critical current, and t is the reduced bath temperature (in units of T_{cw}, the intrinsic transition temperature of the proximity-effect-induced weak region).

Now, if a direct current I_J is injected through the transverse normal-metal strip of the proximity-effect bridge while the bath temperature T is held constant, we observe a decrease in the hysteresis and the *I-V* characteristics obtained under this dc electric current injection (see Fig. 3) are reminiscent of those ob-

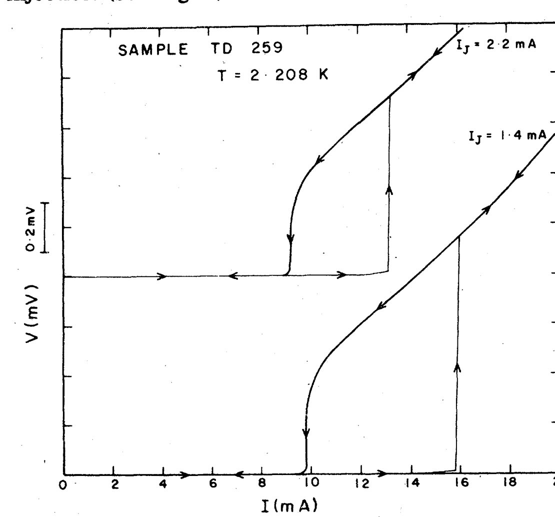

FIG. 3. *I-V* characteristics of a typical proximity-effect bridge (TD 259) for two different injector currents at $T = 2.208$ K.

tained by varying the helium-bath temperature (as seen in Fig. 1).

IV. PRINCIPAL CAUSE OF THE HYSTERESIS

On the basis of a simple model of a localized normal hot spot maintained by Joule heating, Skocpol, Beasley, and Tinkham[16] have shown that the formation of such a hot spot is the dominant cause of the hysteresis observed in the *I-V* characteristics of superconducting thin film microbridges at low temperatures. In the context of the above model (hereinafter called the SBT hot-spot model), which we assume to hold good for proximity-effect bridges also, the second critical current I_{c2} of long[16] bridges can be expressed in the form

$$I_{c2}(t) = I_h = (\alpha w^2 T_{cw} d/\rho)^{1/2}(1-t)^{1/2} \ , \qquad (2)$$

where I_h is the minimum current required to sustain a normal hot spot in a bridge of width w, thickness d with resistivity ρ, intrinsic transition temperature of the proximity-effect-induced weak region T_{cw}, and with α the total-heat-transfer coefficient per unit area of the bridge. Here t is the reduced bath temperature (in units of T_{cw}). The variation of $I_{c2}^2(t)$ with t is reasonably linear for $t \leq 0.9$ and the linear portion when extrapolated passes through the point $t=1$ (see Fig. 4) in agreement with Eq. (2), which demonstrates that the SBT hot-spot model could as well as be successfully applied to an understanding of the electrical behavior of superconducting proximity-effect bridges.

V. EFFECT OF PHONON INJECTION ON HYSTERESIS

Direct current injection through the transverse normal-metal strip of a proximity-effect bridge generates thermal phonons in this heater strip, which in turn creates nonequilibrium quasiparticles in the overlay region by pair breaking.[2] This causes a decrease in the BCS energy-gap parameter Δ of the overlay region which is manifested as an experimentally observed decrease of the critical current.

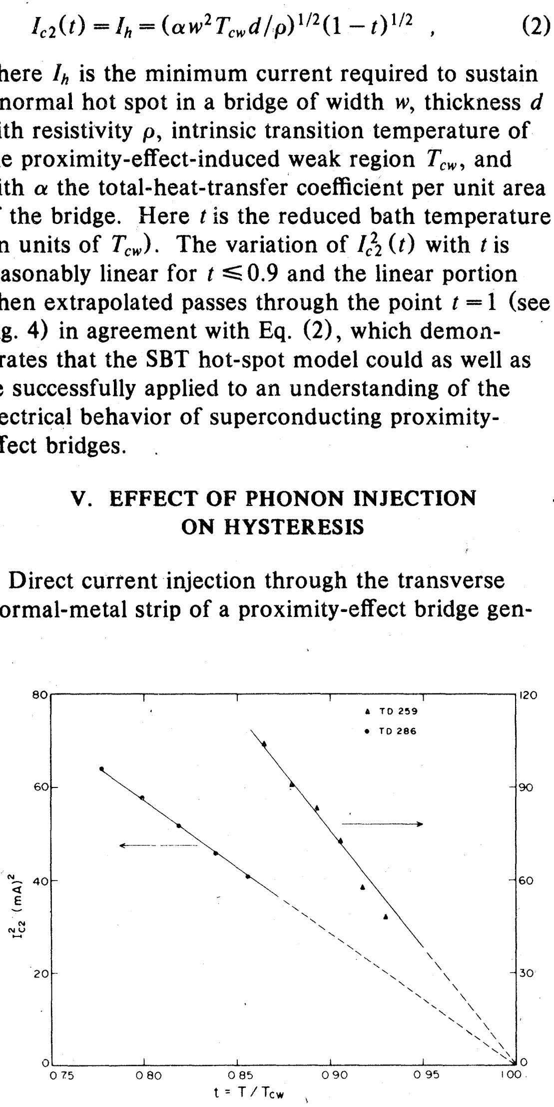

FIG. 4. Variation of the square of the second critical current with bath temperature for two typical proximity-effect bridges.

A. Simple heating model

First, we examine whether the observed phenomenon is the result of a simple heating of the overlay region by thermal phonons injected through the transverse normal-metal heater strip. For this we assume that the only effect of the injected phonons is to raise the temperature of the overlay region which remains in complete thermal equilibrium at the elevated temperature. In a thin-film geometry at low temperatures, it is reasonable to assume that this elevated temperature of the superconductor is determined by the heat input and the thermal conductance between the thin film and substrate and/or the liquid-helium bath. The thermal conductance depends on the difference of the fourth powers of the film and ambient temperatures.[17] Thus, the elevated temperature T^* is given by

$$T^{*4} - T^4 = \eta I_J^2 / I_{J0}^2 \qquad (3)$$

where η is adjusted such that $T^* = T_{cw}$ when $I_J^2/I_{J0}^2 = 1$ and I_{J0} is the critical injector current that just drives normal the overlay region.

At a given bath temperature T, one can determine the values of T^* equivalent to different injector currents I_J using Eq. (3), and then corresponding values of I_{c1} and I_{c2} at this elevated temperature T^* can be estimated using Eqs. (1) and (2). The values of I_{c1} and I_{c2} estimated using this simple heating model are compared with those observed experimentally in Fig. 5. Here we have made a one-parameter empirical fit using the experimental values of $I_{c1}(t)$ and $I_{c2}(t)$ corresponding to zero injection for evaluating the proportionality constants of Eqs. (1) and (2), respectively. The agreement is remarkably good for I_{c2}. This is understandable because the bridge characteristics in the current-induced resistive state are solely decided by the Joule heating[14,16] caused by the injector current and the bridge current. However, the experimentally observed I_{c1} systematically deviates from and is lesser in magnitude than that estimated using the above simple heating model indicating that for a given injector current I_J; the effective temperature of the bridge is slightly greater than the increased temperature resulting from pure heating. This systematic deviation from a simple heating model is in the right direction consistent with Parker's modified heating theory of nonequilibrium superconductors. We feel these observations justify our argument that in the superconducting state, the

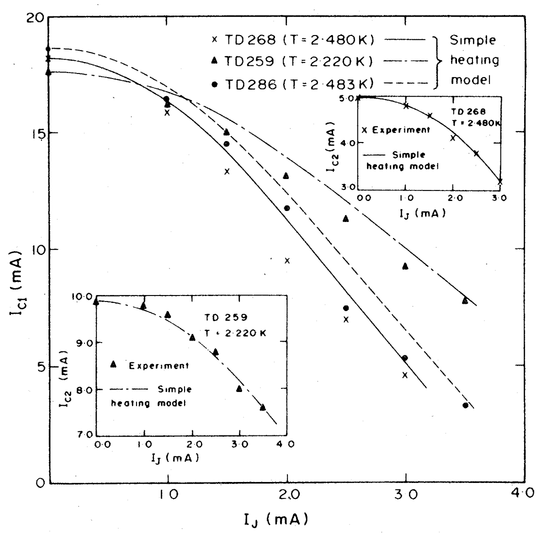

FIG. 5. Injector current dependence of the critical currents for a few proximity-effect bridges.

bridge is driven out of equilibrium if phonons are injected through the transverse normal-metal strip.

B. Nonequilibrium model

The phenomenological equations of Rothwarf and Taylor[18] for quasiparticle injection by an external mechanism can be written

$$\frac{dN}{dt}=I_0+\frac{2N_\omega}{\tau_B}-RN^2 \tag{4}$$

and

$$\frac{dN_\omega}{dt}=\frac{RN^2}{2}-\frac{N_\omega}{\tau_B}-\frac{N_\omega-N_{\omega T}}{\tau_\gamma} . \tag{5}$$

Here, I_0 is the volume rate of creation of quasiparticles by an external mechanism (such as phonon injection), N is the number density of quasiparticles, N_ω is the number density of phonons with energy greater than 2Δ (where Δ is the BCS energy gap of the superconductor), τ_B^{-1} is the mean rate at which these phonons create quasiparticles by pair breaking, R is the intrinsic quasiparticle recombination coefficient, τ_γ is the rate at which phonons of energy greater than 2Δ disappear by processes other than quasiparticle creation, and $N_{\omega T}$ is the thermal-equilibrium number density of phonons with energy greater than 2Δ.

The steady-state solutions of these equations are

$$N_\omega/N_{\omega T}=1+(\tau_\gamma/2N_{\omega T})I_0 \tag{6}$$

and

$$(N/N_T)^2=1+(1+\tau_\gamma/\tau_B)I_0\tau_R/N_T , \tag{7}$$

where

$$N_T=(2N_{\omega T}/R\,\tau_B)^{1/2}$$

is the thermal-equilibrium number density of quasiparticles and $\tau_R=(RN_T)^{-1}$ is the intrinsic recombination time.

An external quasiparticle creation mechanism such as phonon injection produces a steady-state number density of excess quasiparticles described by a normalized quantity

$$n=(N-N_T)/4N(0)\Delta(0) , \tag{8}$$

where $N(0)$ is the single-spin density of states and $\Delta(0)$ is the zero-temperature energy gap of the superconductor. Using this, Eq. (7) can be expressed in the form

$$n(n+2n_T)=4n_TN(0)\Delta(0)\tau_{\rm eff}I_0 , \tag{9}$$

where

$$\tau_{\rm eff}=\tau_R(1+\tau_\gamma/\tau_B)$$

and

$$n_T=N_T/4N(0)\Delta(0) .$$

In the present experiment, the injected phonons are of course divided between the superconductor on one side of the injector and the bath on the other, and the relative numbers in the two directions depend on the poorly understood phonon transmission properties of the interfaces.[19] In this context, we relate the quasiparticle injection rate density I_0 to the injector current I_J by

$$I_0=\beta I_J^2 , \tag{10}$$

where β depends on the phonon transmission properties of the interfaces and is proportional to the fraction of the total dissipated injection energy which appears as the energy of pair-breaking phonons in the bridge region of the superconductor and as excited quasiparticle energies. Substituting this value of I_0 into Eq. (9), we get

$$n(n+2n_T)=4n_TN(0)\Delta(0)\tau_{\rm eff}\beta I_J^2 . \tag{11}$$

By virtue of the experimentally observed critical current dependence on temperature described by Eq. (1) it is possible to directly relate the critical current I_{c1} to an effective energy gap[2,14] by

$$I_{c1}\sim\Delta^3 . \tag{12}$$

Hence, a plot of $(I_{cJ}/I_{c0})^{1/3}$ vs I_J/I_{J0} or I_J^2 (see Fig. 6), where I_{cJ} and I_{c0} are, respectively, the experimentally observed values of I_{c1} for finite values of I_J and $I_J=0$ relates the normalized energy gap to the excess quasiparticle density n [see Eq. (11)]. This plot (Fig. 6) shows a second-order phase transition to the nor-

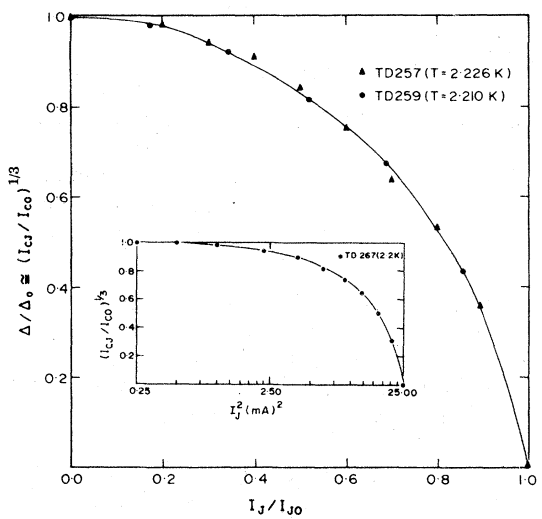

FIG. 6. Injector current dependence of the normalized energy gap exhibiting a second-order phase transition for several proximity-effect bridges.

mal state under excess phonon injection. This observation is in conformity with Parker's T^* model.

VI. CONCLUSIONS

We have demonstrated that superconducting proximity-effect bridges are driven into nonequilibrium states when a direct current is injected through the bridge transverse normal-metal strip. We have also shown that the experimentally observed variation of the critical current hysteresis with bath temperature is accountable by a SBT hot-spot model, whereas the hysteresis of these bridges under phonon injection cannot be satisfactorily understood by a simple heating model and a SBT hot-spot-model ansatz. For a qualitative explanation of the observed deviation of the experimental results from the predictions of the simple heating model, we find that one can invoke Parker's T^* model satisfactorily and further that the observed second-order phase transition to the normal state under excess phonon injection can also be understood within the framework of the same model.

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[1]D. N. Langenberg, in *Proceedings of the Fourteenth International Conference on Low Temperature Physics, Ontaniemi, Finland, 1975*, edited by M. Krusius and V. Vuorio (North-Holland, Amsterdam, 1975), Vol. V, p. 223.

[2]T. Wong, J. T. C. Yeh, and D. N. Langenberg, Phys. Rev. Lett. 37, 150 (1976); IEEE Trans. Magn. 13, 743 (1977).

[3]J. Fuchs, P. W. Epperlein, M. Welte, and W. Eisenmenger, Phys. Rev. Lett. 38, 919 (1977).

[4]R. C. Dynes, V. Narayanamurti, and J. P. Garno, Phys. Rev. Lett. 39, 229 (1977).

[5]S. B. Kaplan, J. R. Kirtley, and D. N. Langenberg, Phys. Rev. Lett. 39, 291 (1977).

[6]I. Iguchi, Phys. Rev. B 16, 1954 (1977); Phys. Lett. A 64, 415 (1978); J. Low Temp. Phys. 31, 605 (1978).

[7]W. H. Parker, Phys. Rev. B 12, 3667 (1975).

[8]C. S. Owen and D. J. Scalapino, Phys. Rev. Lett. 28, 1559 (1972).

[9]J. Chang and D. J. Scalapino, Phys. Rev. B 10, 4047 (1974).

[10]G. A. Sai-Halasz, C. C. Chi, A. Denenstein, and D. N. Langenberg, Phys. Rev. Lett. 33, 215 (1974).

[11]P. Hu, R. C. Dynes, and V. Narayanamurti, Phys. Rev. B 10, 2786 (1974).

[12]G. Dharmadurai and B. A. Ratnam, Proc. Nucl. Solid State Phys. Symp. India C 20, 6 (1977).

[13]H. A. Notarys and J. E. Mercereau, J. Appl. Phys. 44, 1821 (1973).

[14]S. K. Decker and D. W. Palmer, J. Appl. Phys. 48, 2043 (1977).

[15]G. Dharmadurai and B. A. Ratnam, Phys. Lett. A 67, 49 (1978).

[16]W. J. Skocpol, M. R. Beasley, and M. Tinkham, J. Appl. Phys. 45, 4054 (1974).

[17]W. A. Little, Can. J. Phys. 37, 334 (1959).

[18]A. Rothwarf and B. N. Taylor, Phys. Rev. Lett. 19, 27 (1967).

[19]D. N. Langenberg (private communication).