Physics of microwave reflection at a dielectric-ferrite interface

S. S. Gupta* and N. C. Srivastava
Physics Department, Indian Institute of Technology, New Delhi-110029, India
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The reflection of electromagnetic waves from magnetized ferrites has been analyzed from the view point of energy flow and ray propagation. Renard's analysis of reflection from dielectrics has been generalized to study the lateral shift of an electromagnetic beam reflected from a ferrite half-space magnetized transverse to the plane of incidence. The nonreciprocity of total reflection has been explained in terms of unequal lateral shifts when the rays are incident from opposite sides to the normal. The shift can be positive as well as negative and is finite at normal incidence. This study leads to a phenomenological ray model for attenuated total reflection from a ferrite provided that the linewidth is small. The results can be useful for open-resonator measurements of linewidth in the millimeter-wave region.

I. INTRODUCTION

The reflection of electromagnetic waves from magnetized ferrites has been extensively studied in the past.[1-4] Seavey and Tannenwald[1] have investigated the microwave power absorbed by nonconducting as well as conducting semi-infinite transversely magnetized ferrite media, in the case of normal incidence of a plane wave. They also studied the body resonances of a metal-backed slab and an unbacked slab. Suhl and Walker[2] have examined the reflection and transmission of plane electromagnetic waves in a longitudinally magnetized slab. Mueller[3] has studied the reflection and transmission in the case of oblique incidence of plane electromagnetic waves at the interface between arbitrarily magnetized ferrites. However, it was recognized recently[4] that the reflection of electromagnetic waves from biased ferrites is nonreciprocal as regards phase and amplitude; this leads to the possibility of fabricating a reflection-beam phase shifter and isolator for operation in the millimeter-wave region.[4] The standard procedure for the investigation of electromagnetic-wave reflection from ferrities involves the solution of Maxwell's equations in each region of space and the application of appropriate boundary conditions at each interface. While this mathematically precise procedure yields the desired results, it does not provide much physical insight into the phenomena occurring at the interface(s). For instance, it does not clearly stress penetration, propagation, and absorption of energy in the reflecting medium (ferrite). Owing to this lack of physical insight it is difficult to make valid approximations and develop simple models for the analysis of rather complicated problems, e.g., guided propagation in ferrites and ferrite-loaded wave guides involving ferrite boundaries.

This paper presents an investigation of the total reflection of microwaves at a dielectric-ferrite interface from the view point of ray and energy propagation, in the case when the magnetization of the ferrite is transverse to the plane of incidence.

The path of the rays in the case of total reflection of light (electromagnetic waves) at the interface between two dielectrics has been a topic of great interest. Sir Isaac Newton[5] predicted that the rays should enter the second (less dense) medium and trace a parabola before returning to the first medium. Subsequent investigations showed that although the basic concepts and reasoning involved in Newton's treatment are not correct, the displacement of a beam on account of total reflection from dielectrics is a correct prediction; this phenomenon has been extensively investigated theoretically as well as experimentally.[6-9] The maximum shift is of the order of a few wavelengths and requires great precision of measurement when investigated experimentally. In recent years, some interesting and informative experiments to measure this shift have been performed[10-13] at microwave frequencies, where the shift is rather large owing to the large wavelength of the microwaves.

The explanation of the lateral shift from the viewpoint of energy and ray propagation was considered by Renard.[14] His derivation was based on the fact that in the evanescent field set up inside the second medium, there is a time-averaged energy flow parallel to the interface; it is this energy flow which causes displacement of totally reflected rays. A generalization of Renard's procedure to the case of total reflection of arbitrarily polarized incident waves and associated lateral and transverse shifts, was given by Imbert.[15]

We have generalized the Renard-Imbert analysis to investigate the displacement of a plane wave (in practice, a broad collimated beam) due to total reflection from a ferrite magnetized transverse to the plane of incidence. In Sec. II, we have first obtained the energy density and Poynting vector in the evanescent field set up in the ferrite region. The inverse ratio of the two is the velocity of energy flow parallel to the interface. Next, we have obtained an expression for the lateral shift of totally reflected rays using the Renard-Imbert procedure. The dependence of the lateral shift on various relevant parameters has been numerically investigated. In Sec. III, we have proposed an approximate ray model[16] for microwave absorption at a dielectric-ferrite interface, when the ferrite is biased in the region of negative effective permeability. It is seen that the agreement between the ray model and the rigorous expression is excellent for a wide range of parameters. The ray model provides physical insight and also leads to a simple, approximate expression for the linewidth of the ferrite in terms of the reflected amplitude; this formula is useful in open-resonator measurements[17] of the linewidth of ferrites in the millimeter-wave region.

II. LATERAL SHIFT

The permeability tensor for a ferrite magnetized along the z axis is given by

$$\underline{\mu} = \begin{pmatrix} \mu & jk & 0 \\ -jk & \mu & 0 \\ 0 & 0 & 1 \end{pmatrix} , \tag{1}$$

where

$$\mu = [H_0(H_0 + 4\pi M_0) - (\omega/\gamma)^2 + j\Delta H(H_0 + 2\pi M_0)]/\Delta',$$
$$k = 4\pi M_0(\omega/\gamma)/\Delta',$$
$$\Delta' = [H_0^2 - (\omega/\gamma)^2 + jH_0 \cdot \Delta H]; \tag{2}$$

$H_0, 4\pi M_0, \gamma$, and ΔH are the biasing field, saturation magnetization, gyromagnetic ratio, and resonance linewidth, respectively.

A uniform plane wave of frequency ω, initially propagating in a lossless dielectric medium, is incident on the dielectric-ferrite interface $y = 0$. The polarization of the electric-field vector of the incident wave is assumed to be normal to the plane of incidence (TE). The dc magnetization of the ferrite is transverse to the plane of incidence (Fig. 1). The angles of incidence and refraction are ψ_i and ψ_f, respectively. Following earlier analyses,[4] the wave vector, the electric and magnetic fields for incident, reflected, and transmitted waves can be expressed as

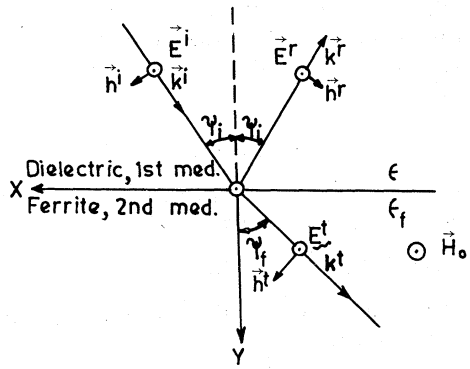

FIG. 1. Geometry of the problem.

$$\vec{K}^i = \beta_0(-\hat{x}\sin\psi_i + \hat{y}\cos\psi_i),$$
$$\vec{K}^r = \beta_0(-\hat{x}\sin\psi_i - \hat{y}\cos\psi_i), \tag{3}$$
$$\vec{K}^t = \beta(-\hat{x}\sin\psi_f + \hat{y}\cos\psi_f),$$

and

$$\vec{E}^i = \hat{z}\exp[j(\omega t - \vec{K}^i\cdot\vec{r})],$$
$$\vec{E}^r = \hat{z}R\exp[j(\omega t - \vec{K}^r\cdot\vec{r})],$$
$$\vec{E}^t = \hat{z}T\exp[j(\omega t - \vec{K}^t\cdot\vec{r})], \tag{4}$$
$$\vec{h}^i = \beta_0(c/\omega)(\hat{x}\cos\psi_i + \hat{y}\sin\psi_i)E_z^i,$$
$$\vec{h}^r = \beta_0(c/\omega)(-\hat{x}\cos\psi_i + \hat{y}\sin\psi_i)E_z^r,$$
$$\vec{h}^t = \beta(c/\Delta\omega)[\hat{x}(\mu\cos\psi_f - jk\sin\psi_f) + \hat{y}(\mu\sin\psi_f + jk\cos\psi_f)]E_z^t,$$

where

$$\beta^2 = \Delta\omega^2\epsilon_f/c^2,$$
$$\beta_0^2 = \omega^2\epsilon/c^2,$$
$$\Delta = \mu^2 - k^2. \tag{5}$$

In the above expressions ϵ and ϵ_f are the dielectric constants of the dielectric and ferrite, respectively. The modified Snell's law, reflection, and transmission coefficients are obtained as[4]

$$\beta_0\sin\psi_i = \beta\sin\psi_f, \tag{6}$$

$$R_\pm = \frac{\Delta\cos\psi_i - (\beta/\beta_0)(\mu\cos\psi_f \mp jk\sin\psi_f)}{\Delta\cos\psi_i + (\beta/\beta_0)(\mu\cos\psi_f \mp jk\sin\psi_f)}, \tag{7}$$

and

$$T_\pm = \frac{2\Delta\cos\psi_i}{\Delta\cos\psi_i + (\beta/\beta_0)(\mu\cos\psi_f \mp jk\sin\psi_f)}.$$

It should be noted that R_- and T_- are the reflection and transmission coefficients in the case when, in Fig. 1, the incident and reflected rays are interchanged.

A. Power flow

The average power flow per unit cross-sectional area for the reflected wave is given by[18]

$$\vec{S}^r = (c/8\pi)\,\mathrm{Re}(\vec{E}^r \times \vec{h}^{r*}). \tag{8}$$

The substitution for $\vec{E}^r$ and $\vec{h}^r$ from Eq. (4), respectively, leads to

$$\vec{S}^r = -(c^2\beta_0/8\pi\omega)(\hat{x}\sin\psi_i + \hat{y}\cos\psi_i)\,|R_\pm|^2. \tag{9}$$

Similarly, the power flow per unit area in the evanescent field set up in the ferrite is obtained as

$$\vec{S}^t = -\hat{x}(c^2\beta/8\pi\omega\Delta)[\mu\sin\psi_f + k(\sin^2\psi_f - 1)^{1/2}] \times |T_\pm|^2 \exp[-2\beta(\sin^2\psi_f - 1)^{1/2}y], \tag{10}$$

where losses have been ignored. Evidently, the power flow in the ferrite region occurs only in the direction parallel to the interface. It is this energy flow in the evanescent field which leads to lateral shift of beams on account of total reflection, as discussed in Sec. II B. Using the modified Snell's law, Eq. (10) is tranformed to

$$S^t_{(\mp x)} = (c^2\beta_0/8\pi\omega\mu_{\mathrm{eff}})[\sin\psi_i + (k/\mu)\alpha] \times |T_\pm|^2 \exp[-2\beta(\sin^2\psi_f - 1)^{1/2}y], \tag{11}$$

where

$$\alpha = (\sin^2\psi_i - \mu_{\mathrm{eff}}\epsilon_f/\epsilon)^{1/2}, \quad \mu_{\mathrm{eff}} = \Delta/\mu \tag{12}$$

B. Generalized Renard formula

Renard's derivation for the lateral shift due to total reflection at the interface separating two dielectric media is based on the fact that in the evanescent-wave field set up in the less dense medium, there is a finite time-averaged flow of energy parallel to the interface. Since the intensity of the reflected beam is the same as the intensity of the incident beam, it follows that the only way of accounting for the flow of energy in the less-dense medium is to assume a lateral shift of the reflected ray. Applying the principle of conservation of energy, it is obvious that the time-averaged flux of energy (u_1) for the plane wave across a strip (Fig. 2) whose width is equal to the lateral shift must be equal to the time-averaged flux of energy (u_2) parallel to the interface in the entire second medium (reflecting medium). This procedure of equating the two energy fluxes can be generalized to the case of total reflection at the dielectric-

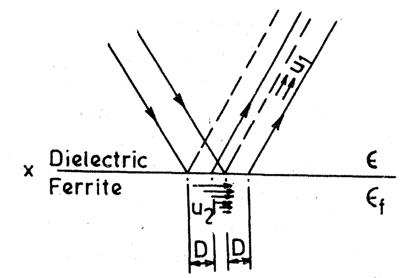

FIG. 2. Lateral shift of a bounded beam due to total reflection.

ferrite interface. The expression for the lateral shift is therefore given by

$$D = \int_0^\infty S^t_{(-x)}\,dy \Big/ S^r_{(-y)}. \tag{13}$$

Substitution for $S^r_{(-y)}$ and $S^t_{(-x)}$ from Eqs. (9) and (11) yields:

$$D_\pm = \frac{2\mu_{\mathrm{eff}}\cos\psi_i}{\beta_0\alpha}\,\frac{\sin\psi_i \pm (k/\mu)\alpha}{\mu^2_{\mathrm{eff}}\cos^2\psi_i + [\alpha \pm (k/\mu)\sin\psi_i]^2}. \tag{14}$$

The losses have been ignored in the derivation of this formula. As such, it is strictly applicable only to a lossless ferrite; in practice, it is a good approximation if the linewidth of the ferrite is small and the region close to resonance is avoided.

It is evident from Eq. (14) that the lateral shift, which is, incidently, the effective distance traversed by a ray in the second medium, is nonreciprocal; its magnitude changes when the wave path is reversed. It follows from Eq. (14) that if, for a given set of parameters, the factor $[\sin\psi_i \pm (k/\mu)\alpha]$ changes sign on reversing the sign of k (i.e., on reversing the wave path), D_+ and D_- would have opposite signs, i.e., either D_+ or D_- would be negative. The meaning of the negative shift has been explained in Fig. 3. At first, this result

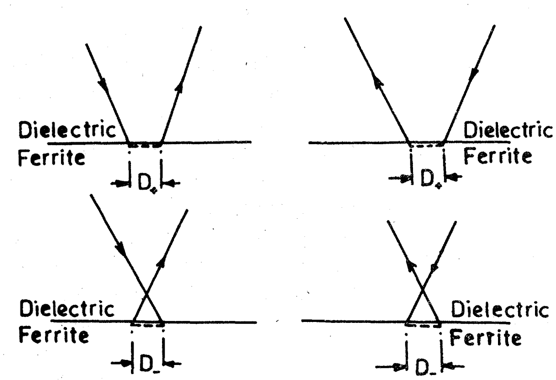

FIG. 3. Illustration of the positive and negative shift in the cases of forward and reverse incidence.

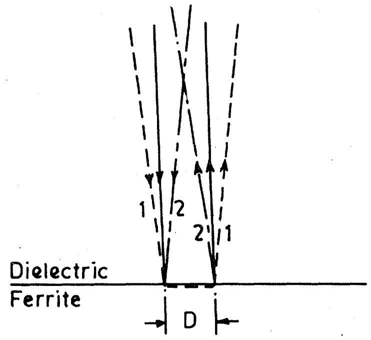

FIG. 4. Illustration of total reflection for a normally incident ray as well as for two more rays incident at small angles from opposite sides of the normal.

may be surprising particularly because in the case of normal incidence[19] ($\psi_i = 0°$), Eq. (14) reduces to

$$D_\pm = \pm 2|k/\mu|/\beta_0(|\mu_{eff}| + \epsilon_f/\epsilon), \tag{15}$$

which implies that D_+ and D_- have equal magnitudes but opposite signs in the case of normal incidence. This might give the impression that there are two distinct shifts for the same incident ray. However, a closer examination reveals that there is no inconsistency and the path of a normally incident ray is as shown in Fig. 4. Apart from the normally incident ray, two more rays incident at small angles (but from opposite sides of the normal) have also been shown. The shift is positive for Ray 1 but negative for Ray 2. In the limit when $\psi_i \to 0°$, it is, therefore, natural that D_+ and D_- should have the same magnitude but opposite signs. The occurrence of a finite beam shift at $\psi_i = 0°$ can be understood in terms (owing to the incident wave) of a circular (in general, elliptical) precession of the magnetization vector about the biasing field; this precession leads to the emergence of an energy packet from a point different from the point at which energy enters. This is a purely magnetic effect and has no analogue in the case of total reflection from a lossless dielectric.[20]

When $\mu_{eff} > 0$, the total reflection occurs only when $\psi_i > \psi_c$; ψ_c can be inferred from Eq. (6). In this case, assuming $\omega > \omega_m$, it can be shown that $[\sin\psi_i \pm (k/\mu)\alpha]$ is positive for all combinations of the relevant parameters. Consequently, D_+ and D_- are both positive although they differ in magnitude. When $\omega = \omega_0 + \omega_m$, i.e., when $\mu_{eff} = 0$, we have $D_\pm = 0$; the lateral shift vanishes at the cutoff limit. On the other hand both D_+ and D_- are large in the vicinity of resonance where μ_{eff} is large. Also, when $\psi_i = 90°$, $D_\pm \equiv 0$, which is self-consistent. Moreover, when $\psi_i \to \psi_c$, $D_\pm \to \infty$ as in the case of total reflection from dielectrics.

Numerical calculations have been performed on DCM microsystem 1101 to investigate the dependence of the lateral shift on the various parameters such as angle of incidence, biasing field, dielectric-constant ratio, and wave frequency. The results for $f = 9$ GHz have been shown in Figs. 5–8. As discussed above, the shifts D_+ and D_- have equal magnitudes, but opposite signs for $\psi_i = 0°$. When the angle of incidence is different from 0°, the resulting lateral shift consists of a magnetic and a dielectric contribution; the latter has a tendency to cause a reciprocal positive shift. Evidently, for a particular direction of incidence, the dielectric and magnetic effects add up, thereby causing the shift to increase with ψ_i. However, for opposite directions of incidence, the magnetic and dielectric effects are of opposite signs and the resultant shift decreases in magnitude as ψ_i increases; this is evident from Fig. 5 in which $D_\pm$ have been plotted as a function of ψ_i for various biasing-field strengths. It is seen that as H_0 increases, the peak value of the lateral shift also increases apart from shifting towards larger ψ_i. Figure 6 shows the variation of D_+ with ψ_i for different values of ϵ/ϵ_f. The shift D_+ exhibits a peak when plotted as a function of the angle of incidence; the peak occurs when the angle of incidence is rather close to $\frac{1}{2}\pi$. The occurrence of these peaks

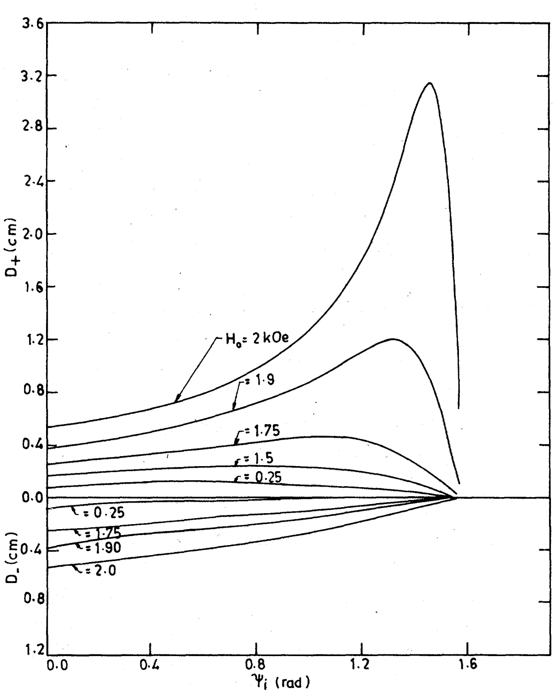

FIG. 5. Variation of the lateral shift $D_\pm$ with ψ_i for different biasing-field strengths, indicated on each curve. Other parameters are $\epsilon/\epsilon_f = \frac{1}{13}$, $4\pi M_0 = 3$kG, and $f = 9$ GHz.

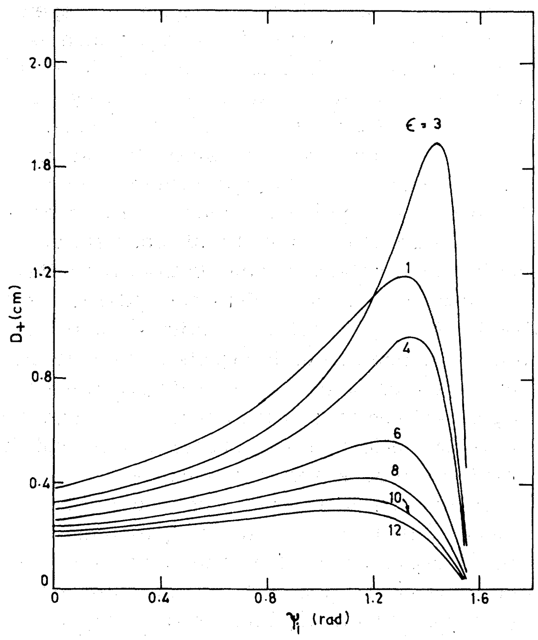

FIG. 6. Variation of the lateral shift D_+ with ψ_i for different values of the dielectric constant ratio ϵ/ϵ_f. ϵ's are indicated next to each curve. Other parameters are $\epsilon_f=13$, $f=9$ GHz, $4\pi M_0=3$ kG, and $H_0=1.9$ kOe.

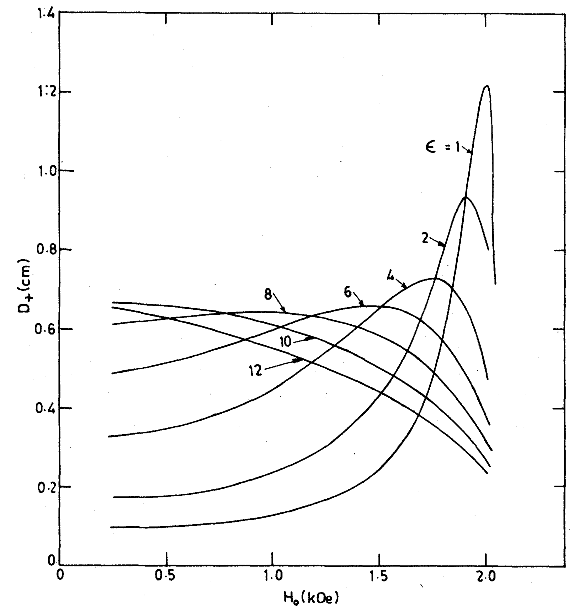

FIG. 7. Variation of D_+ with H_0 for different values of the dielectric constant ϵ, shown near each curve. Other parameters are $\psi_i=1.0$ rad, $\epsilon_f=13.0$, and $f=9$ GHz.

is a purely magnetic effect since, in the case of reflection from dielectrics, the lateral shift monotonically decreases with increasing angle of incidence. It is seen that the position and height of the peaks in D_+ strongly depend on the dielectric-constant ratio. When ϵ is large, the maximum value of D_+ is also large and occurs at higher angles of incidence. However, a critical stage is reached beyond which further increase in ϵ leads to reduction in the overall magnitude of D_+. The variation of the lateral shift D_+ with biasing field for different values of the dielectric constant ratio ϵ/ϵ_f is shown in Fig. 7. It is seen that the maximum value of the lateral shift increases and shifts towards a larger resonant biasing field as the ratio ϵ/ϵ_f decreases. Figure 8 shows the variation of the normalized lateral shift with biasing field for $\psi_i=1.5$ (rad) and for different values of the dielectric-constant ratio, ϵ/ϵ_f. Figure 8 also shows the shift of the peak value towards a lower biasing-field strength as the ratio ϵ/ϵ_f increases. It would be interesting to perform experiments at millimeter-wave frequencies to observe and measure the magnetically tunable, nonreciprocal lateral shift due to total reflection from a biased ferrite. Figure 9 shows the variation of D_+ with H_0 for $f=90$ GHz and $\psi_i=1.5$ rad. The general nature of the curves is essentially the same as for $f=9$ GHz. The absolute lateral shifts for $f=9$ GHz in all cases are

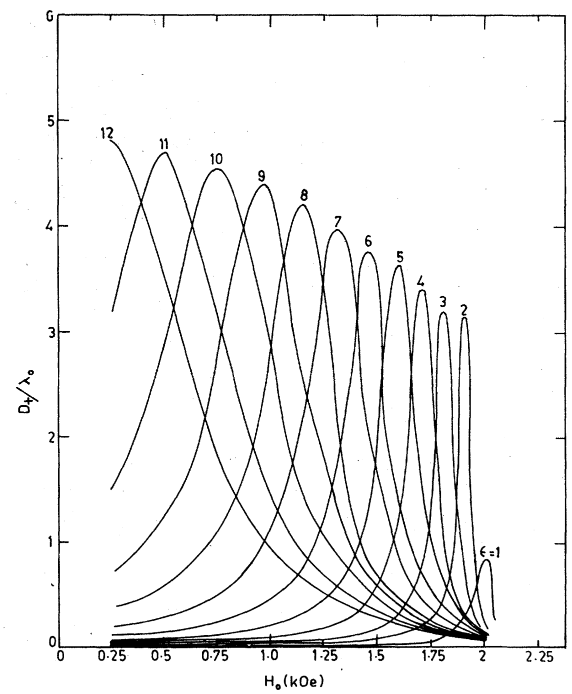

FIG. 8. Variation of normalized lateral shift D_+/λ_0 with H_0 for different values of the dielectric constant ϵ shown near each curve. Other parameters are $\psi_i=1.5$ rad, $4\pi M_0=3$ kG, $\epsilon_f=13.0$, and $f=9$ GHz.

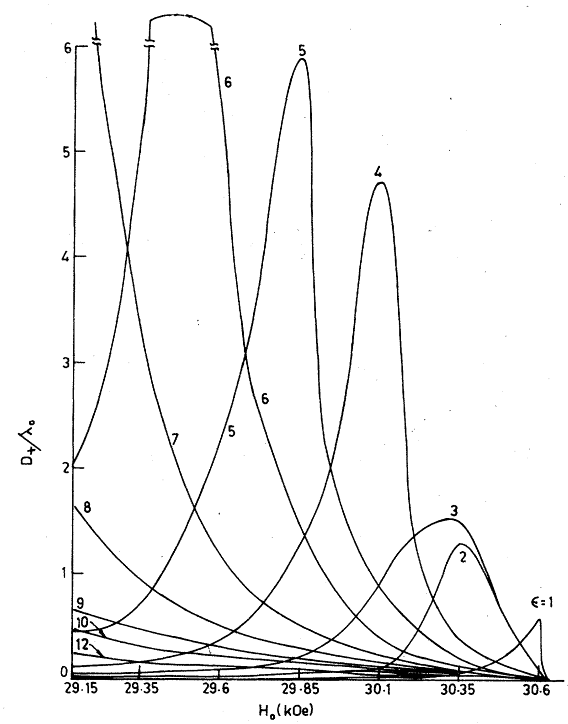

FIG. 9. Variation of the normalized shift D_+/λ_0 with H_0 for wave frequency $f=90$ GHz. Other parameters are the same as for Fig. 8.

larger than the corresponding lateral shifts for $f=90$ GHz, for otherwise the same parameters.

III. RAY MODEL FOR MICROWAVE ABSORPTION

The permeability tensor for weakly lossy ferrites can be expanded as a power series in the linewidth. Neglecting second and higher powers of ΔH, we can write

$$\underline{\mu}^f = \underline{\mu}' - j\underline{\mu}''\Delta H, \tag{16}$$

where

$$\underline{\mu}'' = \begin{pmatrix} \alpha' & j\beta' & 0 \\ -j\beta' & \alpha' & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{17}$$

The parameters α' and β' are defined by

$$\begin{aligned} \alpha' &= (\mu_0/\mu^0_{eff})(\delta + 2k_0\delta'), \\ \beta' &= (1/\mu^0_{eff})[(\mu_0^2 + k_0^2)\delta' + (k_0/\mu^0_{eff})\delta], \end{aligned} \tag{18}$$

where

$$\begin{aligned} \mu_0 &= \frac{H_0(H_0 + 4\pi M_0) - (\omega/\gamma)^2}{H_0^2 - (\omega/\gamma)^2}, \\ k_0 &= \frac{4\pi M_0(\omega/\gamma)}{H_0^2 - (\omega/\gamma)^2}, \\ k/\mu &= (k_0/\mu_0) - j\Delta H\delta', \\ \mu_{eff} &= \mu^0_{eff} - j\Delta H\delta, \\ \mu^0_{eff} &= (\mu_0^2 - k_0^2)/\mu_0. \end{aligned} \tag{19}$$

We now derive the expression for the effective absorption coefficient for rays propagating parallel to the interface, in the ferrite region. The power loss per unit volume is given approximately[21] by

$$\frac{d(S_x^t)}{dx} = -\frac{\omega}{8\pi}\mathrm{Re}[(\underline{\mu}''\cdot\vec{h})\cdot\vec{h}^*]. \tag{20}$$

Using Eqs. (17), (18), and (4), Eq. (20) reduces to

$$\begin{aligned} \frac{d(S_x^t)}{dx} &= -\frac{\beta_0\Delta H}{\mu^0_{eff}}\frac{2\delta}{(\sin\psi_i \pm \alpha k_0/\mu_0)} \\ &\quad\times\left[\sin^2\psi_i + \alpha\sin\psi_i \times\left(\pm\frac{k_0}{\mu_0} \mp \frac{\delta'\mu^0_{eff}}{\delta}\right) - \tfrac{1}{2}\mu^0_{eff}\frac{\epsilon_f}{\epsilon}\right]S_x^t \\ &\equiv -2\alpha^{\pm}_{eff}S_x^t, \end{aligned} \tag{21}$$

where

$$\alpha^{\pm}_{eff} = \frac{\beta_0\Delta H}{\mu^0_{eff}}\frac{\delta}{\sin\psi_i \pm \alpha k_0/\mu_0} \times\left[\sin^2\psi_i + \alpha\sin\psi_i\left(\pm\frac{k_0}{\mu_0} \mp \frac{\delta'\mu^0_{eff}}{\delta}\right) - \tfrac{1}{2}\mu^0_{eff}\frac{\epsilon_f}{\epsilon}\right]. \tag{22}$$

The integration of Eq. (21) yields

$$S_x^t = S_0\exp(-2\alpha_{eff}x), \tag{23}$$

Thus $2\alpha_{eff}$ is equal to the effective power-absorption coefficient for rays in the ferrite region. It follows that the ratio of reflected power (P_R) to incident power (P_i) is, in fact, the fractional power lost by the ray in traversing a distance D in the ferrite region. Thus,

$$P_R/P_i = |R_\pm|^2 = \exp(-2\alpha^{\pm}_{eff}D_\pm). \tag{24}$$

Equivalently,

$$|R_\pm| = \exp(-\alpha^{\pm}_{eff}D_\pm). \tag{25}$$

Numerical calculations have been performed in order to check the validity of Eq. (25). Figures 10–15 display the reflection coefficient $|R|$ as obtained from the rigorous expression [Eq. (7), solid lines] as well as from the ray model [Eq. (25), broken lines]. It is seen that the ray model works

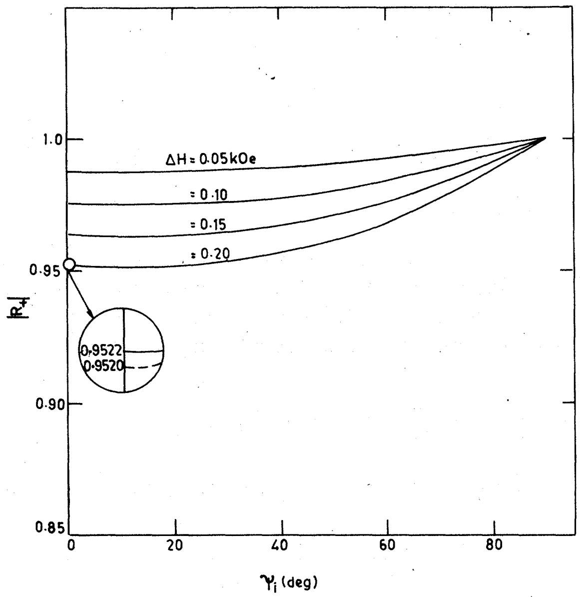

FIG. 10. Variation with angle of incidence ψ_i of $|R_+|$ as obtained from the rigorous formula (solid lines) and from Eq. (25) (broken lines) with ΔH as the parameter. Other parameters are $H_0 = 0.75$ kOe, $\epsilon = 1.0$, $4\pi M_0 = 3$ kG, $\epsilon_f = 13.0$, and $f = 9$ GHz.

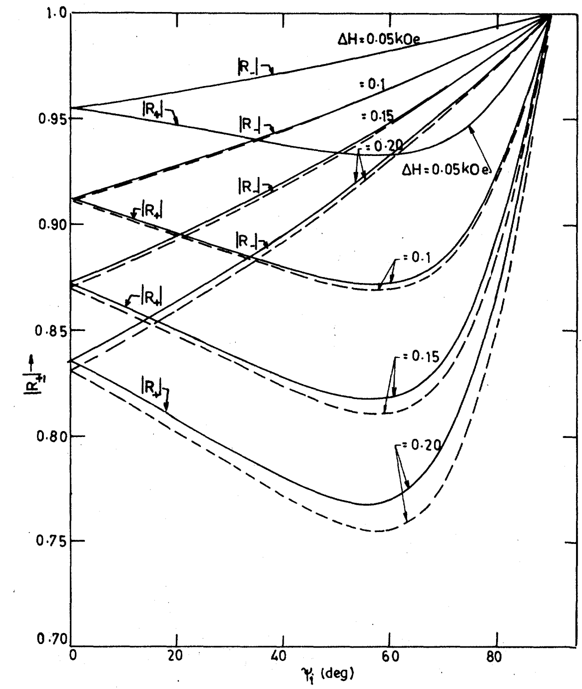

FIG. 12. Variation of $|R_\pm|$ with ψ_i for H_0= kOe. Other parameters are same as for Fig. 10.

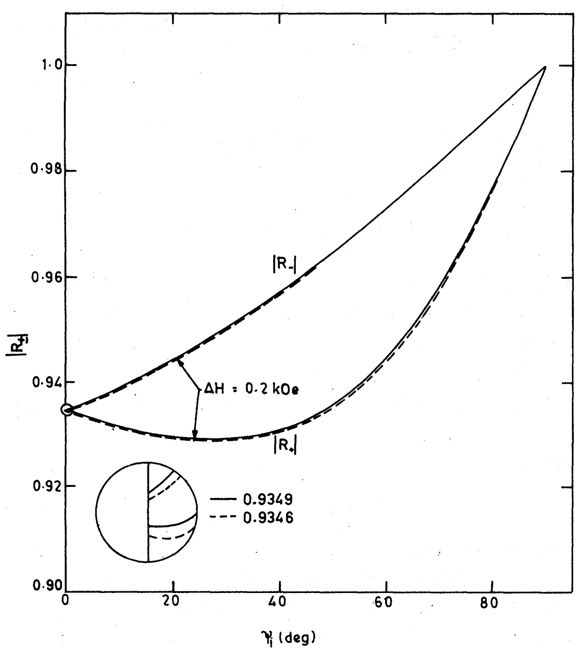

FIG. 11. Variation of $|R_\pm|$ with ψ_i for $H_0 = 1.25$ kOe, $4\pi M_0 = 3$ kG, $\Delta H = 0.2$ kOe, and $f = 9$ GHz.

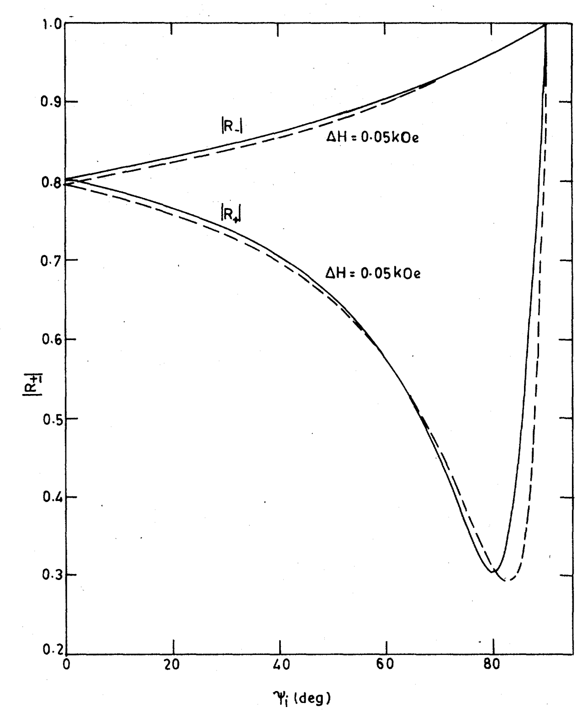

FIG. 13. Variation of $|R_\pm|$ with ψ_i for $H_0 = 2.0$ kOe, $\Delta H = 0.05$ kOe, $4\pi M_0 = 3$ kG, and $f = 9$ GHz.

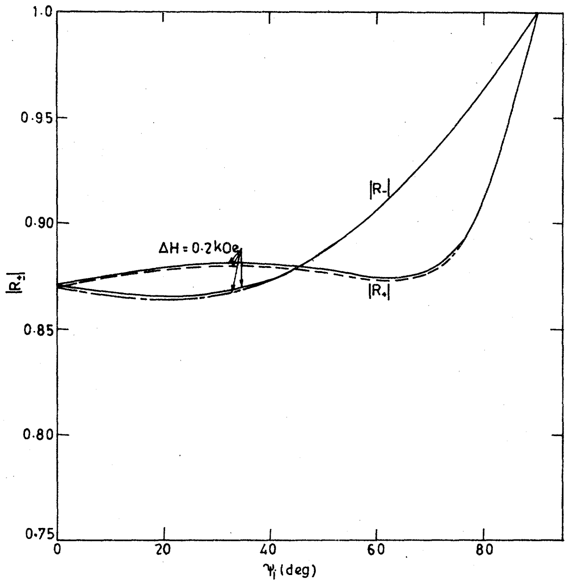

FIG. 14. Variation of $|R_\pm|$ with ψ_i for $\epsilon=\epsilon_f=13.0$, $H_0=1.75$ kOe, $4\pi M_0=3$ kG, $\Delta H=0.2$ kOe, and $f=9$ GHz.

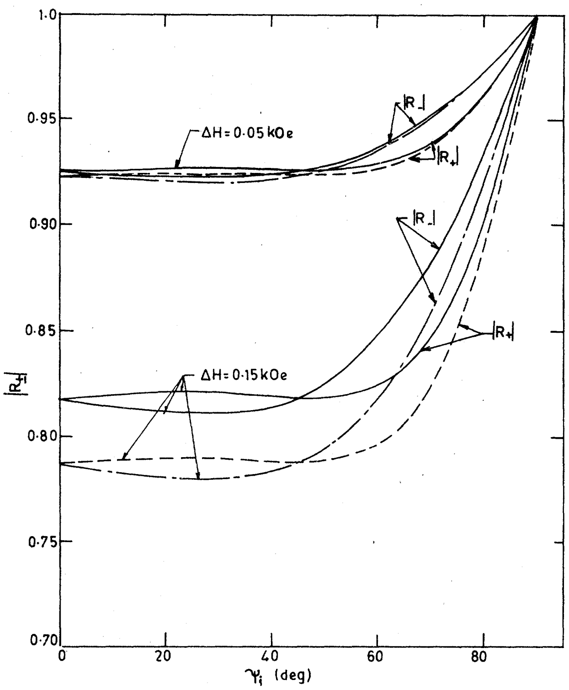

FIG. 15. Variation of $|R_\pm|$ with ψ_i for different ΔH. Other parameters are $\epsilon=\epsilon_f=13.0$, $H_0=2.0$ kOe, $4\pi M_0=3$ kG, and $f=9$ GHz.

very well except in the region close to resonance or when the angle of incidence is close to the critical angle (in the case when $\mu_{eff}>0$). Figure 10 shows the variation of $|R_+|$ with ψ_i for $H_0=0.75$ kOe for different values of ΔH indicated next to each curve. As shown in the inset, the agreement between the rigorous result and the ray model is excellent throughout the range of parameters involved; the maximum error is less than 0.02%. Figures 11–13 show $|R_+|$ for $H_0=1.25$, 1.75, and 2.0 kOe, respectively. The error is negligibly small in most of these cases, but it increases as one approaches the resonance (which is obtained at $H_0\simeq 2.04$ kOe). Figures 14 and 15 show the variation of $|R_\pm|$ with ψ_i for $\epsilon=\epsilon_f=13$, for $H_0=1.75$ and 2.0 kOe, respectively.

The agreement between the rigorous result and the ray model is again excellent except when ΔH is large. The general applicability of the ray model makes it possible to obtain useful expressions for $|R|$ in the case of normal incidence. It follows from Eq. (14) and (22) that

$$\alpha^{\pm}_{eff}D_{\pm}=\frac{2\cos\psi_i\,\delta\Delta H}{\alpha}\;\frac{\sin^2\psi_i+\alpha\sin\psi_i\,(\pm k_0/\mu_0\mp\delta'\mu^0_{eff}/\delta)-\frac{1}{2}\mu^0_{eff}\epsilon_f/\epsilon}{(\mu^0_{eff})^2\cos^2\psi_i+[\alpha\pm(k_0/\mu_0)\sin\psi_i]^2}\,. \tag{26}$$

When $\psi_i=0$, Eq. (26) reduces to

$$\alpha^{\pm}_{eff}D_{\pm}=\frac{(2\epsilon_f/\epsilon)^{1/2}\delta\Delta H}{(-\mu^0_{eff})^{1/2}(\epsilon_f/\epsilon-\mu^0_{eff})}\,. \tag{27}$$

The linewidth is therefore obtained as

$$\Delta H=-\frac{(-\mu^0_{eff})^{1/2}(\epsilon_f/\epsilon-\mu^0_{eff})}{(2\epsilon_f/\epsilon)^{1/2}\delta}\ln|R|\,. \tag{28}$$

This provides a method for the measurement of the linewidth in the millimeter-wave region. It should be mentioned that the phase-shift method for open-resonator measurement of the linewidth has already been used in the recent past[15]; Eq. (28) provides a complementary formula for the linewidth in terms of the reflected amplitude. This formula can be simplified by making appropriate approximations. When $|\mu^0_{eff}|\ll\epsilon_f/\epsilon$ (which is so when the biasing field is appreciably different from the resonance limit), the expression for the linewidth reduces to

$$\Delta H=-(\epsilon_f|\mu^0_{eff}|/2\epsilon)^{1/2}(1/\delta)\ln|R|\,. \tag{29}$$

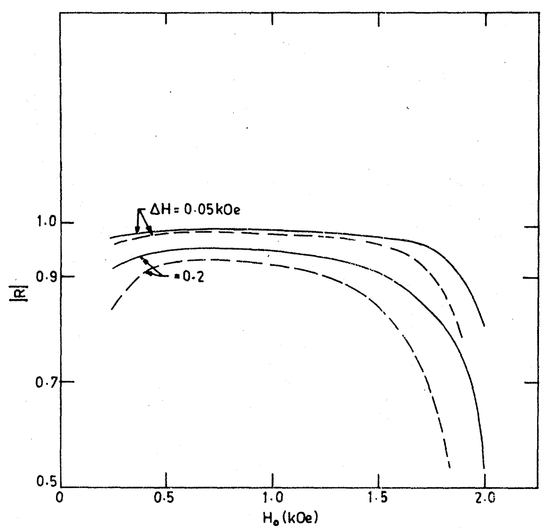

FIG. 16. Variation with biasing-field strengths of $|R|$ as obtained from the exact formula (solid lines) and from Eq. (29) (broken lines) for different ΔH's for the case of normal incidence.

Figure 16 shows the comparison between $|R|$ as obtained from the rigorous expression and as obtained from Eq. (29). It is seen that agreement is very good in the intermediate-biasing-field region and for small linewidths.

When $\epsilon = \epsilon_f$ and $\mu^0_{eff} \gg 1$, i.e., when the biasing field is somewhat closer to resonance, the expression for the linewidth reduces to

$$\Delta H = -\left(|\mu^0_{eff}|^{3/2}/\sqrt{2}\,\delta\right) \ln|R| . \tag{30}$$

It follows that the measurement of linewidths in the millimeter-wave region is possible by using Eqs. (28)–(30). The generalized Renard procedure and the ray model make it possible to analyse a variety of problems involving guided waves in ferrites. As a simple example, electromagnetic wave propagation in a rectangular wave guide loaded with one or two transversely magnetized, thick ferrite slabs placed against the side walls, can be analyzed in terms of multiple reflections. Each reflection can be described by the ray model which would lead to an approximate expression for the attenuation. It should also be possible to study, by ray techniques, magnetostatic bulk waves in ferrite slabs and films; this would be the generalization of the present study, under magnetostatic approximation, to the case of reflection of a wave, initially propagating in a ferrite, at the ferrite-dielectric interface.

Although the present analysis applies only to a medium with permeability tensor of the form (1), the generality of the procedure suggests its validity also for other media similar to an electrically gyrotropic medium, e.g., magneto-optical materials, magnetoplasmas, semiconductors, etc. Consequently, it may be possible to analyze magneto-optical film wave guides, generalizing the present results.

IV. SUMMARY

Ray and energy propagation in the case of electromagnetic-wave reflection at a dielectric-ferrite interface have been investigated. A magnetically tunable, nonreciprocal lateral shift of a collimated beam on account of total reflection from the ferrite is predicted; it can be measured in the millimeter-wave region by techniques already used in the case of microwave reflection at dielectric-air interfaces.[10-13] The dependence of this shift on a variety of parameters has also been studied and physical insight into the interaction of microwaves with the ferrite surface has been provided. An approximate ray model for attenuated total reflection from weakly absorbing ferrites is also proposed. Apart from providing physical insight into the process of microwave absorption, the ray model leads to approximate expressions for $|R|$, which may be useful in linewidth measurements in the millimeter-wave region.

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[1]M. H. Seavey and P. E. Tannenwald, MIT Lincoln Lab. Tech. Report No. 143, 1957 (unpublished).
[2]H. Suhl and L. R. Walker, Bell Syst. Tech. J. 33, 579 (1954).
[3]R. S. Mueller, J. Appl. Phys. 42, 2264 (1971).
[4]N. C. Srivastava, J. Appl. Phys. 49, 3181 (1978).
[5]I. Newton, *Opticks* (Dover, New York, 1952).
[6]R. S. Mueller, J. Appl. Phys. 45, 3587 (1974).
[7]R. S. Mueller, J. Appl. Phys. 47, 4164 (1976).
[8]H. K. V. Lotsch, Optik 32, 116 (1970); 32, 189 (1970); 32, 299 (1971); 32, 553 (1971).
[9]K. Artmann, Ann. Phys. (Leipz) 2, 87–102 (1948).
[10]V. Kyles, J. Kauranel, and T. M. Knossel, Appl. Opt.

13, 742 (1974).
[11]L. Arthur, A. Read, I. R. Dagg, and G. E. Reesor, Can. J. Phys. 50, 52 (1972).
[12]L. Arthur A. Read, I. R. Dagg, and G. E. Reesor, Can. J. Phys. 52, 2088 (1974).
[13]J. J Cowan and B. Anicin, J. Opt. Soc. Am. 67, 1307 (1977).
[14]R. H. Renard, J. Opt. Soc. Am. 54, 1190 (1964).
[15]C. Imbert, Phys. Rev. D 5, 787 (1972).
[16]M. S. Sodha *et al.* (private communication).
[17]B. C. S. Lint and A. C. Priou, IEEE Trans. Microwave Theory Tech. 24, 863 (1976).
[18]L. D. Landau and E. M. Lifshitz, *Electrodynamics of Continuous Media* (Pergamon, New York, 1963), p. 253.
[19]When the ferrite is biased in the region of negative effective permeability (i.e., when $[\omega_0(\omega_0+\omega_m)]^{1/2} < \omega < (\omega_0+\omega_m)$), there is total reflection for all values of ψ_i including $\psi_i = 0°$.
[20]Total reflection from a dielectric does not occur in the case of normal incidence. However, in plasmas the effective permitivity can be negative, in which case total reflection at normal incidence does occur, but there is no lateral shift of the reflected ray.
[21]It is worthwhile to mention that Eq. (20) is strictly valid only when losses are small, which requires that the region of anomalous dispersion (and hence dissipation) is avoided. A biased ferrite is strongly dispersive in the vicinity of ferromagnetic resonance and therefore the present analysis is only valid when frequencies close to the resonance frequency are avoided.