Two-dimensional electron gas in a strong magnetic field

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Some interesting properties of the phase diagram of a two-dimensional electron gas are calculated within the framework of Hartree-Fock picture. We find that the system is unstable to the formation of a charge-density wave at temperatures well above the *classical* Wigner solid transition temperature.

I. INTRODUCTION

Recently there has been a great deal of interest in the properties of quasi-two-dimensional electron gases. In these systems [electrons accumulated at the interface of a metal-vacuum-helium (MHV) structure[1] or at a metal-oxide-silicon (MOS) sandwich],[2] the energy-level spacing for motion perpendicular to the plane of the structure is large compared, for example, to the temperature or Fermi energy of the electrons. In the plane the electrons move like quasi-free-particles, their interactions being dominated by their interactions with the other electrons. Thus we have, to a good degree of approximation, an interacting two-dimensional (2-D) electron gas.

A fascinating aspect of this type of configuration is that the surface electron density in these systems may be varied over many orders of magnitude, thus in effect varying the strength of the Coulomb interactions. The dimensionless strength of this interaction is quite generally the ratio (Γ_0) of mean Coulombic energy $\langle V\rangle$ to mean kinetic energy $\langle K\rangle$ in the system. For large enough values of this parameter, we expect that the system would like to minimize its potential energy and should, in fact, condense into a solid. Such a solid transition (Wigner condensation) has been conjectured theoretically[3,4] and has been shown to exist for reasonable experimental parameters $\Gamma_0 \simeq 95$ in the MHV system by means of a numerical evaluation of the problem of 10^4 interacting classical particles.[5] It has even been suggested[6] that melting of the solid occurs when the free energy for the creation of pairs of line defects become negative. While such a theory seems to give a reasonable estimate of the classical parameter Γ_0,[6] it disagrees in detail with the numerical calculation. A λ-like singularity in the specific heat seems to be present in the numerical results, while the theory predicts a smooth transition. In addition, such a classical theory cannot predict melting at zero temperature as the density is increased. All such classical theories lead to a phase diagram characterized by the parabola

$$n = (k_B T \Gamma_0/\pi^{1/2} e^2)^2 \tag{1.1}$$

in the n-T plane. This parabola, shown as the dashed line in Fig. 1, divides solid from liquid.

Another theoretical model[4] suggests that an instability of the transverse mode in the solid signals the onset of melting. It "predicts" a curve for the phase diagram over the entire range of temperatures, classical and degenerate, shown

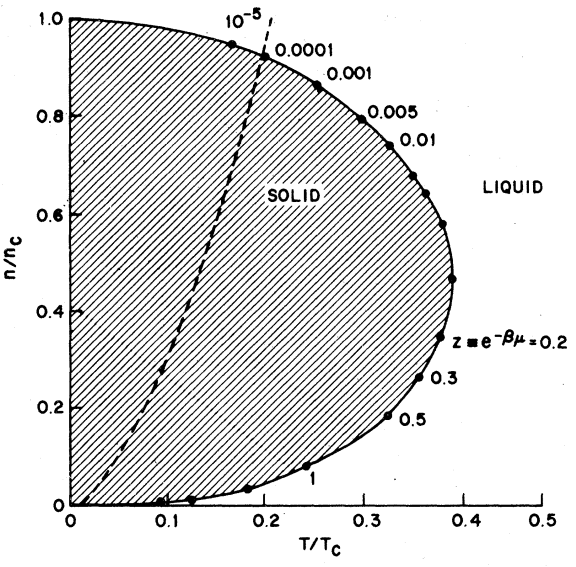

FIG. 1. Schematic phase diagram of the two-dimensional electron gas. The solid curve is obtained by setting $\langle V\rangle/\langle K\rangle = \tilde{\Gamma}_0$ with $\langle V\rangle$ and $\langle K\rangle$ evaluated for a noninteracting Fermi gas. The dashed curve is simply a plot of Eq. (1.1). The ratio $\Gamma_0/\tilde{\Gamma}_0 = 5$ is picked for purposes of illustration ($n_c = 4m^2e^4/\pi\tilde{\Gamma}_0^2$, $T_c = 2e^4m/\tilde{\Gamma}_0^2$).

as the solid curve in Fig. 1. The instability theory is roughly characterized by another value of the parameter $\Gamma_0(\tilde{\Gamma}_0)$ or equivalently ($\hbar = 1$),

$$n_c = \frac{4m^2e^4}{\pi\tilde{\Gamma}_0^2} = \frac{4}{\pi a_0^2}\frac{1}{\tilde{\Gamma}_0^2}, \quad k_B T_c = \frac{2e^4 m}{\tilde{\Gamma}_0^2}, \tag{1.2}$$

where $a_0 = (me^2)^{-1}$ is the three-dimensional Bohr radius and $\tilde{\Gamma}_0 \cong 5$ depends on the details of the theory. In the classical regime, the instability theory seems to seriously overestimate the solid region and the defect theory[6] wins out, although problems with the nature of the transition persist. At low temperatures, where quantum-mechanical effects are important, the situation is reserved.

Assuming $\Gamma_0 > \tilde{\Gamma}_0$, then from the above arguments and from the fact that $2e^4m$ is the only energy in the problem, we see that the temperature scale (the temperature at which the phase diagram begins to deviate from the classical curve) given by Eq. (1.1) in this 2-D electron gas is set by

$$k_B T^* = 2e^4 m/\Gamma_0^2. \tag{1.3}$$

The density scale is still set by

$$n_c = (4/\pi a_0^2)(1/\tilde{\Gamma}_0^2).$$

For an MHV system and for $\Gamma_0 = 100$, $T^* \simeq 30$ K and $n_c \simeq 10^{15}$. On the other hand, we obtain $T^* \simeq 0.05$ K and $n_c \simeq 10^{12}$ if we use $m^* \simeq 0.2m$ and $\epsilon_0 \simeq 10$ to simply characterize MOS devices. It would then seem that for MOS systems the region of Wigner crystallization cannot be reached unless one goes to extremely low temperatures.[7] However, in the presence of a strong magnetic field, the phase diagram may be drastically altered and the Coulomb-localized [Wigner solid (WS)] portion of the phase diagram may in fact become accessible to experiment.[8] In fact, several recent experiments suggest that the magnetotransport of MOS devices at fields of about 100 kG are anomalous.[9] In this paper we would like to theoretically explore the properties of an idealized 2-D system in the presence of a strong magnetic field (B) oriented perpendicular to the surface. For noninteracting electrons and large enough fields ($\omega_c \equiv eB/mc \gg k_B T, E_F$) the electrons are trapped in the lowest Landau level, i.e., they cannot move in the plane and there is no kinetic energy in the problem. Under such circumstances the system is quasi-zero-dimensional, and Coulomb interactions, at all densities, are expected to have drastic effects on its low-temperature properties.

Qualitatively a new energy or temperature scale $e^2/\epsilon_0 l$ is introduced into the problem. Here $l = (\hbar c/eB)^{1/2}$ is the radius of the lowest Landau level and $e^2/\epsilon_0 l$ at 100 KG is about 100 °K even for the Si MOS. Since there is no kinetic energy in the problem, all parameters except the occupation of the lowest Landau level $\nu \equiv 2E_F/\hbar\omega_c \equiv 2\pi n l^2$ are of order 1. Thus for $\nu \cong 1$ the transition temperature in any theory is probably of order $e^2/\epsilon_0 l$.

We will, in fact, be able to show that within a reasonable Hartree-Fock (HF) picture, the system develops a charge-density-wave (CDW) instability as the temperature is lowered. Still within the framework of HF theory, we will be able to estimate T_c and discuss the nature of the transition, i.e., its order. We find that for all fractional occupations of the lowest Landau level except one half that the transition is first order.

II. FREE ENERGY AND THE TRANSITION TEMPERATURE

Our model Hamiltonian is given by

$$H = \tfrac{1}{2}\sum_{\vec{q}} v(\vec{q})[\rho(\vec{q})\rho(-\vec{q}) - e^{-q^2 l^2/2}\rho(0)]. \tag{2.1}$$

Here $v(\vec{q}) = 2\pi e^2/\epsilon_0 qS$, and ϵ_0 and S are the effective dielectric constant and the total area of the system, respectively. The charge-density operator is defined by

$$\rho(\vec{q}) = \int dr\, e^{-i\vec{q}\cdot\vec{r}}\rho(r) = \int dr\, e^{-i\vec{q}\cdot\vec{r}}\psi^\dagger(r)\psi(r), \tag{2.2}$$

$$\psi(\vec{r}) = \sum_X a_X \phi_X(\vec{r}). \tag{2.3}$$

In Eq. (2.3) $\sum_X = (L/2\pi l^2)\int_0^L dX$, where L is the linear dimension of our system and $\phi_X(\vec{r})$ are the Landau wave functions for the lowest level, i.e.,

$$\phi_X(\vec{r}) = (\pi^{1/2} lL)^{-1/2}\exp[-iXy/l^2 - (x-X)^2/2l^2]. \tag{2.4}$$

By the use of Eqs. (2.3) and (2.4), we can rewrite Eq. (2.2) as

$$\rho(\vec{q}) = \sum_X \exp[-iq_\alpha X - \tfrac{1}{4}(ql)^2] a^\dagger_{X_+} a_{X_-}, \tag{2.5}$$

where $X_\pm = X \pm \frac{1}{2}l^2 q_y$ and q_α is the αth component of $\vec{q}$.

The order parameter of the CDW with a wave vector $\vec{q} = \vec{Q}$ is $\langle\rho(\vec{Q})\rangle$, where $\langle\ \rangle$ is the thermal average. From Eq. (2.5) we see that the requirement of finite $\langle\rho(\vec{Q})\rangle$ is equivalent to setting,

$$\langle a^\dagger_{X_+} a_{X_-}\rangle = (2\pi/L)\{\Delta(\vec{Q})e^{iQ_x X}\delta(q_y - Q_y) + [\Delta(\vec{Q})]^* e^{-iQ_x X}\delta(q_y + Q_y)\}, \tag{2.6}$$

Here $\Delta(\vec{Q})$ is a complex constant independent of X and is of the order of ν. The X-dependent phase factor in Eq. (2.6) follows from Eq. (2.5) and the requirement that $\langle\rho(q)\rangle$ be finite for $\vec{q} = \vec{Q}$. Using Eq. (2.6), the CDW in real space is given by

$$\langle \rho(\vec{r}) \rangle = \frac{1}{(2\pi)^2} \int d^2q \, \langle \rho(\vec{q}) \rangle e^{i\vec{q}\cdot\vec{r}}$$
$$= n + 2\mathrm{Re}\left(\frac{\Delta(\vec{Q})}{2\pi l^2} e^{-(Ql)^2/4} e^{i\vec{Q}\cdot\vec{r}} \right). \quad (2.7)$$

In terms of the order parameter $\Delta(Q)$, the Hartree-Fock Hamiltonian for Eq. (2.1) is given as (see the Appendix)

$$H = -\sum_{\vec{Q}_i} (\{ \rho(\vec{Q}_i) e^{(Q_i l)^2/4} U(\vec{Q}_i) [\Delta(\vec{Q}_i)]^* + \mathrm{H.c.} \}$$
$$- u(\vec{Q}_i) |\Delta(\vec{Q}_i)|^2) + \hat{N}_e E_0 , \quad (2.8)$$

where the operator $\hat{N}_e$ represents the total number of electrons. In Eq. (2.8) the existence of several order parameters $\Delta(\vec{Q}_i)$ corresponding to different wave vectors $\vec{Q}_i$ has been assumed. The summation over $\vec{Q}_i$ is restricted so that $\vec{Q}_i$ and $-\vec{Q}_i$ should not be counted twice since $\Delta(-\vec{Q}_i) = [\Delta(\vec{Q}_i)]^*$. In Eq. (2.8) E_0 is the Fock energy in the normal state when the charge density is uniform, and the Hartree-Fock potential $U(\vec{Q})$ is given as (see the Appendix)

$$U(\vec{Q}) = \frac{e^2}{\epsilon_0 l} \left[\left(\frac{\pi}{2} \right)^{1/2} e^{-(Ql)^2/4} I_0\left(\frac{(Ql)^2}{4} \right) - \frac{1}{Ql} e^{-(Ql)^2/2} \right]$$
$$\equiv \frac{2\pi l^2}{S} u(\vec{Q}). \quad (2.9)$$

Here $|\vec{Q}| = Q$ and $I_0(x)$ is the modified Bessel function. The Q dependence of $U(\vec{Q})$ is shown in Fig. 2.

In order to determine the critical temperature of the phase transition, we evaluate the difference of free energies between the CDW state and the normal states with a fixed total electron number. It can be written in terms of the thermodynamic potential Ω as

$$\delta F \equiv F_{\mathrm{CDW}}(n) - F_0(n)$$
$$= \Omega_{\mathrm{CDW}}(\mu) - \Omega_0(\mu_0) + (\mu - \mu_0) N_e , \quad (2.10)$$

where N_e is the total number of electrons and μ and μ_0 are the chemical potentials of the CDW state and the normal state, respectively. We will determine δF to order $\Delta(Q_i)^4$, using $\partial\Omega(\mu_0)/\partial\mu_0 = -N_e$, i.e.,

$$\delta F = \Omega_{\mathrm{CDW}}(\mu) - \Omega_0(\mu) + \tfrac{1}{2}(\mu - \mu_0)^2 \frac{\partial^2 \Omega_0(\mu_0)}{\partial \mu_0^2} + \cdots \quad (2.11)$$

A. Second-order transition

First we estimate the critical temperature under the assumption that the transition is second order. In this case it suffices to consider the contribution to δF to second order in the order parameter. By noting that $\mu - \mu_0$ is proportional to the square

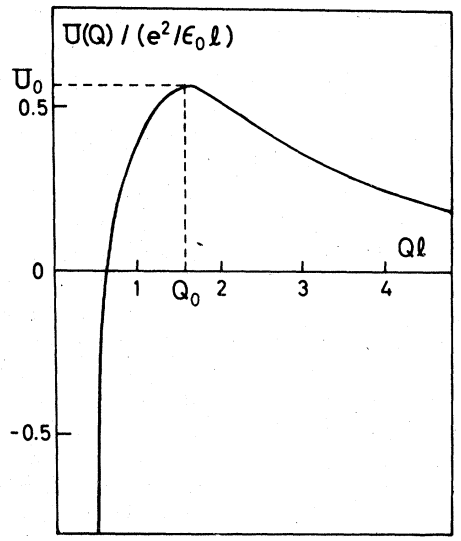

FIG. 2. Plot of the effective HF potential in momentum space (b).

of the order parameter and by considering one kind of order parameter, $\Delta(Q)$, we obtain for $\delta F^{(2)}$

$$\delta F^{(2)} = N[U(Q) + U(Q)^2 f'(E_0)] \Delta(Q)^2 \quad (2.12a)$$
$$= NU(Q) \left(1 - U(Q) \frac{\nu(1-\nu)}{T} \right) \Delta(Q)^2 , \quad (2.12b)$$

where $N = S/2\pi l^2$. In Eq. (2.12a) $f'(E_0)$ is given by

$$f'(E_0) = \frac{d[\exp\beta(E_0 - \mu) + 1]^{-1}}{dE_0} .$$

In obtaining Eq. (2.12b), we used the relation $f'(E_0) \simeq -\nu(1-\nu)/T$. Thus we see that the instability of the normal state is determined by the maximum value of $U(Q)$, $U(Q_0) = U_0$, i.e.,

$$T_c = \nu(1-\nu) U_0 = 0.557 \nu(1-\nu) e^2/\epsilon_0 l , \quad (2.13a)$$
$$|Q| = Q_0 = 1.568\, l^{-1} . \quad (2.13b)$$

Note from Eq. (2.13b) that the wave vector of the CDW is independent of the degree of filling ν if the transition is second order.

Several features of T_c are worth noting. As advertised, T_c is of the order $e^2/\epsilon_0 l$ and the period only depends on l. The density dependence is parabolic and symmetric about $\nu = \frac{1}{2}$. The symmetry of all the physical properties about $\nu = \frac{1}{2}$ is a general property of the Hamiltonian, Eqs. (2.1) and (2.2), which follows from the invariance of this Hamiltonian under a transformation $a_{\vec{x}\lambda} \to b^\dagger_{\vec{x}\lambda}$. The explicit form $\nu(1-\nu)$ for the transition is *not* a general property but a characteristic of the HF equations. At low densities the CDW transition temperature is linear in n, unlike the classical transition temperature which goes like $n^{1/2}$. In Si MOS

$T_c \simeq 20$°K for $B \simeq 100$ kG at $\nu = 0.5$ ($n = 10^{11}$ cm^{-2}). This is a rather high temperature when compared with the classical temperature for the transition to a Wigner solid at the same density in the absence of a magnetic field. Using the numerical results of Hockney and Brown,[5] $T_{c1} \simeq 2$ °K for $\nu = 0.5$ and $B = 100$ kG.

To put the relation between the classical transition temperature T_{c1} and the quantum CDW temperature T_c in the proper perspective, we have plotted both temperatures for a fixed 100-kG field as a function of ν (density) in Fig. 3. The classical temperature is low compared to the CDW temperature unless $\nu < 10^{-4}$.

The classical transition temperature to a Wigner solid (WS) is about one order of magnitude smaller than our estimate of the CDW transition temperature. In fact, careful consideration of the WS phase diagram in a magnetic field amplifies this difference. Using a Lindeman melting criterion designed to reproduce the classical result, we find that the solid region is strongly suppressed in the presence of zero-point fluctuations.[10] In fact, at fields of 100 kG we find that the WS no longer exists for $\nu \geq 10^{-2}$. At these "high" CDW temperatures, the strong short-range order which is not included in our HF description has not had time to build up, and our HF description is a good one.

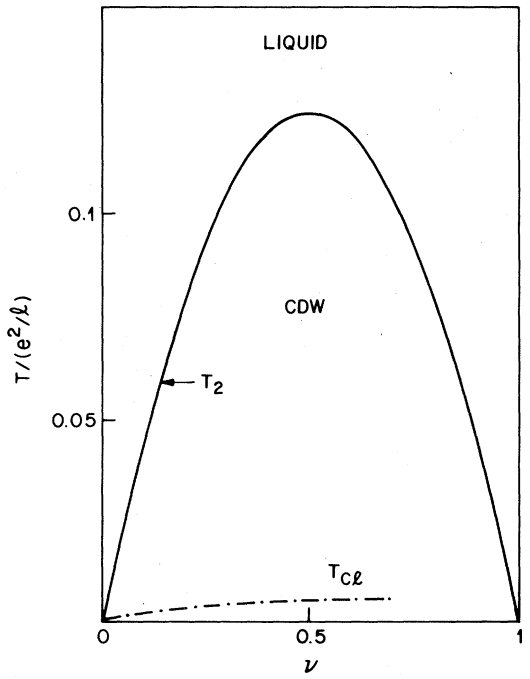

FIG. 3. Transition temperature in this HF theory. The solid curve shows the second-order transition temperature. Thd dashed curve is a plot of the classical transition temperature for a fixed $B = 100$ kg.

B. First-order transition

As seen from Eq. (2.9), $U(\vec{Q})$ is a function of $|Q|$ and the maximum value of $U(\vec{Q})$ occurs on a circle $|\vec{Q}| = Q_0$ in momentum space. This fact introduces the possibility of a first-order transition, since any CDW with wave vector $|Q| = Q_0$ has an equal right to evolve at $T = T_c$. If the charge density with $\vec{Q}_1 (|\vec{Q}_1| = |\vec{Q}_0|)$ were to start building up, those with $\vec{Q}_2$ and $\vec{Q}_3$ which are oriented 120° apart and satisfy $\vec{Q}_1 + \vec{Q}_2 + \vec{Q}_3 = 0$ would also build up (see Fig. 4). Of course, the choice of the direction of Q_1 is arbitrary in our present model, but once $\vec{Q}_1$ is fixed, for example, by a boundary condition, $\vec{Q}_2$ and $\vec{Q}_3$ should also be. In the following, we consider the case where we choose such a particular set of three order parameters.[11]

In order to incorporate the possible fact that the absolute magnitude of the wave vector Q_i is not necessarily equal to Q_0 at the first-order transition, we assume $|Q_i| = Q$ arbitrary and determine Q variationally. For this purpose we evaluate the difference of the free energy δF between the normal state and the CDW state as a power series in $\Delta(Q)$ up to fourth order. The third-order contributions are shown diagramatically in Figs. 5(a) and 5(b).[12] The straight lines represent one-particle Green's functions and the vertices are the order parameters $\Delta(Q_i)$. These two terms are complex conjugates of one another. They give a term

$$\delta F^{(3)} = -2NU(Q)^3 f''(E_0)(\Delta_1 \Delta_2 \Delta_3 + \text{c.c}) \times \cos(\tfrac{1}{2} l^2 \vec{Q}_1 \times \vec{Q}_2 \cdot \hat{e}_z), \quad (2.14)$$

where $\Delta_i \equiv \Delta(\vec{Q}_i)$ and $\hat{e}_z$ is the unit vector in the direction of the magnetic field. In deriving Eq. (2.14) we noted the following fact for the phase factor in Fig. 5(a):

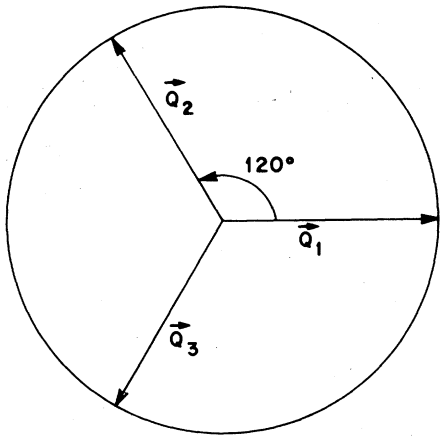

FIG. 4. Three wave vectors of the charge-density waves involved in a first-order transition.

$$\sum_X \exp\left\{-i\frac{1}{2}\left[Q_{1x}\left(X+\frac{l^2Q_{1y}}{2}\right)+Q_{2x}\left(X+l^2Q_{1y}+\frac{l^2Q_{2y}}{2}\right)+Q_{3x}\left(X+\frac{l^2(Q_1+Q_2)_y}{2}\right)\right]\right\}=N\exp\left(-i\frac{l^2}{2}(Q_{1y}Q_{2x}-Q_{1x}Q_{2y})\right).$$

In Fig. 5(b), on the other hand, the phase factor is

$$\sum_X \exp\left\{-i\frac{1}{2}\left[Q_{1x}\left(X+\frac{l^2Q_{1y}}{2}\right)+Q_{3x}\left(X+l^2Q_{1y}+\frac{l^2Q_{3y}}{2}\right)+Q_{2x}\left(X+l^2\frac{Q_{1y}+Q_{3y}}{2}\right)\right]\right\}$$

$$=N\exp\left[-i\frac{l^2}{2}(Q_{1y}Q_{3x}-Q_{3y}Q_{1x})\right]$$

$$=N\exp\left[-i\frac{l^2}{2}(Q_{1y}Q_{2x}-Q_{1x}Q_{2y})\right].$$

The contributions to $\Omega_{\mathrm{CDW}}(\mu)-\Omega_0(\mu)$ in the fourth order in Δ_i are given by

$$U(Q)^4 f''(E_0)\left(\frac{1}{4}\sum_i|\Delta_i|^4+\frac{1}{3}\sum_{i\neq j}|\Delta_i|^2|\Delta_j|^2[2+\cos(l^2\vec{Q}_1\times\vec{Q}_2\cdot\vec{e}_z)]\right). \tag{2.15}$$

Note that the term $|\Delta_i|^4$ does not have any phase factor whereas, regarding the phase factor, the term $|\Delta_i|^2|\Delta_j|^2$ $(i\neq j)$ has two different contributions. These contributions are shown in Figs. 6(a)–6(c). In Fig. 6(a) the phase factor is cancelled. On the other hand Figs. 6(b) and 6(c) contain factors given by

$$\exp\left\{-i\left[Q_{ix}\left(X+\frac{l^2Q_{iy}}{2}\right)+Q_{jx}\left(X+l^2Q_{iy}+\frac{l^2Q_{jy}}{2}\right)-Q_{ix}\left(X+l^2Q_{jy}+\frac{l^2Q_{iy}}{2}\right)-Q_{jx}\left(X+\frac{l^2Q_{jy}}{2}\right)\right]\right\}$$

$$=\exp[il^2(Q_{ix}Q_{jy}-Q_{iy}Q_{jx})],$$

$$\exp\left\{-i\,Q_{ix}\left(X+\frac{l^2Q_{iy}}{2}\right)-Q_{jx}\left(X+l^2Q_{iy}\frac{l^2Q_{iy}}{2}\right)-Q_{ix}\left(X-l^2Q_{jy}+\frac{l^2Q_{iy}}{2}\right)+Q_{jx}\left(X-\frac{l^2Q_{jy}}{2}\right)\right\}$$

$$=\exp[il^2(Q_{iy}Q_{jx}-Q_{ix}Q_{jy})],$$

respectively.

Since

$$\frac{\partial^2\Omega_0(\mu_0)}{\partial\mu_0^2}=-\frac{\partial N_e}{\partial\mu_0}=f',$$

the change of the chemical potential $\mu-\mu_0$ is given by

$$\mu-\mu_0=U(Q)^2f''/f'\sum_i|\Delta_i|^2. \tag{2.16}$$

In Eq. (2.16), $f^{(n)}$ is defined as

$$f^{(n)}=\frac{d^n[\exp\beta(E_0-\mu_0)+1]}{dE_0^n},$$

and the first few are

$$f'=-\nu(1-\nu)/T, \tag{2.17a}$$

$$f''=\nu(1-\nu)(1-2\nu)/T^2, \tag{2.17b}$$

$$f'''=-\nu(1-\nu)[6(\nu-\tfrac{1}{2})^2-\tfrac{1}{2}]/T^3. \tag{2.17c}$$

By using the relation $f'(E_0)=f'-(\mu-\mu_0)f''$ in Eq. (2.12a) and by setting $\Delta(Q_i)=\Delta_0e^{i\theta_i}$, we may finally write

$$\delta F=NU_0(a\Delta_0^2+b\Delta_0^3+c\Delta_0^4), \tag{2.18}$$

$$a=3\left(1-\frac{\nu(1-\nu)}{T}U(Q)\right), \tag{2.19}$$

$$b=-2[U(Q)/T]^2\nu(1-\nu)(1-2\nu)\cos\tfrac{1}{4}\sqrt{3}\,(Ql)^2\times\cos(\theta_1+\theta_2+\theta_3), \tag{2.20}$$

$$c=\tfrac{9}{2}[U(Q)/T]^3\nu(1-\nu)[\tfrac{5}{12}-(\nu-\tfrac{1}{2})^2]+[U(Q)/T]^3\nu(1-\nu)[1-\cos\tfrac{1}{2}\sqrt{3}\,(Ql)^2]\times[6(\nu-\tfrac{1}{2})^2-\tfrac{1}{2}]. \tag{2.21}$$

By noting that $\cos\frac{1}{4}\sqrt{3}(Q_0l)^2=0.48$ at $Q_0l=1.568$,

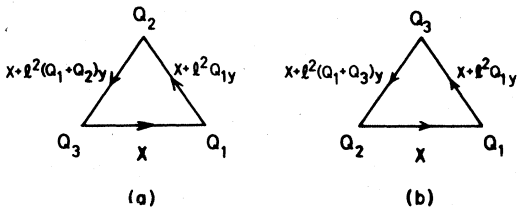

FIG. 5. Contributions to the thermodynamic potential in third order.

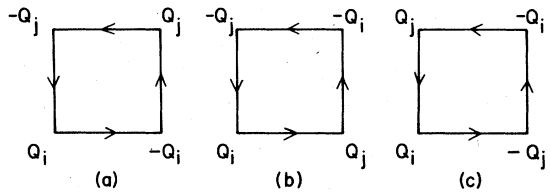

FIG. 6. Contributions to the thermodynamic potential in fourth order.

we see that $\theta_1+\theta_2+\theta_3=0$ (π) if $\nu<\frac{1}{2}$ $(\nu>\frac{1}{2})$. The first-order transition temperature T_1 determined by the equation $a=b^2/4c$ is now given as

$$T_1=T_c\left(\frac{U(Q)}{U(Q_0)}\right)\left(1+\frac{A\cos^2\phi}{B+C\cos^2\phi}\right), \tag{2.22}$$

where $\phi=\frac{1}{4}\sqrt{3}(Ql)^2$, and A, B, and C are defined by

$$A=\tfrac{8}{27}(\nu-\tfrac{1}{2})^2, \tag{2.23a}$$

$$B=\tfrac{7}{36}+\tfrac{5}{3}(\nu-\tfrac{1}{2})^2, \tag{2.23b}$$

$$C=\tfrac{2}{9}-\tfrac{8}{3}(\nu-\tfrac{1}{2})^2. \tag{2.23c}$$

Maximizing T_1 with respect to Q or ϕ, we obtain the deviation of T_1 from T_c [Eq. (2.13a)] as shown in Fig. 7(a). In Fig. 7(b) we show the $Q=Q_M$ which gives this maximum. It is seen that Q_M decreases as ν deviates from $\frac{1}{2}$. This tendency is reasonable in view of our discussion concerning the nature of the transition at low density, where we expect a transition to a solid whose basic periodicity $Q_W=2.69\,\nu^{1/2}l^{-1}$. Q_W is shown as the dotted curve in Fig. 7(b). The variation of Q_M is a reasonable first approximation to such a behavior.

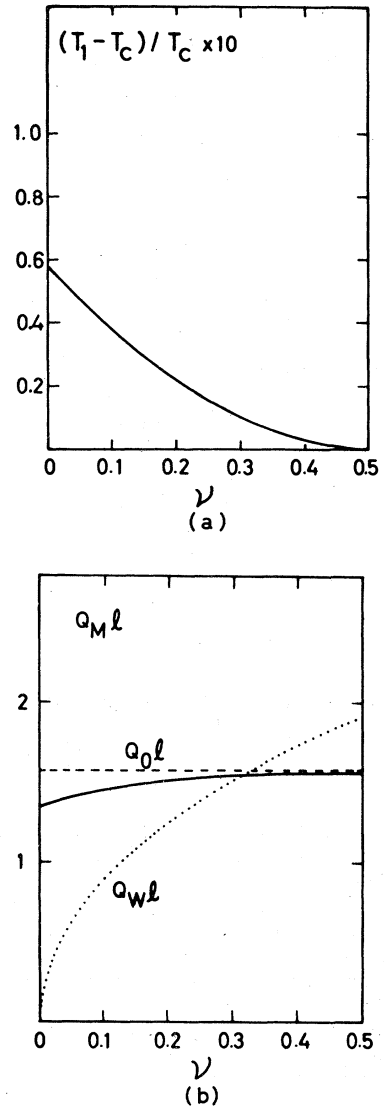

FIG. 7. First-order transition temperature and the wave-vector variation in this HF scheme. The dotted line shows the wave vector for the Wigner solid Q_W, while the dashed straight line is the constant wave vector Q_0 in the HF scheme.

III. DISCUSSIONS

We have examined the CDW phase transition within a Hartree-Fock approximation using the usual delocalized Landau wave functions. The physical picture which emerges from these calculations is that two-dimensional electrons in a large magnetic field will have a variety of phase transitions linked up with the formation of a CDW. This CDW occurs at temperatures high compared to the classical transition temperature and evolves in a second-order fashion with a bit of first-order character. It is harmonic in character near the transition T_c. As the temperature is lowered, we expect the period and harmonic content of the CDW to change, evolving towards an anharmonic Wigner solid. If we solve a nonlinear version of it, the HF theory presented here gives us an appropriate framework within which we can analyze such problems.

We do not believe that our transition represents the actual melting temperature of the CDW, but rather a kind of transition between an essentially gaslike phase and a phase with charge-density waves but no long-range order. The long-range order in angle and position will probably be destroyed by defects near the classical melting point or at least at temperatures low compared to our T_c, according to one version or another of the defect-melting theories.[6] We do, however, expect that our transition may show up as a quantitative change in the degree of short-range order, e.g., that the structure factor may have a rather sharp peak at our Q_M below our T_c. The reason is that the density of dislocations, while finite, may not be very high because as the CDW becomes less solidlike, the core radius a_0 of the dislocation increases, and their entropy, which is proportional to $-b\ln(na_0^2)$ decreases (although T_c, which depends on the coefficients of the logarithms in energy and entropy, will still be low). We propose, in other words, that Hartree-Fock probably gives a good description of the short-range order, which will become very pronounced at our T_c and below.

APPENDIX

The Hartree-Fock decoupling of the Hamiltonian [Eq. (1)] is

$$\frac{1}{2}\sum_q v(q)\rho(q)\rho(-q)$$

$$-\sum_q v(q)\langle\rho(-q)\rangle\rho(q)-\sum_q\sum_{X,X'} v(q)\exp\left(-\frac{(lq)^2}{2}-iq_x(X-X')\right)a^\dagger_{X_+}a_{X'_+}\langle a^\dagger_{X'_-}a_{X_-}\rangle . \tag{A1}$$

The first term of Eq. (A1) is trivially given as

$$Sv(Q)\Delta(Q)e^{-Q^2l^2/4}\rho(Q)+\text{H.c.}$$

while the second term is evaluated by using Eq. (2.6), i.e.,

$$-\Delta(Q)\sum_X a^\dagger_{X_-}a_{X_+}e^{iQ_xX}\sum_q v(q)\exp\left(-\frac{(ql)^2}{2}+il^2(q_xq_y-q_yq_x)\right)-[\Delta(Q)]^*\sum_X a^\dagger_{X_-}a_{X_+}e^{-iQ_xX}$$

$$\times\sum_q v(q)\exp\left(-\frac{(ql)^2}{2}+il^2(q_yQ_x-q_xQ_y)\right)=-\{\Delta(Q)\rho(-Q)+[\Delta(Q)]^*\rho(Q)\}e^{(Ql)^2/4}$$

$$\times\int d^2q\,\frac{e^2}{2\pi\epsilon_0 q}\exp\left(-\frac{(lq)^2}{2}+il^2(q_yQ_x-q_xQ_y)\right)$$

$$=-\{\Delta(Q)\rho(-Q)+[\Delta(Q)]^*\rho(Q)\}\frac{e^2}{\epsilon_0 l}\left(\frac{\pi}{2}\right)^{1/2}I_0\left(\frac{(Ql)^2}{4}\right). \tag{A2}$$

Equations (A1) and (A2) yield the desired results.

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[11]The arbitariness in the direction at the CDW leads to a fluctuation in the angle of the triangular lattice which is formed by three CDW's. This is an interesting problem beyond the scope of this paper.

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